# On the Tilt and Twist of Solar Active Regions

Zachary A. Holder, Richard C. Canfield<sup>1</sup>, Rebecca A. McMullen & Dibyendu Nandy<sup>2</sup>

Department of Physics, Montana State University, Bozeman, MT 59717, U.S.A.

1canfield@mithra.physics.montana.edu

<sup>2</sup>nandi@mithra.physics.montana.edu

Robert F. Howard

National Solar Observatory, Tucson, AZ 85726, U.S.A.

rfhoward@solar.stanford.edu

and

Alexei A. Pevtsov

National Solar Observatory/Sacramento Peak, PO Box 62, Sunspot, NM 88349, U.S.A.

apevtsov@nso.edu

#### ABSTRACT

Tilt and twist are two measurable characteristics of solar active regions which can give us information about subsurface physical processes associated with the creation and subsequent evolution of magnetic flux tubes inside the Sun. Using Mees Solar Observatory active-region vector magnetograms and Mt. Wilson Observatory full-disk longitudinal magnetograms we measure the magnetic twist and tilt angles of 368 bipolar active regions. In addition to two well-known phenomena, Joy's law and the hemispheric helicity rule, this dataset also shows a lesser-known twist-tilt relationship – which is the focus of this study. We find that those regions that closely follow Joy's law do not show any twist-tilt dependence. The dispersion in tilt angles and the dispersion in twist are also found to be uncorrelated with each other. Both of these results are predicted consequences of convective buffeting of initially untwisted and un-writhed flux tubes through

<sup>&</sup>lt;sup>3</sup>National Solar Observatory (NSO) is operated by the Association of Universities for Research in Astronomy (AURA, Inc) under cooperative agreement with the National Science Foundation (NSF).

the  $\Sigma$ -effect. However, we find that regions that strongly depart from Joy's law show significantly larger than average twist and very strong twist-tilt dependence – suggesting that the twist-tilt relationship in these regions is due to the kinking of flux tubes which are initially highly twisted, but not strongly writhed. This implies that some mechanism other than the  $\Sigma$ -effect (e.g., the solar dynamo itself or the process of buoyancy instability and flux tube formation) is responsible for imparting the initial twist (at the base of the solar convection zone) to the flux tubes that subsequently become kink-unstable.

Subject headings: Sun: magnetic fields — sunspots — Sun: activity

#### 1. INTRODUCTION

The strong toroidal fields that ultimately form active regions (ARs) are believed to be created in a strong shear layer in the rotation beneath the base of the solar convection zone (SCZ) by the solar dynamo. These toroidal fields are subject to buoyancy instability once they are brought out into the base of the SCZ – collapsing to form flux tubes and subsequently rising as  $\Omega$ -shaped loops to the surface (Parker 1955) forming sunspots or ARs. During their buoyant rise to the surface the toroidal flux tubes are acted upon by a variety of forces, notably the Coriolis force and turbulent convective buffeting. Both of these physical processes imprint their signatures, in the form of tilt and twist, on the rising magnetic fields. Studying the tilt and twist of solar ARs and their inter-relationship, therefore, can be a diagnostic of the afore-mentioned sub-surface processes. The tilt angle of solar ARs is the angle that its axis (i.e., the line connecting the centroid of the two bipolar spots constituting the AR) makes with the east-west direction. The twist of a AR flux system is a measure of how wrapped the magnetic field lines are around the flux tube axis (i.e., of its helical pitch).

Tilt of solar ARs, which are directly related to the writhe (spatial deformation or turning of the axis) of the flux tube, have been observed to follow a systematic dependence on latitude known as Joy's law (Hale et al. 1919). Theoretical simulations of the buoyant rise of flux tubes from the base of the SCZ have firmly established that this tilt is due to the effect of the Coriolis force on the expanding plasma within these rising tubes (D'Silva & Choudhuri 1993; Fan, Fisher, & McClymont 1994). A large scatter in the tilt values around Joy's law is also observed. This scatter is explained by considering the effect of turbulent convective buffeting of the rising flux tubes during the course of their journey through the SCZ (Fisher, Fan, & Howard 1995; Longcope & Fisher 1996). Tilt of solar ARs constitute a vital ingredient in many models of the solar dynamo, contributing to the creation of the solar poloidal field through the decay of tilted bipolar ARs (see Nandy & Choudhuri 2001 and references therein)

and also influencing the heliospheric field evolution (Wang & Sheeley 2003).

