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Earth's atmosphere and physical environment...

# HIGH ALTITUDE OBSERVATORY

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## Solar Dynamo Models

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SVECSE 2008 June 2 Bozeman, Montana



NCAR

High Altitude Observatory (HAO) – National Center for Atmospheric Research (NCAR)  
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# Outline

- Why is modeling of the solar dynamo so difficult
  - Limitations of different modeling approaches
  - Observational constraints
- Incomprehensive overview of solar dynamo models
  - Overshoot dynamos
  - Distributed dynamos
    - Interface dynamos
    - Flux transport dynamo models
- Current state of 3D simulations
- Solar-Stellar connection

# Why is it so difficult to model the solar dynamo?

## ➤ 3D simulations:

- Large scale solar dynamo simulation not feasible yet – but significant progress over past couple years
- Too large separation in time and length scales (large  $Re$ ,  $Rm$ , strong stratification)
  - Geodynamo: no stratification,  $Rm < 1000$ !
- Solar dynamo: some ingredients, no large scale cyclic dynamo
- Fast rotating solar-like stars: large scale dynamo action (B. Brown, M. Miesch, J. Toomre)

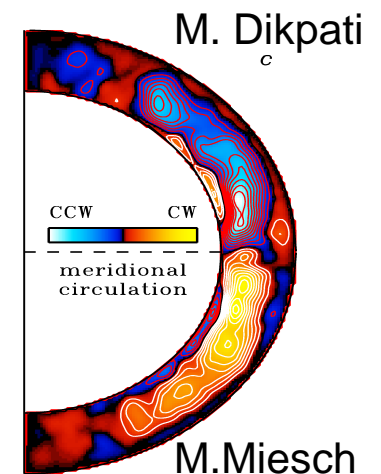
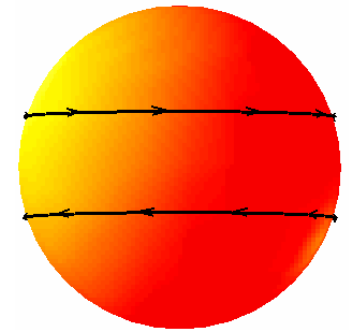
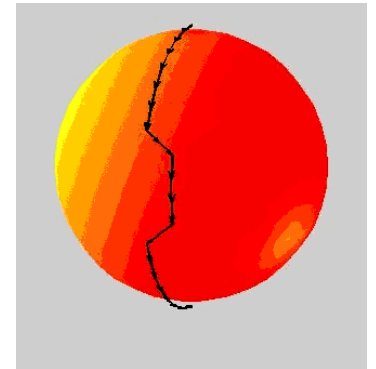
# Why is it so difficult to model the solar dynamo?

## ➤ Mean field models:

- Focus on large scale field (in most cases axisymmetric component)
  - Significant reduction in computational cost
- Main drawbacks
  - Convective (turbulent) induction effects have to be parametrized
  - No general recipe to derive these effects that is guaranteed to yield more than qualitative results
  - Require in general some tuning – Do we get the right answer for the right reason? Robustness of results?
- Rule of thumb: Look for robust features

# Why is it so difficult to model the solar dynamo?

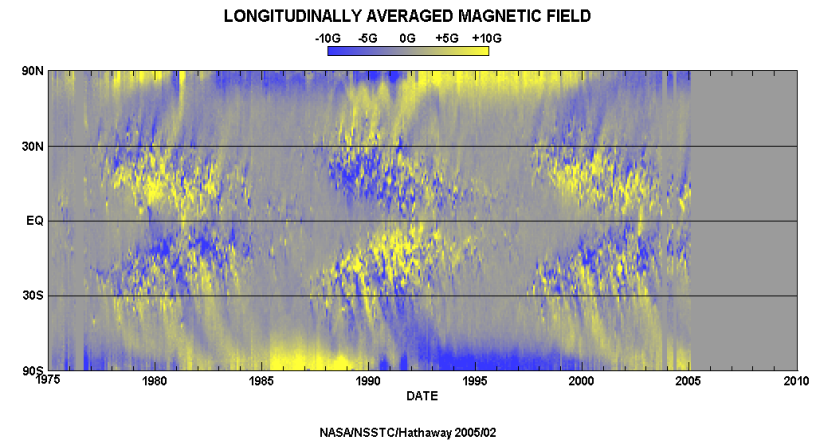
- Major dynamo ingredients are identified
  - Poloidal  $\rightarrow$  toroidal field
    - Differential rotation (observed)
  - Toroidal  $\rightarrow$  poloidal field ( $\alpha$ -effect)
    - Babcock-Leighton (observed)
    - Convective (uncertain)
    - Tachocline (uncertain)
  - Transport
    - Meridional flow (top layers of CZ)
    - Turbulent diffusivity (photosphere)
    - Turbulent pumping (uncertain)
- Relative contribution unknown, can observations help?



# Solar dynamo – observational constraints

## ➤ Butterfly diagram

- 11 year dynamo period
- Equatorward propagation
- Weak overlap between cycles
- Activity in low latitudes
  - Does not necessarily rule out field in high latitudes (provided field is stable there)



D. Hathaway

## ➤ Poloidal surface field

- Polar field strength
- Poleward migration
- Phase relation to toroidal field
  - Result of Joy's law (related to dynamics of rising flux tubes)
  - Weak constraint on dynamo mechanism

# Solar dynamo – observational constraints

- Hemispheric preference of helicity
  - Sign of  $\alpha$ -effect?
  - Formation of flux tubes?
- Hemispheric coupling, symmetry across equator
- Total flux of AR in 11 year cycle  $\sim 10^{25}$  Mx
  - $\Delta\theta \sim 30$  deg,  $\Delta r \sim H_p$ ,  $B \sim 10$  kG  $\rightarrow \phi \sim 10^{25}$  Mx
  - $E_{\text{mag}} = \frac{1}{4} B\phi r \sim 10^{39}$  ergs  $\sim E_{\text{diffrot}}$
- Non-axisymmetric constraints
  - Active longitudes
- Predictability?

