

Toward Data Assimilation Models of Active Regions

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Overview: Using numerical models to understand solar magnetic fields

Numerical models that incorporate data fall into several categories:

- Tracking algorithms and related inversion techniques used to determine surface flows
- Magnetostatic models --- Potential field extrapolations, constant, and non-constant- α force-free extrapolations, steady-state descriptions
- Dynamic models --- Data inspired, data-driven, or fully assimilative MHD

Overview: Using numerical models to understand solar magnetic fields II

“Stand-alone” numerical models:

- Local and global MHD models of the deep convective interior
- Local radiative-MHD models of surface convection and the low atmosphere
- Local and global idealized MHD models of the low atmosphere and corona
- Local semi-realistic MHD models of the upper convection zone and corona

Data-incorporated numerical models: tracking algorithms and velocity inversion techniques

Motivation: A quantitative description of photospheric flows based on a time-series of magnetograms helps determine

- the Poynting flux at the photosphere, and thus the magnetic energy introduced into the corona from below
- the magnetic or current helicity across the photosphere
- whether magnetic flux is emerging, submerging, or undergoing “cancellation”
- how active region flux disperses, giving better estimates of e.g., eddy diffusion coefficients for flux transport models
- the photospheric boundary conditions for MHD models of the solar atmosphere

Tracking algorithms

Tracking algorithms alone (e.g., LCT) directly applied to a sequence of magnetograms do not produce physical flows without additional assumptions and analysis.

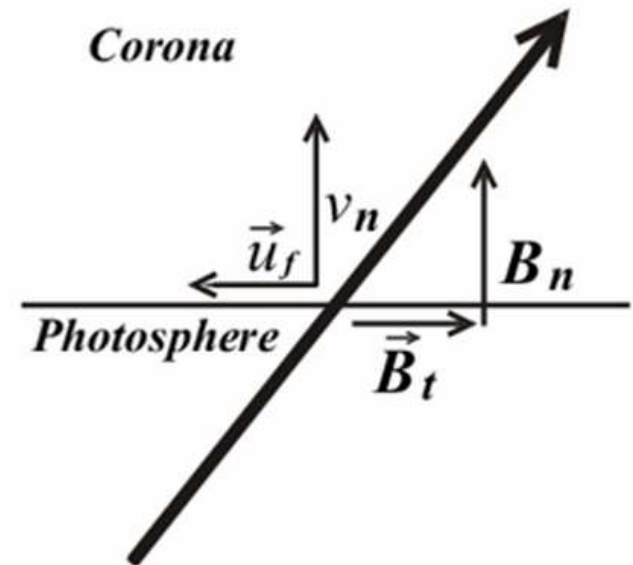
- Thus, tracking velocities do not correspond to the actual flow field

Furthermore, flows are not guaranteed to be physical ; the electric field derived from these flows is unlikely to be consistent with Faraday's law.

Since this is a necessary prerequisite for incorporating sequences of magnetograms into dynamic models of the solar atmosphere, we must address the question

Given a time series of magnetograms, can we determine an electric field consistent with Faraday's law?

$$\mathbf{B}_n \mathbf{u}_f = B_n \mathbf{v}_t - \mathbf{B}_t v_n$$



(Demoulin & Berger 2003)

Velocity inversion techniques

Several groups have tackled this problem in the context of ideal MHD, and developed a number of techniques to solve this underdetermined system for the evolution of the *vertical* component of the magnetic field

- Inductive Local Correlation Tracking, ILCT (Welsch et al. 2003)
- Minimum Energy Fit, MEF (Longcope 2004; Ravindra et al. 2008)
- Differential Affine Velocity Estimator, DAVE (Schuck 2005)
- Minimum Structure Reconstruction Method (Georgoulis & LaBonte 2006).

More recently, there have been an effort to construct velocities or electric fields consistent with all three components of the induction equation

- Ideal Vector Driving, IVD (Welsch et al. 2008 AGU/SPD)
- Poloidal-Toroidal Decomposition, PTD (Fisher et al. 2008 AGU/SPD)

Static models of the coronal magnetic field

Potential field extrapolations (global and local)

$$\mathbf{B} = -\nabla \psi \quad \longrightarrow \quad \mathbf{J} = 0$$

Use data to specify boundary conditions, obtain solution via e.g., Fourier/spherical harmonic decomposition, or a Green's function formalism

Advantages:

Computationally inexpensive, relatively easy to integrate into a pipeline

Disadvantages:

Active region magnetic fields are often not well described by a potential, current-free field