Compared to observations of AR tilt, studies of the magnetic twist of solar ARs are relatively new, though significant advances have been made in the last decade. The twist in the magnetic field lines of a solar AR is a component of the magnetic helicity associated with the AR flux system (writhe being the other component). In practice, observational estimates of the parameter  $\alpha$  associated with the force-free field equation  $\nabla \times \mathbf{B} = \alpha \mathbf{B}$  are taken to be a measure of the twist associated with a AR flux system (for a straight and uniformly twisted cylindrical flux tube, the parameter  $\alpha$  can be shown to be proportional to the twist per unit length). Hereby, we shall refer to the magnetic twist in the field lines of solar ARs (as observationally quantified by the parameter  $\alpha$ ) simply as the AR twist. Like tilt, AR twist also show a hemispheric (or latitudinal) dependence and considerable scatter around the mean values (Pevtsov, Canfield, & Metcalf 1994, 1995; Pevtsov, Canfield, & Latushko 2001). The dynamo itself may create twisted magnetic fields, though preliminary estimates of the twist imparted by the dynamo point to a mean value much less than that observed (Gilman & Charbonneau 1999; Longcope et al. 1999). Since magnetic helicity (having contributions from both twist and writhe of magnetic fields) of a flux system is a conserved quantity, the creation or change in one component of it, necessarily changes the other, if this evolution happens in a close to ideal magnetohydrodynamic system. Twist can be acquired as a consequence of helicity conservation when Coriolis force writhes a rising flux tube. However, the writhing of flux tubes due to Coriolis force is insufficient to explain the observed amplitudes of the twist (Longcope et al. 1999; Fan & Gong 2000). It turns out that the buffeting of rising magnetic flux tubes by helical turbulent convection, a process termed as the  $\Sigma$ -effect, predicts the correct sign as well as amplitude of the observed twist values (Longcope, Fisher, & Pevtsov 1998; Longcope et al. 1999). Twist (which is a measure of the stress associated with a AR flux system and the presence of field-aligned currents) and its evolution in magnetic flux systems play an important role in the explosive activity of solar ARs such as flaring and initiation of CMEs (Canfield, Hudson, & McKenzie 1999; Nandy et al. 2003).

Although tilt and twist have been studied separately for many years, only recently has attention been focussed in studying the inter-relationship between these two parameters. From a theoretical viewpoint, the  $\Sigma$ -effect model for the creation of twist predicts that there is no correlation between twist and tilt, other than a very small one due to the influence of the Coriolis force on plasma material within the rising flux tubes (Fan & Gong 2000) or a larger one due to the mutual dependence of twist and tilt (or writhe) on latitude. The latter is trivial; using the sign conventions discussed in Section 2, the hemispheric helicity rule (Seehafer 1990, Pevtsov, Canfield, & Metcalf 1994; Pevtsov, Canfield, & Latushko 2001) and Joy's law describe the statistical tendencies for both twist and tilt to be negative in the

northern hemisphere and positive in the southern hemisphere (following our sign convention a negative tilt corresponds to right-handed or positive writhe and vice-versa). Hence, the correlation due to mutual latitudinal dependence between tilt and twist would be positive (while that between twist and writhe would be negative). The  $\Sigma$ -effect also predicts that the dispersion in the tilt and twist around their mean values would be un-correlated with each other.

The kink instability can also introduce a dependence between tilt and twist. Un-writhed but twisted flux tubes that are close to a certain critical twist value are susceptible to kink instability during their buoyant rise (Linton, Longcope, & Fisher 1996; Linton et al. 1999). Through this instability, rising flux tubes undergo kinking (developing a bend in its axis), trading some of its twist for writhe. In this case, helicity conservation arguments lead to the conclusion that the twist and writhe would be positively correlated with each other (while twist and tilt would be negatively correlated), thereby distinguishing this process from the Coriolis force and the  $\Sigma$ -effect. The kink instability is important because it is believed to result in the formation of  $\delta$ -spot ARs which show significantly higher flaring activity as compared to normal ones (Tanaka 1980; Sammis & Zirin 1998).

On the observational front, Canfield & Pevtsov (1998) and Pevtsov & Canfield (1999) studied the correlation between tilt and twist of 99 ARs observed by the Haleakala Stokes Polarimeter (HSP). Canfield & Pevtsov (1998) found that regions significantly deviating from Joy's law showed a weak negative correlation, contrary to the expected dependence introduced by the action of the Coriolis force on a rising flux tube. Hence, Pevtsov and Canfield (1999) concluded that at least some twist of the photospheric fields is due to the subphotospheric dynamo. On the other hand, Tian et al. (2001) compared the sign of tilt and twist and found a positive correlation. Hence, Tian et al. (2001) concluded that the twist observed in the photosphere is the result of the Coriolis force distorting the apex of emerging  $\Omega$ -loops – which introduces both twist and writhe in originally untwisted magnetic fields. In a subsequent study of 86 flare-productive solar ARs, Tian & Liu (2003) find evidence of kink instability. More recently, López Fuentes et al. (2003) studied the evolution of 22 ARs and found evidence of kinking in about 35% of these regions, while 41% of these regions showed a twist-tilt relationship indicative of having resulted from the Coriolis force. These contradictory results and the important role played by twist and tilt in solar dynamics warrant a more closer look at the tilt-twist relationship in a much larger data set – which is the basis of our current investigation. In addition, the earlier study by Canfield & Pevtsov (1998) employed HSP magnetograms for determining AR tilt, which could have affected the tilt measurements because of their limited field of view.

In this paper we study the observed relationship between the tilt and twist of active

regions and compare it to that predicted by the various aforementioned mechanisms. In Section 2 we combine two datasets to relate tilt and twist. We recover Joy's law and the hemispheric helicity rule, and document a complex relationship between twist and tilt. In Section 3 we show a statistically significant correlation between tilt and twist, which is not due to their mutual dependence on latitude. We find that the  $\Sigma$ -effect model explains the observed lack of a twist-tilt correlation in regions that closely follow Joy's law, while some other mechanism (arguably kinking) comes into play in anomalous regions. We conclude in Section 4, discussing the implications of our results for subsurface physical processes that govern the creation and subsequent evolution of twist and tilt in solar AR flux systems.