# Solar dynamo – key questions

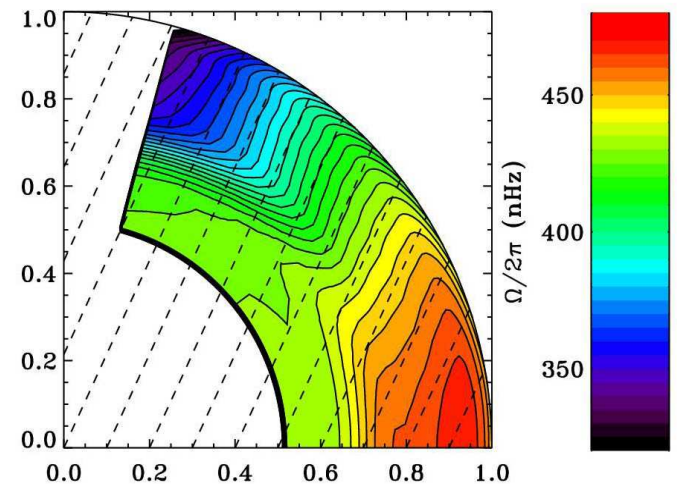
## ➤ Role of tachocline

– Overshoot/tachocline dynamo

- $\Omega$  and  $\alpha$  effect in tachocline

– Distributed dynamo

- Interface dynamo
  - $\Omega$  effect in tachocline
  - $\alpha$  effect in convection zone
- Convection zone dynamo
  - $\Omega$  effect,  $\alpha$  effect in convection zone
  - Base of CZ/Overshoot: Flux storage



R. Howe

## ➤ Role of meridional flow

– Surface: essential ingredient

– Bulk of CZ: flow structure, magnetic diffusivity

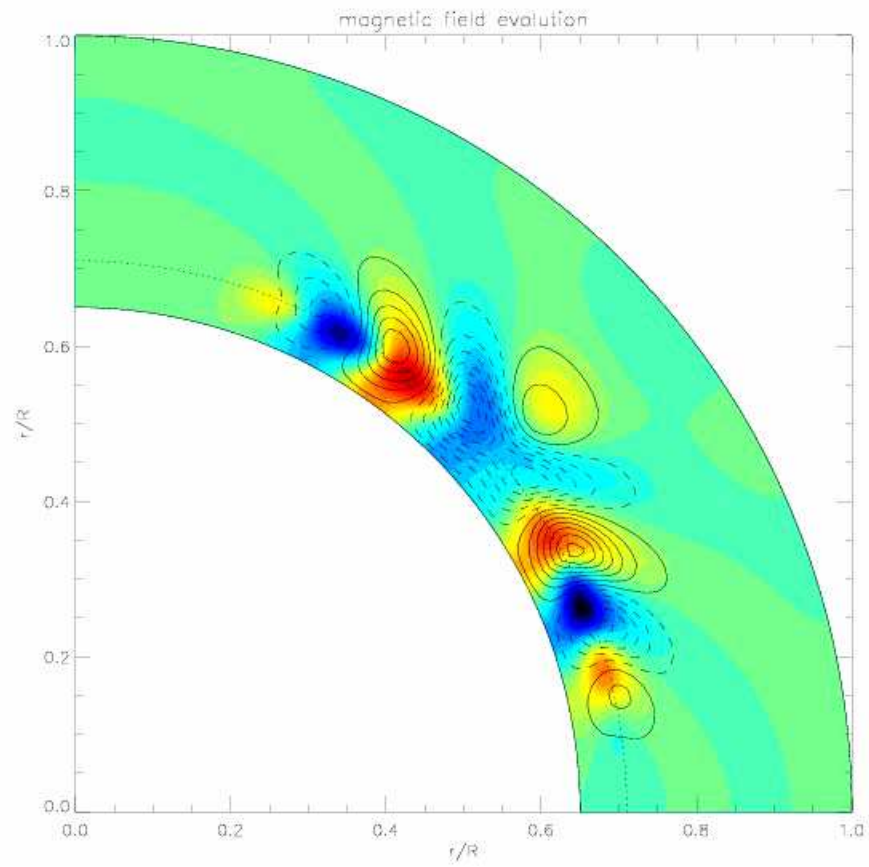


# Solar dynamo – key questions

- Origin of the alpha-effect
  - Convective  $\alpha$  effect (classic mean field theory)
  - Tachocline instabilities (hydro + magnetic)
  - Babcock-Leighton mechanism
    - Rising flux tubes (magn. Buoyancy) + Coriolis force
    - Tilt angle of active regions (only observed  $\alpha$  effect)
- What determines flux and field strength of dynamo
  - Geometric constraints (similar for other stars, independent of rotation rate)
  - Differential rotation
  - Potential energy of stratification ('explosion of flux tubes')

# Overshoot/tachocline dynamos

- Strong radial shear (energy source) + stable stratification (storage of strong magnetic field)
- Strong radial shear + alpha effect
  - Latitudinal propagation (with neg.  $\alpha$ )
    - Equatorward in low latitudes
    - Poleward in high latitudes
  - Strongest activity in high latitudes
    - Not necessarily a problem if field is more stable in high latitudes
  - Multiple overlapping activity belts
    - Intrinsic problem of all dynamos in thin shells
    - Major problem, contradiction to butterfly diagram



# Distributed dynamos

- Interface dynamos (Parker, Charbonneau, MacGregor)
  - $\Omega$  effect in tachocline,  $\alpha$  effect in convection zone
  - Proposed by Parker (1993) to overcome problems with too strong  $\alpha$  quenching
    - Strong toroidal field in tachocline
    - $\alpha$  effects operates on weak poloidal field in CZ
    - Diffusive coupling
  - Overlapping cycles typically not a problem (shell is sufficiently thick)
  - Solutions very sensitive to details such as  $\alpha$  profile and location and tachocline thickness

# Flux transport dynamo

(Durney, Choudhuri, Schuessler, Dikpati, Nandi, Charbonneau, Gilman, Rempel)

