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MAGNETIC TOPOLOGY OF THE SOLAR CORONA

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Ph.D. Thesis

University of St Andrews

Submitted July 8th, 2003.

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Abstract

This thesis examines the magnetic topology of the solar corona. Many of the dynamic processes in the Sun's atmosphere are driven by the magnetic field, and so understanding the structure of such such fields is a key step towards modelling these phenomena.

The technique of Magnetic Charge Topology (MCT) is used to determine the topologies due to various source configurations. The balanced four-source case is completely classified, and seven distinct topological states are found. This is compared to the complete three-source classification performed by Brown and Priest (1999a). A method is described for extending the analysis to greater numbers of sources.

MCT is also used to discuss the creation of magnetic null points in the solar corona. Until recently, it was tacitly assumed that any coronal nulls would have to be created by means of a local double-separator bifurcation in the photospheric source plane. A counterexample - the new, coronal local separator bifurcation - with five unbalanced sources is found and analysed, and several seven-source scenarios are also discussed.

We also find that this new bifurcation plays a critical role in the Magnetic Breakout Model for solar flares and coronal mass ejections (Antiochos *et al.*, 1999). We provide a simple MCT model for a flaring delta-spot region and find that a 'breakout' can be provoked in several different ways.

Finally, a Monte Carlo variation on MCT is used to determine the proportion of upright nulls in a field due to a large number of sources. By overlaying two plane topologies, we find also the number of separators and use the result to calculate typical sizes for elemental flux loops in the corona.

Acknowledgements

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Lastly, this thesis would probably never have been completed without my girlfriend Emma Felber.

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Chapter 1

Introduction

In the beginning, when the world was new, there was no sun and the humans and animals had to hunt and gather by the light of the dim moon. One day the brolga and the emu had a huge argument over whose babies were best. The brolga got so furious that she stole one of the emu's eggs which she threw into the sky. As she threw it into the air it smashed on a few sticks. The yellow yolk burst into flames and lit up the earth.

Indigenous Australian creation myth, retold by Sarah Steele

1.1 Introduction

The solar corona is a complicated and constantly-changing layer of the Sun's atmosphere. Lying above the Sun's lower atmospheric regions, the photosphere and chromosphere, it extends far beyond even the furthest planets and into interstellar space.

Many of the Sun's most spectacular sights are seen in the lower part of the corona: for instance, the gigantic loop structures shown by the TRACE and Yohkoh satellites, massive explosions such as solar flares and the eruptions of prominences that lead to huge Coronal Mass Ejections.

All of these phenomena are magnetic in nature - that is to say, they are mainly driven

by the coronal magnetic field. This field arises from a large number of intense, isolated flux sources in the photosphere, locations where flux tubes originating in the solar interior break through the surface.

This field, even from a handful of stationary sources, is immensely complicated. In reality, there are many thousands of sources constantly moving around, emerging and disappearing, combining and fragmenting and growing or shrinking in strength and size. We are a long way from even a basic understanding of such a complex field.

Our approach is to try to understand the structure of relatively simple fields, in the hope that these can be used to build up pictures of more complicated structures. We do this by examining the topological features described later in this chapter.

In most parts of the lower corona, the magnetic energy density far exceeds any other form of energy. From this it follows that many of the dynamic coronal events, such as flares are driven by the magnetic field. In particular, these events are often linked to complex configurations where several topologically distinct regions interact (Lau, 1993; Aulanier *et al.*, 1998; Fletcher *et al.*, 2001).

1.2 Equations

The magnetic field is governed by the equations of magnetohydrodyamics (MHD), the details of which can be found in any reputable MHD textbook such as Priest (1982). We will be using in particular

• The equation of motion:

$$\rho \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\nabla p + \mathbf{j} \times \mathbf{B} + \rho \mathbf{g},\tag{1.1}$$

where ρ is the plasma density, **v** the plasma velocity, *p* the plasma pressure, **j** the electric current density, **g** gravity, and **B** the magnetic field;

• Ampère's law:

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$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j},\tag{1.2}$$

where μ_0 is magnetic permeability (assumed to be constant);

• The solenoidal condition:

$$\nabla \cdot \mathbf{B} = 0; \tag{1.3}$$

• The induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{1.4}$$

where η is the magnetic diffusivity, taken to be uniform.

We can then define the magnetic Reynolds number R_m by comparing the dimensions of the terms in Equation 1.4:

$$R_m \sim |\nabla \times (\mathbf{v} \times \mathbf{B})| / \eta |\nabla^2 \mathbf{B}| \sim \frac{L_0 v_0}{\eta}, \tag{1.5}$$

where L_0 and v_0 are typical length and velocity scales.

When the magnetic Reynolds number $R_m \gg 1$, as is true nearly everywhere on the Sun, Alfvén's theorem applies, and the plasma is 'frozen in' to the magnetic field, and can effectively move only along field lines (e.g., Priest, 1982).

Reconnection occurs when plasma is allowed to move across field lines with different connectivity, which occurs when $R_m \leq 1$. In the corona, $\eta \sim 1 \text{m}^2 \text{s}^{-1}$. For R_m to be sufficiently small for reconnection to occur, either the velocity or the length scale must be very small indeed. Since coronal velocities are generally less than or of the order of the Alfvén velocity $v_A \sim 10^6 \text{ms}^{-1}$, it would seem that minuscule length scales are required. In two dimensions, null points are the only locations for reconnection; in three dimensions, reconnection is not confined to null points although it can occur there.

Photospheric elements, however, do not move so quickly. Most agree on velocities of the order of $v_0 \sim 10^3 \text{ms}^{-1}$, so that $v_0 \ll v_A$. That is to say, coronal structures (which are thought to have velocities of the same order as the photospheric movements causing them) move in most cases far slower than the Alfvén speed, and can be considered to be in *quasi-static equilibrium* - effectively, in force balance. If we neglect also gravity and plasma pressure (reasoning that they are generally far smaller than the Lorentz force), the equation of motion (Equation 1.1) reduces to

$$\mathbf{j} \times \mathbf{B} = \mathbf{0} \tag{1.6}$$

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This assumption (the *force-free* assumption) breaks down in highly dynamic events such as the explosive phase of a flare, although it is valid for the slow build-up of energy beforehand.

Where it is valid, it implies that the current flow is everywhere parallel to the magnetic field, or $\mathbf{j} = \alpha(\mathbf{r})\mathbf{B}$, where α is a scalar function of position. Using Equation 1.2, this becomes:

$$\nabla \times \mathbf{B} = \alpha(\mathbf{r})\mathbf{B},\tag{1.7}$$

generally a non-linear partial differential equation.

The form of α can, however, be chosen so as to linearise this equation. The simplest example is $\alpha(\mathbf{r}) = 0$, which gives (in conjunction with Equation 1.3) a potential field. Another possibility is $\alpha(\mathbf{r}) = \alpha_0$, which gives a *linear force-free field*. Analytical solutions to this do exist for a given set of boundary conditions, but to discuss them here would be something of a digression; force-free fields are discussed only in passing in Chapter 4.

1.3 Magnetic charge topology

1.3.1 Introduction to MCT

The purpose of this thesis is to study the possible topologies of (largely) simple magnetic fields. To do so, we use the technique of *Magnetic Charge Topology* or *MCT* (e.g. Longcope, 1996). This involves making three main simplifying assumptions:

- Elements of photospheric flux are taken to be point sources (magnetic charges).
- The charges are assumed to lie in a plane; the corona is considered to be the halfspace where z > 0.
- The field due to the charges is assumed to be potential.

These assumptions warrant further examination, not least because two of them seem unphysical at first sight. The first assumption appears to contravene the solenoidal condition

 $\nabla \cdot \mathbf{B} = 0$ at such a source; the third seems unphysical because in a potential field $\nabla \times \mathbf{B} = 0$ and hence (in view of Ampère's law $\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$) no current can flow.

It could also be argued that the second condition is unphysical because the Sun isn't flat.

However, with a little work, all three of the assumptions can be justified. In the first instance, the magnetic charges aren't true monopoles, but instead representations of flux tubes passing through the solar surface and spreading out into the corona. At a distance r much greater than the radius of the flux tube, the magnetic field due to it will be effectively indistinguishable from that of a point source.

The second assumption is also permissible, as long as the area of the solar surface considered is small enough that the Sun's curvature can be neglected. In order to obtain some topological results, it is convenient to use the mirror corona in the half-space z < 0 as if it were real, although the physical conclusions apply only in the corona with z > 0.

The final assumption, that of a potential field, such that $\nabla \times \mathbf{B} = 0$ is more contentious. It is believed, however, (e.g., Longcope, 1996) that the magnetic field in the solar corona is quite close to potential - although at low altitudes, and in certain structures such as prominences, this is not true. A more valid approach would be to consider a force-free field satisfying Equation 1.7; however, this is computationally much more complicated and in any case, using a weakly force-free field rather than a potential field is not expected to give any new topological behaviour, although the parameter values at which bifurcations (changes between topological states - see Section 1.4) occur will naturally change depending on the exact form of $\alpha(\mathbf{r})$ (Brown and Priest, 2000).

One of the computational problems with using a force-free field with boundary conditions at z = 0 is that it is possible for more than one field to satisfy the equations. This is a topic we will return to briefly in Chapter 3.

Our other concession to $\nabla \cdot \mathbf{B} = 0$ is an insistence on flux balance. This is not always made explicit. For instance, in the 'five-source' example of Chapter 3, a sixth, balancing source is assumed to exist a great distance away.

Having made the above assumptions, we can then write the magnetic field explicitly at any point in space. If there are n sources at positions \mathbf{r}_i , (i = 1...n) with strengths

 $\epsilon_i, (i = 1 \dots n)$, then the field strength at a point **r** is:

$$\mathbf{B}(\mathbf{r}) = \sum_{i=1}^{n} \epsilon_i \frac{\mathbf{r} - \mathbf{r}_i}{|\mathbf{r} - \mathbf{r}_i|^3}$$
(1.8)

Armed with this information, we can consider the relative positions and orientations of the field's topological features: null points, spine field lines, separatrix surfaces and separator field lines, as described in the following Section. Table 1.1 shows how these are depicted in diagrams throughout this thesis.

1.3.2 Topological features: the magnetic skeleton

Null points are locations at which the magnetic field vanishes. Their local structure has been examined in detail, for instance by Parnell *et al.* (1996), and is depicted in Figure 1.3.1. A co-ordinate system can be chosen such that the first-order linear field near a magnetic null can normally be written as $\mathbf{B} = \mathbf{M} \cdot \mathbf{r}$, where $\mathbf{r} = (x, y, z)^T$ and

$$\mathbf{M} = \begin{pmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}(q-j_{\parallel}) & 0 \\ \frac{1}{2}(q+j_{\parallel}) & p & 0 \\ 0 & j_{\perp} & -(p+1) \end{pmatrix}, \quad (1.9)$$

where j_{\parallel} and j_{\perp} represent components of the current parallel and perpendicular to the spine, respectively, while p and q are parameters of the potential field. For nearly all cases in this thesis, we will be considering the potential situation, where j_{\parallel} and j_{\perp} are equal to zero. The solenoidal condition $\nabla \cdot \mathbf{B} = 0$ implies that the trace of the matrix **M** in Equation 1.9 vanishes, and hence so does the sum of its eigenvalues. Ignoring the degenerate cases when one or more of the eigenvalues is equal to zero, it is clear that one of the eigenvalues (λ_1) is of the opposite sign to the other two (λ_2 and λ_3). We label their corresponding eigenvectors as \mathbf{e}_1 , \mathbf{e}_2 and \mathbf{e}_3 , respectively. These eigenvectors are crucial to the skeleton.

