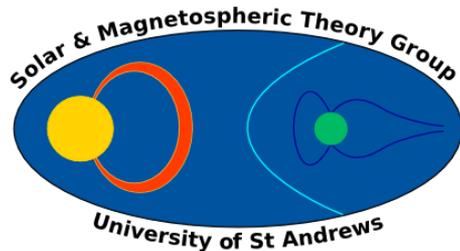




University of
St Andrews

600
YEARS

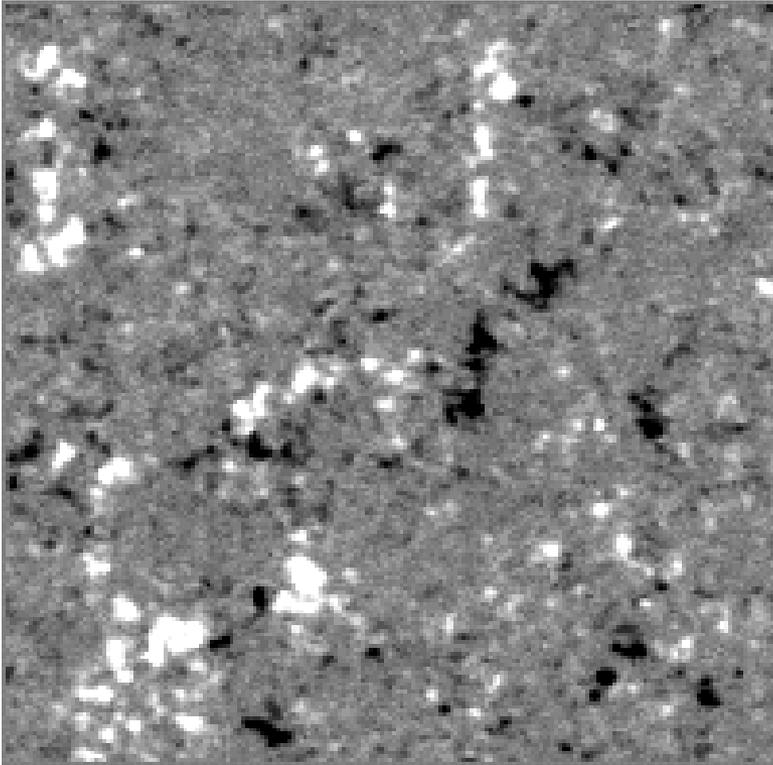


A Non-Linear Force-Free Field Model for the Solar Magnetic Carpet

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Magnetic Carpet: Observations



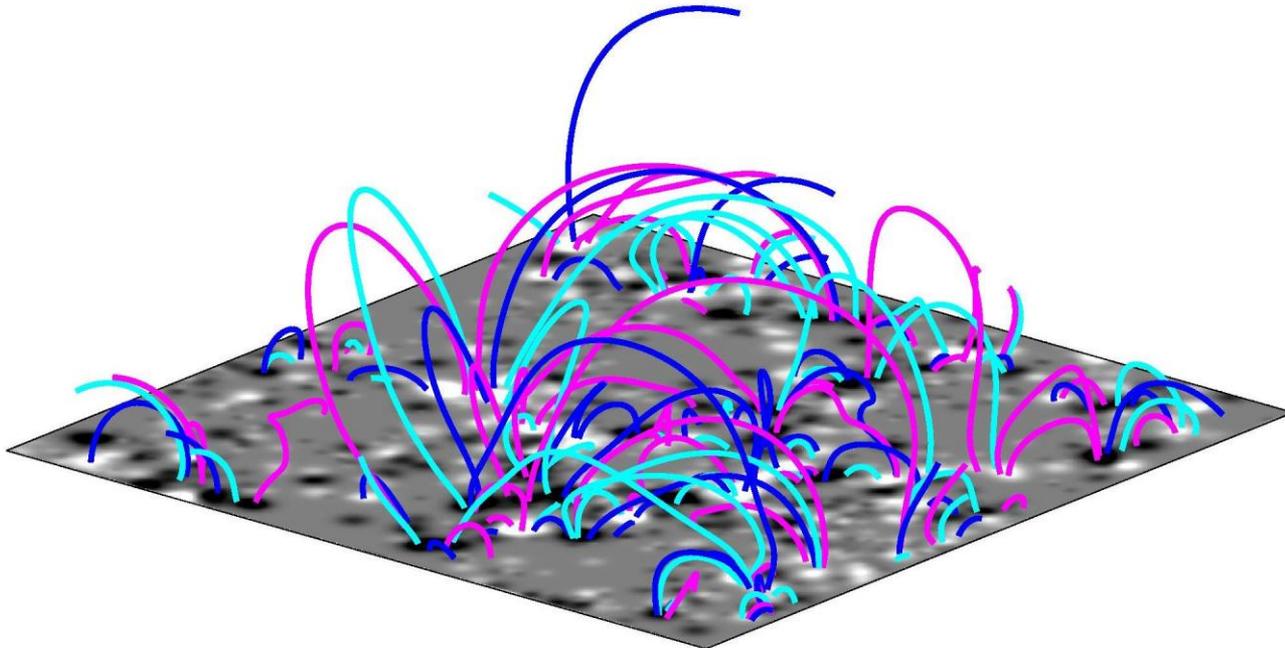
SDO/HMI: 73,000x73,000 km, 6 hours

- Small-scale photospheric magnetic field of quiet Sun.
- Motions dominated by supergranulation.
- Flux evolves via **emergence**, **cancellation**, **coalescence**, **fragmentation**.
- Flux: $10^{15} - 10^{19}$ Mx
Diameter: **few thousand km**
Lifespan: **minutes to hours**

Magnetic Carpet: Model

A two-component model:

- **2D model:** realistic model for photospheric evolution of magnetic carpet.
- **3D model:** continuous evolution of non-linear force-free corona, driven by photospheric boundary motions.