Static models of the coronal magnetic field II

Force-free field extrapolations (global and local)

$$\mathbf{J} \times \mathbf{B} = 0 \quad \longrightarrow \quad \mathbf{J} = 4\pi\alpha\mathbf{B}$$

Use data to specify boundary conditions, obtain solution via e.g., relaxation techniques, ---- see the tests and comparisons of Schrijver et al. (2008)

Advantages:

Provides an improved representation of active region magnetic fields over potential extrapolations

Disadvantages:

A non-constant α force-free extrapolation is a challenging mathematical problem (ill-posed), and for large datasets can be computationally expensive

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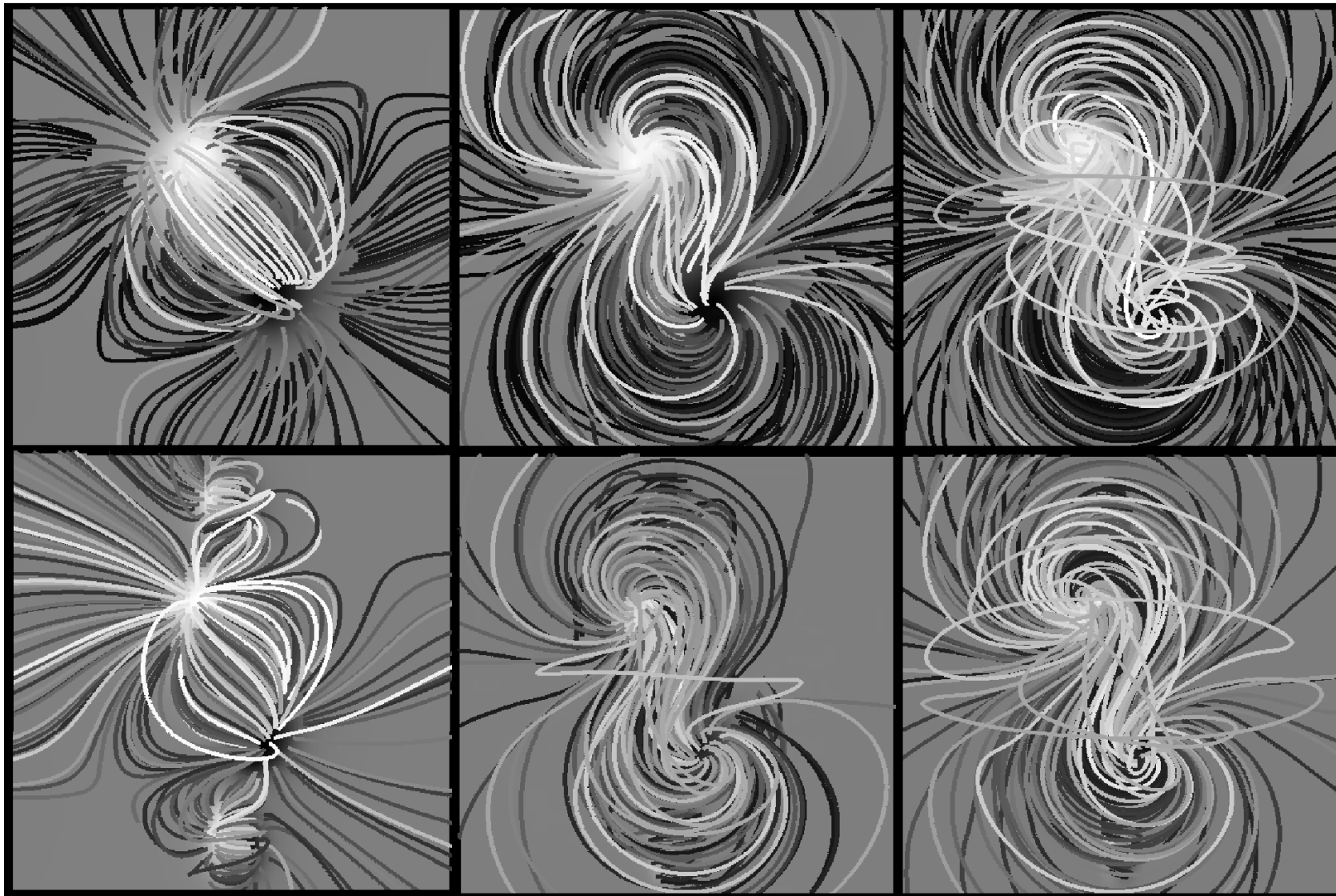
Potential field