The eigenvector associated with the odd-signed eigenvalue, \mathbf{e}_1 , defines two isolated field lines known as *spines* (Priest and Titov, 1996). If $\lambda_1 > 0$, these are directed away from the null point, and if $\lambda_1 < 0$, they are directed towards it. These field lines end (or begin) in sources called *spine sources*. If a null has two distinct spine sources, it is called "weethesis" 2003/7/11 page 10

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heterovertebraic; if both spines connect to the same source, the null is *homovertebraic*. These types are also known as *boundary* and *internal* nulls (respectively) in the literature (e.g., Longcope and Klapper, 2002).

Together, \mathbf{e}_2 and \mathbf{e}_3 define a *fan plane*. Points lying in this plane near to the null define field lines which form a *separatrix surface* (also called the *fan*) dividing space into regions of different *connectivity*: field lines on different sides of the surface either start from or end at different sources, in fact the spine sources of the null.

If λ_2 and λ_3 are positive, the fan field lines diverge from the null point; if the eigenvalues are negative, these field lines converge on the null. The null is called *positive* if λ_2 and λ_3 are both positive, or *negative* if both are negative. When all of the sources are located on a plane (the *photosphere*), there will be a population of nulls which lie in this plane, called *photospheric nulls*. A photospheric null point whose spine lies in the plane of the sources is described as *prone*, whereas a photospheric null with a spine directed vertically is called *upright*.

In a situation with flux balance, the field at a great distance from the sources is approximately dipolar. On a contour of sufficiently large diameter, the Kronecker-Poincaré index of the field χ will be two (Molodenskii and Syrovatskii, 1977). The Euler characteristic equation $M - c + m = \chi$ then holds in the photospheric plane. M is the number of potential maxima (see, for instance, Inverarity and Priest, 1999); m is the number of minima, and c is the number of saddle points. Saddle points of the potential correspond to prone nulls; maxima (respectively, minima) correspond either to positive (respectively, negative) sources or to positive (respectively, negative) upright nulls.

This allows us to relate the numbers of sources (S), prone nulls (n_p) and upright nulls (n_u) by the two-dimensional Euler characteristic,

$$S + n_u = n_p + 2,$$
 (1.10)

when the net flux in the source plane is zero. The properties of nulls in 3D space are governed by the 3D Euler characteristic,

$$S_{+} - n_{+} = S_{-} - n_{-}, \tag{1.11}$$

where S_{\pm} represents the number of positive or negative sources and n_{\pm} the number of positive or negative nulls. In both of these equations, flux balance is assumed: for an

unbalanced case, it is necessary to add a balancing source at a great distance and increase S, S_+ and S_- accordingly.

Separators are field lines which begin at one null point and end at another. They are the three-dimensional analogue of a two-dimensional X-point and are prime locations for reconnection (Greene, 1988; Lau and Finn, 1990; Priest and Titov, 1996; Galsgaard and Nordlund, 1997). Separators can also be seen as the boundary of four different regions of connectivity - although the two definitions aren't quite equivalent. An example will be discussed in Section 2.6.1 in the upright null state, where the eponymous upright null has both of its spines connectivity regions, also called *flux domains*. Such separators will be given the name *half separators* as opposed to *proper separators* which lie on the boundary of four regions.

Continuity arguments can be used to show that a separator connects two nulls if and only if the fan of one null is bounded in part by the spine of the other (as in Figure 1.3.3). The proof is given in Appendix A.

A useful tool in calculating even a fairly simple topology is the *domain graph* (Longcope, 2001). In this, each source S_i is represented by a node s_i on the graph; if any field lines connect two sources S_i and S_j then the corresponding nodes s_i and s_j are connected. In conjunction with knowledge about the number of nulls, it is possible to catalogue quite complex topologies with some confidence. The method for doing so is explained in Chapter 2.

Longcope and Klapper (2002) found a relationship between the number of flux domains (N_d) , separators (N_{nn}) , null points (N_0) and sources (S):

$$N_d = N_{nn} - N_0 + S, (1.12)$$

although this applies to the whole of space rather than to the coronal half-space. For a result in this region, we must differentiate between *photospheric domains*, which contain field lines which lie in the photosphere, and *purely coronal domains*, which do not. Making this distinction, we can modify the equation to:

$$N_{\phi d} + 2N_{cd} = 2N_{nn} - N_{\phi n} - 2N_{cn} + S, \qquad (1.13)$$

where $N_{\phi d}$ is the number of photospheric domains, N_{cd} the number of purely coronal



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Figure 1.1.1: Loop structures imaged by the TRACE satellite. Any document which mentions TRACE is legally required to include such a picture.

Feature	Depicted as	Colouring
Null point	Filled circle	Red [blue] if positive, [negative].
Flux source	Star	Red [blue] if positive, [negative].
Spine field line	Heavy solid line	Red [blue] if due to positive [negative] null.
Fan field line	Thin solid line	Red [blue] if due to positive [negative] null.
Separator field line	Heavy dashed line	Various, often magenta.

Table 1.1: Legend for all topology diagrams in this thesis.

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Figure 1.3.2: The local structure of a magnetic null. In one direction, the field lines cluster around an isolated field line known as the spine; perpendicular to this, the lines spread out in a fan plane. The field lines of this fan plane form a separatrix surface, which generally divides space into regions of different connectivity.

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Figure 1.3.3: Schematic diagram of a separator (dashed black line) joining two nulls (red and blue dots). Each separatrix (thin plane) is partly bounded by the spine (thick solid line) of the other. A proof of this is found in Appendix A.

domains, $N_{\phi n}$ the number of photospheric nulls, N_{cn} the number of coronal nulls and S the number of sources.

By changing the source strengths and positions of the sources, it is possible to force a change from one topological state to another - for instance by creating a pair of null points, or by allowing two separatrix surfaces to intersect, giving rise to a separator.

In this work, we will examine several different types of bifurcation, in two distinct classes:

- Local bifurcations in which the number of nulls changes.
- Global bifurcations in which the structure of the field changes.

1.4 Bifurcations

In this section, we look in detail at some of the elementary bifurcations considered in this thesis, although we will leave some of the new, more complicated bifurcations until Chapter 3.

1.4.1 Local bifurcations

A local bifurcation is one in which a pair of nulls is created or destroyed. There are two known simple examples, discussed by Brown and Priest (1999a) and Brown and Priest (2001): the local separator bifurcation and the local double-separator bifurcation.

Local separator bifurcation

The local separator bifurcation (LSB) was studied in detail, and modelled analytically, by Brown and Priest (1999a). During such a bifurcation, two null points either spontaneously appear or collide and annihilate each other. The three-dimensional Euler characteristic equation (Equation 1.11) insists that the two nulls be of opposite sign. If the bifurcation

takes place in the plane - which is more usual, although Chapters 3 and 4 discuss this further - then the two-dimensional Euler characteristic equation (Equation 1.10) forces one of the nulls to be positive and the other negative.

The process is illustrated in Figure 1.4.4. A second-order null appears out of nothing in the second frame; it then splits into two nulls. Eventually, the blue null will annihilate the black null in the reverse process, leaving only the red null.

Although we have yet to find a proof, it seems likely that a local separator bifurcation requires the black and red nulls (of the same type) to share exactly one of their spine sources. This is based only on the absence of a counter-example. It certainly appears to be always true.

Local double-separator bifurcation

The local double-separator bifurcation (LDSB) was analysed by Brown and Priest (2001), who provided an analytical model for it. In it, a null point becomes a third-order null before splitting into three first-order nulls. This type of bifurcation requires a high degree of symmetry, such as that provided by the photosphere, which provides a mirror corona for z < 0. It seems unlikely that an LDSB would take place anywhere other than on the photosphere, creating one *coronal null* (one lying above the photosphere) and a mirror image null below the photosphere. By symmetry, the coronal null and its mirror image must be of the same sign; the three-dimensional Euler characteristic equation 1.11 insists that both of these nulls be of the sign of the original photospheric null, and that the photospheric null change sign.

The process is illustrated in Figure 1.4.5. A single null becomes three, creating two new separators.

We believe this bifurcation requires at least two sources of both signs to take place. Again, we have no proof, although the counter-example would require an unlikely-looking topology, discussed in Appendix C.

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Figure 1.4.4: Local separator bifurcation. In the first frame (left), a single null point (black dot) exists. In the second frame (centre), a second-order null (purple dot) comes into existence. This splits into two nulls (red and blue) in the third frame. These two nulls are linked by a separator (purple dashed line). Thick and thin solid curves represent spine and fan field lines, respectively; the dashed black line is also a separator created by the bifurcation, but is not strictly part of it.



Figure 1.4.5: Local double-separator bifurcation. In the left-hand frame, a single null (red dot) exists; in the centre, it becomes a third-order null. On the right, the null has split into three: a red null above the photosphere; a blue null on the photosphere; and a pink null below the photosphere. The two new separators are marked by light and dark purple dashed lines.

1.4.2 Global bifurcations

Global bifurcations differ from local bifurcations in that null points are not created or destroyed. Instead, they involve a change in the global structure of the field - a realignment of separatrix surfaces and spine field lines, for example. There are four simple instances of global bifurcations: the global spine-fan bifurcation (Brown and Priest, 1999a), the global separator bifurcation (Brown and Priest, 1999a), the global separatrix quasi-bifurcation and the global spine quasi-bifurcation (Beveridge *et al.*, 2002).

Global spine-fan bifurcation

The global spine-fan bifurcation is discussed in Brown and Priest (1999a). It allows a spine field line connecting to one source and a separatrix connecting to another swap connectivities. This process is shown in Figure 1.4.6. The spine and fan involved in the bifurcation originally connect to different sources (left); the two approach, until the spine lies in the fan surface. At the moment of bifurcation (centre) the spine technically forms a separator because it connects two null points; however, this configuration is highly unstable. As the process continues, the spine passes through the fan to connect to a different source; likewise, the fan now connects to the source originally connected to the spine.

Global separator bifurcation

The global separator bifurcation, in which a separator is destroyed or created, is wellunderstood (Brown and Priest, 1999a). Figure 1.4.7 shows an example of this. On the left there are two separatrix domes intersecting in a separator. As the two domes move apart, the separator falls in height until, at the moment of bifurcation (middle), it reaches the plane and vanishes, to leave the detached topology (right).

Global separatrix quasi-bifurcation

In the global separatrix quasi-bifurcation, discussed in Beveridge *et al.* (2002), a separatrix grows infinitely large and wraps around to the other side of the configuration. The "weethesis" 2003/7/11 page 19

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Figure 1.4.6: Global spine-fan bifurcation. The red spine initially connects to the left of the configuration, and the blue fan connects to the right. The two approach each other until (centre) the red spine lies in the blue fan plane (hence the name 'spine-fan'). By this process, the fan and spine swap connectivities. The dotted black line is not a field line, but simply a reference line connecting the two nulls. This bifurcation requires two nulls of the same sign.



Figure 1.4.7: Global separator bifurcation. The intersecting separatrix surfaces approach each other (left), and the separator drops in height. At the point of bifurcation, the separator lies in the plane (centre) before vanishing (right); there are now two detached separatrix surfaces.



Figure 1.4.8: Global separatrix quasi-bifurcation. One of the separatrix domes (the blue one) grows in size (left) until it becomes a separatrix wall (centre) and eventually wraps around the other (bottom).