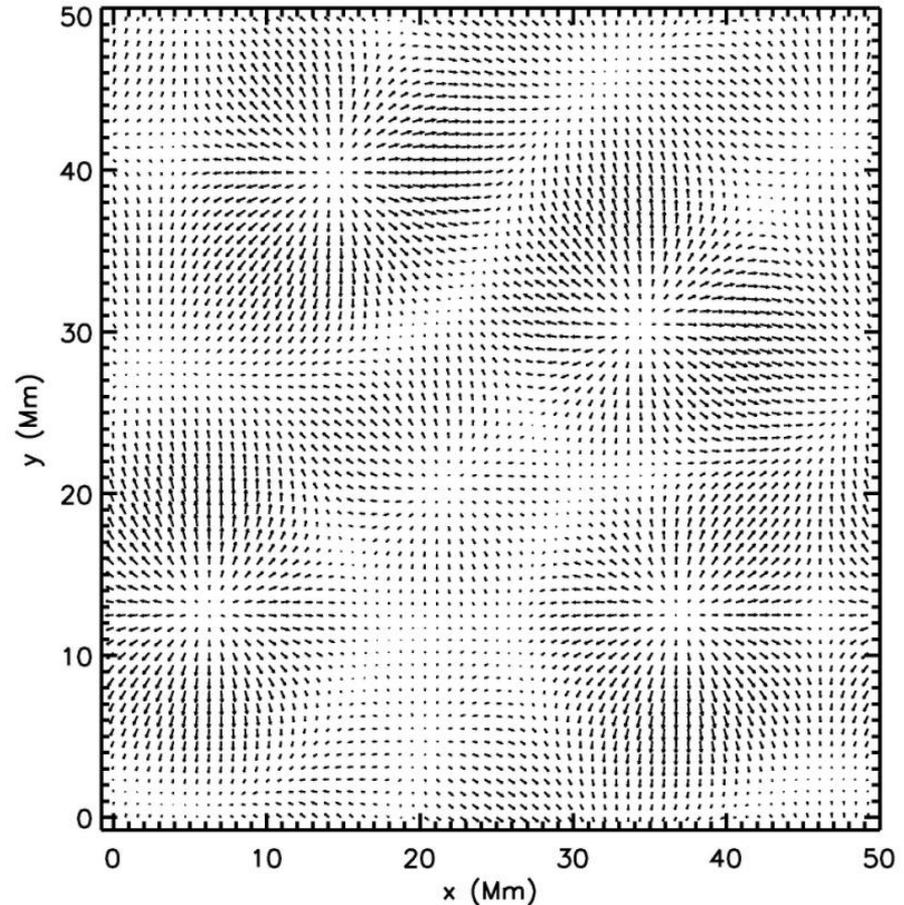


Photospheric Model: Ingredients

- Steady supergranular flow profile:

$$v(r) = A_0 r \exp\left\{\frac{-r^2}{r_0^2}\right\}$$

- Peak flow **0.5 km/s**.
- Random motions representing **granulation**.
- Periodic in x and y.
- Includes **emergence**, **cancellation**, **coalescence** and **fragmentation**.



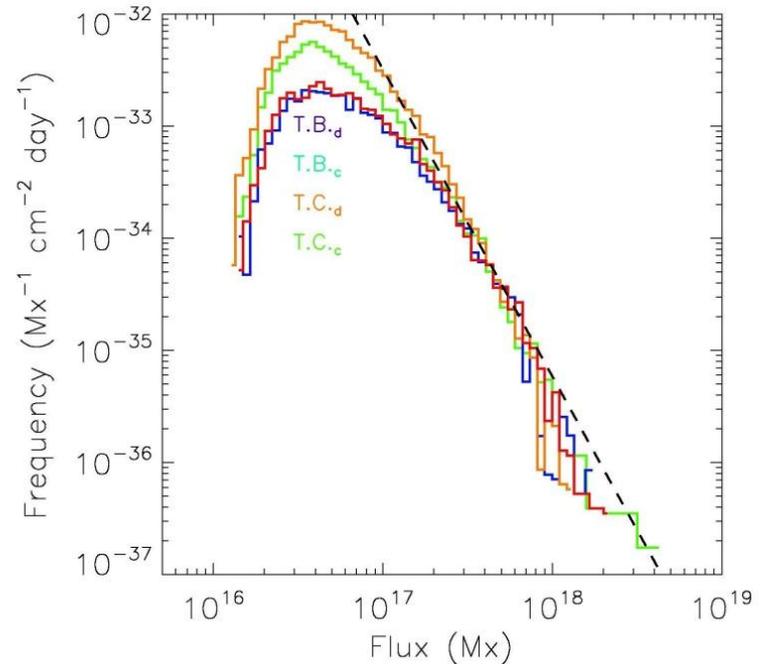
Photospheric Model: Ingredients

- Probability distribution for newly emerging bipoles (Thornton and Parnell, 2010) ($\text{Mx}^{-1} \text{cm}^{-2} \text{day}^{-1}$):

$$N(\phi) = \frac{n_0}{\phi_0} \left(\frac{\phi}{\phi_0} \right)^{-\alpha}$$

- Number of bipoles emerging in the range $[\phi_1, \phi_2]$ Mx , ($\text{cm}^{-2} \text{day}^{-1}$):

$$\int_{\phi_1}^{\phi_2} \frac{n_0}{\phi_0} \left(\frac{\phi}{\phi_0} \right)^{-\alpha} d\phi = \frac{n_0}{1-\alpha} \left[\left(\frac{\phi}{\phi_0} \right)^{1-\alpha} \right]_{\phi_1}^{\phi_2}$$