#### 2. Datasets

We have compiled a combined dataset, consisting of 368 active regions, from two independent datasets, one for twist and the other for tilt. The Mees Solar Observatory dataset consists of vector magnetograms from the Haleakala Stokes Polarimeter (Mickey 1985) obtained in 1988-1995. From these vector magnetograms, values of the best-fit overall twist of active regions  $\alpha_{best}$  were extracted following Pevtsov, Canfield, & Metcalf (1995). Their method is based on the deduction of the constant  $\alpha$  in the force-free field equation  $\nabla \times \mathbf{B} = \alpha \mathbf{B}$ , though it does not explicitly assume the field to be force-free. From the observed vertical magnetic field, the x and y components of the horizontal magnetic field are computed using the force-free magnetic field equation assuming a particular value of  $\alpha$ . The parameter  $\alpha$  is then varied until the difference between the x and y components of the computed (horizontal) force-free field and the observed horizontal field is minimized (using only those pixels for which  $|B_{trans}| > 300$  Gauss because the noise level in the magnetograms is typically less than 100 Gauss for the transverse field), giving the best fit value  $\alpha_{best}$ .

The limited field of view of the Haleakala Stokes Polarimeter (HSP) is adequate for the calculation of  $\alpha_{best}$ , since the strong-field regions on which it must be based (Leka & Skumanich 1999) are not spatially extended. However, for calculation of active region tilt, a larger field of view is desirable. Hence, we use the Mt. Wilson dataset for this purpose. This dataset (Howard 1989) consists of line of sight, full disk magnetograms collected from 1967 to 1995. The Mt. Wilson dataset identifies the magnetic flux-weighted centroids of the two polarities within each active region. We define the tilt angle  $\theta$  to be the angle (positive counterclockwise up to 180°) between the local parallel of latitude and the line joining the centroids of the two polarities, and d to be the distance between them. This definition of tilt is similar to Pevtsov and Canfield (1999), but it is opposite (in sign) to Tian et al. (2001).

We associate corresponding active regions in the HSP and Mt. Wilson datasets using

the daily NOAA active region lists, checking for consistency in latitude ( $\pm 5^{\circ}$ ) and central meridian distance ( $\pm 5^{\circ}$ ) after correction for solar rotation. Knowing that the twist patterns within solar ARs evolve on characteristic time scales of about 27 hours (Pevtsov, Canfield, & Metcalf 1994), we accepted up to 12 hours time difference for matching purposes. We further sort the data by excluding regions that do not obey the Hale-Nicholson polarity law (Hale & Nicholson 1925) because we cannot unambiguously measure tilt in these non-Hale regions. Regions that obey the Hale-Nicholson polarity law comprise 91.9% of the Mt. Wilson dataset. The final combined dataset contains overall active-region twist  $\alpha_{best}$ , tilt  $\theta$ , polarity separation d, and latitude  $\phi$  for 368 regions.

As expected, our combined dataset shows various well-known relationships. Figure 1 shows the dependence of  $\theta/d$  on latitude, reflecting what is commonly called Joy's law (Hale et al. 1919); positive tilt values dominate in the south, and negative in the north, with considerable scatter. The heavy solid line in Figure 1 shows a linear fit to our results. Our dataset is much smaller than the full Mt. Wilson dataset studied by Wang & Sheeley (1991), and in comparison contains fewer small active regions, because the HSP data acquisition strategy favored larger (flare-productive) ones. Nevertheless, our fit to the latitude dependence of tilt agrees with Fan's (1993) fit to Wang & Sheeley (1991) at the  $\sim 3\sigma$  level.

Hereby, our analysis is motivated by a simple model of a straight and uniformly twisted cylindrical flux tube where the force-free field parameter  $\alpha$  is related to the twist per unit length. Since we are interested in the relationship of twist to tilt, we choose a measure  $(\theta/d)$  for the tilt that has the same dimensions as  $\alpha$ . Note that geometric considerations also point to  $\theta/d$  as a better proxy for writhe, containing more information about the morphology of the flux tube than  $\theta$  alone. As discussed in Section 4, the fact that  $\theta/d$  is better correlated with  $\alpha$  than  $\theta$  alone, supports the interpretation given here.

In Figure 2, where we plot the twist parameter  $\alpha_{best}$  versus latitude, we see that our dataset shows the well-known hemispheric helicity rule (Pevtsov, Canfield, & Latushko 2001), where negative (left-handed) values dominate in the northern hemisphere and positive (right-handed) in the southern, again with considerable scatter. The solid line in Figure 2 shows a linear fit to our results. This fit agrees with Pevtsov, Canfield, & Latushko (2001) at the  $\sim 3\sigma$  level.

In addition to these well-known trends, Figure 3 shows that  $\theta/d$  and  $\alpha_{best}$  are negatively correlated with each other. Active regions that are positively tilted tend to have negative twist. This is surprising given that their mutual latitudinal dependence would point to a positive correlation between twist and tilt. The  $\pm 3\sigma$  error bands show that although the scatter is large, the correlation is significant. The scatter is real, which led Longcope et al. (1998) to attribute it to the stochastic nature of turbulent convection. This scatter does not

originate uniquely in either the  $\theta/d$  or  $\alpha_{best}$  data. It is not reduced when regions with values of separation much less than the average value ( $< d > \sim 9 \times 10^7 m$ ) are removed from the combined dataset (d has earlier been used as a proxy for flux and the observed scatter in tilt values is known to be larger for lower flux, see e.g., Fisher, Fan, & Howard [1995]). We now concentrate on the twist-tilt correlation, which is the main focus of this paper.