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Figure 1.4.9: Global spine quasi-bifurcation. One separatrix surface containing a spine (the blue one), grows until it forms a separatrix wall (centre) and eventually wraps around to the other side of the configuration (right).

process is shown in Figure 1.4.8. One separatrix dome grows progressively larger until it extends to infinity and becomes a separatrix 'wall'. The separatrix wall still divides the space into two distinct regions, but does not enclose either of them. After the bifurcation, the field lines connect again with the same source, but on the other side of the system, in such a way that the separatrix dome now encloses a different source.

We refer to this as a quasi-bifurcation because one of the features of the skeleton (in this case the separatrix surface) moves off to infinity, as opposed to regular bifurcations where the skeleton is altered within a bounded region. When this movement to infinity happens, there may be a change of topological state from one type to another (as in the change from an enclosed state to a nested state in the three-source case (Brown and Priest, 1999a)); or, as in the present case, there may be a change of handedness from one state to another distinct state of the same type. Here the left and right states in Figure 1.4.8 are indeed distinct because the separatrix domes enclose different sources. However, there is no regular bifurcation behaviour in any bounded region.

Global spine quasi-bifurcation

The global spine quasi-bifurcation (Figure 1.4.9) is effectively identical to the global separatrix quasi-bifurcation except that the separatrix involved contains the spine field line of the other null.

1.5 Outline

The aim of this thesis is to use the technique of Magnetic Charge Topology to examine certain configurations of the magnetic field in the solar corona. Some of these configurations are relatively simple, such as the four-source systems. Others, like the seven-source scenario or the Monte Carlo experiments are far more complicated. In some sense, the methods used to find, understand and communicate these, often Byzantine, structures are just as important as the mathematical results.

In the following chapter, we will consider the possible topologies due to a situation with four balanced sources. We begin by considering previous analysis undertaken in particular by Brown and Priest (1999a) on unbalanced three-source systems, and on balanced four-source scenarios by Gorbachev *et al.* (1988).

We then discuss a systematic method for finding which topologies are possible, before applying it first to a simple system of two bipoles. This corresponds to the fairly common solar occurrence of the emergence of a new bipole into an existing bipolar region. We find four distinct topologies are possible in this case, and produce a bifurcation diagram for this scenario.

We generalise the analysis to a less-restricted case with four balanced sources. We discover that three further topologies are possible. We conclude Chapter 2 with a discussion of the bifurcations between the various states, and a comparison to the unbalanced three-source catalogue of Brown and Priest (1999a).

In Chapter 3, the unexpected result that local bifurcations can take place outwith the source plane is discovered. Until now, it was tacitly assumed that local bifurcations could take place only in the same plane as the sources. While this is almost certainly true for the local double-separator bifurcation (Brown and Priest, 2001), which relies to a great degree on symmetry, we show that a local separator bifurcation can take place above the plane.

This can be achieved with as few as five unbalanced sources, although we go on to consider some seven-source configurations. We look in some detail at the bifurcation process which is relatively simple with five sources, but still involves four separators becoming five. With seven sources, it is possible for such a local bifurcation to have an additional global effect, adding two separators at some distance from the bifurcation. This is a pre"weethesis" 2003/7/11 page 22 cursor to a cartoon model of Magnetic Breakout in Chapter 4.

This model relies on a slightly simpler coronal bifurcation which involves only two separators. One of the coronal null points and one of the photospheric nulls then undergo a global spine-fan bifurcation which allows previously enclosed flux in a delta-sunspot configuration to connect to distant flux systems; this is the topological analogue to breakout.

In Chapter 5, a topological model is used to analyse the properties of elemental flux loops. These are defined as all of the flux joining two photospheric flux sources. We consider the end regions of a *superloop*, as considered by Longcope and van Ballegooijen (2002), made up of many elemental loops. Each of our end regions consists of 1000 sources arranged according to a planar poisson point process, with a specified flux imbalance, and a specified distribution of fluxes. It is possible to use a gradient map in conjunction with the Euler characteristic equations (Equations 1.10 and 1.11) to determine the fraction of photospheric nulls which are upright in a particular scenario.

We continue by finding the density and distribution of separators in a superloop, by overlaying pairs of end regions. There is a tendency for the separatrices of the prone nulls to form 'trunks', analogous to river valleys in a geographical map. We find there are approximately 18 separators for each source; this implies that each source connects to about 20 sources in the other end region. This leads to the conclusion that a typical elemental loop has a diameter of around 200km, agreeing with the estimate of Priest *et al.* (2002). We also find that the arrangement of separators is consistent with a concentration into clusters of about 130, most likely due to the tendency of separatrices to form trunks. This leads us to believe that many of the elemental loops will contain very little flux, while others will compensate by being much larger than this estimate.

We conclude with a discussion of our results and their significance for the world of solar physics.

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Chapter 2

Topologies due to four balanced sources

Knowing that we are looking for something we already have and are does not, of course, mean that the journey is unnecessary, only that there is a vast and sublime joke waiting to be discovered at its end.

Andrew Harvey, The Direct Path

2.1 Abstract

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The Sun's atmosphere contains many diverse phenomena that are dominated by the coronal magnetic field. To understand these phenomena it is helpful to determine first the structure of the magnetic field, i.e. the magnetic topology. In this chapter, we study the topological structure of the coronal magnetic field arising from the interaction of four magnetic point sources in flux balance. We find that seven distinct, topologically stable states are possible: four in the case where there are two positive and two negative sources, and three states when one source is of opposite sign to the other three.

We show by means of bifurcation diagrams how the magnetic configuration can change as the parameters are altered; we also examine the possible bifurcations between the states.

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In Section 2.2, we introduce the problem. We outline our assumptions and the model adopted in Section 2.3. In Section 2.4, we show the method we will use to catalogue the topologies. Section 2.6 details the different types of topology that can be created with this model, and Section 2.5 examines the bifurcations between them. We conclude with a discussion of our results.

The work in this chapter relating to two bipoles was published in Vol. 209 of Solar Physics, September 2002 (Beveridge *et al.*, 2002).

2.2 Introduction

An important long-term project is to categorise and study the different types of topology of the coronal magnetic field as a prerequisite for a full understanding of the mechanisms which control dynamic phenomena such as flares and loop structures.

In this chapter, our aim is to focus at first on the simplest class of complex topologies that occurs in practice in a solar active region, namely the field due to two dipoles, before extending the analysis to a more general balanced four-source case. This first scenario is of some importance, since it arises reasonably frequently, for instance, when a new bipole emerges into a pre-existing bipolar region.

We consider the magnetic *skeleton* of the field as described in Section 1.3.2. This consists of the positions of the sources and any null points along with their spine curves and fan, or separatrix surfaces, as well as any separators.

The arrangement of these structures determines their topology. We examine here the topologies due to a small number of discrete point sources in the photosphere, following for instance Gorbachev *et al.* (1988). They gave a preliminary treatment of four sources and found that a coronal null can exist in such a configuration, and that separators do not occur in every case. They also showed that a null line can exist in a non-co-linear configuration, but made no mention of stability. Their bifurcation analysis was also somewhat limited, since they concerned themselves with existence proofs rather than a quantitative analysis.

Further work on coronal nulls has been carried out by Inverarity and Priest (1999) and Brown and Priest (2001), who consider general solutions for such nulls and how they can

bifurcate out of the photosphere into the corona.

This study is similar to work undertaken by Priest *et al.* (1997) on two-source and simple three-source cases, and by Brown and Priest (1999a) who completely classified the three-source scenario. They found that eight topologies are possible in that case, and analysed the bifurcations between them.

They divide the scenarios into three classes: those with three sources of the same sign, those with two sources of one sign outweighing a single source of the other, and those with one source outweighing two sources of the opposite sign. Without loss of generality, we assume the majority of the sources are positive.

With three sources of the same sign, two topologies are possible (see Figure 2.2.1). In the divided state, two unconnected separator walls exist, dividing space into three flux domains. Each of these connects a source to a balancing source at infinity. In the triangular state, an upright null and an additional prone null exist. There are now three separatrix walls dividing space into three flux domains as before. These walls meet in the spine of the upright null. The separatrix surface of the upright null lies in the plane, and is bounded by the spines of the three prone nulls.

When two sources of the same sign outweigh one of the opposite sign, there are three possible topologies (Figure 2.2.2). Firstly, there is the nested state, in which both of the separatrix surfaces are domes. These do not touch, and one lies entirely inside the other. There are three regions of connectivity. Secondly, in the intersecting state (Figure 2.2.4), four regions of connectivity exist; one of the separatrix surfaces forms a dome, while the other is a wall which intersects it. Lastly, in the detached state (Figure 2.2.2), there are two disconnected surfaces. Again, one is a wall and the other a dome; there are three flux domains.

When the odd source outweighs the two sources of the same sign, there are also three possible topologies (Figure 2.2.3). In the separate state, there are two separatrix domes which meet at the negative source, allowing three flux domains. The enclosed state is quite similar, although one of the domes now encloses the other. Lastly, in the touching state, an upright null and an additional prone null exist. Both spines of the upright null connect to the odd source, and bound the separatrix of the new prone null. The separatrices of the two original prone nulls are now also bounded by this spine; a three-dimensional view of this more complicated topology can be seen in Figure 2.2.5.

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Figure 2.2.1: Possible topologies with three positive sources: left, the divided state; right, the triangular state. The stars represent sources and the dots null points; thick solid lines are spine field lines, thin solid lines are fan field lines, while dashed lines represent separators.



Figure 2.2.2: Possible topologies with two strong positive sources: left, the nested state; centre, the intersecting state, and right, the detached state.



Figure 2.2.3: Possible topologies with two weak positive sources: left, the separate state; centre, the touching state; right, the enclosed state. All topology pictures in this chapter follow the legend in Table 1.1.

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Figure 2.2.4: A typical three-source topology - the intersecting case. The red and blue crosses represent positive and negative sources, respectively; the large dots are null points. The dashed line is a separator, which is the line of intersection between two separatrix surfaces (containing the lighter solid field lines) which here form a dome and a wall. The thick solid lines are spine field lines.

2.3 Assumptions, model and method

As described in Sections 1.2 and 1.3, the coronal magnetic field is often considered to be force-free (since $\beta \ll 1$ and the coronal motions are much slower than the Alfvén speed). As we are studying the topology of the field, we will make the further assumption that the field is potential, for the sake of simplicity. Linear force-free fields are unlikely to have any different topological states, particularly for non-extreme values of α . The precise parameter values that produce changes between them will certainly differ depending on how far from potential the field is (Brown and Priest, 2000). This would introduce an extra set of parameters into the already complicated analysis presented here. The same is most likely true of non-linear force free fields where the photospheric flux patches are discrete.

Our aim is to produce diagrams to show where bifurcations occur in parameter space (*bifurcation diagrams*). To do this, we find the null points of the magnetic field at certain locations in parameter space, before calculating numerically the field's skeleton. We then classify the topology into one of the types found by the method described in the following section. To do this, we require a model and a parameterisation of the magnetic field.

We consider four flux sources situated in the photosphere. Included in this set-up is the fairly common scenario of two bipoles, which might model a new sunspot pair emerging into an existing sunspot region.

