

# Of Tilt and Twist

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## ABSTRACT

Using Mees Solar Observatory active-region vector magnetograms and Mt. Wilson Observatory full-disk longitudinal magnetograms, we measure both the twist and tilt of the magnetic fields of 368 active regions. This dataset clearly shows two well-known phenomena, Joy's law and the hemispheric helicity rule, as well as a lesser-known twist-tilt relationship, which is the point of this work. Those regions that closely follow Joy's law show no twist-tilt relationship, which is a predicted consequence of convective buffeting of initially untwisted and unwrithed flux tubes through the  $\Sigma$  effect. Those regions that strongly depart from Joy's law show significantly larger than average twist and a very strong twist-tilt relationship. These properties suggest that the twist-tilt relationship in these regions is due to kinking of flux tubes that are highly twisted but not strongly writhed.

*Subject headings:* Sun: magnetic fields—Sun: activity

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## 1. INTRODUCTION

The magnetic flux bundles that form active regions are thought to originate at the base of the solar convection zone, through which they rise as  $\Omega$ -shaped loops. There is reason to expect these flux bundles to be twisted by the dynamo itself (Gilman and Charbonneau 1999). As they rise they acquire twist and writhe (Moffatt and Ricca 1992) through the combined effects of the Coriolis force on large-scale flows within them, buffeting by helical turbulent convection, and kinking (Linton *et al.* 1996). The writhe, driven by the Coriolis force (Fan *et al.* 1994), successfully explains the latitude dependence of the tilt of active regions (Joy’s law, Hale *et al.* 1919), but not the amplitude of the twist (Longcope *et al.* 1999; Fan and Gong 2000). Buffeting of flux bundles by helical turbulent convection has been shown to produce twist whose amplitude is much greater than that produced by the dynamo, and comparable to what is observed (Longcope *et al.* 1998; Longcope *et al.* 1999). This mechanism (called the  $\Sigma$  effect) predicts that there is no correlation between twist and tilt, other than a very small one due to the influence of the Coriolis force on rising flux tubes (Fan and Gong 2000) or a trivial one due to the mutual dependence of twist and tilt on latitude (Tian *et al.* 2001).

In this paper we study the observed relationship between the tilt and twist of active regions and compare it to that predicted by these models. In Section 2 we combine two datasets to relate tilt and twist. We recover Joy’s law and the hemispheric helicity rule, and document a complex relationship between twist and tilt. In Section 3 we compare our results to previous work on this topic (Canfield and Pevtsov 1997; Pevtsov and Canfield 1999; Tian *et al.* 2001). We show that the  $\Sigma$ -effect model explains the observed lack of a twist-tilt correlation in regions that closely follow Joy’s law, while some other mechanism (arguably kinking) comes into play in anomalous regions.

## 2. Datasets

We have compiled a combined dataset, consisting of 368 active regions, from two independent datasets, one for twist and the other for tilt. The Mees Solar Observatory dataset consists of vector magnetograms from the Haleakala Stokes Polarimeter (Mickey 1985) obtained in 1988-1995. From these vector magnetograms, values of the best-fit overall twist of active regions  $\alpha_{best}$  were extracted following Pevtsov *et al.* (1995). The limited field of view of the Haleakala Stokes Polarimeter (HSP) is adequate for the calculation of  $\alpha_{best}$ , since the strong-field regions on which it must be based (Leka and Skumanich 1999) are not spatially extended. However, for calculation of active region tilt, a larger field of view is desirable. Hence, we use the Mt. Wilson dataset for this purpose. This dataset (Howard 1989) consists

of line of sight, full disk magnetograms collected from 1967 to 1995. The Mt. Wilson dataset identifies the magnetic flux-weighted centroids of the two polarities within each active region. We define the tilt angle  $\theta$  to be the angle (positive counterclockwise) between the local parallel of latitude and the line joining the centroids of the two polarities, and  $d$  to be the distance between them.