## 3. Analysis: The Twist-Tilt Relationship

Let us begin by dismissing the trivial relationship between twist and tilt that arises due to the mutual dependence of these quantities on latitude. Consider the dependence of  $\theta/d$  and  $\alpha_{best}$  on latitude shown in Figures 1 and 2. In the northern hemisphere both  $\theta/d$  and  $\alpha_{best}$  tend to be negative, and in the southern hemisphere, positive. Hence, we expect hemispheric dependence to produce a positive correlation. On the contrary, the correlation shown in Figure 3 is negative. Thus, the mutual dependence of  $\theta/d$  and  $\alpha_{best}$  on latitude cannot explain their observed relationship. We now turn to two different techniques to quantitatively remove the effect of mutual latitudinal dependence to see how this affects the correlation.

We first take a strictly statistical approach to the interrelationships among  $\theta/d$ ,  $\alpha_{best}$  and latitude. Kendall's rank correlation coefficient ( $\tau$ ) tells us the strength of the correlation between two quantities (Daniel 1990). Table 1 shows values of  $\tau$  for  $\theta/d$  vs latitude,  $\alpha_{best}$  vs latitude and  $\theta/d$  vs  $\alpha_{best}$ . From this analysis we can see that the correlation between  $\theta/d$  and latitude holds the highest confidence (the chance of occurrence in an uncorrelated dataset is  $\sim 10^{-22}$ ). This is hardly surprising given the past work on Joy's law cited earlier. Twist and latitude have a weaker, but still very significant correlation. Significantly, the confidence level for the correlation between  $\theta/d$  and  $\alpha_{best}$  is very high; the chance of occurrence in an uncorrelated dataset is  $\sim 10^{-6}$ . We use Kendall's partial correlation coefficient,  $\tau_p$ , to quantify this correlation between  $\theta/d$  and  $\alpha$  while holding latitude constant (Daniel 1990). If the correlation between  $\theta/d$  and  $\alpha_{best}$  were due to their mutual latitude dependence,  $|\tau_p|$  would be significantly less than  $|\tau|$ . In fact, we find that  $|\tau_p| > |\tau|$  (see Table 1.)

We next take a more intuitive approach to eliminating the influence of the mutual dependence of  $\theta/d$  and  $\alpha_{best}$  on latitude – we simply subtract the fitted latitudinal trends of Figures 1 and 2 from our individual  $\theta/d$  and  $\alpha_{best}$  values. Figure 4 shows the relation between these  $\theta/d$  and  $\alpha_{best}$  residuals. As a check, recalculating  $\tau$  using these residuals (Table 1) confirms that (a) the  $\theta/d$  and  $\alpha_{best}$  residuals hold no significant correlation with latitude and (b) the  $\theta/d$  residuals show a strong correlation with the  $\alpha_{best}$  residuals, of even greater magnitude than in the raw data, and  $\tau_p \simeq \tau$ . This more intuitive approach further

confirms that the negative correlation between  $\theta/d$  and  $\alpha_{best}$  is not due to their mutual latitudinal dependence and originates via some mechanism other than the Coriolis force.

Now we turn to the relationship between the dispersion in twists and tilts about their mean values. One of the predictions of the  $\Sigma$ -effect model for the creation of twist is that the dispersion in twist would be un-correlated with the dispersion in the tilt. This is because the amount of dispersion in twist and tilt depends on the amplitude of turbulent fluctuation, which is believed to be spatially isotropic with respect to latitude. Thus if one calculates the dispersion in twist and tilt across different latitudes one would expect these two quantities to be un-correlated with each other. We now explore this prediction in our data set. To do this we first bin the combined data into 10 latitudinal bins containing equal number of points and then calculate the mean and standard deviation of twist (denoted by  $\langle \alpha_{best} \rangle_i$  and  $\Delta(\alpha_{best})_i$ , respectively) and tilt (denoted by  $\langle \theta/d \rangle_i$  and  $\Delta(\theta/d)_i$ , respectively) in each bin (here i is an index for the bins, with i = 1, 10). The dispersion around the mean twist and tilt are the quantities  $\Delta(\alpha_{best})_i$  and  $\Delta(\theta/d)_i$ . We plot these two quantities with respect to the median latitude of each bin in Figure 5 (top) to show how the binning was done. When these two quantities are plotted against each other in Figure 5 (bottom) we find that they don't appear to correlate very well. The linear-correlation coefficient between these two quantities is found to be insignificant at 89.95%. However, there could be a weak non-linear dependence between these two quantities. To check for this possibility, we also calculate the Spearman's rank correlation coefficient (which measures the correlation based on a distribution of ranks and hence is not limited to only linear relationships [Press et al. 1986]) between  $\Delta(\alpha_{best})_i$ and  $\Delta(\theta/d)_i$  and find it to be 94.13%. Therefore we rule out any significant correlation between the dispersion in twists and tilts in our dataset.

We return now to the twist-tilt correlation and search for clues that tell us how this relationship might arise. We know that the statistical basis for Joy's law is very strong. How does the twist-tilt relationship depend on the extent to which individual regions obey this law? The first two rows of Table 2 compare two subsets of our combined dataset; those with  $\theta/d$  values that follow the best-fit "Joy's law" line in Figure 1 to within  $\pm 6\sigma$  and those that deviate by greater amounts. We find that although the subsets are of comparable size, there is no statistically significant relationship between twist and tilt for those regions that obey Joys law to this extent (first row in Table 2), whereas there is a statistically strong relationship for the regions that depart more from it (second row in Table 2). This means that the main contribution to the observed negative correlation between twist and tilt comes from ARs that significantly depart from Joy's law (at the  $6\sigma$  level) – also characterized by a higher average twist (see second row in Table 2).