For a set of *n* discrete sources placed at \mathbf{r}_i with strengths ϵ_i (i = 1..., n), the field is given by Equation 1.8:

$$\mathbf{B}(\mathbf{r}) = \sum_{i=1}^{n} \frac{\epsilon(\mathbf{r} - \mathbf{r}_i)}{|\mathbf{r} - \mathbf{r}_i|^3}$$
(2.1)

We examine the case with n = 4. Without loss of generality, we can re-scale the geometry by choosing two of the source locations as $\mathbf{r_1} = (0, 0, 0)$ and $\mathbf{r_2} = (1, 0, 0)$. We can also re-scale the source strengths so that $\epsilon_1 = 1$. In general, then, we have four free coordinate parameters ($\mathbf{r_3}$ and $\mathbf{r_4}$), and two free strength parameters (ϵ_2 and ϵ_3 ; flux balance ensures that $\epsilon_4 = -(1 + \epsilon_2 + \epsilon_3)$). In other words, by re-scaling, we can reduce the twelve dimensional parameters of Equation 2.1 to just six dimensionless parameters. "weethesis" 2003/7/11 page 29 Our expression for $\mathbf{B}(\mathbf{r})$ is then

$$\mathbf{B}(\mathbf{r}) = \frac{(\mathbf{r})}{|\mathbf{r}|^3} + \frac{\epsilon_2(\mathbf{r} - \hat{\mathbf{x}})}{|\mathbf{r} - \hat{\mathbf{x}}|^3} + \frac{\epsilon_3(\mathbf{r} - \mathbf{r_3})}{|\mathbf{r} - \mathbf{r_3}|^3} + \frac{-(1 + \epsilon_2 + \epsilon_3)(\mathbf{r} - \mathbf{r_4})}{|\mathbf{r} - \mathbf{r_4}|^3}.$$
 (2.2)

Since six parameters is still too many to permit a comprehensive study, we decide to fix ϵ_2 and the position of $\mathbf{r_3}$ so as to reduce the number of parameters to three. For certain values of ϵ_3 we then vary the position of the fourth source $\mathbf{r_4}$ and find where the bifurcations occur.

To do this, though, we need to know which topologies are possible.

2.4 Domain graph method

We find the possible topologies by calculating which domain graphs (see Section 1.3) are allowable under the following rules:

- A positive source may only connect to negative sources and vice versa.
- The graph must be connected that is to say, any two sources are joined by at least one path.
- Multiple connections between two sources are not permitted.

This last restriction is a little contentious: in a situation with many sources, multiple connections are indeed permitted (Longcope, 2001). However, these are quite unlikely in scenarios with few sources.

In the four-source scenario, three domain graphs are possible, as shown in Figure 2.4.6: three sources of one sign all connect to a single source of the other; or if there are two sources of each polarity, either all possible connectivities occur or one is excluded.

These graphs correspond to three classes of topology, each with its own connectivity pattern. Within each class, the topology can change only by means of a bifurcation with no effect on connectivity: all of the elementary bifurcations described in Section 1.4 apart from the global separator bifurcation (in which a flux domain is created or destroyed) are possible, subject to their normal restrictions. For each class, it suffices to find a sample

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Figure 2.2.5: A complicated three-source topology - the touching case. The separatrices of all three prone nulls (red dots) are bounded by the blue spine; the separatrix of the central prone null is a bounded wall, while the other two are part-domes. The separatrix of the upright null (blue) is bounded by the three red spines, and stretches to infinity.



Figure 2.4.6: Possible domain graphs for four sources.

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Left: with three positive and one negative source, the only possibility is that the negative source connects to all three positive sources.

Centre and right: with two sources of each polarity, there are two possibilities; either each positive source connects to both negative sources and vice versa, or a negative source and a positive source are disconnected.

topology and consider how it can bifurcate. For instance, the third scenario, where three flux domains exist, is satisfied by the detached state (Figure 2.6.12).

Consider the possible bifurcations:

- Local separator: Impossible, since it requires at least three sources of same polarity.
- Local double-separator: Impossible, because it requires a separator.
- Global spine-fan: Impossible, since it requires two nulls of the same sign.
- Global separatrix quasi-bifurcation: Possible, as it changes to the nested state (Figure 2.6.13).
- Global spine quasi-bifurcation: Possible, but doesn't change the topology.

Repeating the analysis for the nested state, we find that these are the only two possible topologies for the third class.

Applying this method to the three classes gives us seven topologies, as described in Section 2.6. First, though, we will put these into context by means of bifurcation diagrams.

2.5 Bifurcation diagrams

Let us consider the arrangements of sources that produce the various topological states. We begin by fixing three sources and allowing a fourth, balancing source to move freely around the source plane; its co-ordinates are $(x_4, y_4, 0)$.

From each such configuration, we find the null points, and follow fan field lines from each of the nulls numerically. By analysing the connectivity of these field lines, it is possible to determine the topology for a given set of sources. In so doing, we find the parameters (x_4, y_4) where bifurcations occur and join them with smooth curves, as described in Appendix B.

If Figure 2.5.7, we analyse a balanced four-source case with three positive sources. We find that when the moving source is far from the fixed sources, the topology is invariably in the upright null state; closer in, the field adopts a separate or enclosed topology. The

local separator bifurcation (marked by a solid line) forms the boundary between these two regions.

Each enclosed region touches a source, and is bounded on either side by a global spine (dotted line) and a global separatrix (dashed line) quasi-bifurcation.

Lastly, the boundaries between the three separate regions, which occur when the fourth source is (in some sense) between the three others, are formed by the global spine-fan bifurcation.

In Figure 2.5.8, we do the same thing for a balanced four-source case with two positive and two negative sources. When the moving source is distant from the sources, or between then, the field is in the intersecting state. A global separator bifurcation (solid line) separates these regions from the nested and detached regions, which in turn are separated by a global separatrix quasi-bifurcation.

There is also a region in which the topology has a coronal null; this touches two of the nulls and is divided from the intersecting region by a local double-separator bifurcation marked by a dashed line.

There is a further global separatrix quasi-bifurcation line which surrounds the sources and divides one intersecting state from another. Lastly, a global spine quasi-bifurcation line (dash-dot-dotted line) passes through the source at the origin; outwith the intersecting region, this becomes a global separatrix quasi-bifurcation.

These two scenarios, between them, allow all seven permissible topologies, and all six permissible bifurcations, as described in Section 1.4.

The resulting bifurcation diagrams (Figures 2.5.7 and 2.5.8) are rather complicated and include six different types of bifurcation (namely, a global separator bifurcation, a global separator bifurcation, a local separator bifurcation, a local double-separator bifurcation, a global separatrix quasi-bifurcation and a global spine quasi-bifurcation). They allow changes of topology between several distinct states: in a situation with three sources of one sign and one of the other, three topologies (namely, the separate, enclosed and upright null states) are possible; if there are two sources of each polarity, then four states (the detached, nested, intersecting and coronal null states) are possible.

Calculating the bifurcation diagrams is made particularly difficult by the global separatrix
and the global spine quasi-bifurcations, in which parts of the skeleton move off to infinity and are not easily found by automatic computational algorithms.

2.6 Topologies

It is the different possible connectivities of the fan and spine field lines which define the different topologies of the overlying coronal magnetic configuration.

By using the Euler characteristic equations detailed in Section 1.3, we see from Equation 1.10 that there must be two more prone nulls than upright nulls. This implies that the number of photospheric nulls is even for a four-source setup. In most cases, there are two photospheric nulls, but a case with four photospheric nulls (three prone and one upright) does exist (the upright null state).

In a situation with three sources of one polarity (say, positive) and one of the other, the three-dimensional Euler characteristic (Equation 1.11) dictates that there be two more positive nulls than negative; if there are two sources of each polarity, there must be as many positive nulls as negative.

2.6.1 Three positive sources and one negative

Three topologies are possible in this class: the separate state, the enclosed state and the upright null state.

Separate and enclosed states

In both of these cases (Figures 2.6.9 and 2.6.10), two separatrix domes exist, each surrounding one of the positive sources and connecting to the negative source. In the separate state, the two domes are independent; in the enclosed state, one of the domes surrounds the other.

These are very similar to the three-source separate and enclosed states; the only difference is that, where a spine in the three-source scenario connected to infinity, here it connects

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Figure 2.5.7: Bifurcation diagram for three positive sources. Three sources are fixed at $\mathbf{x}_1 = (0,0)$, $\mathbf{x}_2 = (1,0)$ and $\mathbf{x}_3 = (0.7,0.6)$, with strengths $\epsilon_1 = 1$, $\epsilon_2 = 0.6$ and $\epsilon_3 = 0.5$. A fourth balancing source with strength $\epsilon_4 = -2.1$ is allowed to move freely. The different regions on the plot indicate where the fourth source must be placed to give these topologies. The lines represent bifurcations: the solid line represents a local separator bifurcation, and the dotted line a global spine-fan bifurcation. The dashed lines are global separatrix quasi-bifurcations, while the dot-dashed lines represent global spine quasi-bifurcations. The topological states are represented by letters: U is upright, E enclosed and S separate.

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Figure 2.5.8: Bifurcation diagram for a bipolar case. Three sources are fixed at $\mathbf{x}_1 = (0,0)$, $\mathbf{x}_2 = (1,0)$ and $\mathbf{x}_3 = (0.7, 0.6)$, with strengths $\epsilon_1 = 1$, $\epsilon_2 = 0.6$ and $\epsilon_3 = -0.6$. A fourth balancing source with strength $\epsilon_4 = -1.0$ is allowed to move freely. The different regions on the plot indicate where the fourth source must be placed to give these topologies. The lines represent bifurcations: the solid line represents a global separator bifurcation, and the dashed line a local double-separator bifurcation. The dotted line represents a global separatrix quasi-bifurcation, and the dash-double-dotted line a global spine quasi-bifurcation. The four possible topologies are denoted by letters I (intersecting), D (detached), N (nested) and CN (coronal null).

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to a third positive source. These two cases also subsume the divided state; whereas both separatrices in that state connect to infinity, here, both connect to a source.

Upright null state

In the upright null state (Figure 2.6.11), three prone nulls and an upright null exist. The fan of the upright null lies in the plane and is bounded by the spines of the prone nulls; its spine connects to the negative source above and below the plane. The spine bounds all of the separatrix surfaces from the prone nulls, two of which form part-domes, and the other a bounded wall.

This is similar to both the touching and triangular states in the three-source scenario; in both of those cases, either separatrices or spines were connected to infinity. Here, they connect to a source.

2.6.2 Two positive sources and two negative: three flux domains

As previously discussed, only two topologies are possible in this case: the detached and nested states.

Detached and nested states.

If all of the fan field lines from one null connect to one source and all those in the other fan connect to the other, the state is either detached (Figure 2.6.12) or nested (Figure 2.6.13). The only difference between the two states is that in the nested state, one of the separatrix domes envelops the other, while the detached state is topologically identical to two independent and unbalanced pairs of sources.

These two states are very similar to the three-source detached and nested states; in those situations, one of the separatrix surfaces connected to infinity; here, the surfaces connect to a source.

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Figure 2.6.9: Separate state. Two independent separatrix domes meet only at the negative source.



Figure 2.6.10: Enclosed state. Two separatrix domes meet only at the negative source; one is entirely enclosed by the other.



Figure 2.6.11: Upright null state. There are three separatrix surfaces from positive nulls: two form part-domes bounded by the spine of the upright null, while the other is a wall, also bounded by the blue spine. The separatrix of the negative null lies entirely in the plane and is bounded by the spine field lines of the positive nulls.

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Figure 2.6.12: Detached state. The two separatrix domes do not intersect. There is no separator and only three regions of connectivity.



Figure 2.6.13: Nested state. One separatrix dome surrounds the other. There are three regions of connectivity and no separator exists. These are schematic plots; in practice, both separatrix domes are often much larger and are far from circular.

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2.6.3 Two positive sources and two negative: four flux domains

In this scenario, two topologies are possible: the intersecting state and the coronal null state.