We associate corresponding active regions in the HSP and Mt. Wilsons datasets using the daily NOAA active region lists, checking for consistency in latitude ( $\pm 5^\circ$ ) and central meridian distance ( $\pm 5^\circ$ ) after correction for solar rotation. Knowing that the twist patterns with active regions evolve on characteristic time scales of about 27 hours (Pevtsov *et al.* 1994), we accepted up to 12 hours time difference for matching purposes.

We further sort the data by excluding regions that do not obey the Hale-Nicholson polarity law (Hale and Nicholson 1925; Hale and Nicholson 1938) because we cannot confidently (uniquely) measure tilt. Regions that obey the Hale-Nicholson polarity law comprise 91.9% of the Mt. Wilson dataset. Finally, close inspection of the Mt. Wilson dataset found that some active regions have the same tabulated latitude for the two polarities to within  $\pm 0.1^\circ$ , resulting in zero values for tilt. In most cases, independent measurements showed significantly non-zero values. As well, zero-tilt measurements were found in the midst of a sequence of significantly non-zero measurements. Hence, we eliminated from the combined dataset all individual measurements for which tilt values are identically zero. The final combined dataset contains overall active-region twist  $\alpha_{best}$ , tilt  $\theta$ , polarity separation  $d$ , and latitude  $\phi$  for 368 regions.

As expected, our combined dataset shows various well-known relationships. Figure 1 shows the dependence of  $\theta/d$  on latitude, reflecting what is commonly called Joy’s law (Hale *et al.* 1919): positive tilt values dominate in the south, and negative in the north, with considerable scatter. The heavy solid line in Figure 1 shows a linear fit to our results. Our dataset is much smaller than the full Mt. Wilson dataset studied by Wang & Sheeley (1991), and contains many fewer small active regions, because the HSP data acquisition strategy favored larger (flare-productive) ones. Nevertheless, our fit to the latitude dependence of tilt agrees with Fan’s (1993) fit to Wang & Sheeley (1991) at the  $\sim 3\sigma$  level.

In this and subsequent figures including tilt, our analysis is guided by a simple nonlinear force-free field – a uniformly twisted cylindrical tube (Priest 1984). It is straightforward to show that the force-free field parameter  $\alpha$  for this field is simply the twist per unit length. Since we are interested in the relationship of twist to tilt, we choose a measure ( $\theta/d$ ) that has the same dimensions as  $\alpha$ . As discussed in Section 4, the fact that  $\theta/d$  is better correlated with  $\alpha$  than  $\theta$  alone supports the interpretation given in that section.

In Figure 2 we see that our dataset shows the well-known hemispheric helicity rule (Pevtsov *et al.* 2001), where negative (left-handed) values dominate in the northern hemisphere and positive (right-handed) in the southern, again with considerable scatter. The solid line in Figure 2 shows a linear fit to our results. This fit agrees with Pevtsov *et al.* (2001) at the  $\sim 3\sigma$  level.

In addition to these well known trends, Figure 3 shows the negative correlation between  $\theta/d$  and  $\alpha_{best}$  that is the main focus of this paper. Active regions that are positively tilted tend to have negative twist. The  $\pm 3\sigma$  error bands show that although the scatter is large, the correlation is significant. The scatter is real, which led Longcope *et al.* (1998) to attribute it to the stochastic nature of turbulent convection (Longcope *et al.* 1998). It does not originate uniquely in either the  $\theta/d$  or  $\alpha_{best}$  data. It is not reduced when regions with values of separation much less than the average value ( $\langle d \rangle \sim 9 \times 10^7 m$ ) are removed from the combined dataset.