We further explore this trend by considering those regions that depart so much from

Joy's law that they disobey the Hale-Nicholson polarity law. To do this we assume that the absolute magnitudes of the tilt angles of such regions range only between 90° and 180° (i.e., regions whose polarity orientation is opposite to that given by the Hale-Nicholson polarity law). The last row of Table 2 shows that for these non-Hale regions the Kendall  $\tau$  coefficient is relatively high and has opposite sign. Unfortunately, this subset comprises only 12 regions, and the confidence level of this correlation is barely significant. A much larger dataset will be required to determine whether these trends are real. However, it is noteworthy that the mean value of overall twist  $|\alpha_{best}|$  is atypically large in this subset – a result that is more robust than the correlation analysis, since it does not rely on our assumption about the range of tilt values for non-Hale regions.

## 4. Discussion

We have analyzed the tilt and twist of ARs from a combined dataset of Mt. Wilson full disk magnetograms and HSP vector magnetograms. In this dataset we recover the well known trends found in larger datasets of tilts and twists, namely their respective hemispheric and latitudinal dependence. Our main emphasis in this paper has been to study the tilt-twist relationship to explore what physical process (or processes) determines this dependence. We find a complex relationship between the tilt angle and overall magnetic twist of solar active regions. Although mutual latitudinal dependence would point to a positive correlation between twists and tilts of solar ARs, we recover a negative correlation in their values. However, of the large majority of regions that obey the Hale-Nicholson polarity law, those that most obediently conform to Joy's law show no significant interdependence of twist and tilt. Nor are the dispersion in twists and tilts related to each other in the whole dataset. These results confirm the predictions of the  $\Sigma$ -effect model (Longcope, Fisher, & Pevtsov 1998), which describes the creation of twist due to the buffeting of initially untwisted and rising flux tubes by helical turbulent convection, taking into account helicity conservation.

Those regions that significantly depart from Joy's law (by more than  $6\sigma$ ) show both stronger than average twist and a strong interdependence of twist and tilt and it is this latter subset of ARs that mainly contribute to the negative correlation between twist and tilt that we find in our data. Such properties would most plausibly result from the kinking of buoyantly rising flux tubes that are initially highly twisted but un-writhed. Simulations of the dynamics of such flux tubes (Linton, Longcope, & Fisher 1996; Linton et al. 1999) have shown that an initial twist is required for them to be susceptible to kink instability, but the origin of this initial twist remains unknown. As a demonstration of the kink instability scenario, consider a flux tube at the base of the convection zone which has a high negative

helicity H in the form of twist  $\mathcal{T}$  but no writhe  $\mathcal{W}$  (say e.g., in the northern hemisphere). If the tube kinks, it trades some of that twist for writhe, and helicity conservation requires that  $\delta \mathcal{T} = -\delta \mathcal{W}$ . In the example above,  $\delta \mathcal{T} > 0$  so  $\delta \mathcal{W} < 0$ . Thus this will result in the flux tube axis acquiring a left-handed or negative writhe and following our convention this will correspond to tilt/separation  $\theta/d > 0$ . Thus we see that kinking of strongly twisted but unwrithed flux tubes will produce a negative correlation between  $\theta/d$  and  $\alpha_{best}$ , as observed. Note that this flux tube in the northern hemisphere which gains a negative writhe and hence positive tilt (in general, flux tubes that follow Joy's law have a right-handed writhe corresponding to a negative tilt [in our sign convention] in the northern hemisphere) would display an anomalous tilt highly departing from Joy's law, as also we recover from our dataset.

A final bit of evidence for kinking comes from the effect of including the active region characteristic length scale in the analysis. Table 1 shows that  $\theta/d$  is extremely well correlated with  $\alpha_{best}$  ( $\tau = -0.156$ ; chance of random occurrence  $10^{-8}$ ). Although we find that  $\theta$  is also well correlated with  $\alpha_{best}$  ( $\tau = -0.145$ ), the chance of random occurrence ( $10^{-7}$ ) is an order of magnitude larger. Theoretical investigations of kink instability show that flux tubes are most unstable to kink perturbations having scales close to the twist in the field lines (Linton, Longcope, & Fisher 1996; Linton et al. 1999). Therefore, when the tube kinks, it picks up a writhe which is of similar scale to that of the twist. This may be the underlying cause of why twist correlates more significantly with the parameter tilt per unit length ( $\theta/d$ ) – that has the same dimensions as twist.

In summary, we conclude that a substantial fraction of ARs in our dataset show evidence of having been subject to kink instability. We note that Tian et al. (2001) drew an opposite conclusion having found a positive correlation between tilt and twist (after correcting for their definition of tilt which results in a sign opposite to that of ours) in their dataset of 286 ARs. The Tian et al. (2001) analysis did not take out the mutual latitudinal dependence between twist and tilt and therefore we believe that any signatures of kink instability would have been suppressed in their dataset.