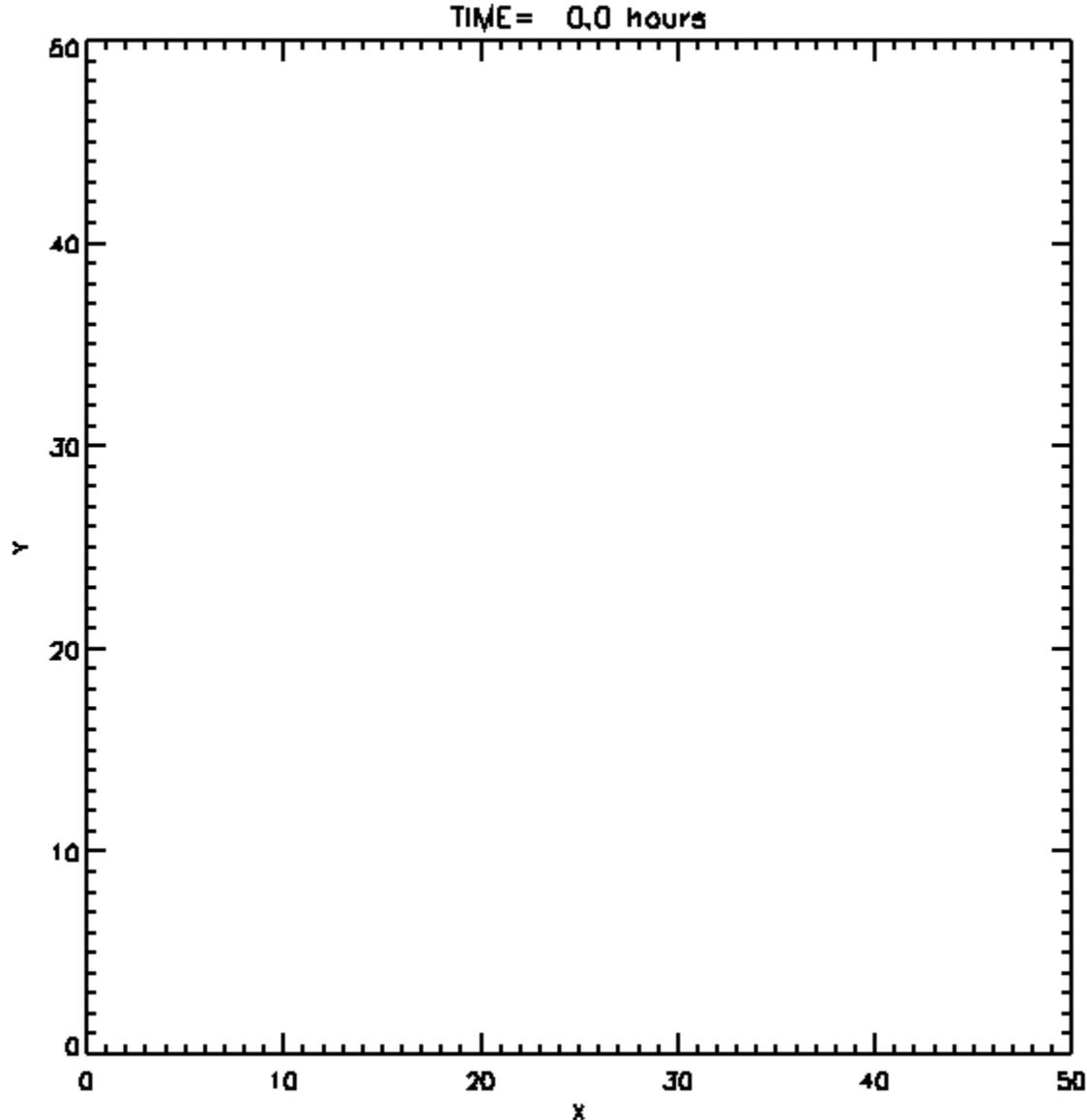


Log-log plot of frequency of emergence against flux emerged for a 5 hour sequence of Hinode/SOT magnetograms.

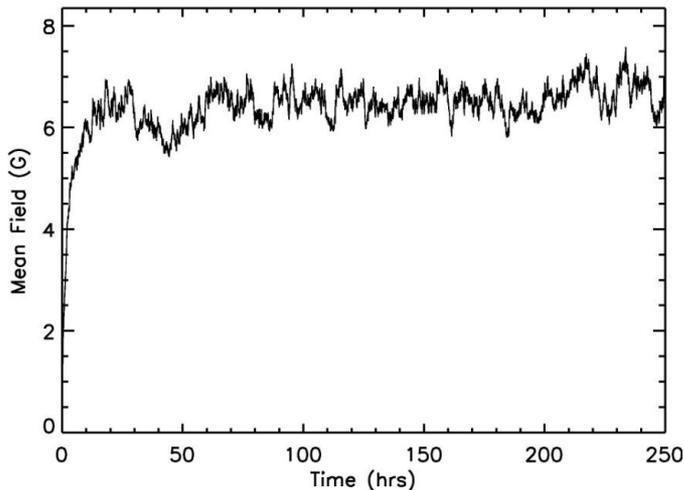
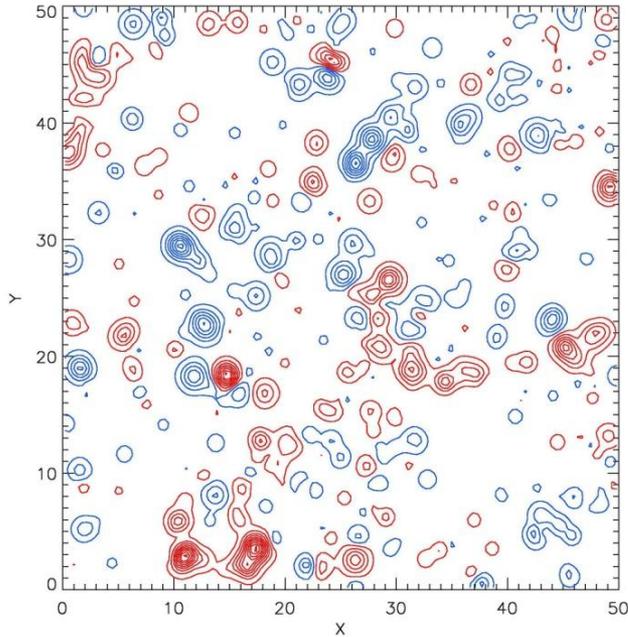
Slope: $-\alpha = -2.74$

2D Magnetic Carpet Model

- 50 x 50 Mm computational box.
- 250 hour simulation.
- 1 min between frames.
- Range for newly emerging bipoles:
 $4 \times 10^{16} - 10^{19} \text{ Mx}$
- Unit flux:
 $\phi_0 = 10^{16} \text{ Mx}$



Photospheric Model: Results



- Model reaches steady state after ~ 24 hours.
(Emergence rate \approx cancellation rate, $Mx \text{ cm}^{-2} \text{ s}^{-1}$)
- Formation of a magnetic network.
- Mean magnetic field $\sim 6 - 7$ G.
- Photospheric recycle time:
1.48 hours
(1 – 2 hours, Hagenaar et al. 2008)
- Average lifespan of flux element:
10 minutes
(1 – 20 mins, mean 2.9 ± 2.0 mins, Zhou et al. 2010)
- Max. lifespan of flux element:
3 – 4 hours
(Ephemeral regions, mean lifespan ~ 4.4 hours, Harvey and Martin, 1973)

Coronal Model

- Magnetofriction ([van Ballegooijen, Priest and Mackay, 2000](#)).
- Photospheric boundary driven by synthetic magnetograms from 2D model.

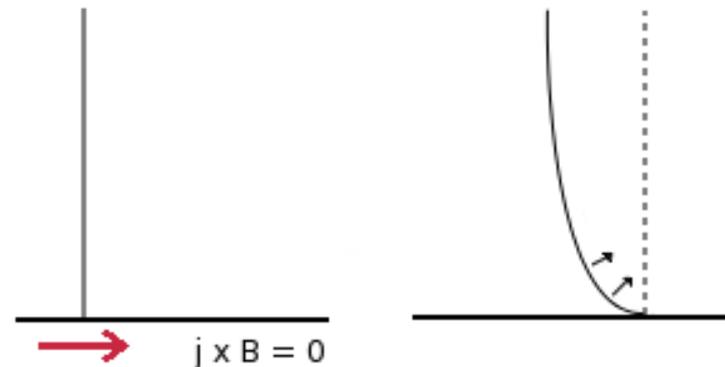
Coronal Field
Induction Equation:

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{v} \times \mathbf{B} + \boldsymbol{\epsilon} \quad \text{where} \quad \boldsymbol{\epsilon} = \frac{\mathbf{B}}{B^2} \nabla \cdot (\eta_4 B^2 \nabla \alpha)$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} = \alpha \mathbf{B}$$