### 3. Analysis

Let us begin by dismissing the trivial relationship between twist and tilt that arises due to the mutual dependence of these quantities on latitude. Consider the dependence of  $\theta/d$  and  $\alpha_{best}$  on latitude shown in Figures 1 and 2. In the northern hemisphere both  $\theta/d$  and  $\alpha_{best}$  tend to be negative, and in the southern hemisphere, positive. Hence, we expect hemispheric dependence to produce a positive correlation. In fact, the correlation shown in Figure 3 is negative. Thus, the mutual dependence of  $\theta/d$  and  $\alpha_{best}$  on latitude (Tian *et al.* 2001) cannot explain their observed relationship. We turn to two different techniques to quantitatively remove the effect of mutual dependence.

We first take a strictly statistical approach to the interrelationships among  $\theta/d$ ,  $\alpha_{best}$  and latitude. Kendall’s rank correlation coefficient ( $\tau$ ) tells us the strength of the correlation between two quantities (Daniel 1990). Table 1 shows values of  $\tau$  for  $\theta/d$  vs latitude,  $\alpha_{best}$  vs latitude and  $\theta/d$  vs  $\alpha_{best}$ . From this analysis we can see that the correlation between  $\theta/d$  and latitude holds the highest confidence (the chance of occurrence in an uncorrelated dataset is  $\sim 10^{-22}$ ). This is hardly surprising, from the past work on Joy’s law cited above. Twist and latitude have a weaker, but still very significant correlation. Significantly, the confidence level for the correlation between  $\theta/d$  and  $\alpha_{best}$  is very high; the chance of occurrence in an uncorrelated dataset is  $\sim 10^{-6}$ . We use Kendall’s partial correlation coefficient,  $\tau_p$ , to quantify this correlation between  $\theta/d$  and  $\alpha$  while holding latitude constant (Daniel 1990). If the correlation between  $\theta/d$  and  $\alpha_{best}$  were due to their mutual latitude dependence,  $|\tau_p|$  would be significantly less than  $|\tau|$ . In fact, we find that  $|\tau_p| > |\tau|$  (see table 1), confirming

that the correlation is not due to mutual dependence on latitude.

We next take a more intuitive approach to eliminating the influence of the mutual dependence of  $\theta/d$  and  $\alpha_{best}$  on latitude – we simply subtract the fitted latitudinal trends of Figures 1 and 2 from our individual  $\theta/d$  and  $\alpha_{best}$  values. Figure 4 shows the relation between these  $\theta/d$  and  $\alpha_{best}$  residuals. As a check, recalculating  $\tau$  using these residuals (Table 1), confirms that (a) the  $\theta/d$  and  $\alpha_{best}$  residuals hold no significant correlation with latitude and (b) the  $\theta/d$  residuals show a strong correlation with the  $\alpha_{best}$  residuals, of even greater magnitude than in the raw data, and  $\tau_p \simeq \tau$ . This more intuitive approach further confirms that the relationship between  $\theta/d$  and  $\alpha_{best}$  is not due solely to mutual latitude dependence.

Finally, we search for clues that tell us how the relationship might arise. We know that the statistical basis for Joy’s law is very strong. How does the twist-tilt relationship depend on the extent to which individual regions obey this law? The first two rows of Table 2 compare two subsets of our combined dataset: those with  $\theta/d$  values that follow the best-fit “Joy’s law” line in Figure 1 to within  $\pm 6\sigma$  and those that deviate by greater amounts. We find that although the subsets are of comparable size, there is no statistically significant relationship between twist and tilt for those regions that obey Joy’s law to this extent, whereas there is a statistically strong relationship for the regions that depart more from it.

We further explore this trend by considering those regions that depart so much from Joy’s law that they disobey the Hale-Nicholson polarity law. To do this we assume that the absolute magnitudes of the tilt angles of such regions range only between  $90^\circ$  and  $180^\circ$ . The last row of Table 2 shows that for these non-Hale regions the Kendall  $\tau$  coefficient is atypically large and has opposite sign. Unfortunately, this subset comprises only 12 regions, and the confidence level of this correlation is far less than the others. A much larger dataset will be required to determine whether these trends are real. However, it is noteworthy that the mean value of overall twist  $|\alpha_{best}|$  is atypically large. This result is more robust than the correlation analysis, since it does not rely on the questionable assumption about the range of tilt values.