## ➤ Motivation:

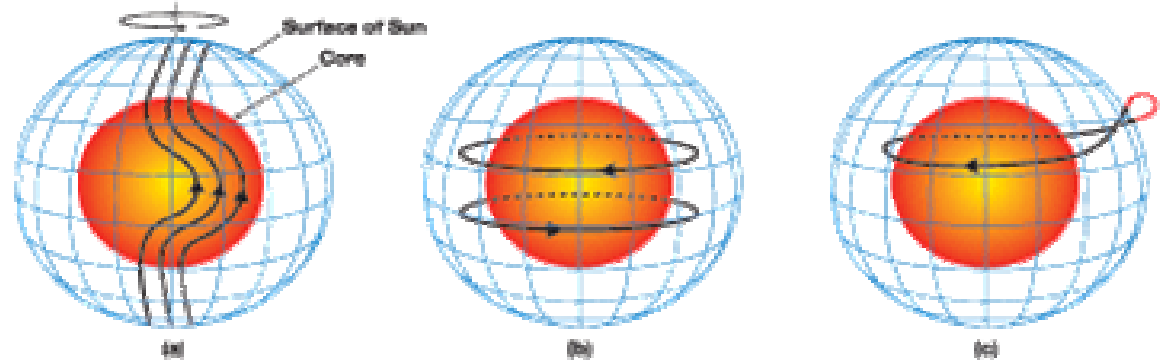
- Meridional flow plays essential role in the evolution of the surface magnetic field
- Bulk of convection zone has little radial shear
  - Impossible to have dynamo waves propagating in latitudinal direction
- Meridional return flow of a few m/s expected at base of CZ from continuity arguments
  - Similar to propagation speed observed in butterfly diagram
  - Overturning time scale  $\sim$  cycle

