

Detection of a Taylor-like Plasma Relaxation Process in the Sun

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ABSTRACT

The relaxation dynamics of a magnetized plasma system is a subject of fundamental importance in magnetohydrodynamics – with applications ranging from laboratory plasma devices like the Toroidal Field Pinch and Spheromaks to astrophysical plasmas, stellar flaring activity and coronal heating. Taylor in 1974 proposed that the magnetic field in a plasma, subject to certain constraints, relaxes to a minimum energy state such that the final magnetic field configuration is a constant α (linear) force-free field – where α is a quantity describing the twist in magnetic field lines. While Taylor’s theory was remarkably successful in explaining some intriguing results from laboratory plasma experiments, a clear signature of this mechanism in astrophysical plasmas remained undetected. Here we report observational detection of a relaxation process, similar to what Taylor envisaged, in the magnetic fields of flare-productive solar active regions. The implications of this result for magnetic reconnection and the coronal heating problem are discussed.

Subject headings: MHD – Sun: magnetic fields – sunspots – flares – coronal heating

1. Introduction – Taylor’s Theory of Plasma Relaxation

Often in astrophysical and laboratory plasma systems we encounter situations where the plasma- β parameter (the ratio of the gas to magnetic pressure) is low (< 1). In such a scenario, the magnetic stresses in the field lines permeating the plasma become unbalanced and the magnetic field system evolves in a self-organized manner to remove these stresses. Two important theorems in this context were derived by Woltjer (1958) who showed that for a perfectly conducting plasma (the ideal MHD limit) the total magnetic helicity ($\int \mathbf{A} \cdot \mathbf{B} dV$) remains invariant during the evolution of any closed flux system and the minimum energy state of this system corresponds to a linear force-free magnetic field configuration satisfying the equation

$$\nabla \times \mathbf{B} = \alpha \mathbf{B} \quad (1)$$

where α is a constant from one field line to another or over the spatial scale of the flux system (as opposed to the non-linear case where $\alpha(x, y)$ varies from one field line to another). The parameter α is a manifestation of the helicity of a flux system and indicates the twist in the magnetic field lines. In a paper of remarkable insight, Taylor (1974) applied these concepts to the subject of plasma relaxation and recognized that while in the ideal MHD limit, the helicity of each field line will be an invariant of motion, for a plasma of small but finite resistivity, only the total magnetic helicity of the flux system will be invariant during its evolution to a minimum energy state. From Woltjer’s theorem it follows that the final state is a linear force-free magnetic field configuration. Taylor’s hypothesis provides an elegant way of circumventing the complex plasma relaxation dynamics (involving turbulence and reconnection) itself and predicting the final state of the system. This theory was immediately able to explain the puzzling phenomenon of reversal of field in Reversed Field Pinch experiments (Bodin & Newton 1980) and has since then been applied to other laboratory plasmas such as the Spheromak and plasma fusion devices (Taylor 1986; Yamada 1999; Bellan, Yee, & Hansen 2001).

The lower solar corona is a low β plasma and in its typical quiet state its magnetic field is believed to be nearly force-free. Coronal magnetic fields or loops are associated with solar active regions (ARs) with their foot points anchored slightly beneath the photosphere. Parker (1983) recognized that the kinetic energy of the anchored foot point motions (derived from convective turbulent flows) would be transmitted into tangled magnetic loops, the subsequent reconnection (and flaring activity) of which would be able to heat and maintain

the solar corona. Heyvaerts and Priest (1984) adapted Taylor’s hypothesis to this scenario and suggested that the corona could be heated by a Taylor-like relaxation process towards linear force-free states in response to being driven to non-linear force-free states by the photospheric foot point motions. The energy available for heating would be the difference between the energy of the non-linear and linear force-free configurations. AR flux loops are not closed systems. However, introducing the concept of relative magnetic helicity (Berger & Field 1984) – i.e., relative to the helicity of the potential field with the same normal magnetic field component at the boundary, it can be shown that Taylor’s theory would still hold in such a scenario. While some 3-D numerical MHD simulations of relaxation processes in the corona indeed show such a relaxation towards a constant- α field (Kusano et al. 1994), other simulations show a more complicated behavior at variance with the prediction of Taylor’s theory (Amari & Luciani 2000). This is not surprising, given our lack of complete understanding of the microphysics and the inherent uncertainties in modelling a complicated relaxation mechanism such as this. Taylor’s relaxation is also believed to play a role in the emission from extragalactic radio jets (Königl & Choudhuri 1985). However, a Taylor-like relaxation process had not been observed in any astrophysical system until now.