Intersecting state

If the fan field lines for a null in the plane connect to different sources, and the nulls are of different sign, then we have the intersecting state (Figure 2.6.14). The fans of the two nulls here form two separatrix domes, which intersect in a separator field line.

This is quite similar to the three-source intersecting state; there, the separatrix wall connected to infinity; here, it connects to a fourth source.

Coronal null state

Finally, if both nulls are of the same sign, a further two nulls of the opposite sign are required to satisfy the three-dimensional Euler characteristic. Because of the symmetry in the plane, one must be above the photosphere (i.e. a coronal null) and the other in the region z < 0, considered only for the mathematics; for the physical interpretation, this region is disregarded. This is the coronal null state (Figure 2.6.15.)

This state has no three-source analogue; in fact, it can be shown that coronal nulls exist only in highly unstable null rings in the unbalanced three source case; see Theorem 2 in Appendix A for a proof.

2.7 Discussion

In reality, the solar surface contains many thousands of flux 'sources' in the form of sunspots, ephemeral regions, network elements and intense flux tubes, which are constantly appearing, fragmenting, merging, cancelling and disappearing. The overlying coronal magnetic field has therefore an incredibly complex nature.

However, studying simpler topologies due to three or four sources is important, since

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Figure 2.6.14: Intersecting state, as a projection in the plane (left) and in three dimensions (right) produced by two positive sources (pink stars) and negative sources (red stars). The fan of each null (heavy dot) defines a separatrix surface (thin solid lines). In this case, the separatrices form domes which intersect in a separator (dashed line). There are four distinct regions of connectivity.



Figure 2.6.15: Coronal null state. There are four regions of different connectivity and two separators, each of which is a field line joining the coronal null to a null in the photospheric plane.

these act as building blocks for the whole corona. So far, a complete study of the topology of three sources has been undertaken (Brown and Priest, 1999a), and an analysis of the two-bipole case made by Beveridge *et al.* (2002). An exhaustive study of the topology due to four balanced sources is far more difficult to complete since it contains two more parameters, namely the position co-ordinates of the fourth source. (If there is a flux imbalance, a third extra parameter, namely the strength of the imbalance, would be included.) Until now, only a cursory analysis has been reported of a few special cases with four sources.

Here we catalogue all of the possible topological states due to four balanced sources, and provide a method to extend the analysis to greater numbers of sources. We also find where bifurcations between the states take place.

It is interesting to compare the results of the balanced four-source case with those of the unbalanced three-source case obtained by Brown and Priest (1999a). Their analysis centred on one source with strength 1, and two having strength ϵ . Examining the range $\epsilon < -0.5$ (so that a source added to give flux balance would have to be positive, giving two sources of either polarity), they find three types of topology: the (three-source) nested, intersecting and detached states. We find analogues to all of these and, in addition, a state with a coronal null. In the Brown and Priest analysis, it seems there is no bifurcation between the nested and detached states. This is because there is no three-source analogue to the global separatrix and global spine quasi-bifurcations. The assumed balancing source at infinity in the three-source case prevents the infinite growth of the separatrix domes and spine loops.

In the range $-0.5 < \epsilon < 0$, so that the balancing source would have to be negative (giving three sources of the same polarity, and one of the other), they again find three topological scenarios: the separate, enclosed and touching states. Again, we find analogues to all three.

Finally, they dealt also with three sources of the same sign (with a balancing source of the opposite sign), finding two topologies: the divided and the triangular states. These are simply disguised versions of the separate, enclosed and touching states.

We expect that extending the analysis to force-free fields or changing the values of our fixed parameters - the positions of the three central sources and the strengths of two of them - would change the size and shape of the regions produced, but is highly unlikely to

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produce any fundamentally different topologies or bifurcations.

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Although understanding these topologies is an important task in its own right, it will be interesting in the future to undertake numerical MHD experiments on various bifurcations that we have identified in order to determine their dynamical consequences for the Sun's atmosphere.

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Chapter 3

Genesis of coronal null points

Thirty spokes are made one by holes in a hub By vacancies joining them for a wheel's use The use of clay in moulding pitchers Comes from the hollow of its absence; Doors, windows in a house, Are used for their emptiness: Thus we are helped by what is not To use what is. Lao Tzu, *Tao te ching* (Trans. Witter Brynner)

Abstract

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Coronal null points are among magnetic topology's most significant features. They are important locations for reconnection and are believed to play a substantial role in the triggering of solar flares.

Yet little is known about the creation and destruction of coronal nulls. Until recently, the only known mechanism for such creation and destruction was local double-separator bifurcation, which takes place in the photosphere. In this chapter, we show that there exist other mechanisms which create or destroy nulls in the corona, well above the photosphere.

Two examples will be presented to illustrate a new, coronal local separator bifurcation.

In the first example, with five unbalanced sources, we show that such a bifurcation can occur. The second, with seven unbalanced sources, shows that comparatively large numbers of nulls can be created in this way in a reasonably realistic scenario.

We analyse these new bifurcations, in preparation for the following chapter which will deal with the Magnetic Breakout Model for flares.

3.1 Introduction

One of the earliest papers to mention coronal null points was Molodenskii and Syrovatskii (1977). They offered a 'natural' method for finding coronal nulls which, in essence, involved finding the intersection of curves satisfying $B_x = B_y = 0$ with the surface $B_z = 0$, where B_x , B_y and B_z are the x-, y- and z-components of the magnetic field **B**.

The first real consideration of coronal nulls, though, was in Gorbachev *et al.* (1988), who examined the balanced four-source case and spoke of null points as possible triggers for solar flares.

Inverarity and Priest (1999) considered a scenario in which a single positive source was surrounded by positive sources on a hexagonal network. They found at most one coronal null, even with large numbers of positive sources. We explain why this should be so in Section 3.6.

Questions about the creation of coronal nulls began to be answered only when the careful analysis of small number of sources was undertaken. Brown and Priest (2001) found that nulls could be created in an unbalanced four-source case through a local double-separator bifurcation, as described in Section 1.4, and further detailed in the following section.

There the story appears to have ended as far as MCT is concerned: more recent work, such as Schrijver and Title (2002) and Longcope *et al.* (2003), concentrated on the density of coronal null points above a photosphere containing many flux sources, although the latter did hint (in passing) that local bifurcations might be possible in the corona.

We will be considering in the following sections extensions of Brown and Priest's work, by adding a second ring of sources to the first. In each of these we will attempt to un-

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derstand the nature of the bifurcations, as well as analysing them somewhat in terms of $B_x = B_y = 0$ curves and $B_z = 0$ surfaces, before concluding with a discussion.

3.2 Four unbalanced sources: the Brown and Priest case

We begin by considering the work of Brown and Priest (2001) from a topological standpoint. They surround a positive source with three negative sources, such that the positive source is outweighed by the others, as in Figure 3.2.1.

The skeleton of this configuration is fairly simple: the separatrix of the coronal null forms a dome based in the ring of photospheric spines, while its own spine connects to the central, positive source and to the balancing source at infinity. The separatrices of the photospheric, negative nulls form walls which meet in the coronal null's spine, dividing the outlying volume into three segments. There are three separators, each linking a photospheric null to the coronal null.

Moving the central source away from the centre, they find that the coronal null falls in height towards one of the the photospheric nulls, and eventually coalesces with this and its image below the corona, leaving one photospheric null (of the same sign as the original coronal null) in its place.

Naturally, this alters the skeleton. The positive null now lies in the photosphere; its spine still connects to the central positive source and to the balancing source, but it now lies in the source place as well. Its separatrix is still a dome bounded by the spines of the two remaining negative nulls, whose separatrices still form walls bounded by the positive spine. Now, though, there are only two separators.

It is instructive to examine the surfaces where various magnetic field components are equal to zero (*zero surfaces*), as plotted in Figure 3.2.3. In this configuration, we have disturbed the symmetry to avoid structurally unstable effects. Surfaces where $B_x = 0$ and $B_y = 0$ pass through each source and each null point. In the source plane, symmetry considerations imply that $B_z = 0$ everywhere, so null points occur where lines of $B_x = 0$ and $B_y = 0$ (in the z = 0 plane) cross. For z > 0, we must consider where the surfaces of $B_x = 0$, $B_y = 0$ and $B_z = 0$ intersect.

In Figure 3.2.3, the $B_y = 0$ surfaces (dashed lines) form a dome (upper) and a wall

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Figure 3.2.1: Brown and Priest's four-source configuration in two (left) and three (right) dimensions. The central, positive source (red star) is outweighed by the three negative sources (blue stars) surrounding it. A coronal null (red dot) exists).



Figure 3.2.2: The configuration after a positive source has been perturbed and a local doubleseparator bifurcation has occurred. The positive null now lies in the photosphere and there is no coronal null.

(lower); the $B_x = 0$ surfaces (dotted lines) do likewise, with a dome to the right and a wall to the left. It is clear that in this case, each null point is linked to exactly one source by a three-dimensional curve above the plane on which $B_x = B_y = 0$ (a zero curve). The null near (0, -1), for instance, is linked in this manner to the source near (0.8, -0.5). It is not the case that every source is linked to a null; the source near (-0.8, -0.5) is a counter-example, as would be the balancing source at a great distance.

Of particular note is the zero curve between the null near (0.8, 0.5) and the central source. The two end-points of this curve lie on different sides of the $B_z = 0$ surface and hence the B_z component along the line must vanish somewhere on it. This point is the coronal null.

If the central source is moved, for example, to the right, the $B_z = 0$ surface also changes shape and, at a critical value, coincides with the null and eventually encloses it. After this, there is no further link crossing the zero surface, so there is no coronal null. So what has happened?

Simply, the null has bifurcated into the photosphere by means of the local double-separator bifurcation as previously described in Section 1.4.1. The process is depicted in Figure 3.2.4. In fact, this provides an intuitive means for understanding the local double-separator bifurcation as a zero curve passing through the intersection of two zero surfaces of the same component. It seems unlikely that this component would ever be anything other than B_z , except in highly symmetric cases.

3.3 Five unbalanced sources: a coronal bifurcation

3.3.1 The double coronal null case

Consider a configuration set up as follows: a positive central source is flanked by two nearby negative sources. Perpendicular to this, and at a greater distance, two more positive sources are placed as shown in Figure 3.3.5. A sixth, balancing source is included at a great distance.

More precisely, the sources are located at:

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Figure 3.2.3: Zero surfaces of the magnetic field for an unbalanced four-source case. The purple dome represents the $B_z = 0$ surface, while the green lines are curves upon which $B_x = B_y = 0$. The coronal null point (dot) occurs when a zero-curve and zero-surface intersect.



Figure 3.2.4: Schematic picture of local double-separator bifurcation. In the left-hand picture, the curve of $B_x = B_y = 0$ (dotted) intersects $B_z = 0$ surfaces in three points (dots). As the curve moves left, these nulls coalesce as a third-order null (centre) before leaving just one first-order null (right).

Source	Position	Strength
S_0^+	(0,0)	ϵ
S_1^-	(1,0)	-1
S_2^+	(0, -3)	+1
S_3^-	(-1, 0)	-1
S_4^+	(0,3)	+1
S_{∞}^{-}	far distant	$-\epsilon$

In this unbalanced case, there are four photospheric null points. Their exact location depends on ϵ , but they will lie somewhere near $(\pm 1, 0)$ and $(0, \pm 5)$. For ϵ smaller than some critical value ϵ_0 , two coronal null points exist on the *z*-axis. This topology is depicted in Figure 3.3.6 and described in Table 3.1.