NLFF field

MHD solution

Photosphere

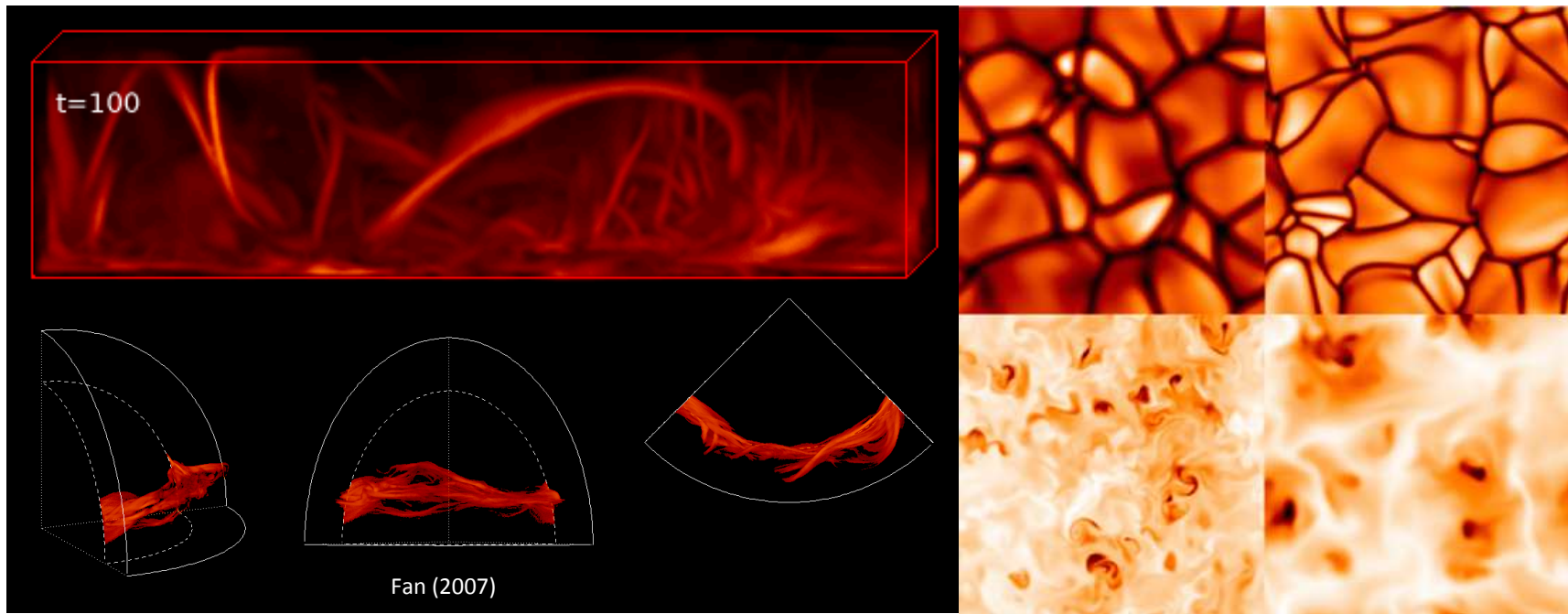
Chromosphere



(Abbett et al. 2004)

Examples of different classes of MHD models

The anelastic approximation: appropriate for low acoustic Mach number plasmas where the Alfvén speed is much less than the local sound speed - -- valid in the high- β solar interior.