Magnetofrictional
velocity:

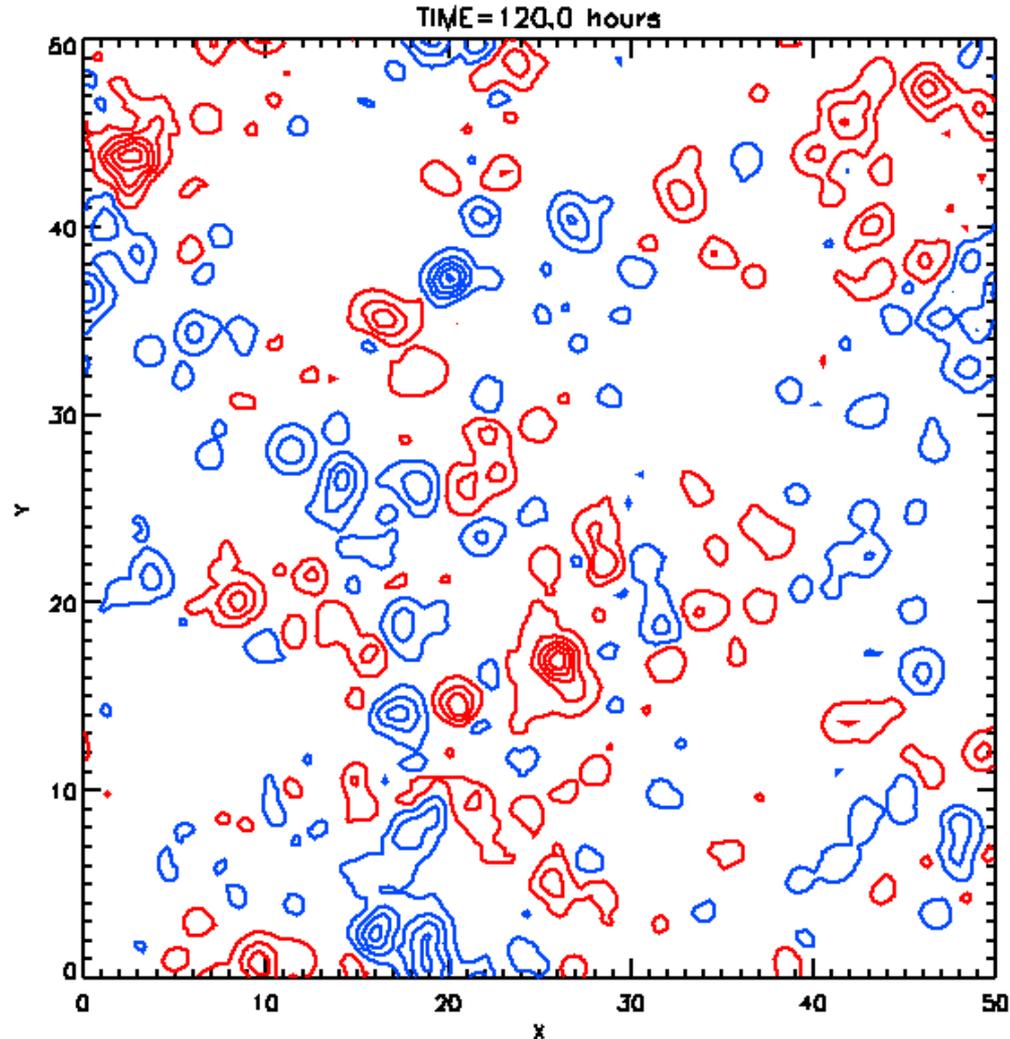
$$\mathbf{v} = \frac{1}{\nu} \frac{\mathbf{j} \times \mathbf{B}}{B^2}$$



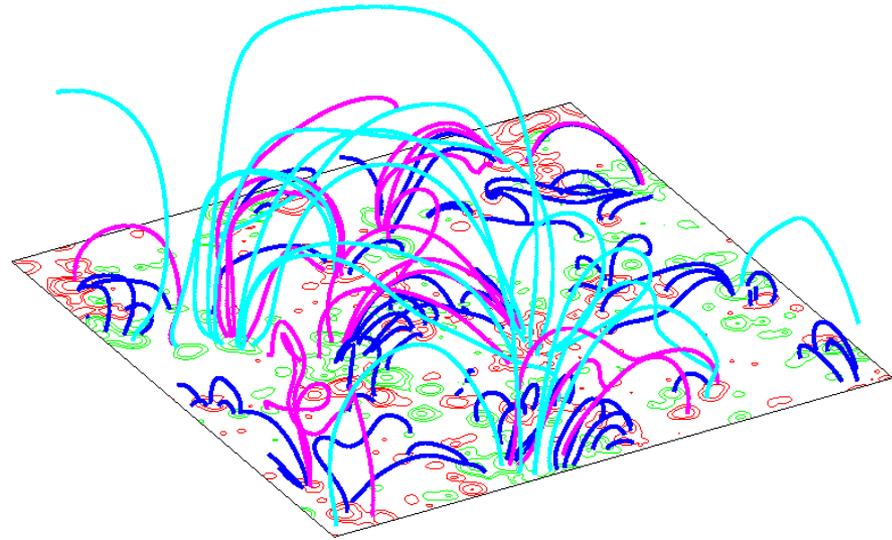
- Other studies: global corona ([Yeates et al. 2008](#)), active region ([Mackay et al. 2011](#), [Gibb et al. 2013](#)), small-scale coronal field ([Meyer et al. 2012](#)).

Coronal Model: Setup

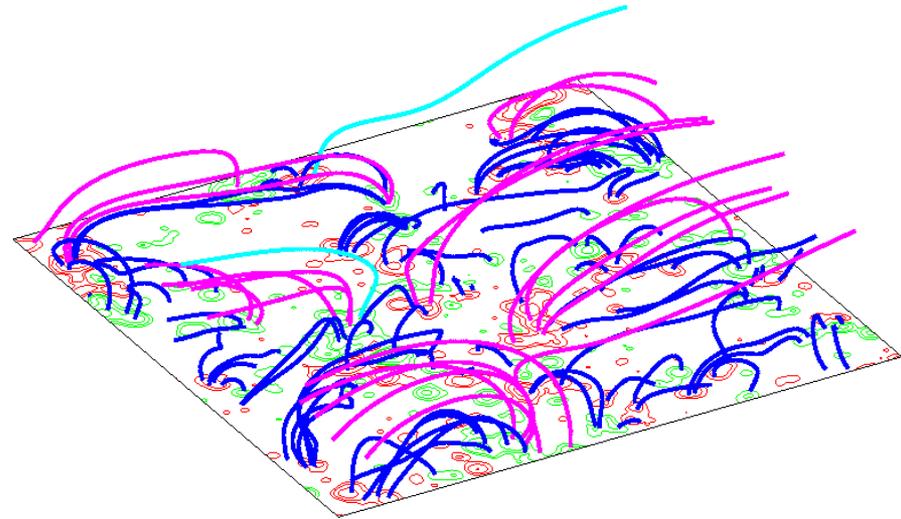
- 48 hr series of synthetic magnetograms
- 120 – 168 hr - model has reached a steady state
- 1 min between magnetograms
- 50 x 50 x 25 Mm (resolution: 512 x 512 x 256)
- 500 relaxation steps between each frame (0.12 s)
- Four strengths of overlying field: 0 G, 1 G, 3 G and 10 G