In the present work we do a statistical study of the evolution of twists (quantified by the parameter α) in a sample of flare-productive solar ARs to seek possible signatures of relaxation towards a Taylor-like linear force-free state. In Section 2 we outline our observations and data analysis procedure and present the results. We find that AR flux systems tend to evolve (as a result of flaring activity) towards a linear force-free state as would be expected from the foregoing discussion. We also derive the timescale of this relaxation process to be of the order of a week. We conclude in Section 3 with a discussion on the implications of our results.

2. Observations, Data Analysis and Results

The primary criterion for data selection of solar ARs for this study was the availability of both photospheric vector magnetic field measurements (magnetograms) from the Haleakala Stokes Polarimeter (HSP) and measurements of flare X-ray flux from the Geosynchronous Operational Environmental Satellites (GOES). We further pared down our data set by selecting only those ARs which had multiple HSP observations (at least 3) spanning days over which the flaring activity was observed. A master list containing 82 ARs with such concurrent observations was generated.

As part of its Space Environment Monitoring system the GOES satellite carries X-ray sensors (XRS) for detecting and measuring solar flare X-ray flux. For our analysis of the

X-ray flux from flares we selected the 1–8 Å wavelength band of XRS which is the one used to assign the B, C, M and X classification to solar flares. NOAA makes these data available with a time resolution of 3.06 s and reports the X-ray flux reaching the detectors. The NOAA AR number with which the detected flares are associated is also listed. For any given AR in our master list, we determined the total number of flares it generated, the start and end time of each flare and the X-ray flux measured by XRS over each of these flares. By integrating the X-ray flux over time for each flare (and subtracting out the basal flux present in the data) and adding the X-ray energy output from each individual flare, we obtained the total integrated flare X-ray energy (E_f) output (in units of Joules per square meter) for each AR. The E_f of any AR calculated in this manner was found to correlate well with the total number of flares and also the maximum X-ray flare energy flux for that AR, throughout the whole data set of 82 ARs. Thus, E_f is a good proxy for the flare productivity of ARs and we use this quantity for our subsequent statistical analysis.

Vector magnetograms can measure the vertical current density $J_Z(x, y)$ and from this one can calculate the force-free parameter α quantified by equation (1). This takes the form

$$\alpha_Z(x, y) = \mu_0 \frac{J_Z(x, y)}{B_Z(x, y)} \quad (2)$$

where B_Z is the vertical magnetic field (in the heliographic coordinate system) and $\mu_0 = 4\pi \times 10^{-3} \text{ G m A}^{-1}$. For calculating the twist parameter α_Z we use vector magnetograms obtained by the HSP instrument (Mickey 1985) located at Maui, Hawaii and operated by the Mees Solar Observatory. HSP vector magnetograms are obtained at two spatial resolutions (with 5".6 and 2".8 pixel spacing) from the Stokes profiles of the spectral lines Fe I $\lambda 6301.5$ and $\lambda 6302.5$ using the nonlinear least-squares Unno-fitting scheme of Skumanich & Lites (1987). Faraday rotation and magnetic filling factor effects are corrected for. The 180° azimuthal ambiguity in the transverse field (B_{trans}) is resolved and J_Z computed using the techniques described in Canfield et al. (1993). The noise level in the magnetograms is less than 100 G for the transverse and 10 G for the longitudinal fields. For the main study we calculate α_Z only for pixels for which $|B_{trans}| > 300 \text{ G}$ (i.e., with a cut-off at the 3σ level) thereby ensuring a high degree of confidence in the distribution of α_Z that we obtain (Pevtsov et al. 1994). The end result of the above procedure is a distribution of $\alpha_Z(x, y)$ for a given magnetogram observation of any AR, with a value of α_Z measured at each pixel in the high magnetic field region ($> 300 \text{ G}$) of the AR. The mean of this distribution, hereafter $\langle \alpha_Z \rangle$, describes the net twist of the AR and its variance, hereafter $\text{Var}(\alpha_Z)$ (or its square root – the standard deviation), describes the non-linearity or non-uniformity of this distribution over the spatial scale of the AR (Pevtsov et al. 1994; Leka & Skumanich 1999). The parameter α is theoretically expected to be constant along a field line from the photospheric level up to the corona and such correspondence has recently been observed between photospheric and

coronal measurements of α (Burnette et al. 2003). However, at the coronal level, current observations allow for the measurement of the net twist only.