Finally, we outline here a framework that binds together our observations with past theoretical work on flux tube dynamics. Numerical investigations of flux tube dynamics have shown that a certain amount of initial twist is required for flux tubes to maintain their integrity (against fragmentation) during their buoyant rise from the base of the SCZ (Emonet & Moreno-Insertis 1998; Fan, Zweibel, & Lantz 1998). Coupled with this, theoretical studies of kink instability point out that for the flux tubes to be kink unstable, they should have a high twist close to or greater than a certain critical twist value (Linton, Longcope, & Fisher 1996; Linton et al. 1999). Presumably, not all flux tubes that form at the base of

the SCZ have these properties. Consequently some of them do not survive the buoyant rise to the surface and instead fragment. There would be a subset of flux tubes which have a small initial twist to make them rise coherently to the surface, but not enough twist to make them kink. We hypothesize that this subset (possibly those constituting the first row in Table 2), which is weakly twisted initially, picks up a more significant amount of twist from the  $\Sigma$ -effect during their rise, therefore exhibiting properties predicted by the  $\Sigma$ -effect model. There would be a second subset that have high enough twist to be kink unstable and exhibit properties of having undergone kinking, e.g., show a negative correlation between tilt and twist (those contributing to the second row in Table 2). Going back to our demonstrative scenario for kink instability discussed earlier in this section, one can deduce that a flux tube (in the northern hemisphere) having a atypically large negative twist (initially) will be writhed much more than usual in the left-handed sense and hence pick up positive tilt, if it undergoes kink instability. If the initial twist is large enough, then it may rotate the flux tube axis counter-clockwise by an angle 180° more than the local Joy's law angle. It is straightforward to deduce then that this angle corresponds to a negative sign (in our convention) and also to a non-Hale region with the leading and following polarities reversed. This demonstrates that non-Hale regions, which were very highly twisted initially, may show a positive correlation between tilt and twist after having undergone a severe kink instability. We believe that these properties are exhibited by the subset in the third row of Table 2. Note however, that this last explanation for the positive correlation found for non-Hale ARs is only one of the possible scenarios. Some non-Hale regions are also likely to have tilt angles falling within the 90° to 180° range, in which case one would still find a negative correlation between tilt and twist. Perhaps our subset of 12 non-Hale regions do not span the full distribution of possible tilt angles and mainly contains highly kinked regions. For a comprehensive study of the evolution of a non-Hale AR, and alternative interpretations, see also López Fuentes et al. (2000).

In conclusion, our dataset lends indirect observational evidence for the presence of an initial twist in flux tubes at the base of the SCZ, even before they begin their buoyant rise. We may note here that the  $\Sigma$ -effect imparts twist to rising flux tubes mainly during the late phase of their rise at the upper-half of the SCZ (Longcope, Fisher, & Pevtsov 1998). Certainly therefore, there must be some alternative physical mechanism for imparting twist to flux tubes at the base of the SCZ and this begs the question; what is this mechanism? While the solar dynamo can in principle generate twist at the core-envelope interface where it creates the strong toroidal flux ropes (Gilman & Charbonneau 1999), it has not yet been convincingly demonstrated that the dynamo can produce sufficiently high twist required to make the flux tubes kink-unstable (Longcope et al. 1999). An alternate mechanism that may introduce twist is the formation process of the flux tube itself, involving buoyancy instability

and helical motion – a process believed to be able to produce a dynamo  $\alpha$ -effect at the base of the SCZ (Ferriz-Mas, Schmitt, & Schüssler 1994). However, the flux tube formation process is even less understood and the ideas discussed in this paragraph remain mere speculation. While we can say confidently at this point in time that there must be some mechanism for the creation of twist at the base of the SCZ, what mechanism, is a question that remains to be answered.

We thank Dana Longcope and James Robison-Cox for numerous discussions of IDL, physics and statistics and the referee for his helpful suggestions. We are also grateful to the Mees Solar Observatory for providing the HSP vector magnetogram data and the Mt. Wilson Observatory for the full disk magnetogram data. The Mt. Wilson sunspot data were obtained by several generations of observers at the Mount Wilson Observatory, supported by the Carnegie Institution of Washington. The NOAA active region numbers used in this study were provided by the National Geophysical Data Center, Boulder, Colorado 80303 USA. This research has been supported by NASA through SR&T grant NAG5-11873 to Montana State University.

### REFERENCES

Canfield, R. C., Hudson, H. S., & McKenzie, D. E. 1999, Geophysics Research Letters, 26(6), 627

Canfield, R. C., & Pevtsov, A. A. 1998, in Helicity of Solar Active Region Magnetic Fields, ed. K. S. Balasubramaniam, J. W. Harvey, & D. M. Rabin (Synoptic Solar Physics, ASP), 131

Daniel, W. W. 1990, Applied Nonparametric Statistics (PWS-KENT Publising Company)

D'Silva, S., & Choudhuri, A. R. 1993, A&A, 272, 621

Emonet, T., & Moreno-Insertis, F. 1998, ApJ, 492, 804

Fan, Y. 1993, Ph.D. Thesis

Fan, Y., Fisher, G. H., & McClymont, A. N. 1994, ApJ, 436, 907

Fan, Y., & Gong, D. 2000, Sol. Phys., 192, 141

Fan, Y., Zweibel, E. G., & Lantz, S. R. 1998, ApJ, 493, 480

Ferriz-Mas, A., Schmitt, D., & Schüssler, M. 1994, A&A, 289, 949

Fisher, G. H., Fan, Y., & Howard, R. F. 1995, ApJ, 438, 463

Gilman, P. A., & Charbonneau, P. 1999, in Magnetic Helicity in Space and Laboratory Plasmas, ed. M. R. Brown, R. C. Canfield, & A. A. Pevtsov (Washington, DC; Geophys. Monogr. 111; AGU), 75