-> Advection dominated dynamo

# Babcock-Leighton flux transport model

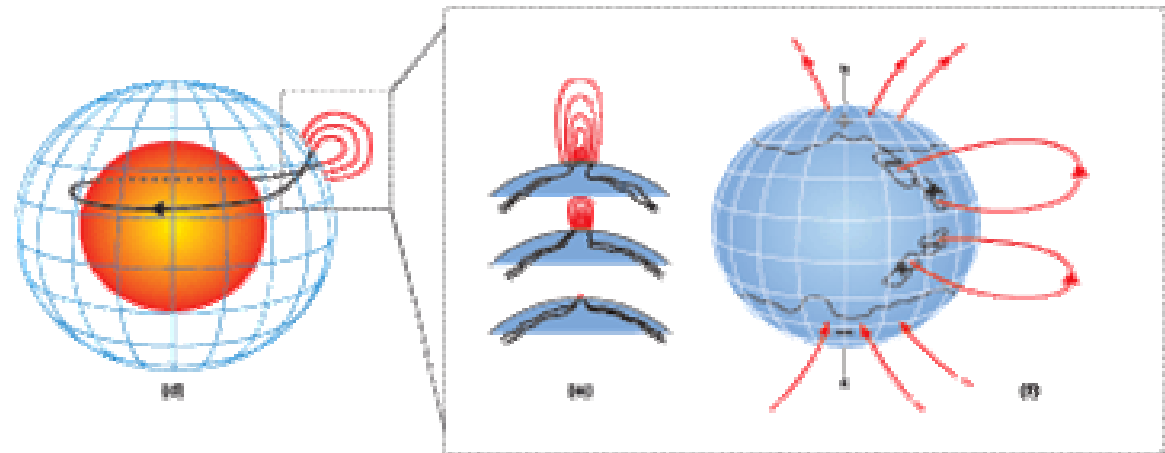
## ➤ Differential rotation

- Toroidal field production
- Stored at base of CZ
- Rising flux tubes

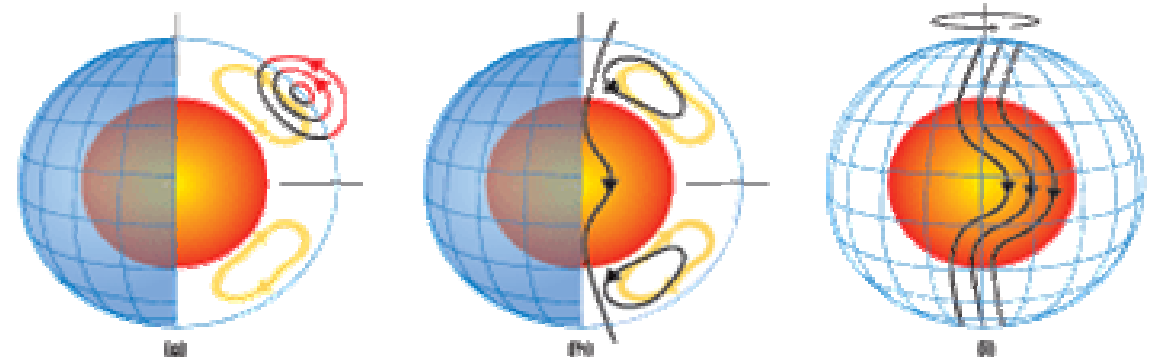


## ➤ Babcock-Leighton $\alpha$ effect

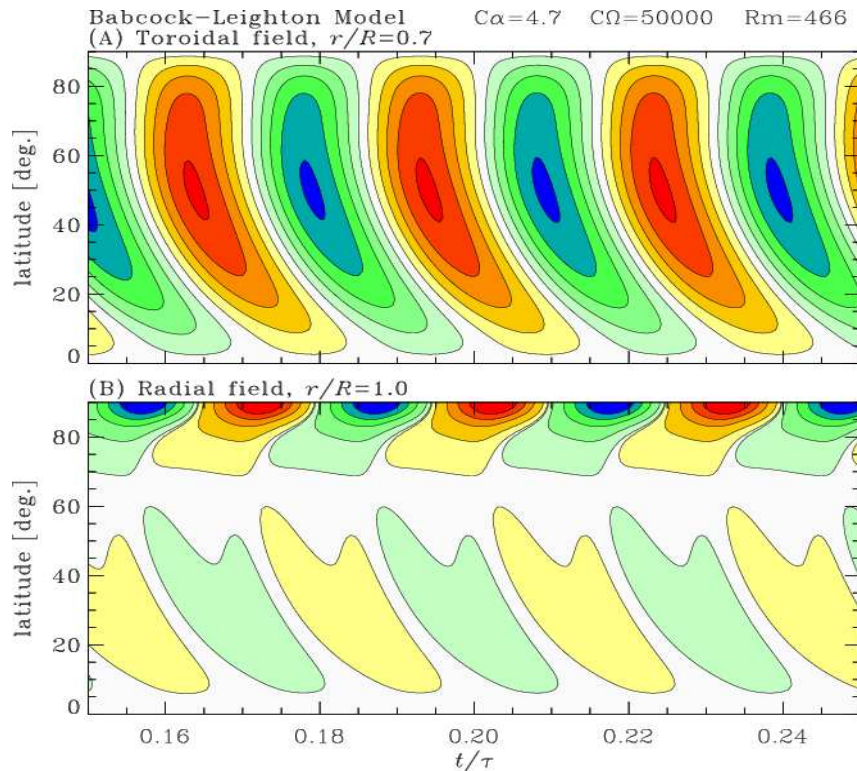
- Tilt angle of AR
- Decay active regions  $\rightarrow$  poloidal field regeneration



## ➤ Transport of magnetic field by meridional flow



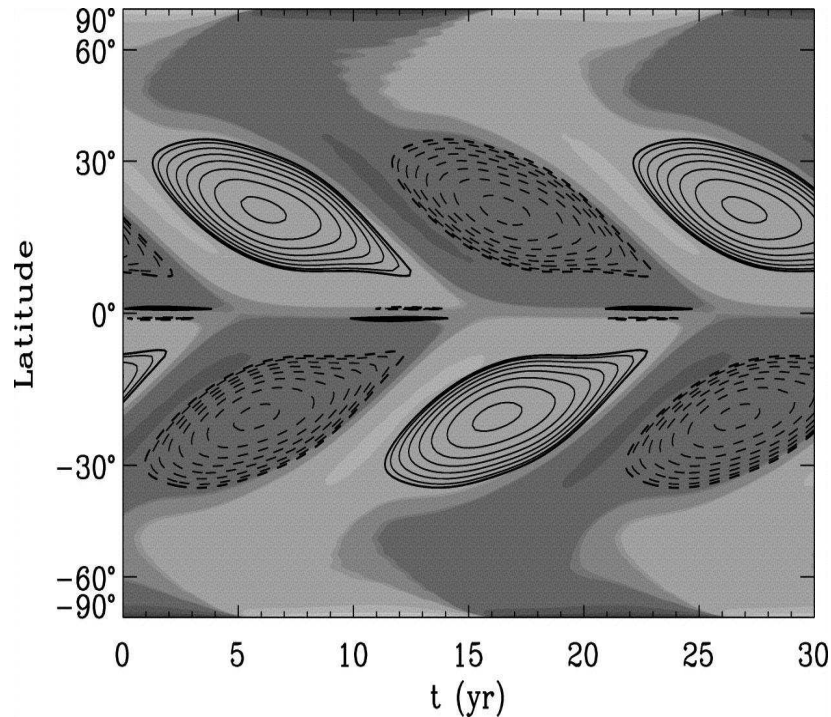
# Solution properties



Dikpati & Charbonneau (1999)

- Butterfly diagram
  - Equatorward propagation
  - Weak cycle overlap
  - Peak amplitudes in mid latitudes
  - Sometimes also polar branch
- Surface field
  - Correct phase relation
  - Polar field in general too strong
    - Toroidal / polar field  $\sim 50$ 
      - 10kG  $\rightarrow$  200G
- Solution symmetry
  - Quadrupole in general preferred
  - Difference between dipole and Quadrupole growth rate very small

# Tachocline $\alpha$ + strong $\eta$ contrast



Dikpati et al. (2004)

- **Butterfly diagram**
  - Activity in low latitudes  $< 35$  deg
  - Peak around 20 deg
- **Surface field**
  - 3 – 4 orders of magnitude weaker than toroidal field
- **Solution symmetry**
  - Dipole preferred, however difference to quadrupole small
- **Solution very sensitive to profile, amplitude and location of tachocline alpha effect!**