Two domes (each comprising two separator surfaces) prevent any flux linking S_0^+ and S_{∞}^- . The separatrices of the coronal nulls divide the flux from either of these sources between the two remaining photospheric sources. There are five separators: each of the photospheric nulls is connected to the coronal null of opposite sign, and the two coronal nulls are also linked by a separator.

Again, it is worth examining the zero surfaces of this configuration (in fact, we displace the central source slightly to break the symmetry). These are shown in Figure 3.3.7.

The null points N_2^+ and N_4^+ (left and right) are clearly linked to the sources S_2^+ and S_4^+ , respectively. The linkage of the other two nulls is less clear: the detail of the centre shows that N_1^- (top) is linked to the central source; the combination of the pictures makes it apparent that N_3^- connects to S_3^- , and that this link appears to cross the $B_z = 0$ surface twice, giving two coronal nulls.

The picture alone is insufficient to conclude that coronal nulls exist, because it is possible that the zero curve simply loops over the zero surface without crossing it. However, it suffices to examine the B_z component along this particular $B_x = B_y = 0$ curve to determine whether coronal nulls exist.

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Figure 3.3.5: Photospheric skeleton of an unbalanced five-source case.



Figure 3.3.6: Five-source configuration with two coronal nulls (blue and red dots). There are five separators.

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Null	Position	Spine	Separatrix
		connections	
N_1^-	(0,5)	S_1^- and S_∞^-	Forms half-dome bounded by spine of CN^+
N_2^+	(-1, 0)	S_2^+ and S_0^+	Forms half-dome bounded by spine of CN^-
N_3^-	(0, -5)	S_3^- and S_∞^-	Forms half-dome bounded by spine of CN^+
N_4^+	(1, 0)	S_4^+ and S_0^+	Forms half-dome bounded by spine of CN^-
CN^-	(0,0,z)	S_1^- and S_3^-	Forms wall bounded by source plane and
			spine of CN^+
CN^+	$(0, 0, z_+)$	S_2^+ and S_4^+	Forms wall bounded by spine of CN^- and
			reaching to balancing source.

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Table 3.1: Separatrix surfaces of five-source, two-null case.



Figure 3.3.7: Zero surfaces for five-source, two-null case with $\epsilon = 0.7$. The left-hand picture shows the surfaces globally, while the right-hand picture shows a detail of the centre.

3.3.2 The four-separator case

As ϵ increases, the two coronal nulls approach and eventually annihilate; the bifurcation process will be examined in the following section. For now we will simply examine the topology.

The photospheric topology remains unchanged; there are four photospheric nulls, whose spines are also unaffected. The separatrices do change, however, as depicted in Figure 3.3.8 and described in Table 3.2

The separatrix surfaces that previously defined the outer dome now form two distinct domes that meet in the spine field lines running across the middle of the configuration. By contrast, the separatrices that formed the inner dome have become walls instead.

The combination of these two changes allows magnetic flux to link the central source S_0^+ to the balancing source S_{∞}^- , as shown in the connectivity diagram Figure 3.3.9. There are now four separators, with each positive null connected to each negative null.

The zero surfaces diagram, Figure 3.3.10, shows that there are now two $B_z = 0$ surfaces and, as one would expect in a case without coronal nulls, none of the $B_x = B_y = 0$ curves cross either of them.

3.3.3 Bifurcation behaviour

Before we discuss the fairly complicated behaviour of this bifurcation, it is worthwhile to examine the local separator bifurcation in the plane by way of comparison. (Through the rest of this chapter, all local bifurcations are described in the sense that null points are created. It is, of course, possible for them to work in reverse, destroying null points.)

This behaviour is shown in Figure 3.3.11. There are initially two separatrix surfaces which move closer to each other. At some critical point, they meet and form a second-order null point. This then splits into two nulls; the (vertical) spine of one constitutes the boundary of the two original fans and the fan of the other new null. The fan of the upright null is bounded by the spine of the new prone null and, (not pictured) the spines of the two original nulls. In this instance there are three new separators where none existed before.

The bifurcation behaviour in the five-source case is somewhat similar to this, although it

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Figure 3.3.8: A five-source configuration with no coronal null.

Null	Separatrix (before)	Separatrix (after)
N_1^-	Half-dome bounded by spine of CN^+	Dome bounded by spines of N_2^+ and N_4^+
N_2^+	Half-dome bounded by spine of CN^-	Wall bounded by spines of N_1^- and N_3^-
N_3^-	Half-dome bounded by spine of CN^+	Dome bounded by spines of N_2^+ and N_4^+
N_4^+	Half-dome bounded by spine of CN^-	Wall bounded by spines of N_1^- and N_3^-

Table 3.2: Separatrix surfaces for five-source, four-separator case.

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Figure 3.3.9: Change in connectivity between the two-coronal-null state (left) and the no-coronal-null state (right). The bifurcation allows flux to connect the central source S_0^+ to the distant balancing source S_{∞}^+ .



Figure 3.3.10: Zero surfaces for five-source, no-null case with $\epsilon = 1.3$.

now requires four separatrix surfaces. Both pairs of surfaces buckle towards each other, and form a second-order null where they meet. This null then splits into two, each new spine bounding two of the original fans and the fan due to the other new null.

The effect in this case, as we hinted before (Figure 3.3.9), is to allow or forbid connectivity between S_0^+ and S_{∞}^- . If the bifurcation destroys two nulls, it allows flux that was previously contained by the separatrix domes to 'break out' and connect to a distant flux system.

In terms of separators, as shown in Figure 3.3.13, the four separators that originally linked the four photospheric nulls in a loop have now been redirected by way of the two new nulls. There are now five separators: four connect photospheric nulls to coronal nulls, and the other connects the two coronal nulls.

3.4 Seven sources: more coronal bifurcations

A more complicated and more interesting case is a direct extension of Brown and Priest's example. In this scenario, we surround their four-source case with three sources of the same sign as the central source, as shown in the following table and in Figure 3.4.14.

Source	Position	Strength
S_0^+	(δ_x,δ_y)	ϵ
S_1^-	$(\sqrt{3}/2, 0.5)$	-1
S_2^+	$(3\sqrt{3}/2, -1.5)$	+3
S_3^-	(0, -1)	-1
S_4^+	$(-3\sqrt{3}/2, -1.5)$	+3
S_5^-	$(-\sqrt{3}/2, 0.5)$	-1
S_6^+	(0,3)	+3
S_{∞}^{-}	far distant	$-(6+\epsilon)$

It is necessary to offset the central source a small distance (δ_x, δ_y) from the origin to avoid the spines of two coronal nulls coinciding. This configuration is highly dependent on the symmetry of the sources and hence topologically unstable.

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Figure 3.3.11: Bifurcation behaviour for local separator bifurcation. Two separatrix surfaces (blue and red) move close to each other (left and centre left). Eventually they meet and form a second-order null (green dot, centre right) which then splits into two (right).



Figure 3.3.12: Bifurcation behaviour for coronal local separator bifurcation. Four separatrix surfaces (blue, red, maroon and dark green) move close to each other (left and centre-left). Eventually they meet and form a second-order null (black dot, centre right) which then splits into two (right).

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Figure 3.3.13: Separator changes for five-source bifurcation. Each edge separator is redirected via the two new coronal nulls.



Figure 3.4.14: Photospheric topology for seven-source state. There are six photospheric nulls, three of each sign.

3.4.1 Two coronal null case

This topology is shown in Figure 3.4.1. There are six photospheric nulls. Each of them has spines connecting to adjacent nulls of the same sign, forming an inner ring of negative spines and an outer ring of positive spines. Their separatrix surfaces are walls or half-domes bounded by the spines of coronal nulls.

For small values of ϵ , there are two coronal null points close to the z-axis. The lower, positive null has a spine connecting to the central source S_0^+ and to one of the outer, positive sources; its separatrix forms an inner dome bounded by the ring of negative spines.

The upper, negative null has spine field lines connecting to the balancing source S_{∞}^{-} and to one of the inner, negative sources. Its separatrix forms an outer dome, bounded by the ring of positive spines.

There are eight separators: each of the coronal nulls is connected to the three photospheric nulls of opposite sign; additionally, the separatrices of N_1^+ and N_2^- intersect, as do those of N_5^+ and N_4^- .

This is summarised in Table 3.3.

3.5 Four coronal nulls

As ϵ increases, two new coronal nulls are created (Figure 3.5.16). This leads to a yet more complicated topology.

There are now two positive coronal nulls, CN_6^+ as before and CN_4^+ , the spines of which connect to S_0^+ and S_4^+ . Their separatrices together form a wine-bottle shape, whose base coincides with the ring of negative spines. The two parts of the bottle are bounded on either side by the spines of the two negative coronal nulls CN_3^- and CN_5^- . These spines connect S_3^- (respectively, S_5^-) to the balancing source S_∞^- .

The separatrices of the negative coronal nulls now form a torus with its hole at S_0^+ and its outer edge on the ring of positive spines. The boundaries between the two separatrices are the spines of the two positive coronal nulls, which form arches.

The separatrices of photospheric nulls N_4^- and N_5^+ have also changed. The separatrix of

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Figure 3.4.15: Seven source, two null state. Left, view from the left; right, a view from the right.

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Null	Approx.	Spine	Fan description
	Position	connections	
N_1^+	(2.5, 1.5)	S_6^+ and S_2^+	Wall connecting to
			$S_1^-, N_2^-, S_3^-, CN_3^-$ and S_∞^-
N_2^-	(0.8, -0.5)	S_1^- and S_3^-	Part-dome connecting to
			$S_0^+, CN_6^+, S_6^+, N_1^+$ and S_2^+
N_3^+	(0, -3)	S_2^+ and S_4^+	Wall connecting to
			S_3^-,CN_3^- and S_∞^-
N_4^-	(-0.8, -0.5)	S_3^- and S_5^-	Part-dome connecting to
			$S_0^+, CN_6^+, S_6^+, N_5^+$ and S_4^+
N_5^+	(-2.5, 1.5)	S_4^+ and S_6^+	Wall connecting to
			$S_5^-, N_4^-, S_3^-, CN_3^-$ and S_∞^-
N_6^-	(0,1)	S_5^- and S_1^-	Wall connecting to
			S_0^+, CN_6^+ and S_6^+
CN_3^-	$(0,0,z_+)$	S_3^- and S_∞^-	Dome connecting to
			$N_1^+, S_2^+, N_3^+, S_4^+, N_5^+$, and S_6^+
CN_6^+	(0,0,z)	S_0^+ and S_6^+	Dome connecting to
			$S_1^-, N_2^-, S_3^-, N_4^-, S_5^-$, and N_6^-

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Table 3.3: Separatrix details for seven-source, two-null case.

 N_4^- is now bounded by the spine of CN_4^+ to form a wall; that of N_5^+ remains a wall, but is now bounded by the spine of CN_5^+ .

The most significant changes, though, are in terms of connectivity (there is now flux connecting S_0^+ to S_∞^-) and in terms of separators. Previously, no separator connected the coronal nulls; now, there is a separator ring connecting CN_3^- to CN_4^+ to CN_5^- to CN_6^+ and back to CN_3^- , a total of four coronal separators. Meanwhile, the separator that connected N_4^- to N_5^+ has vanished, as have those connecting N_4^- to CN_6^+ and N_5^+ to CN_3^- ; in their place, N_4^- and N_5^+ are connected to CN_4^+ and CN_5^- (respectively) by new separators, of which there are now eleven.

This new configuration is summarised in Table 3.4.