Hale, G. E., Ellerman, F., Nicholson, S. B., & Joy, A. H. 1919, ApJ, 49, 153

Hale, G. E., & Nicholson, S. B., 1925, ApJ, 62, 270

Howard, R. F., 1989, Sol. Phys., 123, 271

Leka, K. D., & Skumanich, A. 1999, Sol. Phys., 188, 3

Linton, M. G., Longcope, D. W., & Fisher, G. H. 1996, ApJ, 469, 954

Linton, M. G., Fisher, G. H., Dahlburg, R. B., & Fan, Y. 1999, ApJ, 522, 1190

Longcope, D. W., & Fisher, G. H. 1996, ApJ, 458, 380

Longcope, D. W., Fisher, G. H., & Pevtsov, A. A. 1998, ApJ, 507, 417

Longcope, D. W., Linton, M. G., Pevtsov, A. A., Fisher, G. H., & Klapper, I. 1999, in Magnetic Helicity in Space and Laboratory Plasmas, ed. M. R. Brown, R. C. Canfield, & A. A. Pevtsov (Washington, DC; Geophys. Monogr. 111; AGU), 93

López Fuentes, M. C., Démoulin, P., Mandrini, C. H., & van Driel-Gesztelyi, L. 2000, ApJ, 544, 540

López Fuentes, M. C., Démoulin, P., Mandrini, C. H., Pevtsov, A. A., & van Driel-Gesztelyi, L. 2003, A&A, 397, 305

Mickey, D. L. 1985, Sol. Phys., 97, 223

Moffatt, K. H., & Ricci, R. L. 1992, Proc. R. Soc. Lond. A 439, 411

Nandy, D., & Choudhuri, A. R. 2001, ApJ, 551, 576

Nandy, D., Hahn, M., Canfield, R. C., & Longcope, D. W. 2003, ApJ, 597, L73

Parker, E. N. 1955, ApJ, 121, 491

Pevtsov, A. A., & Canfield, R. C. 1999, in Magnetic Helicity in Space and Laboratory Plasmas, ed. M. R. Brown, R. C. Canfield, & A. A. Pevtsov (Washington, DC; Geophys. Monogr. 111; AGU), 103

Pevtsov, A. A., Canfield, R. C., & Latushko, S. M. 2001, ApJ, 549, L261

Pevtsov, A. A., Canfield, R. C., & Metcalf, T. R. 1994, ApJ, 425, L117

Pevtsov, A. A., Canfield, R. C., & Metcalf, T. R. 1995, ApJ, 440, L109

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1986, Numerical Recipes in Fortran 77 (Cambridge; Cambridge University Press)

Sammis, I. R., & Zirin, H. 1998, paper given at American Geophysical Union, Spring Meeting, SH22A, 4

Seehafer, N. 1990, Sol. Phys., 125, 219

Tanaka, K. 1980, in Solar-Terrestrial Predictions Proc., ed. R. Donnelly, 3 (NOAA-ERL), C1

Tian, L., Bao, S., Zhang, H., & Wang, H. 2001, A&A, 374, 294

Tian, L., & Liu, Y. 2003, A&A, 407, L13

Wang, Y.-M, & Sheeley, N. R. 1991, ApJ, 375, 761

Wang, Y.-M, & Sheeley, N. R. 2003, ApJ, 599, 1404

This preprint was prepared with the AAS LATEX macros v5.2.

Table 1. Data Correlation

Quantities	au	Confidence (%)	$ au_p$
Tilt/Separation-Latitude	-0.336	99.99	
Twist-Latitude	-0.077	97.33	
Tilt/Separation-Twist	-0.156	99.99	-0.193
Tilt/Separation Residuals-latitude	-0.017	36.66	
Twist Residuals-Latitude	-0.018	39.8	
Tilt/Separation Residuals-Twist Residuals	-0.204	99.99	-0.205

Table 2. Tilt-Twist correlation

Data subset	τ	Confidence (%)	$\frac{\text{Mean }  \alpha_{best} }{(10^{-8}m^{-1})}$	Active regions
$Joy \pm \le 6\sigma$ $Joy \pm \ge 6\sigma$ Non-Hale		86.2 99.9 95.8	1.1 1.6 2.5	194 174 12

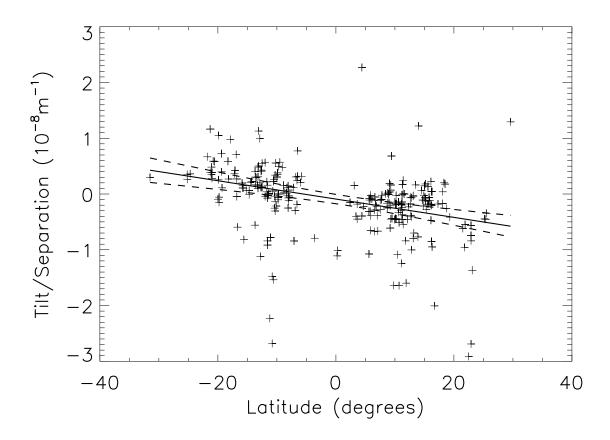


Fig. 1.— The latitude dependence of active region tilt/separation (reflecting Joy's Law) in our combined dataset. In this and all subsequent figures the solid line indicates the linear least-squares fit to our data and the dashed lines show the  $\pm 3\sigma$  error bands. Fit:  $\theta/d=-0.0164268\phi-0.0907248$ 