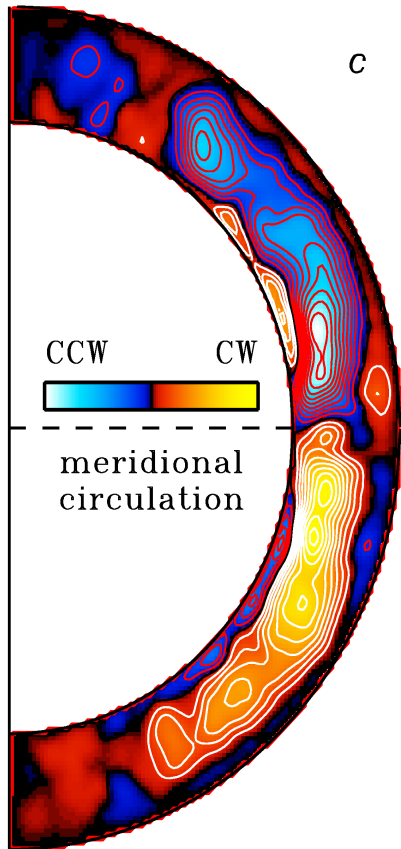


# How justified are the key assumptions ?

## ➤ Meridional flow

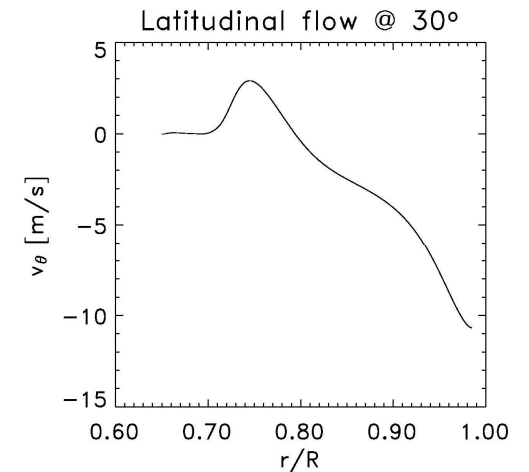
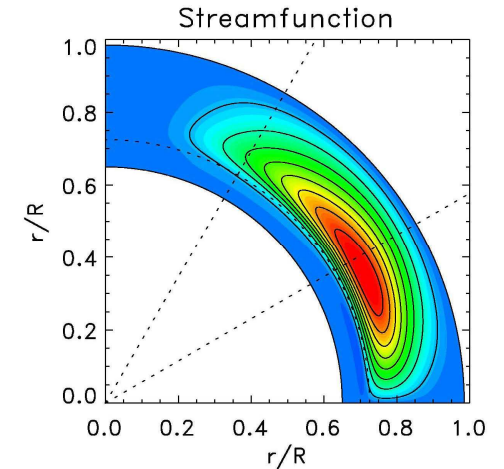
- 1 dominant flow cell in each hemisphere
  - Second flow cell in high latitudes OK
  - Several flow cells in radius problematic
- Lorentz force feedback on meridional flow negligible
  - Kinetic energy of meridional flow very small, non-trivial assumption
- Advective transport dominates over turbulent transport
  - Low magnetic diffusivity  $< 10^{11}$  cm<sup>2</sup>/s
  - Typical mixing length estimate 10 – 100 times larger!

# Meridional flow structure



3D simulation  
Miesch et al. (2008)

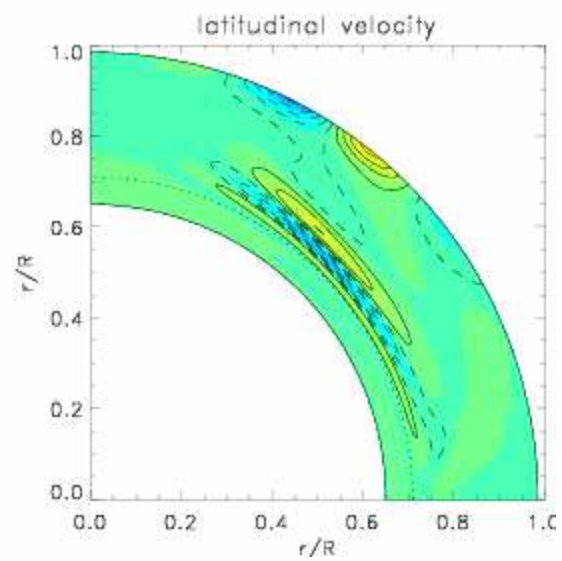
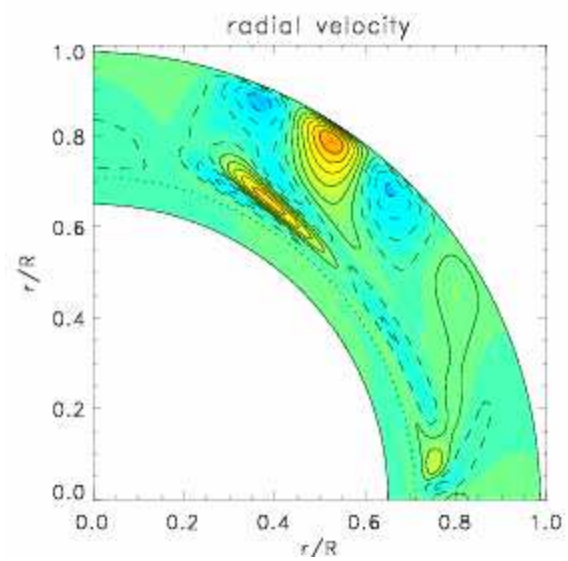
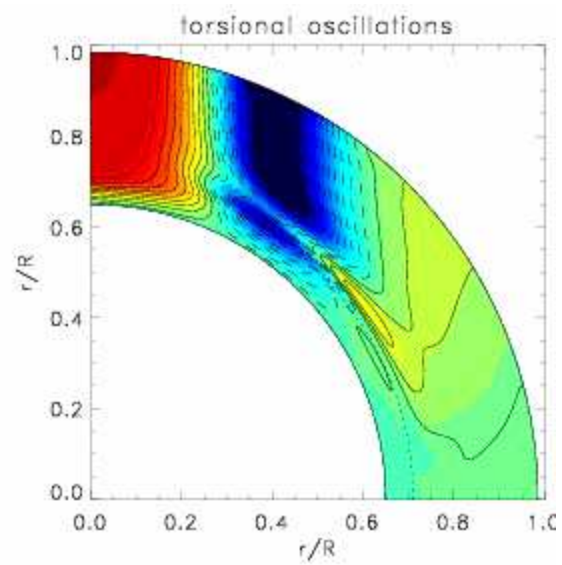
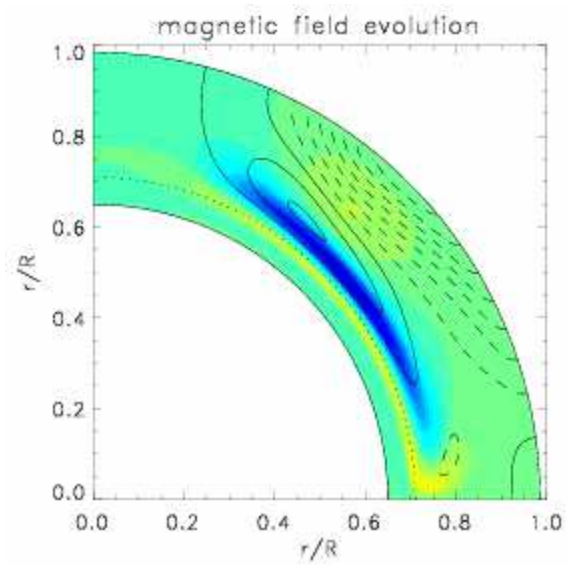
- Poleward at surface (observed)
- Return flow not observable through helioseismology
  - 50 Mm depth still poleward (Gizon & Rempel 2008)
- Mass conservation
  - Equatorward at base
- Theory:
  - Meanfield models
    - Typically equatorward
  - 3D simulations
    - Most recent (high res.): Equatorward
    - Low resolution runs: Multiple flow cells
- Overall: Equatorward flow at base of CZ very reasonable