3.5.1 Six coronal nulls

Increasing ϵ still further allows a second bifurcation to take place, creating two more coronal nulls, CN_1^- and CN_2^+ . The spines and separatrices of these nulls, and of the photospheric nulls below them, are (effectively) mirror images of CN_5^- and CN_4^+ , respectively. The six coronal nulls are linked by a loop of separators, and each photospheric null is linked to exactly one coronal null, giving a total of twelve separators. The topology is summarised in Table 3.5 and depicted in Figure 3.5

In this topology, S_6^+ and S_3^- are no longer linked by magnetic flux. The changes in connectivity engendered by the bifurcations are shown in Figure 3.5.23.

Figure 3.5.19 shows the zero-surfaces in the source plane for cases with two and with six nulls. Qualitatively, the two are identical, although the number of nulls is patently different. Does this spell the end of our cherished zero-surface method for finding nulls?

Not in the least. The *xy*-zero lines still cross the *z*-zero surfaces; they simply do so in some cases twice above the plane. To find all coronal nulls, we must follow the zero lines from each photospheric null to a source, noting if and where the sign of B_z changes, as shown in Figure 3.5.20.

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CN6+ CN3-₩^{S3-} **★**^{S0+} S6+ N6 N3+ ⋇ N2-S1-**★**_{S2+} N1+ CN3-CN6+ CN4+ CN5-9 N6-₩_{S0+} **₭** 86+ ★ N3+ S3-S5-N4-N5+ ¥ 54+

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Figure 3.5.16: Seven source, four null state. Left, view from the left with the two new nulls; right, a view from the right.

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Null	Approx.	Spine	Fan description
	Position	connections	
N_1^+	(2.5, 1.5)	S_6^+ and S_2^+	Wall connecting to
			S_1^-,N_2^-,S_3^-,CN_3^- and S_∞^-
N_2^-	(0.8, -0.5)	S_1^- and S_3^-	Part-dome connecting to
			$S_0^+, CN_6^+, S_6^+, N_1^+$ and S_2^+
N_3^+	(0, -3)	S_2^+ and S_4^+	Wall connecting to
			S_3^-, CN_3^- and S_∞^-
N_4^-	(-0.8, -0.5)	S_3^- and S_5^-	Wall connecting to
			S_0^+, CN_4^+ , and S_4^+
N_5^+	(-2.5, 1.5)	S_4^+ and S_6^+	Wall connecting to
			S_5^-, CN_5^- and S_∞^-
N_6^-	(0,1)	S_5^- and S_1^-	Wall connecting to
			S_0^+, CN_6^+ and S_6^+
CN_3^-	$(0,0,z_3)$	S_3^- and S_∞^-	Part-dome (torus) connecting to
			$CN_6^+, S_6^+, N_1^+, S_2^+, N_3^+, S_4^+, CN_4^+, \text{ and } S_0^+$
CN_4^+	$(0,0,z_4)$	S_0^+ and S_4^+	Wall (bottle) connecting to
			$S_3^-, N_4^-, S_5^-, CN_5^-, S_\infty^-$ and CN_3^-
CN_5^-	$(0,0,z_5)$	S_5^- and S_∞^-	Part-dome (torus) connecting to
			$S_0^+, CN_6^+, S_6^+, N_5^+, S_4^+$ and CN_4^+
CN_6^+	$(0,0,z_6)$	S_0^+ and S_6^+	Dome connecting to
			$S_1^-, N_2^-, S_3^-, CN_3^-, S_\infty^-, CN_5^-, S_5^-$ and N_6^-

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Table 3.4: Separatrix details for seven-source, four-null case.

CN6+ CN3-9 CN2+ CN1 Ć) N6-____**≯** N3+ N2-N1+ ***** _{S2+} CN3-CN6+ CN4+ CN5 N6-₩₅₀₊ ≯ \$6+ N3+ S3-S5-N4-N5+ ***** S4+

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Figure 3.5.17: Seven source, six null state. Left, view from the left; right, a view from the right with two new nulls.

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Null	Approx.	Spine	Fan description
	Position	connections	
N_1^+	(2.5, 1.5)	S_6^+ and S_2^+	Wall connecting to
			S_1^-, CN_1^- and S_∞^-
N_2^-	(0.8, -0.5)	S_1^- and S_3^-	Wall connecting to
			S_0^+, CN_2^+ and S_2^+
N_3^+	(0, -3)	S_2^+ and S_4^+	Wall connecting to
			S_3^-, CN_3^- and S_∞^-
N_4^-	(-0.8, -0.5)	S_3^- and S_5^-	Wall connecting to
			S_0^+, CN_4^+ , and S_4^+
N_5^+	(-2.5, 1.5)	S_4^+ and S_6^+	Wall connecting to
			S_5^-, CN_5^- and S_∞^-
N_6^-	(0,1)	S_5^- and S_1^-	Wall connecting to
			S_0^+, CN_6^+ and S_6^+
CN_1^-	$(0,0,z_1)$	S_1^- and S_∞^-	Part-dome (torus) connecting to
			$N_1^+, S_2^+, CN_2^+, S_0^+, CN_6^+$ and S_6^+
CN_2^+	$(0,0,z_2)$	S_0^+ and S_2^+	Wall (bottle) connecting to
			$N_2^-, S_3^-, CN_3^-, S_\infty^-, CN_1^-$ and S_1^-
CN_3^-	$(0,0,z_3)$	S_3^- and S_∞^-	Part-dome (torus) connecting to
			$N_3^+, S_4^+, CN_4^+, S_0^+, CN_2^+$ and S_2^+
CN_4^+	$(0,0,z_4)$	S_0^+ and S_4^+	Wall (bottle) connecting to
			$S_3^-, N_4^-, S_5^-, CN_5^-, S_\infty^-$ and CN_3^-
CN_5^-	$(0,0,z_5)$	S_5^- and S_∞^-	Part-dome (torus) connecting to
			$S_0^+, CN_6^+, S_6^+, N_5^+, S_4^+$ and CN_4^+
CN_6^+	$(0,0,z_6)$	S_0^+ and S_6^+	Dome connecting to
			$S_1^-, CN_1^-, S_\infty^-, CN_5^-, S_5^-$, and N_6^-

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Table 3.5: Separatrix details for seven-source, six-null case

3.5.2 Bifurcation behaviour

The first seven-source local coronal bifurcation behaviour (between the two-null and fournull case) is, in some ways, similar to its five-source counterpart - it creates two nulls connected by a separator and allows previously enclosed flux from S_0^+ to connect to S_{∞}^- .

However, it is qualitatively different. Where the five-source bifurcation results from the pinching together of four separators, in this case, only three separators are originally involved. There is an added peculiarity in that a separator connecting the two original coronal nulls is created - a global twist to a local bifurcation.

This is depicted in Figures 3.5.18 and 3.5.21.

The second bifurcation, converting a four-null state into a six-null state, appears topologically identical to the five-source bifurcation, as shown in Figure 3.5.22.

3.6 Discussion

In this chapter, we have found and examined surprising examples of local bifurcations taking place above the source plane. The first of these allowed a five-source configuration to switch between having two nulls and having none; the second changed a seven-source setup with two nulls into one with four nulls and thence into one with six nulls.

These particular configurations are particularly important ones to analyse since they have the form of δ -spots, prolific producers of flares. While our analysis remains, for the moment, potential, we expect the topologies produced in a sheared scenario to be little (if any) different.

That said, a potential field is insufficient to model magnetic breakout; the breakout, is due to shearing around a photospheric neutral line (or $B_z = 0$ surface) rather than a simple increase of flux in the central source. Moreover, our form of increase - implicitly, the distant balancing source grows with the S_0^+ - is pretty unrealistic.

However, we can lay foundations for the application of the seven-source bifurcations to the Magnetic Breakout Model, considered in the following chapter. We conjecture that the topological changes produced by shearing the field near a neutral line in a δ -spot

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Figure 3.5.18: Behaviour for first coronal bifurcation, between two coronal nulls and four. The initial configuration is similar to the five source case (Figure 3.3.12), except that only three separators are present (top left). As before, the fan surfaces buckle in (top right) and form a second-order null (bottom left) which splits into two new nulls (bottom right). The peculiar global behaviour is not shown.

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Figure 3.5.19: Zero surfaces in the plane for seven-source cases. Left, a two-null case and right, a six-null case. There are no qualitative differences in the plane.



Figure 3.5.20: Detail of seven-source zero-surfaces at z = 0.25 (left), z = 0.6 (centre) and z = 0.75 (right). The zero line (where the dotted and dashed lines cross) is outside the z-zero surface (solid line) at z = 0.25 but outside at z = 0.75; it crosses the boundary near z = 0.6.
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Figure 3.5.21: Separator changes for first seven-source bifurcation. There are originally three separators involved; these become five in the same way as the five-source coronal bifurcation. Additionally, a sixth separator is created between the two original coronal nulls.



Figure 3.5.22: Separator changes for second seven-source bifurcation. There are originally four separators involved; these become five in the same way as the five-source coronal bifurcation.



Figure 3.5.23: Connectivity graph for seven-source bifurcations

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are similar to the changes produced by the seven-source bifurcations, even if produced by a different means. We plan, in the medium term, to develop simulations to test this hypothesis.

We also develop the method of Molodenskii and Syrovatskii (1977) for finding coronal nulls using zero surfaces. This process in fact explains why Inverarity and Priest (1999) were unable to find any more than one coronal null with their configurations of a single positive source surrounded by negative sources on a hexagonal network. The *z*-zero surface must lie between the positive source and the each negative source - in fact, is lies between the positive source and the nearest photospheric null (all of which lie approximately on the network boundaries). Since there is only one xy-zero surface, and hence there is only one coronal null.

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Chapter 4

The Magnetic Breakout Model

But through the flood-gates breaks the silver rain, And with his strong course opens them again.

William Shakespeare, Venus and Adonis, 959-60

4.1 Abstract

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The Magnetic Breakout Model describes how the application of shear to a neutral line in a delta-sunspot can cause low-lying field to 'break out' through an overlying coronal arcade, allowing the release of magnetic energy. It has been suggested as a process which could trigger an eruptive flare. We construct a simple topological representation of a deltasunspot region and show that magnetic breakout can be achieved by way of topological bifurcations in the corona. We also find a new bifurcation, the coronal local separator bifurcation, in which two new null points are created well above the source plane.

4.2 Introduction

4.2.1 Delta sunspots and magnetic breakout

A *delta sunspot* is defined as two opposite-polarity sunspot umbrae sharing a common penumbra. It is well-known that such configurations are prolific producers of flares. Antiochos (1998) found that delta sunspots are the simplest configurations with sufficient complexity to allow a *magnetic breakout* to occur.

The Magnetic Breakout Model was proposed by Antiochos *et al.* (1999) and developed by Aulanier *et al.* (2000) as a mechanism for the initiation of a coronal mass ejection (CME). In this model, shear is applied to initially low-lying field which lies beneath an unsheared coronal arcade. The shear causes the low-lying field to rise, and the overlying arcade to reconnect with other flux systems. Crucially, this reconnection reduces the amount of magnetic flux restraining the low-lying field, and eventually weakens it to the extent that the low-lying field can 'break out' and connect to distant flux systems. This is shown in Figure 4.2.1.

In this chapter, we will model a delta sunspot region using MCT and find that adjusting certain parameters can effect just such a breakout. We make particular use of an equation (Equation 1.13) adapted from Longcope and Klapper (2002) connecting the number of photospheric flux domains $(N_{\phi d})$, coronal flux domains (N_{cd}) , separators (N_{nn}) , photospheric null points (n_{ϕ}) , coronal null points (n_c) and sources S:

$$N_{\phi d} + 2N_{cd} = 2N_{nn} - N_{\phi n} - 2N_{cn} + S, \tag{4.1}$$

as described in Section 1.3.