#### 4. Discussion

In our analysis we find a complex relationship between the tilt angle and overall magnetic twist of solar active regions. Of the large majority of regions that obey the Hale-Nicholson polarity law, those that most obediently conform to Joy’s law show no significant interdependence of twist and tilt. This is a prediction of the  $\Sigma$ -effect model (Longcope *et al.* 1998),

which describes buffeting by helical convective turbulence of initially untwisted straight flux tubes, taking into account helicity conservation. On the other hand, those regions that significantly depart from Joy’s law show both stronger than average twist and a strong interdependence of twist and tilt. Such properties would most plausibly arise as the result of kinking of highly twisted flux tubes (Linton *et al.* 1996; Lopez Fuentes *et al.* 2003). Suppose that a flux tube is created at the base of the convection zone with high positive helicity  $H$  in the form of twist  $\mathcal{T}$  but no writhe  $\mathcal{W}$ . When the tube kinks, it trades some of that twist  $\delta\mathcal{T}$  for writhe  $\delta\mathcal{W}$ , and helicity conservation requires that  $\delta\mathcal{T} = -\delta\mathcal{W}$ . In the example above,  $\delta\mathcal{T} < 0$  so  $\delta\mathcal{W} > 0$ , and hence tilt/separation  $\theta/d < 0$ . Thus we see that kinking of strongly twisted but unwrithe flux tubes will produce a negative correlation between  $\theta/d$  and  $\alpha_{best}$ , as observed.

A final bit of evidence for kinking comes from the effect of including the active region characteristic length scale in the analysis. Table 1 shows that  $\theta/d$  is extremely well correlated with  $\alpha_{best}$  ( $\tau = -0.156$ , chance of random occurrence  $10^{-8}$ ). Although we find that  $\theta$  is also correlated very well with  $\alpha_{best}$  ( $\tau = -0.145$ ), the chance of random occurrence ( $10^{-7}$ ) is an order of magnitude larger.

We speculate that the dependence of  $\theta/d$  on  $\alpha_{best}$  for the regions that depart by more than  $\sim 6\sigma$  results from the kink instability acting on flux tubes that are highly twisted by the dynamo itself.

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Table 1. Data Correlation

Quantities	$\tau$	Confidence (%)	$\tau_p$
Tilt/Separation-Latitude	-0.336	99.999999...	...
Twist-Latitude	-0.077	97.33	...
Tilt/Separation-Twist	-0.156	99.999999	-0.193
Tilt/Separation Residuals-latitude	-0.017	36.66	...
Twist Residuals-Latitude	-0.018	39.8	...
Tilt/Separation Residuals-Twist Residuals	-0.204	99.999999...	-0.205

Table 2. Tilt-Twist correlation

Data subset	$\tau$	Confidence (%)	Mean $ \alpha_{best} $ ( $10^{-8}m^{-1}$ )	Active regions
Joy $\pm \leq 6\sigma$	-.071	86.2	1.1	194
Joy $\pm \geq 6\sigma$	-.178	99.9	1.6	174
Non-Hale	.45	95.8	2.5	12