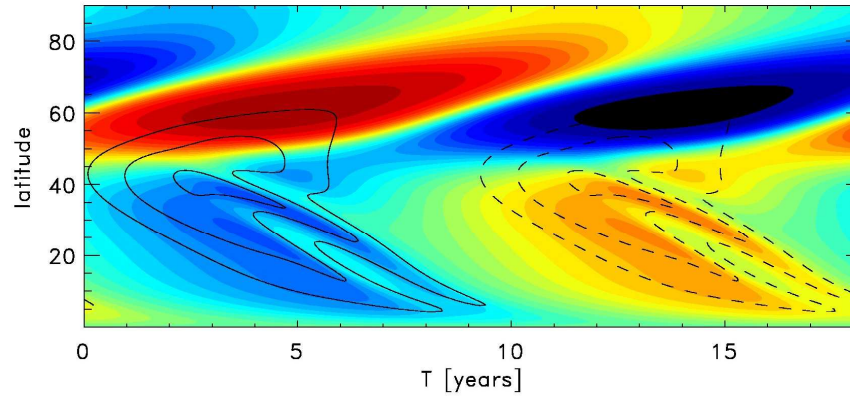
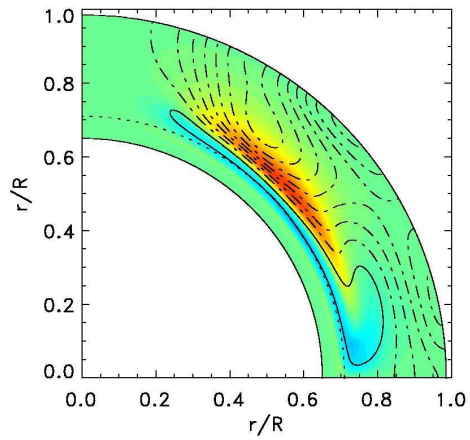


Meanfield model  
Rempel (2005)

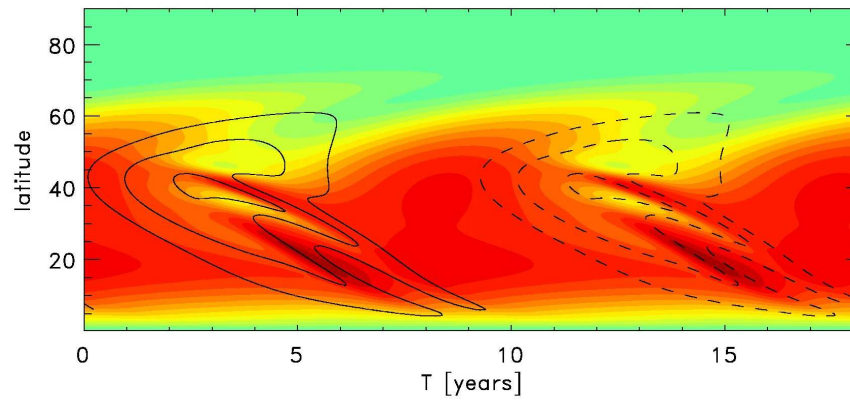
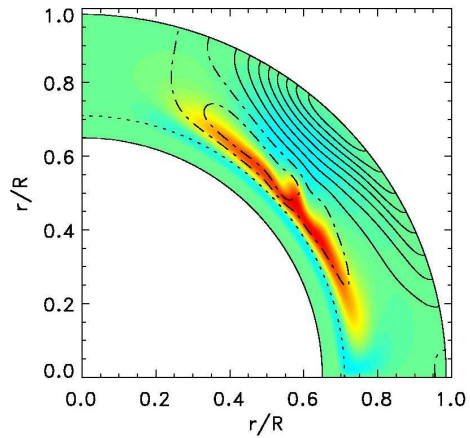
# Lorentz force feedback: Non-kinematic flux-transport dynamo

- Axisymmetric MHD equations
- Differential rotation model:
  - Parameterization of turbulent angular momentum transport drives DR and MF
  - Entropy perturbation originating in tachocline causes deviation from Taylor-Proudman state
- Dynamo model:
  - Flux-transport dynamo, only Babcock-Leighton  $\alpha$ -effect
  - Macroscopic Lorentz-force feedback on DR and MF

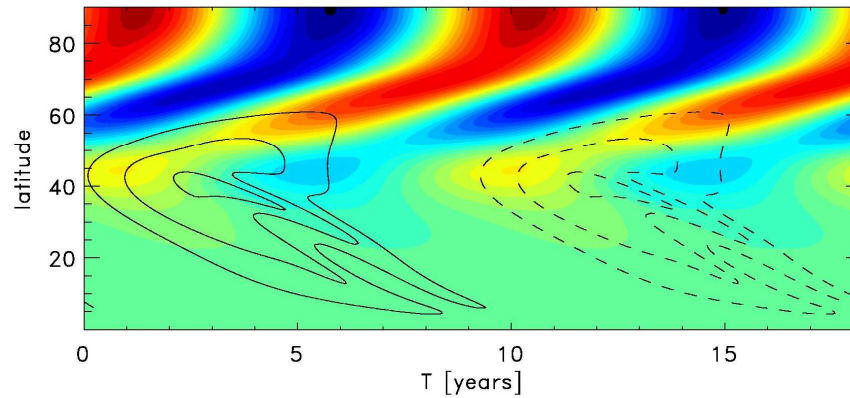
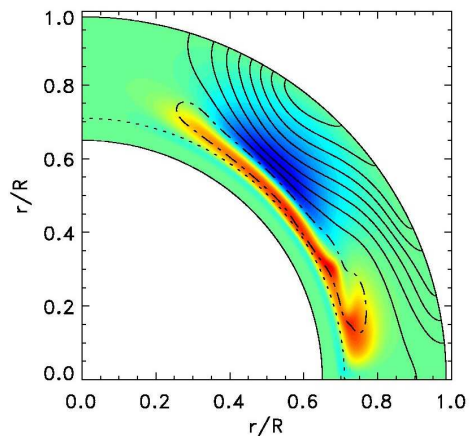