We develop an additional scheme for determining the change in the $\langle\alpha_Z\rangle$ and $\text{Var}(\alpha_Z)$ values for any given AR from its multiple magnetogram observations spanning days over which it generated flares. The number of such magnetogram observations in our data set ranged from 3 to 12 per AR, typically having one observation per AR per day. The change in $\langle\alpha_Z\rangle$ over this period was defined as $|\langle\alpha_Z\rangle|_{final} - |\langle\alpha_Z\rangle|_{initial}$. We work with absolute values of the signed quantity $\langle\alpha_Z\rangle$ because we are interested in how the magnitude of the twist evolves over the flaring period (whether it increases or decreases irrespective of its sign). For ARs with 3 observations we used just the difference between these quantities calculated from the last and the first observation. For ARs with 4 and 5 observations we used the average of the last 2 (i.e., average of last observation $|\langle\alpha_Z\rangle|$ value and its previous observation $|\langle\alpha_Z\rangle|$ value) minus the average of the first 2. For ARs with 6 or more observations we used the average of the last 3 minus the average of the first 3 observations. Similarly, the change in $\text{Var}(\alpha_Z)$, an unsigned quantity, was defined as $\text{Var}(\alpha_Z)_{final} - \text{Var}(\alpha_Z)_{initial}$ with an identical scheme of averaging. This scheme of averaging may be somewhat *ad-hoc*. However, it is a more faithful indicator of the change in the twist and its variance that may result from flaring activity lasting several days, implicitly incorporating a correction for fluctuations which may arise from the injection of helicity from beneath the photosphere – having a typical ramp-up time of 1 day (Pevtsov et al. 2003). To summarize, each AR in our data set (of 82 ARs) is characterized by its total integrated flare X-ray energy output E_f and the corresponding change in its $|\langle\alpha_Z\rangle|$ and $\text{Var}(\alpha_Z)$ values over a time concurrent with its flaring activity.

In Figure 1a (top panel) we plot the change in the absolute mean value of the α_Z distribution for each AR versus its total integrated flare X-ray energy output, E_f . Note that in this plot as well as in the bottom panel, we have zoomed into the y-axis appropriately to make the changes visible to the eye. Consequently some of the outlying data points have been left out. However, for the data analysis and the correlations we report, we have used all the 82 ARs in our data set. The *Spearman's* rank correlation coefficient (which measures the correlation based on a distribution of ranks for the two variables and is thus not limited to measuring only *linear* trends [Press et al. 1986]) for the relationship between the change in $|\langle\alpha_Z\rangle|$ and E_f is -0.1694 and has a low confidence level of 87.21%. This basically means that although there is a tendency for the magnitude of the mean twist $|\langle\alpha_Z\rangle|$ to decrease with increasing flaring activity this relationship is not statistically significant in our data set and we cannot exclude the possibility that this is from random chance.

Next, we plot the change in variance of the α_Z distribution versus E_f for each AR in Figure 1b (bottom panel). The *Spearman's* rank correlation coefficient in this case turns out

to be -0.2809 with an acceptably high confidence level of 98.95%. This is a statistically significant result which indicates that $\text{Var}(\alpha_Z)$ decreases with increasing flaring activity. A visual inspection of Figure 1 (bottom panel) conveys the same information. It is seen that for less active ARs (towards the left of the x-axis) the $\text{Var}(\alpha_Z)$ doesn't change much (sometimes showing a slight increase), while the $\text{Var}(\alpha_Z)$ of ARs which were highly flare-productive tends to decrease. Recall that $\text{Var}(\alpha_Z)$ is a measure of the non-uniformity of the distribution of the force-free parameter α over the AR flux system. A decrease in this quantity implies that the flux system is being driven towards a more uniform distribution, resembling a constant- α (linear) force-free configuration, as envisaged by Taylor (1974, 1986) in his theory of plasma relaxation. As a test of our analysis, we re-evaluated the quantities in our data set with different cutoff values of 150 G and 450 G for B_{trans} . The trends were found to be similar, moreover for the higher cut-off (with less noisy data), the correlation between changes in $\text{Var}(\alpha_Z)$ and E_f was found to be stronger and more statistically significant than the 300 G results. This is in agreement with expectations and compels us to believe in the robustness of our data analysis procedure.