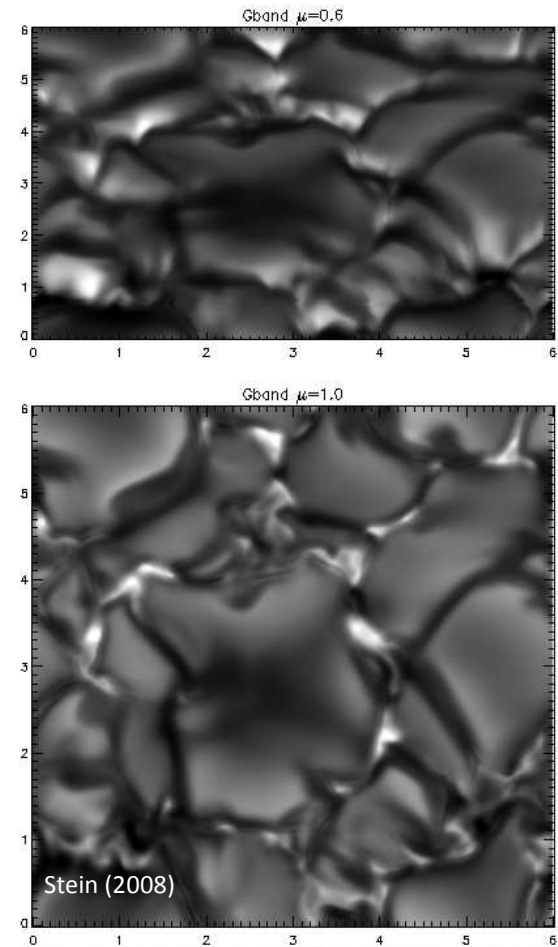
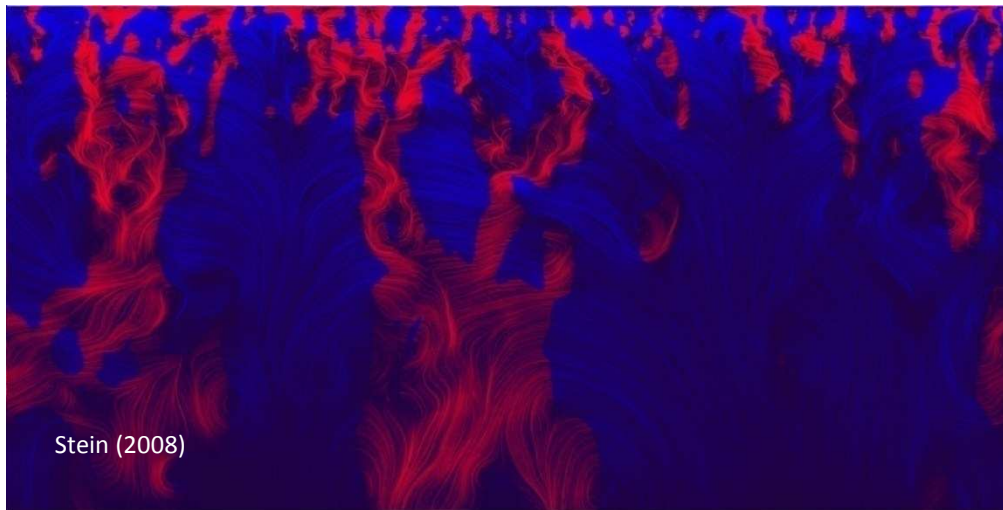
Advantage: efficient

Disadvantage: approximation breaks down near the surface

Implementation: spectral, finite difference

Examples of different classes of MHD models

Fully compressible, realistic radiative-MHD: appropriate for the turbulent surface layers and low-atmosphere. The LTE radiative transfer equation is solved as a part of an MHD system that is closed with a non-ideal equation of state.



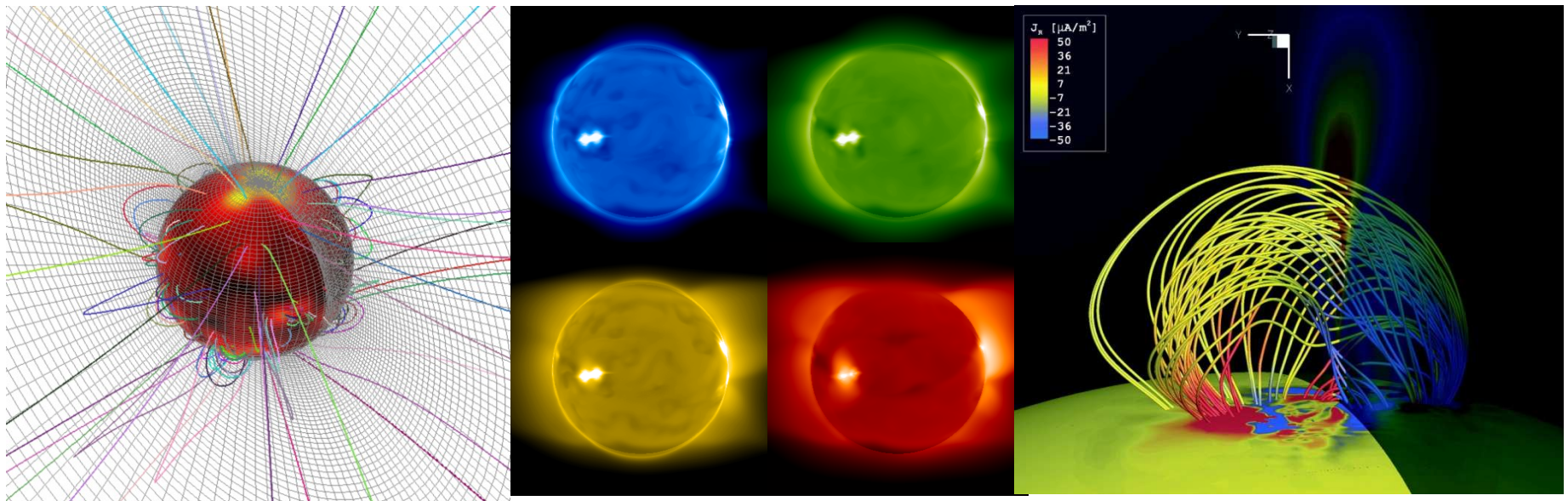
Advantage: can compare directly with observational data