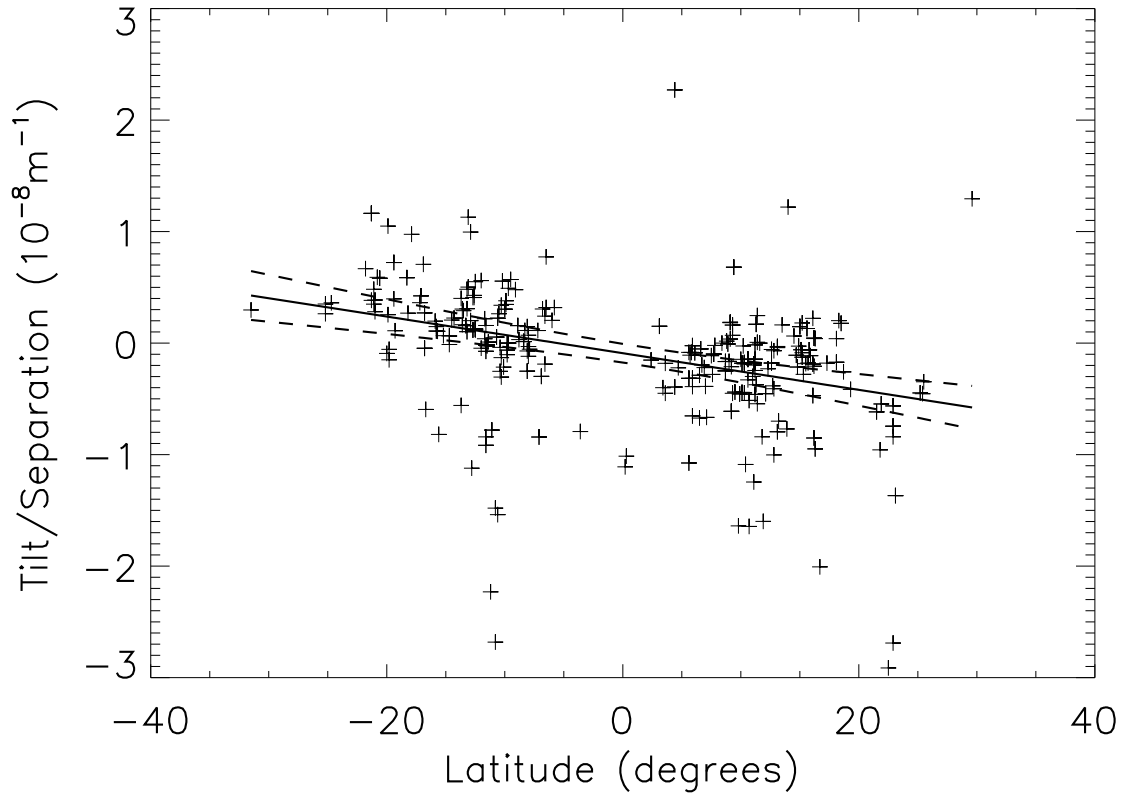


Fig. 1.— The latitude dependence of active region tilt/separation (reflecting Joy’s Law) in our combined dataset. In this and all subsequent figures the solid line indicates the linear least-squares fit to our data and the dashed lines show the  $\pm 3\sigma$  error bands.