Coronal Model: Results

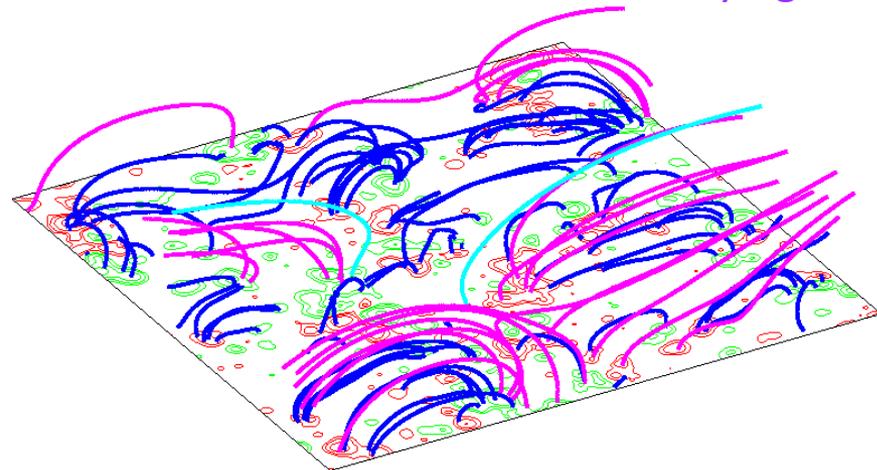


t=128 hr – no overlying field



t=128 hr – 3 G overlying field

Dark Blue field lines: < 5 Mm
Magenta field lines: 5-10 Mm
Light Blue field lines: > 10 Mm

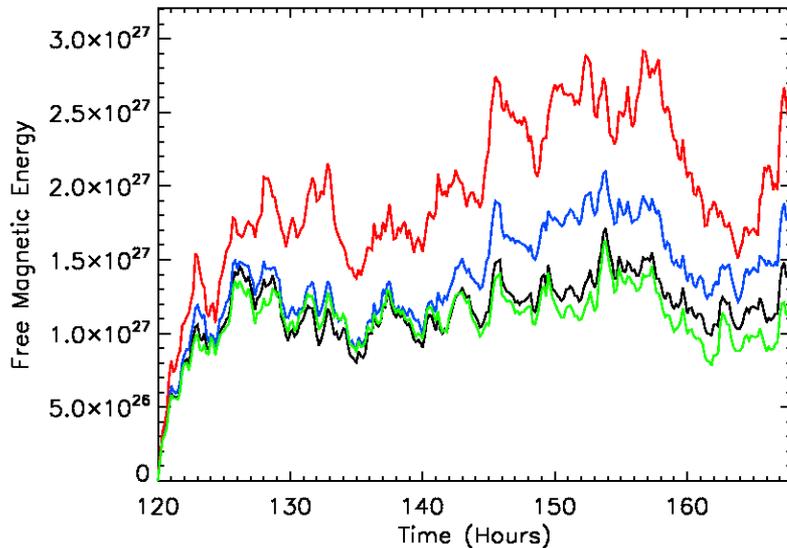


t=128 hr – 3 G overlying field (potential field)

Free Magnetic Energy

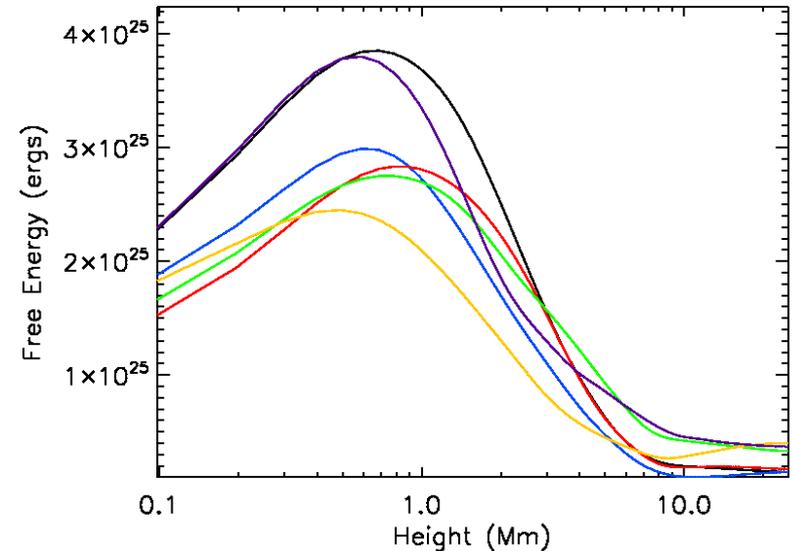
- Free magnetic energy:

$$E_f(t) = \int_V \frac{B_{np}^2 - B_p^2}{8\pi} dV$$



- Green: 0 G
- Black: 1 G
- Blue: 3 G
- Red: 10 G

3 G overlying field

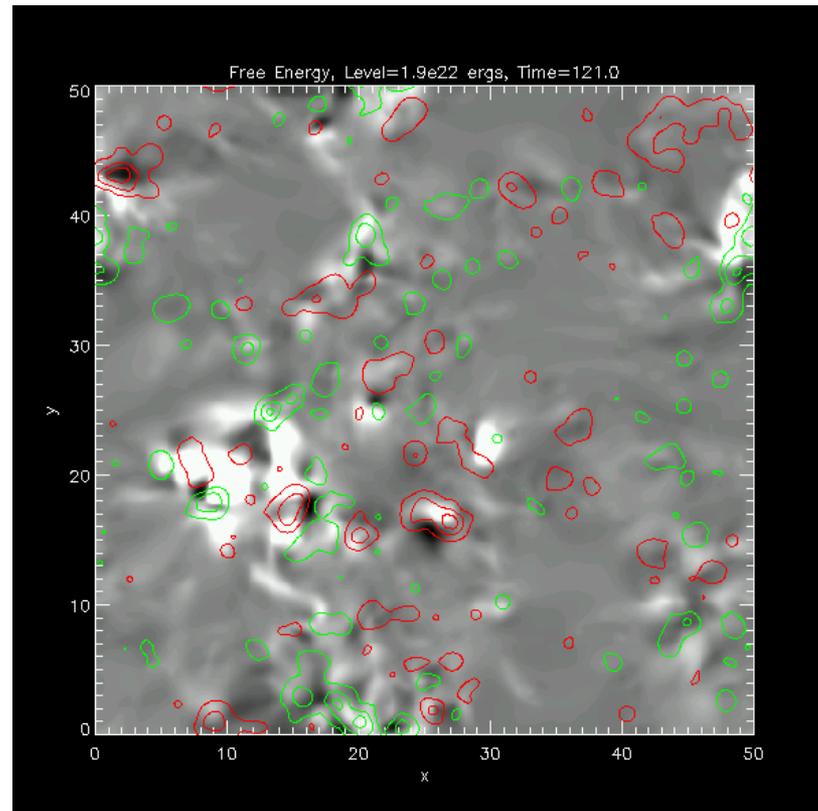


- 128 hr
- 136 hr
- 144 hr
- 152 hr
- 160 hr
- 168 hr

Free Magnetic Energy Density

- Free magnetic energy density:

$$E_f(x, y) = \Delta x \Delta y \int_{z_{\min}}^{z_{\max}} \frac{B_{np}(x, y, z)^2 - B_p(x, y, z)^2}{8\pi} dz$$



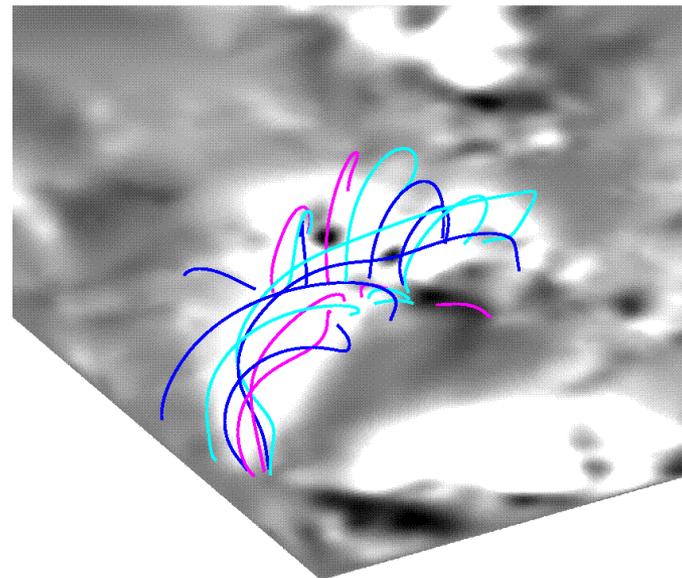
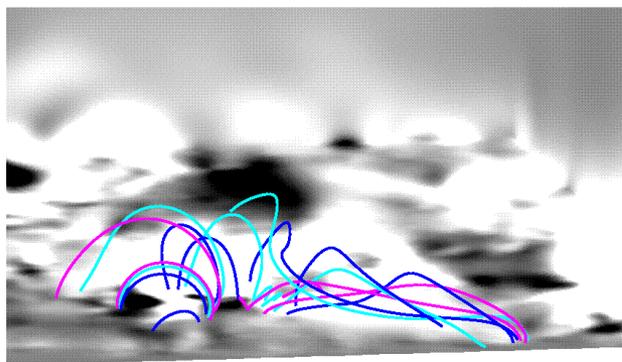
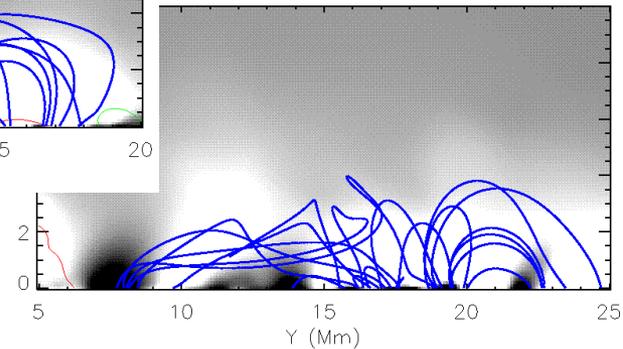
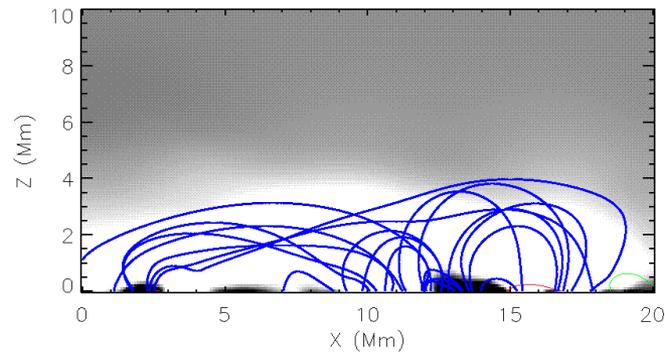
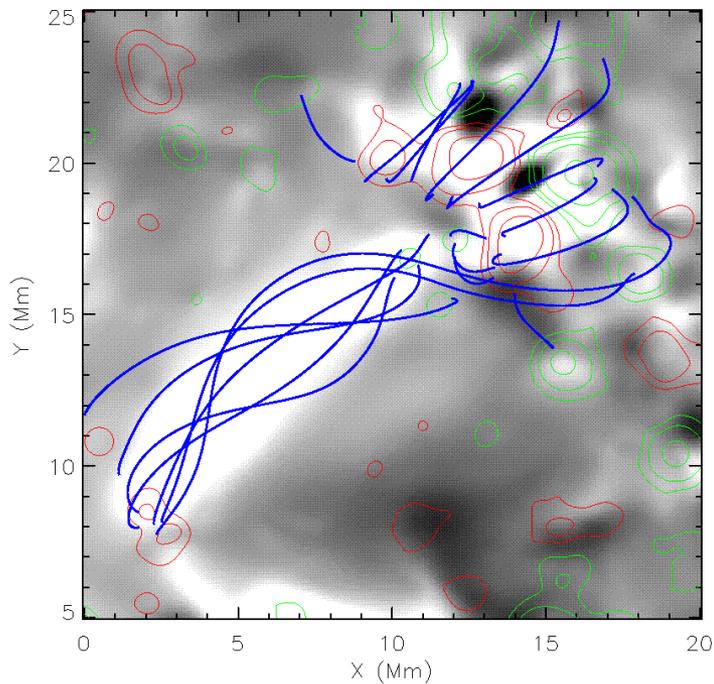
White patches:

$$B_{np}^2 > B_p^2$$

Black patches:

$$B_{np}^2 < B_p^2$$

Free Magnetic Energy Density



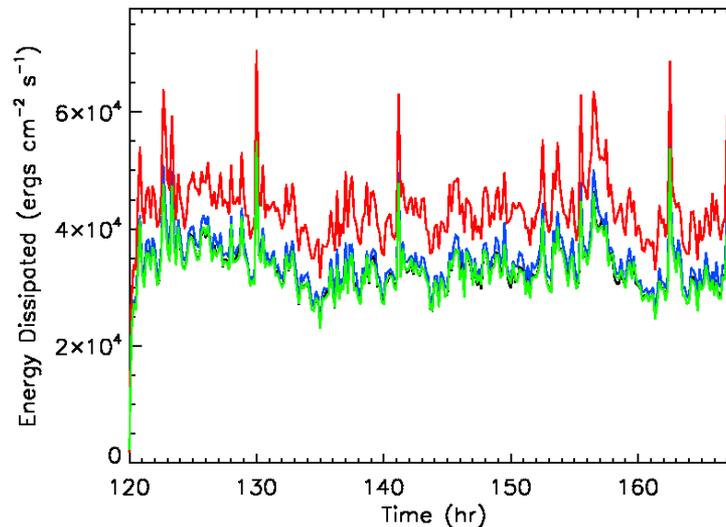
Energy Dissipated

- Rate of change of magnetic energy:

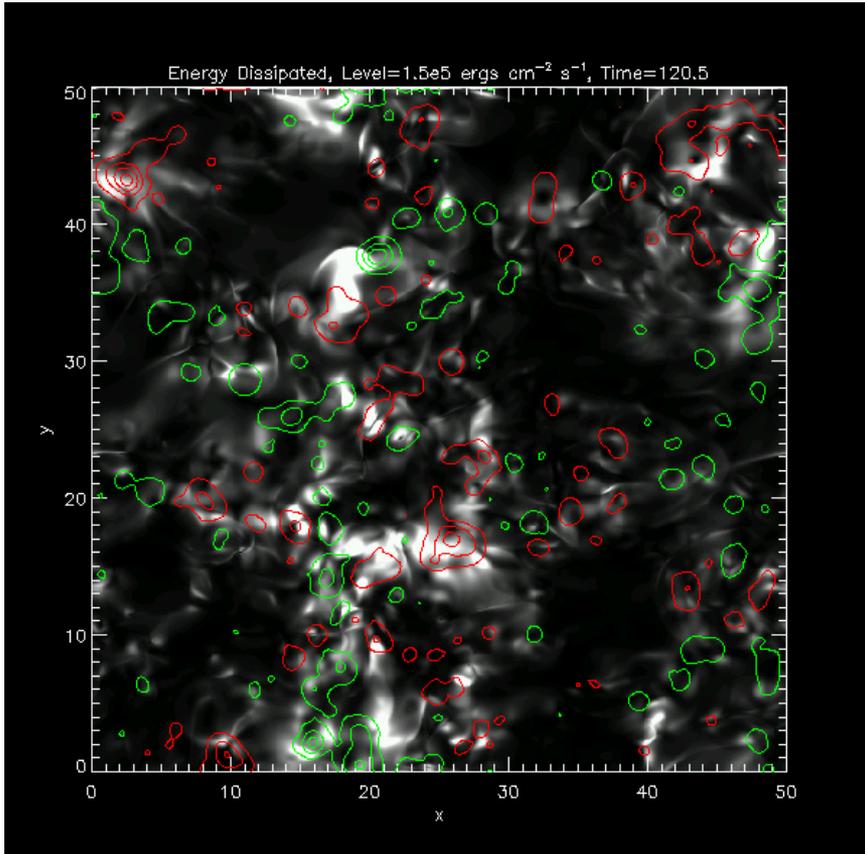
$$\frac{dW}{dt} = \frac{d}{dt} \left(\int_V \frac{B^2}{8\pi} dV \right) = \int_S \mathbf{I} \cdot d\mathbf{S} - \int_V Q dV$$

- Energy dissipated: $Q = \frac{B^2}{4\pi} (\nu |\mathbf{v}|^2 + \eta_4 |\nabla \alpha|)$

(see e.g. van Ballegooijen & Cranmer, 2008; Meyer, Mackay & van Ballegooijen, 2012)