Disadvantage: computationally expensive, spatially restricted

Implementation: finite difference

Examples of different classes of MHD models

MHD models of the corona: different levels of idealization, ranging from adiabatic, polytropic, ideal, to semi-realistic.



SAIC global model corona

Roussev et al. (2007)

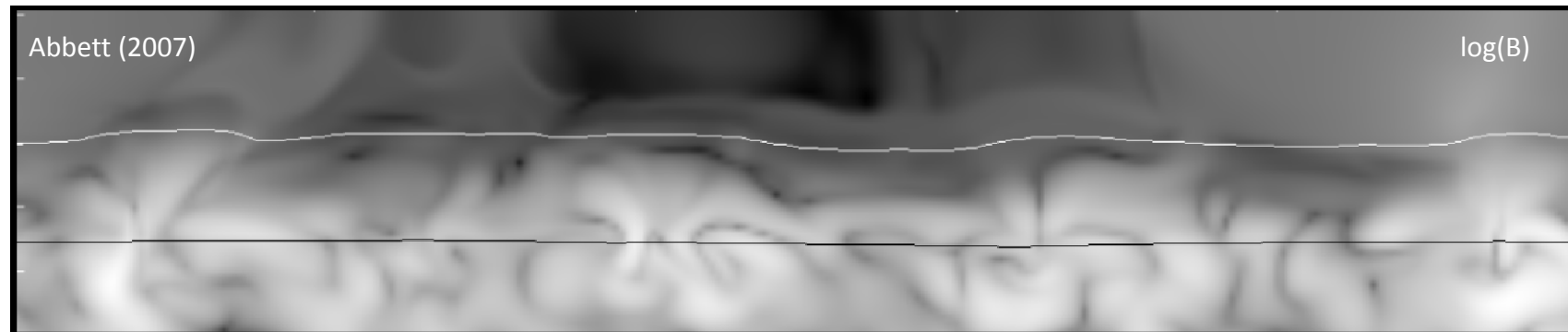
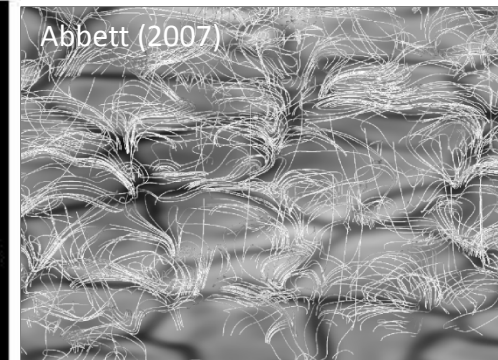
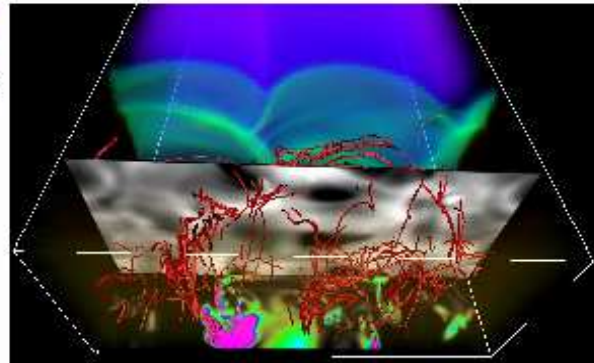
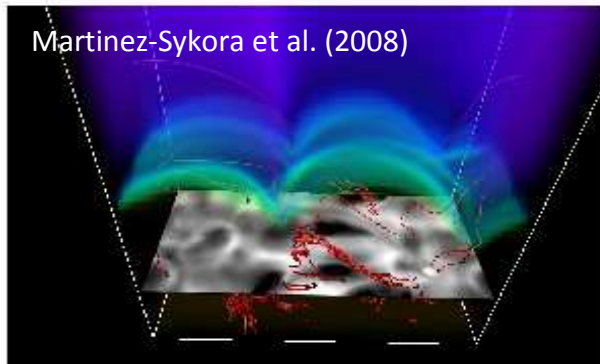
Advantage: efficient, semi-realistic models can incorporate and be compared with data