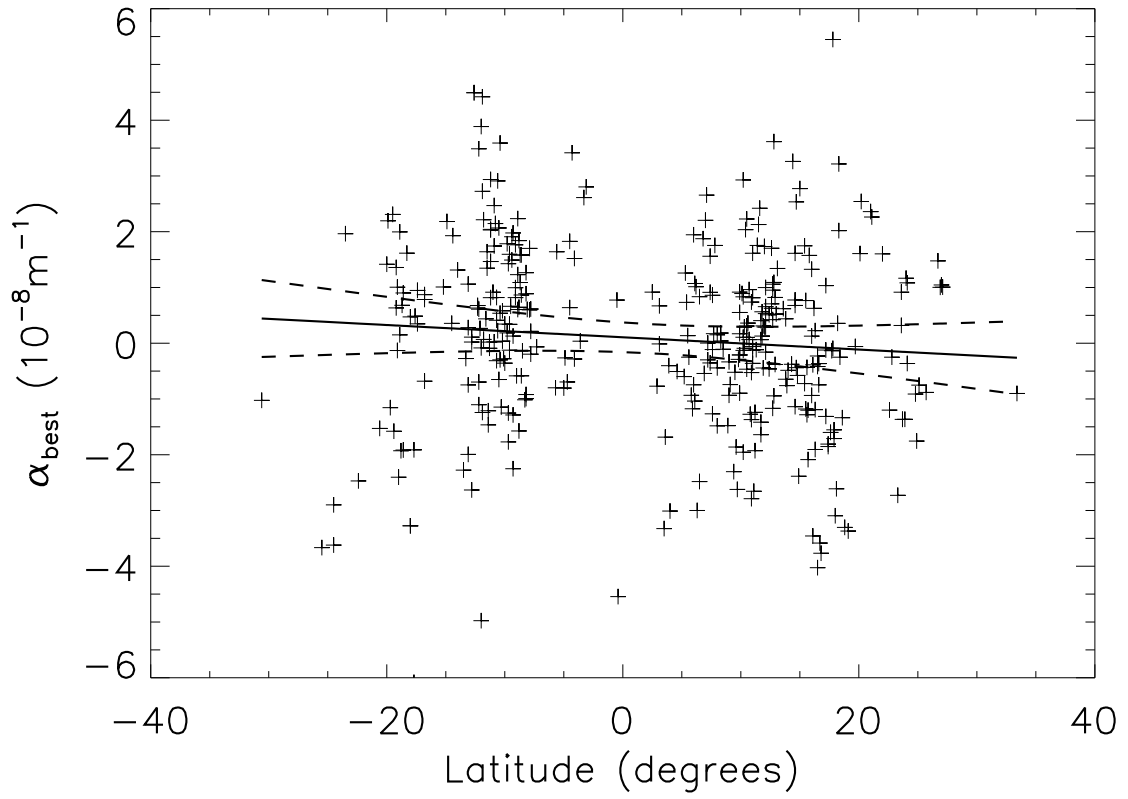


Fig. 2.— The latitude dependence of the overall twist  $\alpha_{best}$  of the magnetic fields of the active regions in our combined dataset.

