## Derivation

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The circle of unit radius is given by:

$$
\begin{equation*}
f=x\left(1-1 / r^{2}\right) \tag{1}
\end{equation*}
$$

Where $z$ and $\zeta$ are complex numbers

$$
\begin{gather*}
z=x+\imath y \quad \text { and } \quad \zeta=\chi+\imath \gamma  \tag{2}\\
J^{\prime} \circ z=\frac{1}{2 A}\left(z \pm \sqrt{z^{2}-4 A B}\right) . \tag{3}
\end{gather*}
$$

This equation is special because it satisfies the LaPlacian Equation:

$$
\begin{equation*}
\nabla^{2} f(z)=0 \tag{4}
\end{equation*}
$$

This implies,

$$
\begin{equation*}
\operatorname{grad}^{2}[f \zeta]=0 \tag{5}
\end{equation*}
$$

So,

$$
\begin{align*}
& g=f(\zeta)=f\left(J^{\prime} \circ z\right)  \tag{6}\\
& \operatorname{grad} g=\operatorname{gradf}\left(J^{\prime} \circ z\right) \tag{7}
\end{align*}
$$

Since $z$ is an element of the complex number set, we must go back to this definition using real numbers. So let us redefine $f$ as,

$$
\begin{equation*}
f=x\left(1-1 / x^{2}+y^{2}\right) \tag{8}
\end{equation*}
$$

Now we will require useful facts that simplify the problem:

At the line $y=0$

$$
\begin{equation*}
\operatorname{grad}=\operatorname{partial}_{x} g \tag{9}
\end{equation*}
$$

This implies that on this line

$$
\begin{equation*}
\operatorname{Im}(z)=\operatorname{Im}(\zeta)=0 \tag{10}
\end{equation*}
$$

This allows us to assume

$$
\begin{equation*}
f(x)=x\left(1-1 / x^{2}\right) \tag{11}
\end{equation*}
$$

and that

$$
\begin{equation*}
\left(J^{\prime} \circ z\right)=\left(J^{\prime} \circ x\right)=\frac{1}{2 A}\left(x \pm \sqrt{x^{2}-4 A B}\right) . \tag{12}
\end{equation*}
$$

This all leads us to the conclusion that,

$$
\begin{equation*}
|B|=\partial_{x}\left[f\left(J^{\prime} \circ x\right)\right] \tag{13}
\end{equation*}
$$