Having found a correlation between the decrease in variance of α and flare productivity, we next quantify this relaxation towards a constant- α force-free field by estimating a timescale for this mechanism. Clearly, what we have found is a statistical trend, which most ARs follow but some do not. The latter may be due to different dynamics and conditions (for example, emerging magnetic and helicity flux from beneath the photosphere or significant loss of helicity from the AR flux system to the corona) which makes these AR flux systems deviate from the implicit assumptions in Taylor's conjecture. Therefore some ambiguity is bound to creep in when one selects a set of ARs for deriving a timescale for the relaxation process. Keeping this caveats in mind we proceed by considering only those ARs which showed a decrease in their variance over the flaring period (with negative values for change in $\text{Var}(\alpha_Z)$ from the data set used for Figure 1). For each of these ARs, we calculate the relative time of each of its magnetogram observations, by defining the time of its first observation to be $t = 0.0$. The end result is that we have the measurements of $\text{Var}(\alpha_Z)$ and the corresponding time of this measurement relative to the first observation of any given AR, throughout the observation period of this AR. The standard deviation ($[\text{Var}(\alpha_Z)]^{1/2}$) has the same units as the mean of α and therefore this is the parameter that should be used for estimating how fast the system evolves towards a constant- α force-free field. We seek an exponential decay in the value of $[\text{Var}(\alpha_Z)]^{1/2} \sim e^{-t/\tau}$, characteristic of a decay phenomenon with the constant τ .

A plot of $\log_e[\text{Var}(\alpha_Z)]^{1/2}$ versus time (Figure 2) has a negative *linear* correlation coefficient with a significance of 99.99% – making a linear fitting meaningful at a high confidence level (thus in essence justifying our modelling this relaxation phenomenon as a exponential

decay process). The solid line denotes the best linear fit to the data. From the slope of this line we calculate the characteristic timescale of this decay process to be $\tau = 8.1$ days. A similar analysis in a subset of ARs characterized by $E_f > 10^{-1} \text{ J m}^{-2}$ (see Figure 1b) yielded a timescale of 15.9 days (linear correlation coefficient of a similar fit being 98.42%). Note however that the latter timescale should be considered as an upper limit, as this subset includes some ARs whose behaviors were at variance with Taylor’s hypothesis ($\text{Var}(\alpha_Z)$ increased) – perhaps due to ongoing helicity and magnetic flux emergence in these ARs.

3. Discussion

We have found that magnetic fields in highly flare-productive AR flux systems relax towards a constant- α linear force-free field configuration. However, they never achieve the completely linear state within the time frame for which magnetogram observations were available. In a real system, encompassing many complexities such as continuous foot point motions, helicity and magnetic flux injection and loss, this is perhaps what is to be expected. In fact some theoretical studies of the Taylor’s relaxation process in the context of the Sun hint that the coronal field may be in a state of partial relaxation (Vekstein, Priest, & Steele 1993).

We have estimated the timescale of the relaxation process to be 8.1 days. Keeping the earlier caveats in mind we stress that this should be taken in the spirit of an order of magnitude estimate (that is of the order of a week). This timescale may be relevant to existing theories for coronal heating. While the heat flux required to heat the corona above ARs to the observed temperatures is about $5000 \text{ W m}^{-2} \text{ s}^{-1}$ (Withbroe & Noyes 1977), the energy available from photospheric foot point motions is of the order of $10^4 \text{ W m}^{-2} \text{ s}^{-1}$ (Parker 1983; Priest 1999). This is certainly enough, but the point is how much of this available energy can be tapped into to heat the corona. Vekstein, Priest, & Steele (1991) show that the coronal heating from this available energy source is maximized when the relaxation or reconnection timescale (assuming that the relaxation is due to a series of microscopic reconnection events or small-scale flares as envisaged by Parker) is higher than the timescale of photospheric foot point motions (thus in effect allowing enough stresses to build up before reconnection can release it). The latter is of the order of 1 day for AR flux systems (Priest 1999) and our estimated relaxation time is higher than that. Thus our result may be taken as observational evidence that the plasma relaxation mechanism, in response to photospheric foot point motions, is a viable mechanism for coronal heating.