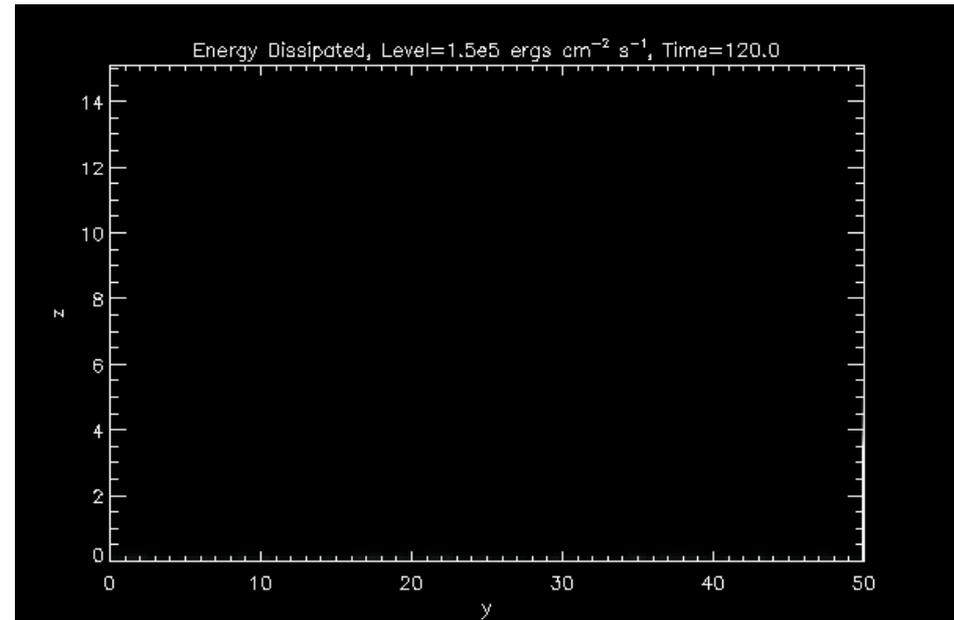
Energy Dissipated



XY plane

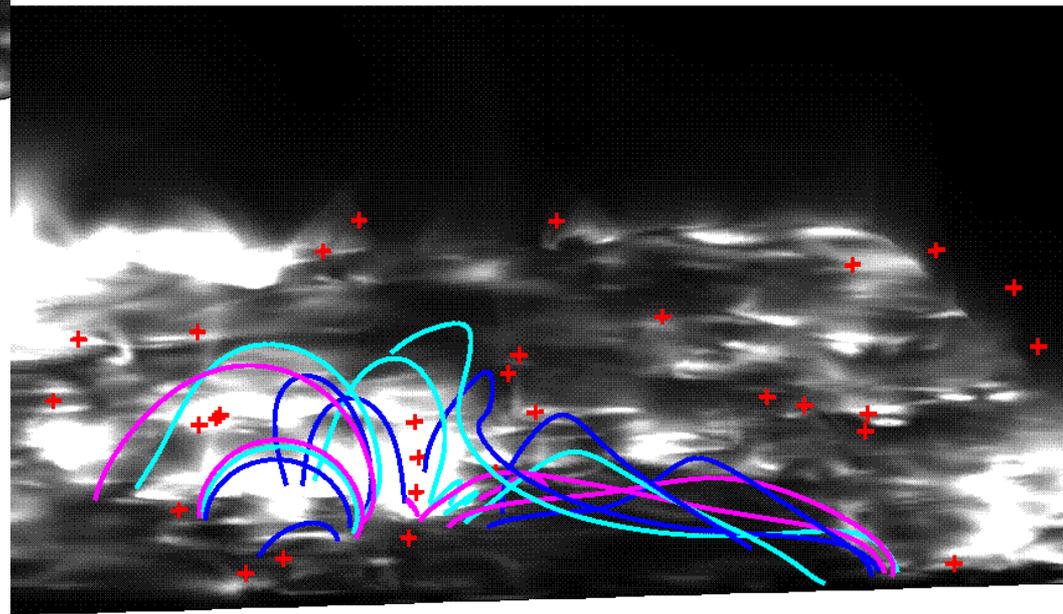
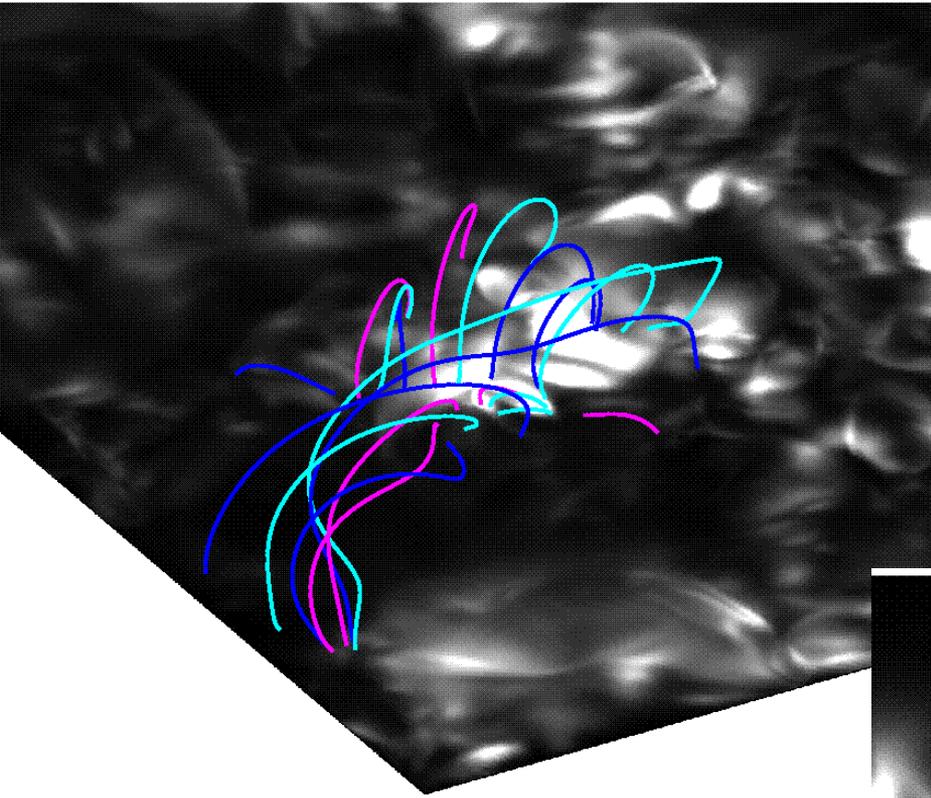
$$Q = \frac{B^2}{4\pi} (v|\mathbf{v}|^2 + \eta_4|\nabla\alpha|)$$

integrated along the line of sight:



YZ plane

Energy Dissipated



Conclusions

- 2D model successfully reproduces many observed properties of magnetic carpet.
- Continuous evolution of coronal magnetic field -> connectivity maintained -> build-up of free magnetic energy.
- Sufficient free energy to explain small-scale, impulsive events such as nanoflares ($\sim 10^{24}$ erg).
- Free energy stored low down, along twisted, closed connections between many magnetic features.
- Energy dissipation is greatest where magnetic field is strong (i.e. near the sources) and at sites of changing magnetic topology.

Photospheric Model: Meyer, Mackay, van Ballegooijen & Parnell, *Solar Phys.*, 2011.

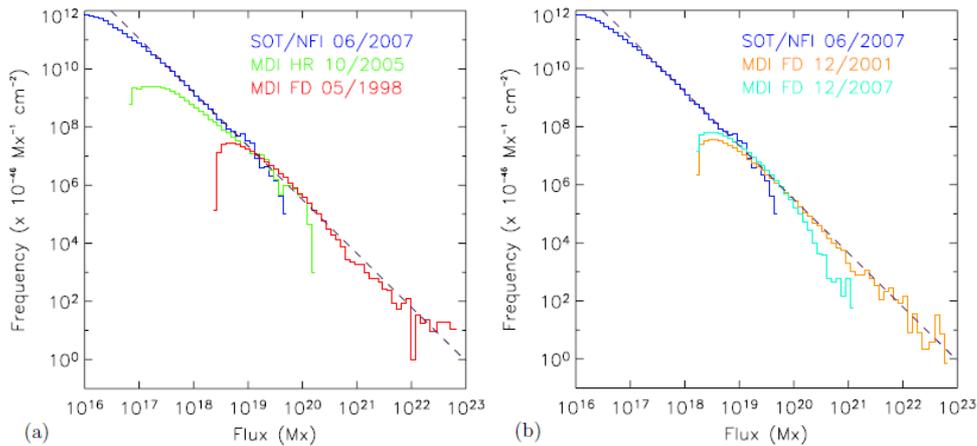
Small-scale Corona: Meyer, Mackay & van Ballegooijen, *Solar Phys.*, 2012.

Coronal Model (synthetic mag.): Meyer, Mackay, van Ballegooijen and Parnell, *Solar Phys.*, 2013.

HMI magnetograms: Meyer, Sabol, Mackay & van Ballegooijen, *ApJL*, 2013.

Current/Future Work

- Improve 2D model: e.g. time-evolving supergranules, fragmentation dependent on underlying flow...



- Check synthetic magnetograms satisfy power-law flux distribution.
- Emergence down to much smaller scales: do we still obtain a power-law distribution?

Parnell et al. 2009

- Global quiet Sun simulations: grand minimum Sun.
- Coronal simulations: how do photospheric events (e.g. cancellation) affect the model corona?