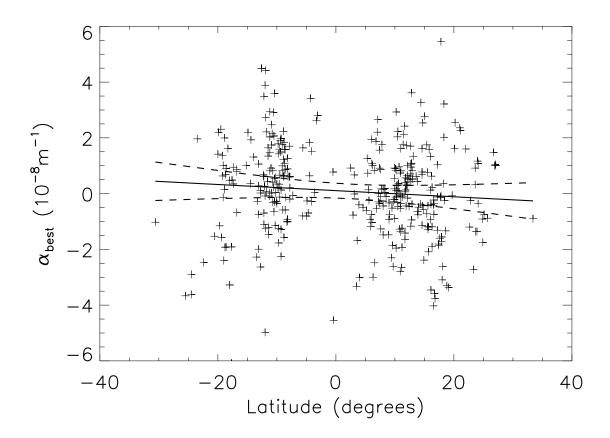


Fig. 2.— The latitude dependence of the overall twist  $\alpha_{best}$  of the magnetic fields of the active regions in our combined dataset. Fit:  $\alpha_{best} = -0.0110040\phi + 0.106473$ 

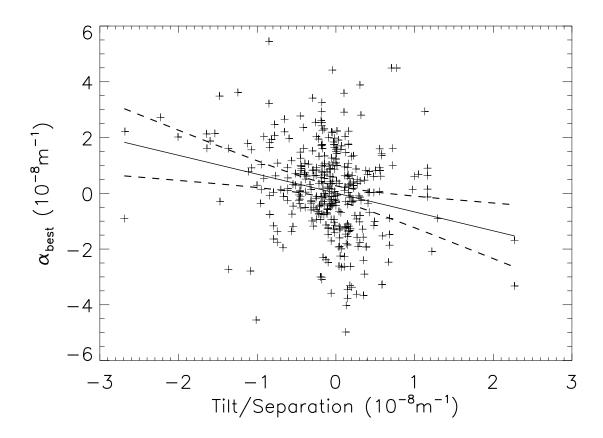


Fig. 3.— The correlation between raw twist  $(\alpha_{best})$  and tilt/separation values in our combined dataset. Fit:  $\alpha_{best}=-0.677077\theta/d+0.00802131$ 

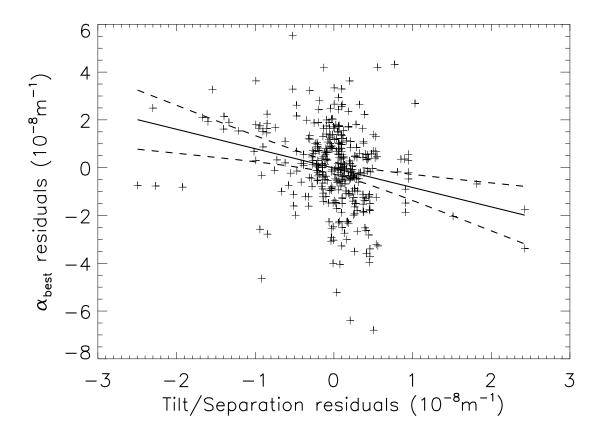


Fig. 4.— The correlation between values of twist  $\alpha_{best}$  and tilt/separation in our combined dataset, after subtraction of the trends shown in Figures 1 and 2. Fit:  $\alpha_{best}(residuals) = -0.812015\theta/d(residuals) - 0.0171826$ 

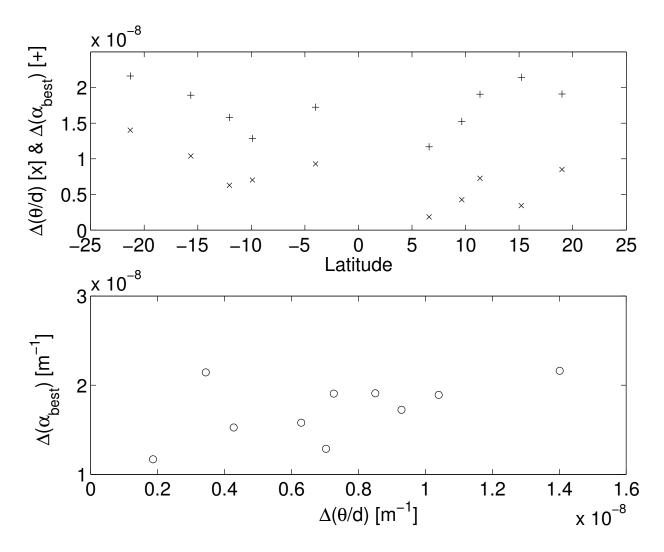


Fig. 5.— Top: Shows the binned values of the dispersion in tilt  $\Delta(\theta/d)_i$  (denoted by cross signs) and twist  $\Delta(\alpha_{best})_i$  (denoted by plus signs) plotted against the median latitude of each latitudinal bin. Bottom:  $\Delta(\alpha_{best})$  in each bin plotted against its corresponding  $\Delta(\theta/d)$  value – the correlation recovered is not significant (see Section 3).