Several theoretical arguments suggest that the twist parameter α evolves diffusively or hyper-diffusively, at least over large spatial and temporal scales (Bellan 2000; Diamond &

Malkov 2003). Diffusive evolution is a well known path to a uniform distribution and is a common effect of randomization. The randomization most often invoked for evolution of magnetic fields near equilibrium is small-scale reconnection between neighboring flux bundles. Turbulent fast reconnection between bundles of transverse scale w would permit α to be diffused with an effective diffusivity as large as $D \sim 0.1v_A w$, assuming a reconnection velocity $0.1v_A$ as in Petschek’s model (Petschek 1964). Diffusion would achieve Taylor relaxation on a time-scale $\tau \sim L^2/D$ over a region of characteristic dimension L . An active region has $L \sim 10^{10}$ cm and Alfvén speeds typically $v_A \sim 10^8$ cm/sec. Assuming the reconnecting flux bundles have widths $w \sim 10^8$ cm, comparable to coronal loops observed in EUV or SXR images, gives a relaxation time scale $\tau \sim 10^5$ s, roughly one day. For this kind of reconnective transport to yield the slower relaxation observed would require that each flux bundle undergoes reconnection occasionally rather than continuously.

We conclude by mentioning that alternative theoretical approaches exist which treat this relaxation phenomenon as a self-organized critical (SOC) avalanche model – the prime example of the latter being the sandpile model (Bak, Tang, & Howard 1987; Kadanoff et al. 1989; Jensen 1998). The evolution of the magnetic topology through a series of force-free equilibria and successive minimum energy states (equivalent to small-scale avalanches) having conceptual similarity to Parker’s theory of nano-flares (Lu & Hamilton 1991; see the review by Charbonneau et al. 2001; for a related treatment see Browning & Van der Linden 2003). While an immediate connection between our findings (especially the timescale that we find) and these alternative concepts cannot be made, we hope that the presented results will motivate further inquiry into the physics of plasma relaxation and connections with observations when established, will unravel some of the hidden intricacies of this process.

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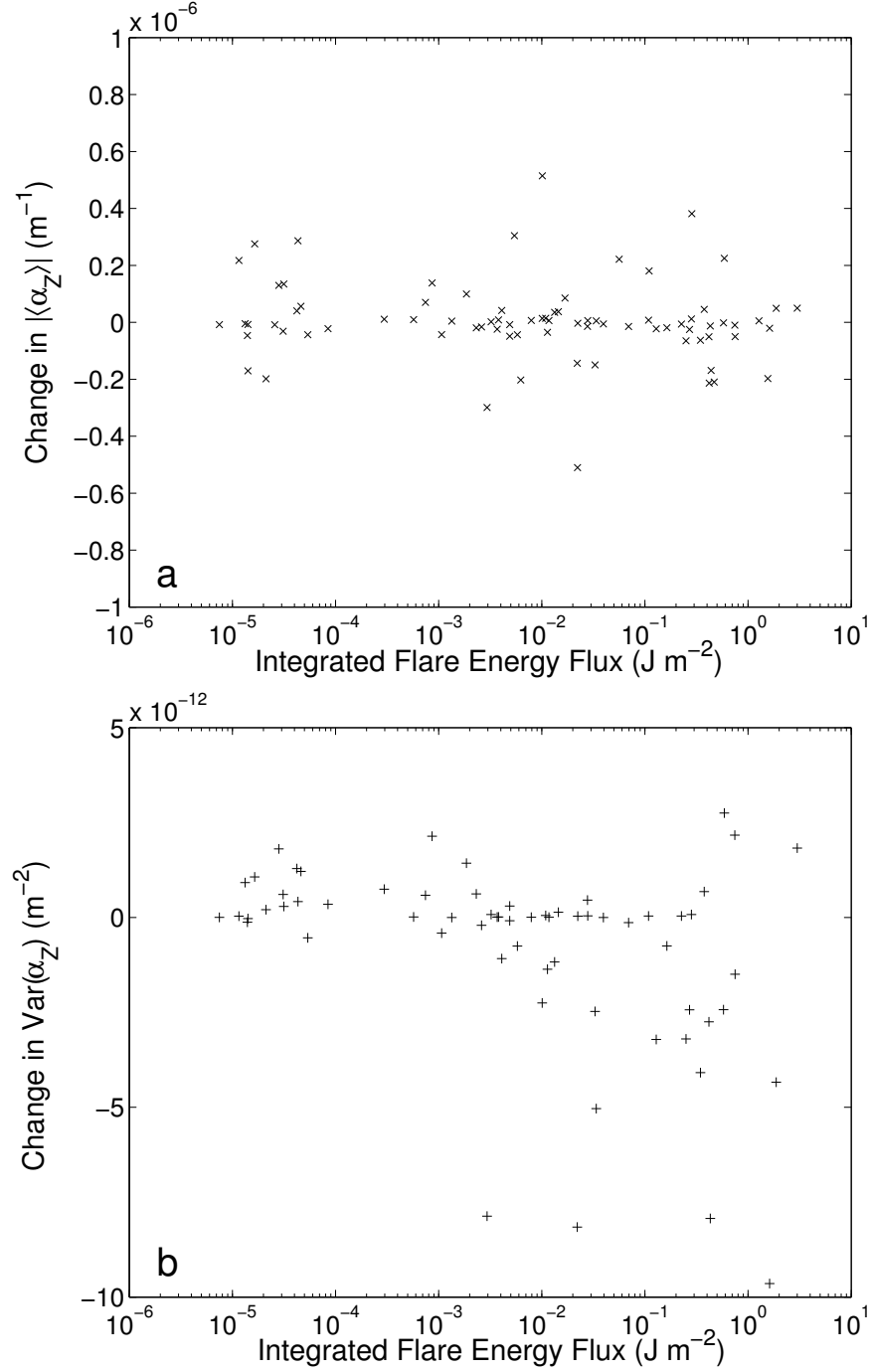