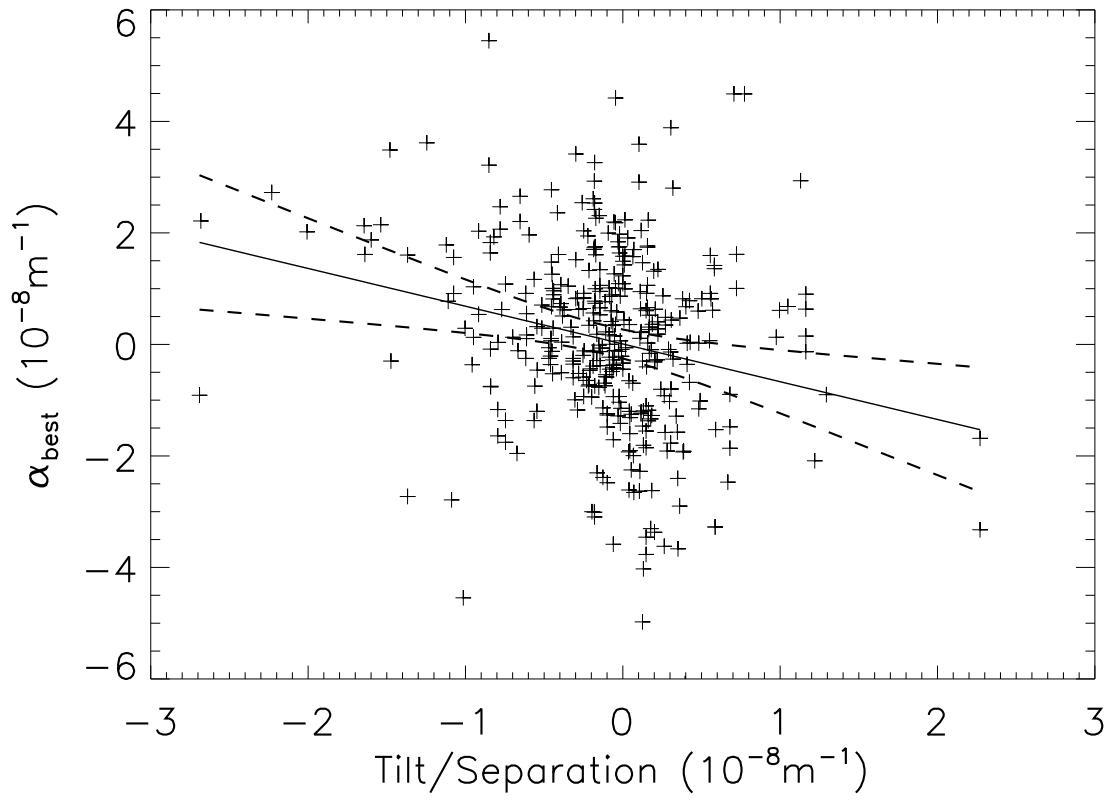


Fig. 3.— The correlation between raw twist ( $\alpha_{best}$ ) and tilt/separation values in our combined dataset.

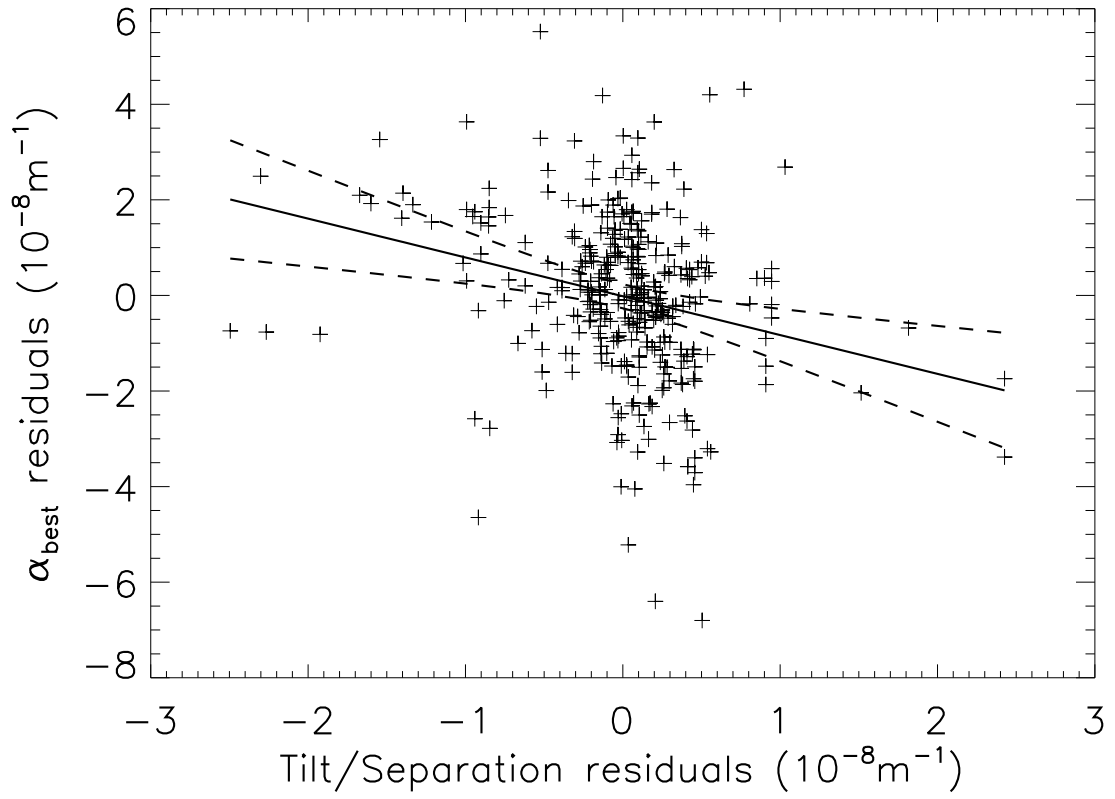


Fig. 4.— The correlation between values of twist  $\alpha_{best}$  and tilt/separation in our combined dataset, after subtraction of the trends shown in Figures 1 and 2