Radial surface field

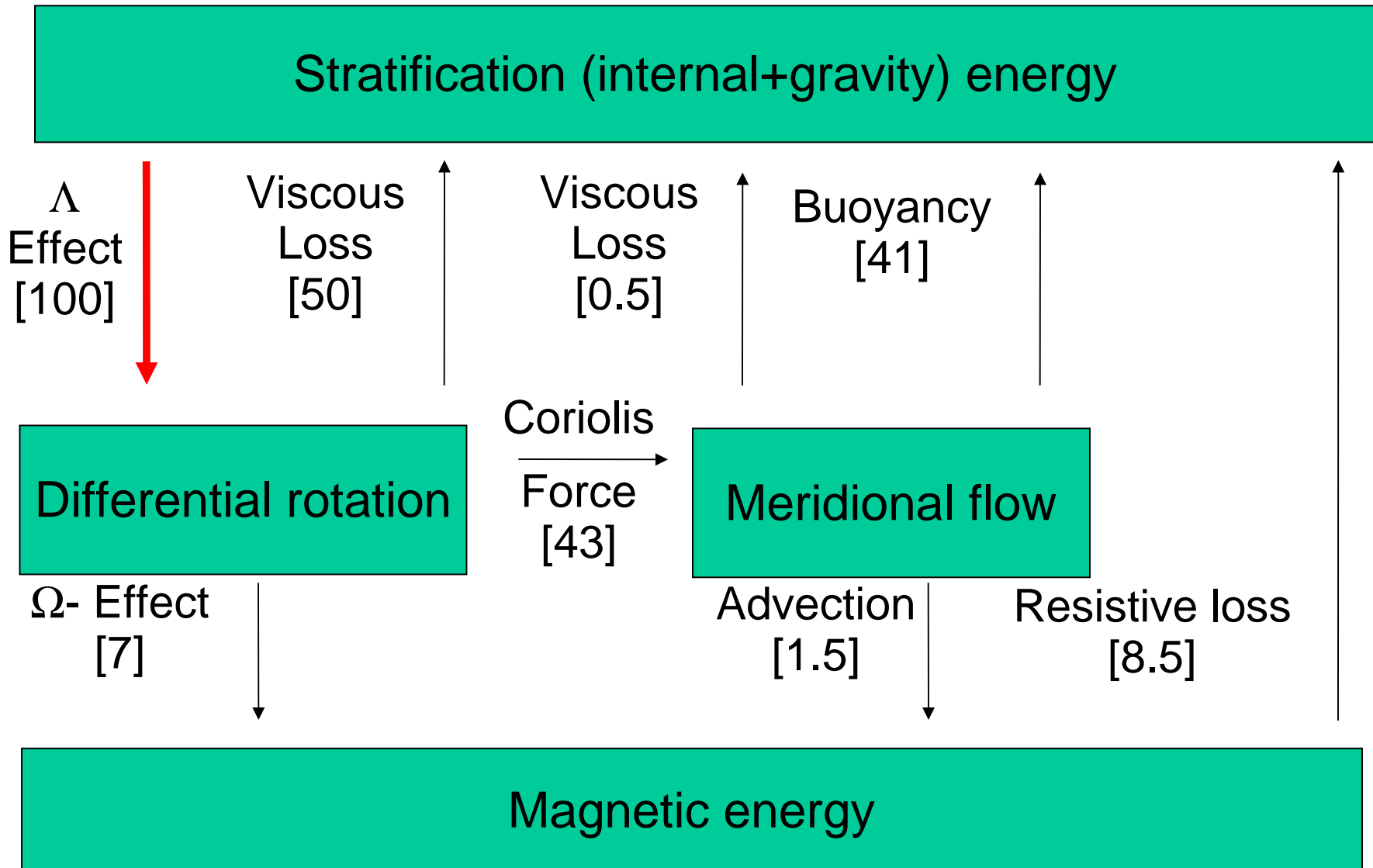


Meridional flow at base of CZ



Torsional oscillation at surface

# Energy balance of solar flux-transport model



# Energetics of dynamo

- Dynamo saturates through reduction of DR
  - Saturation field strength  $\sim 10$  to  $15$  kG
  - Magnetic flux @ BC  $\sim 10^{24}$  Mx
- Feedback on meridional flow not significant for  $B < 15$  kG
  - Operation of flux-transport dynamo not significantly impacted
  - Tachocline  $\alpha$ -effect allows for stronger toroidal field, but meridional flow cannot transport it anymore
- About  $1\%$   $L_{\text{sun}}$  required to maintain differential rotation and meridional flow
- Dynamo converts  $\sim 0.1\%$   $L_{\text{sun}}$  ( $\sim 10\%$  of flux required to maintain DR)
  - Close to observed irradiance variation (coincidence?)