Fig. 1.— The top panel (a) shows the change in the values of $|\langle \alpha_Z \rangle|$ versus E_f . The bottom panel (b) shows the change in $\text{Var}(\alpha_Z)$ values for the same ARs versus E_f .

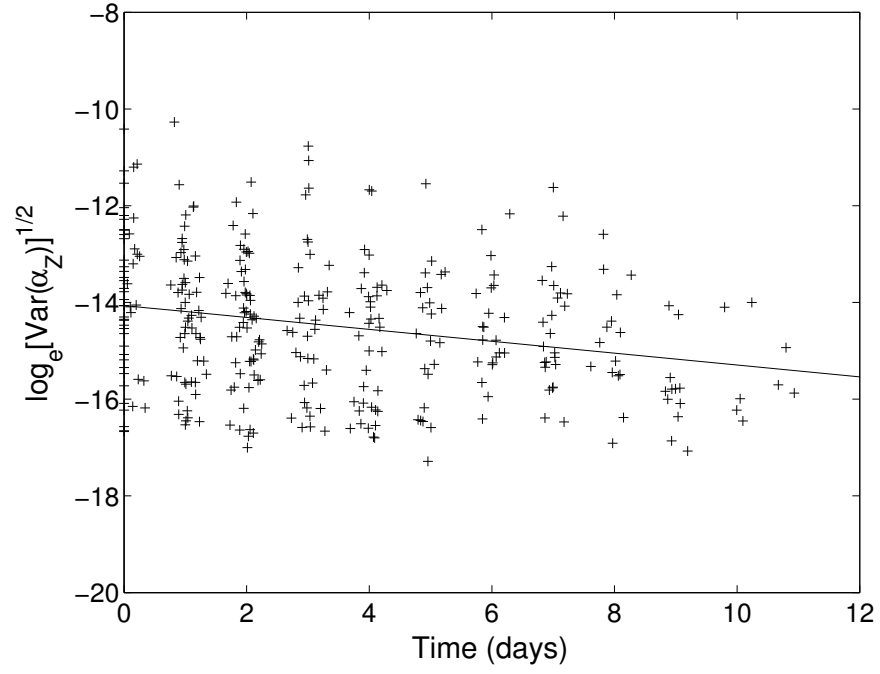


Fig. 2.— Evolution of $\log_e[\text{Var}(\alpha_Z)]^{1/2}$ values versus normalized time of observations in days. The solid line shows the best linear fit to the data.