Disadvantage: thermodynamics not well described in more idealized models

Implementation: shock capture; e.g., upwind differencing, approximate Riemann solvers, FCT

Examples of different classes of MHD models

MHD models of the upper convection zone to corona system: different levels of sophistication ranging from realistic (RT equation solved) to semi-realistic (optically thick RT approximated)



Advantage: can *directly* incorporate photospheric data into a model atmosphere

Disadvantage: realistic models computationally expensive, combined models are research codes that would be difficult to envision as a pipeline product

Implementation: finite difference, hybrid CWENO / JFNK

RADMHD (Abbett 2007): Numerical techniques

- We use a semi-implicit, operator-split method.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left[\rho \mathbf{u} \mathbf{u} + \left(p + \frac{B^2}{8\pi} \right) \mathbf{I} - \frac{\mathbf{B} \mathbf{B}}{4\pi} - \Pi \right] = \rho \mathbf{g}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{u} \mathbf{B} - \mathbf{B} \mathbf{u}) = -\nabla \times \eta (\nabla \times \mathbf{B})$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (e \mathbf{u}) = -p \nabla \cdot \mathbf{u} + \frac{\eta}{4\pi} |\nabla \times \mathbf{B}|^2 + \phi + Q$$

- **Explicit sub-step:** We use a 3D extension of the **semi-discrete** method of Kurganov & Levy (2000) with the third order-accurate central weighted essentially non-oscillatory (**CWENO**) polynomial reconstruction of Levy et al. (2000).
- **CWENO interpolation** provides an efficient, accurate, simple shock capture scheme that allows us to resolve shocks in the transition region and corona without refining the mesh. The solenoidal constraint on \mathbf{B} is enforced implicitly.

RADMHD: Numerical techniques

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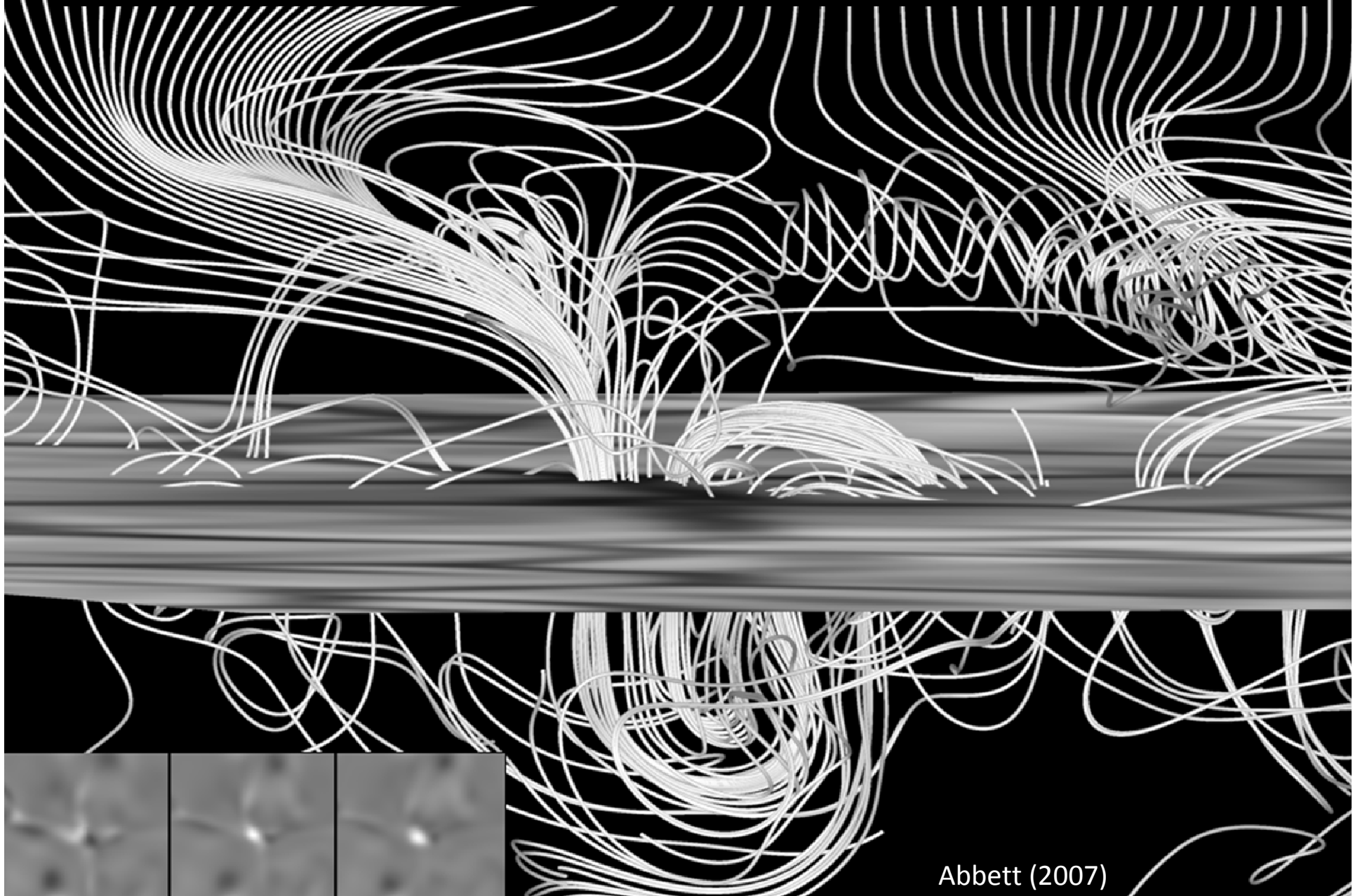
$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left[\rho \mathbf{u} \mathbf{u} + \left(p + \frac{B^2}{8\pi} \right) \mathbf{I} - \frac{\mathbf{B} \mathbf{B}}{4\pi} - \mathbf{\Pi} \right] = \rho \mathbf{g}$$