# The role of turbulent transport

- Parametrized in mean field models through scalar magnetic diffusivity:
  - $\eta < 10^{11}$  cm<sup>2</sup>/s (advection dominated regime)
  - $\eta \sim 1/3 v_{\text{Hp}}$  ( $10^{12} - 10^{13}$  cm<sup>2</sup>/s)
  - Flux transport dynamo requires at least factor of 10 less magnetic diffusivity to operate
    - Not impossible, but also not evident from any theoretical or numerical investigation
    - Theory of DR:  $v \sim 1/3 v_{\text{Hp}}$  require reasonable
- Additional turbulent transport
  - Turbulent pumping (downward and equatorward)
    - Can partially compensate for effects of high diffusivity
    - Advection like behavior without bulk flow of plasma (amplitude comparable to meridional flow)
    - Substantial effect on flux transport dynamos (Guerrero, Dal Pino 2008)

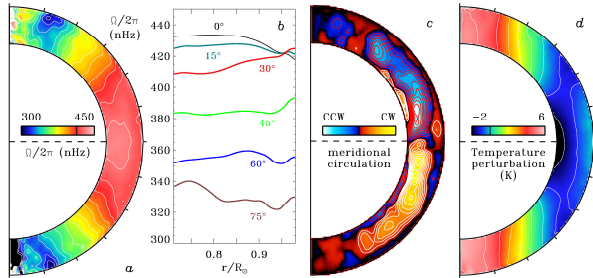


# Summary: Flux transport dynamo

- Babcock-Leighton flux transport dynamo
  - most robust and promising mean field scenario for solar cycle
  - Becomes more sensitive to details if fine tuning of cycle features is pursued
- Meridional flow structure seems to be a very reasonable assumption (limited observations + theoretical investigations)
- Can tolerate Lorentz force feedback
  - $B_{\max} < 15\text{kG}$
  - 100 kG impossible to transport by meridional flow
- Unknown amplitude of turbulent transport processes biggest weakness

# 3D dynamo simulations

(Miesch, Brun, Brown, Browning, Toomre)



Miesch et al.  
(2008)

- Convection zone dynamo
  - Turbulent field, meanfield < 0.03%

## ➤ Tachocline

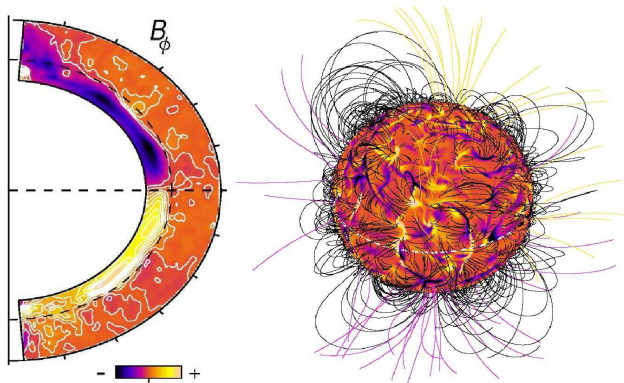
- Strong mean field ~10 kG

## ➤ Faster rotators (3x)

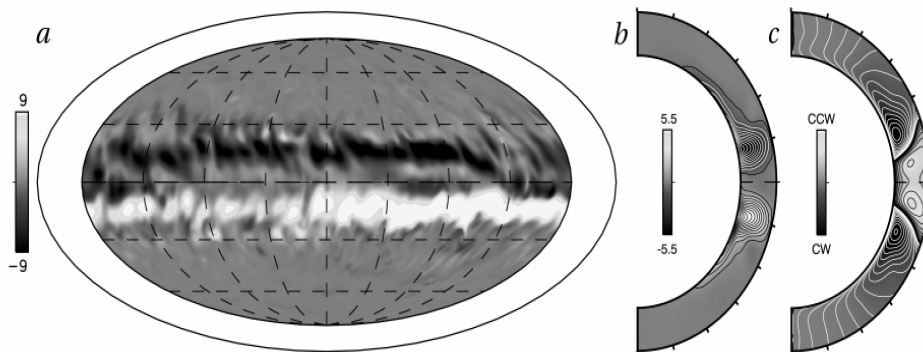
- Strong (~ 5 – 10 kG) field in convection zone
- Antisymmetric over equator
- Activity confined to low latitudes

## ➤ No cyclic dynamo yet

- Difficult to evolve 3D runs for > 10 years



Browning et al.  
(2006)



Brown et al. (2008)

# Implications for solar-like stars

- Assume flux transport dynamo is also operating in other stars, what are the predictions
  - Activity rotation relationship
  - Dynamo period
- $P_{\text{dyn}} \sim 0.1 P_{\text{DR}} \sim 0.001 L$ 
  - $P_{\text{dyn}} \sim 0.1 P_{\text{DR}}$ : Unlikely to change with  $\Omega$
  - $P_{\text{DR}} \sim 0.01 L$ : At best scale up  $\sim \Omega$
  - Star with 10 times solar rotation at best 10 times more energy for dynamo
    - Insufficient to explain activity-rotation relationship
      - $\phi \sim \Omega^2$ ,  $B \sim \Omega$  (at least)  $\rightarrow P \sim \Omega^3$
    - Change of surface topology?

# Meridional flow in a nutshell

Differential rotation and meridional flow driven by Reynolds stresses:

$$\mathbf{F}_{\text{RS}} = \rho r \sin \theta (\overline{v_r' v_\phi'} \hat{\mathbf{r}} + \overline{v_r' v_\phi'} \hat{\boldsymbol{\theta}})$$

$$\mathbf{F}_{\text{MC}} = \rho (r \sin \theta)^2 \Omega (\bar{v}_r \hat{\mathbf{r}} + \bar{v}_\theta \hat{\boldsymbol{\theta}})$$

Stationary state requires:

$$\nabla \cdot (\mathbf{F}_{\text{RS}} + \mathbf{F}_{\text{MC}}) = 0$$

Rough estimate of meridional flow amplitude:

$$v_m \sim \frac{\overline{v_i' v_\phi'}}{\Omega d} \ll \frac{v_{rms}^2}{\Omega d}$$

Decline of meridional flow energy with increasing omega also seen in 3D simulation (Brown et al. 2008)

- Meridional flow constant with Reynolds stress scales up linear with  $\Omega$ 
  - Certainly the case for slow rotators
- Fast rotators: Saturation of Reynolds stress
  - Decline of meridional flow  $\sim 1/\Omega$
- $V_m \sim \Omega$  as often found in literature requires RS  $\sim \Omega^2$ 
  - very unlikely for fast rotators
- Flat cycle period, potentially increase for fast rotators
- Problem: Flow topology could change (as indicated in 3D simulations)
  - Multi cellular flow
    - shorter cycle period
    - Flux transport dynamo might not work at all