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- **Implicit sub-step:** We use a “**Jacobian-free**” **Newton-Krylov (JFNK)** solver (see Knoll & Keyes 2003). The Krylov sub-step employs the generalized minimum residual (**GMRES**) technique.
- **JFNK** provides a memory-efficient means of implicitly solving a non-linear system, and frees us from the restrictive CFL stability conditions imposed by e.g., the electron thermal conductivity and radiative cooling.

Flux submergence in the quiet Sun and the connectivity between an initially vertical coronal field and the turbulent convection zone



Abbett (2007)

Progress toward data driving:

- Numerical models of the corona can now extend into the photosphere where vector magnetic fields are measured.
- It may now be possible to routinely derive a photospheric flow field from a time series of vector magnetograms that will provide the forces necessary to drive an MHD model atmosphere in a manner fully consistent with the observed evolution of the vector magnetic field.

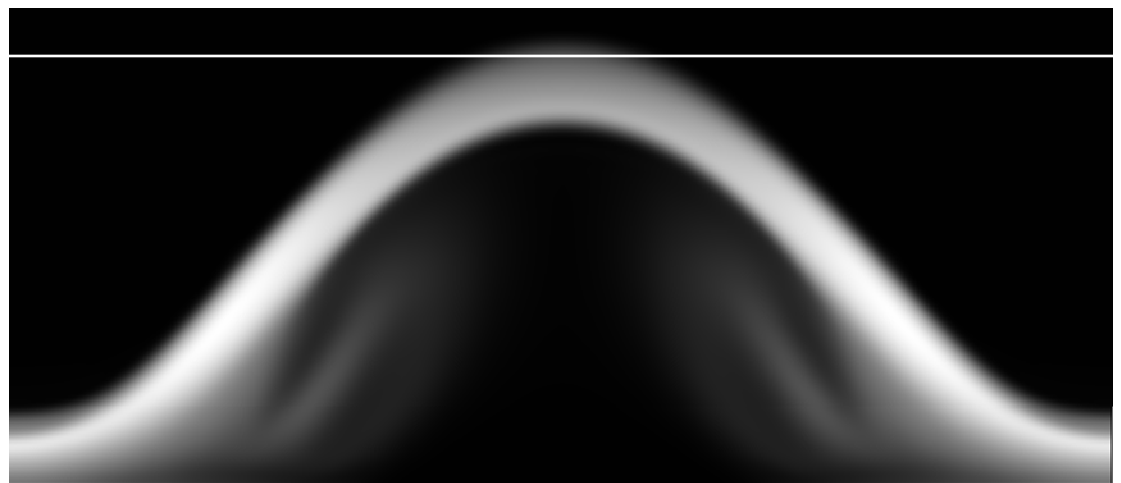
Remaining challenges:

- Determine the most physically-consistent means to dynamically couple the kinematic model photosphere to the MHD model atmosphere.
- Apply these methods to simulated datasets, and compare the atmospheric evolution (a crucial “hare and hound” exercise)
- Determine a means to disambiguate the magnetogram data in such a way that ensures consistency between subsequent magnetograms.



Above: The magnetic structure of the RADMHD model atmosphere

Right: The ANMHD sub-surface omega loop (Run SS3 from Abbett et al. 2000) used to emerge magnetic flux into the model atmosphere from below.



What would an “Assimilative Model” of the solar atmosphere consist of?

A time-evolving physical model of the Sun’s atmosphere, or a portion of the Sun’s atmosphere, which can be corrected by time-dependent measurements that can be related in some manner to properties of the solar atmosphere.

In particular, this means a 3D-MHD model of an active region, from photosphere to corona, that is updated by means of vector magnetograms.

What are the most important elements of a physics-based model of the Sun?

- Nearly all transient phenomena, such as solar-initiated “space weather” events, are driven by, or strongly affected by, magnetic fields.
- A fluid treatment (MHD) is reasonable most of the time (except, probably, during solar flares).
- Magnetic fields thread all layers of the Sun’s convection zone and atmosphere.
- Maps of the estimated solar magnetic field (line-of-sight component) can be performed regularly in the photosphere.
- In the near future, maps of all 3 components of the estimated magnetic field (vector magnetograms) will be taken regularly.
- Vector magnetograms are essential for determining the free energy available in the solar atmosphere to drive violent phenomena. Without vector magnetograms, solar models are not meaningfully constrained.

Needed ingredients for an assimilative model of the solar atmosphere:

1. A reasonably good physical model Good progress!
2. Measurements with a good enough time cadence and accuracy to be useful Rapidly improving!
3. A well-understood connection between physical and measured variables Reasonably good
4. A good understanding of the data and model errors
Data errors - reasonably good; Model errors – unknown!

Where do we stand with respect to these requirements?

Formal data assimilation: the Kalman filter

- M - State Transition Matrix
- P - Model Error Covariance
- y - Data Vector
- R - Data Error Covariance
- X - Model State Vector
- η - Transition Model Error
- Q - Transition Model Error Covariance
- H - Measurement Matrix
- ϵ - Observation Error
- K - Kalman Gain

$$\mathbf{x}^f = \mathbf{M}\mathbf{x} + \eta \quad (1)$$

$$\mathbf{P}^f = \mathbf{M}\mathbf{P}\mathbf{M}^T + \mathbf{Q} \quad (2)$$

$$\mathbf{y}^o = \mathbf{H}\mathbf{x} + \epsilon \quad (3)$$

$$\mathbf{K} = \mathbf{P}^f \mathbf{H}^T (\mathbf{H} \mathbf{P}^f \mathbf{H}^T + \mathbf{R})^{-1} \quad (4)$$

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}(\mathbf{y}^o - \mathbf{H}\mathbf{x}^f) \quad (5)$$

$$\mathbf{P}^a = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}^f \quad (6)$$

Formalism is appropriate for linear systems; covariance matrices can be prohibitively large. For complex non-linear systems, an “ensemble” Kalman filter is likely required.

RADMHD + data assimilation?

- Collaboration with NSO GONG team members F. Hill, R. Komm, I. González-Hernández to incorporate GONG magnetograms and/or subsurface velocity maps into RADMHD
- Data assimilation tutorial by Utah State GAIM group
 - Use ensemble Kalman filter
 - However, computational cost to run many instances of RADMHD prohibitive
 - Start simple!

Simpler model: flux transport

- Passive scalar transport of radial magnetic field (global)

$$\frac{\partial B_r}{\partial t} = -\nabla_{\perp} \cdot (B_r \mathbf{v}) + \eta \nabla_{\perp}^2 B_r + \nabla_{\perp} \eta \cdot \nabla_{\perp} B_r$$

- Parameterized global flows and turbulent diffusivity
 - Differential rotation
 - Meridional circulation
- Planned tests
 - Fix flows, assimilate magnetic field --- use model to improve poor polar data, use data to improve model flux emergence
 - Take magnetic field data as “error-free” --- vary flow parameters, tune meridional flow for solar cycle variations?
 - Quantify data and model errors
 - How to handle large data gaps? Can we incorporate far-side images?

Summary

- Solar data assimilation models are in their infancy.
- But we have reached a point where models and data overlap on spatial and temporal scales , so assimilative techniques are worth detailed investigation
- Vector magnetograms are the best data option, but subsurface flow maps may also be viable
- Full MHD models are still computationally expensive, but simpler models may be more amenable to data assimilation. Data driving be the best approach for more complex models