4.2.2 Our model

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We model a delta sunspot in MCT in the following way: a central, positive source (the parasitic spot) is surrounded by a ring of negative sources (the parent spot). Flanking this is a second positive region, corresponding to the partner sunspot of the parent and consisting of two sources. This is shown in Figure 4.2.2.

We perform three MCT experiments. In the first, the strength of the central source is

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Figure 4.2.1: Representation of magnetic breakout, after Aulanier *et al.* (2000). The flux near the neutral line between P_{in} and N2 is subjected to shear, which forced it to rise and 'break out' explosively through the overlying field, represented by the dashed line.



Figure 4.2.2: Initial source configuration. The stars represent sources and the solid lines contours of zero vertical flux at a small height δz above the photosphere.

Source	Position	Strength
P0	$(\delta x, \delta y, 0)$	ϵ
N1	(0,1,0)	-1
N2	$(-\sqrt{3}/2, -0.5, 0)$	-1
N3	$(\sqrt{3}/2, -0.5, 0)$	-1
P4	(0, -3, 0)	2.5
P5	$(3\sqrt{3}/2, 1.5, 0)$	2.5

Table 4.1: Initial positions and strengths of sources.

increased and the changes in topology tracked. The second experiment is similar, except that instead of increasing the central source strength, we change the location of certain sources; lastly, we find the bifurcations due to changing the force-free field parameter α .

Of course, none of these experiments represent the true behaviour of a flaring delta sunspot, in which the field near the polarity inversion line around the parasitic spot is sheared. However, increasing the source strength mimicks the rising of the field, while increasing α reproduces the change in helicity such shearing causes.

In the following section, we analyse the changes in the magnetic skeleton brought about by altering these parameters. We then discuss the nature of the bifurcations involved before we conclude with a discussion of these results.

4.3 Results

4.3.1 Source strength experiment

In this experiment, we vary the strength ϵ of the central source, located near the origin, between $\epsilon = 1.3$ and $\epsilon = 1.8$. For lower values of ϵ , a coronal null can exist, but it has no relevance to our breakout model.

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Figure 4.3.3: Topologies for $1.3 < \epsilon \leq 1.33$ (top), $1.33 \leq \epsilon \leq 1.68$ (middle) and $1.68 \leq \epsilon$ (bottom). Full captions overleaf.

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Figure 4.3.3: Top: Initial topology for $1.3 < \epsilon \leq 1.33$. Five null points (dots) exist; their spines represented by thick lines. The fan of A1 forms a dome following the spine of null B2. The fan of A3 runs along the spines of B2 and B4. The fan of A5 forms an outer dome following the spine of B4. The fan of B2 is a dome which follows the spines of A1 and A3; finally, the fan of B4 forms a wall running along the spines of A3 and A5. Separators (dashed lines) connect A1 to B2; B2 to A3; A3 to B4 and B4 to A5.

Centre: Topology after coronal local bifurcation, $\epsilon = 1.5$. Two new nulls (B7 and A6) appear. The fans of A1 and A5 are unchanged; that of A3 is now bounded above by the spine of B7. The fan of new null A6 forms a dome bounded by the spines of B7, B4 and B2. The fan of B2 is bounded by the spines of A1 and A3; in combination with the fan of B7 (which is bounded by the fans of A6 and A3), it forms a dome enclosing the flux from central source P0. Finally, the fan of B4 is bounded by the spines of A5 and A6. There are six separators.

Bottom: Topology after spine-fan bifurcation. The spine of coronal null A6 now connects to the balancing source. The fan of A5 no longer forms a dome, and connects to P0. It is bounded by the spines of B2 and B7. The fans of A1, A3 and A6 are unchanged. The fans of B2 and B7 remain bounded in part by the spine of A6 and now connect to the balancing source. The fan of B4 is now bounded only by the spine of A6.





Figure 4.3.4: Connectivity graphs initially (left), after a coronal local separator bifurcation (middle) and after a global spine-fan bifurcation (right). There are initially ten flux domains, with four independent circuits. The local bifurcation does not change the domain graph; however, the global spine-fan bifurcation allows flux to connect P0 to $N\infty$. This adds a new gv theflux domain and a new circuit into our reckoning.

Range	$1.3 \lesssim \epsilon < 1.33$	$1.33 \leq \epsilon < 1.63$	$1.63 \le \epsilon$
Purely coronal domains (N_{cd})	0	0	1
Separators (N_{nn})	4	6	7
Coronal null points (N_{cn})	0	2	2

Table 4.2: Numbers of coronal domains, separators and coronal null points for different stages of our model. The number of domains, sources and null points in the photosphere remain constant at $N_{\phi d} = 10$, S = 7 and $N_{\phi n} = 5$, respectively. All three situations satisfy Equation 4.1: $N_{\phi d} + 2N_{cd} = 2N_{nn} - N_{\phi n} - 2N_{cn} + S$.

For 1.3 < $\epsilon \leq 1.33$, the topology has no coronal nulls. The configurations for all three states are shown in Figure 4.3.3, and the domains in Figure reffig:mbo:domains. There are five photospheric null points, comprising three negative and two positive nulls. There are four separators connecting these, and ten flux domains (of which none are coronal) exist. Flux from the central source P0 is prevented from connecting to the balancing source $N\infty$ by the fan of null B2, which forms an inner dome; this is in turn contained by an outer dome comprising the fan of the null A5.

At $\epsilon \sim 1.33$, a local bifurcation occurs. Two null points, one of either sign, are created well above the source plane. The bifurcation is analysed in some detail in Section 4.4.

The photospheric arrangement of spines and fans remains unchanged, although the coronal topology does change significantly. Now six separators connect the seven nulls, although the domain graph is unchanged. However, this bifurcation is a necessary step towards a breakout, although the outer dome remains intact. The inner dome now con-

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sists of the fans of B2 and B7, which meet at the spine of A6. Equation 4.1 remains valid, as shown in Table 4.2.

At $\epsilon \sim 1.63$, a global spine-fan bifurcation occurs. One of the spine field lines of the negative coronal null (*B*7) changes its connectivity; before the bifurcation, it connects to source N2; afterwards, it is connected to the balancing source N ∞ , a large distance away. In this case, there are seven separators and eleven flux domains, one of which is coronal. Equation 4.1 remains true, as shown in Table 4.2.

The new flux domain connects the central source P0 to balancing source $N\infty$ - the flux originally constrained by two domes has 'broken out' to connect to distant flux systems.

4.3.2 Source location experiment

In this experiment, we begin with a configuration identical to the previous subsection, except that ϵ is fixed at a value of 1.5. The topology is that shown in the centre of Figure 4.3.3.

We will begin by moving the source P_5 in various directions to try to recreate the spinefan bifurcation found in the previous subsection. Secondly, we will analyse the topological behaviour caused by moving the other outer positive source P_4 .

We find that moving the source P5 in the negative x-direction causes the same spine-fan bifurcation at approximately (1.89, 1.5). Many different directions also give the same bifurcation; the line of bifurcation is plotted in Figure 4.3.5.

Moving P4, though, has a somewhat different effect, although with similar overall results. We choose to examine the effects of moving the source 'north-west' (that is to say, in the $(-\hat{x}, \hat{y})$ direction). Instead of the spine of A6 bifurcating with the spine of A5 immediately, three bifurcations take place. Firstly, the spine of A5 and the fan of A1 undergo a spine-fan bifurcation, before the spine of A6 and the fan of A1 do the same thing. This leaves the topology effectively a reflection of that in the centre of Figure 4.3.3 along a line through A5 and B4; continuing to move P4 in the same direction allows A6 and A5 to bifurcate as before.



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Figure 4.3.5: Bifurcation diagram for spine-fan bifurcation between A5 and A7. If source P5 is placed to the left of the line, the spine of A6 connects to the balancing source at infinity; to the right, it connects to source N2.

4.3.3 Force-free field experiment

In this experiment, we fix $\epsilon = 1.6$ and the source positions, while changing the parameter α of a linear force-free field. We find that decreasing α results in three spine-fan bifurcations, in an identical sequence to moving P4, with the final bifurcation occurring at $\alpha \simeq -0.087$. If instead we increase α , we find that a single spine-fan bifurcation between A5 and A6 takes place when $\alpha \simeq 0.215$. This, however, is a large value for α to take, and the region of validity for the force-free approximation becomes small.

4.4 Bifurcation analysis

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Until recently, it was tacitly assumed that all local bifurcations - ones in which null points are created or destroyed - occur in the source plane. Coronal nulls in particular were thought to owe their existence to the local double-separator (or pitchfork) bifurcation (Brown and Priest, 2001), in which one null in the source plane becomes three.

The local bifurcation found at $\epsilon \sim 1.33$, however, clearly does not take place in the source plane. This is a new type of bifurcation, which we will call the *coronal local separator bifurcation*. In particular, this is a three-separatrix variety; a similar (although

more complex) bifurcation is possible involving four separatrices, as discussed in the previous chapter.

The bifurcation process is depicted in Figure 4.4.6. Two fan surfaces (from B2 and B4) buckle together and eventually meet. At this point, a second-order null point is formed, which subsequently splits into two new coronal nulls. The fans are now bounded by the spine of the new negative null A7; their lower parts are replaced by the fan of the new positive null B6. The other fan involved, that of A3, is now bounded by the spine of B6, with its upper part replaced by the fan of A7. In this way, the three separators originally involved become four.

It is worth stressing that the dome enclosing the central flux (from P0) now consists of two separatrices (from A3 and A7), which are both bounded by - and inextricably linked to - the spine of B6.

It is, however, the global spine-fan bifurcation that allows the breakout to take place. This bifurcation was studied by Brown and Priest (1999a). It involves two null points of the same sign (in this case, negative nulls A7 and A5); the spine of one passes through the fan of the other, changing the connectivity of both.

In particular, the fan of A5 connects to the central source P0, and no longer forms a dome enclosing all of the central flux. In addition, the separatrices of A3 and A7, which previously combined to form a dome, are opened up by the connection of B6's spine to infinity.

In this way, there is no longer a barrier separating P0 and $N\infty$, and flux can connect these two sources. This connection is the topological analogue to breakout.

4.5 Discussion

In this chapter, we have shown that a simple model of a delta sunspot region can be made to display behaviour topologically similar to the breakout model proposed by Antiochos *et al.* (1999).

Increasing the flux of the parasitic spot, moving flux sources, and altering the force-free field parameter α all lead to a global spine-fan bifurcation allowing flux to connect the

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Figure 4.4.6: Bifurcation behaviour for coronal local separator bifurcation. Two fan surfaces buckle together (top right) before joining together in a second-order null point. This splits into two first-order nulls; the original buckled fans and the fan of the new positive null B7 are now bounded by the spine of the new, negative null A6. The fans of A6 and A3 are both bounded by the spine of B7. Where there were originally three separators, there are now four.



Figure 4.4.7: Global separator bifurcation. Initially, the spine of the lower null (A7) connects to the right (N2), while the fan of the upper null (A5) connects to the left (to P4 and P5). The two approach each other until at the point of bifurcation, the spine lies in the fan plane. After bifurcation, the spine of A7 connects to the balancing source $N\infty$ and the fan of A5 connects, additionally, to P0.

parasitic spot to distant flux systems.

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It is worth re-examining Figure 4.2.1 in this new, topological light. It is clear to see that the thick solid lines represent the skeleton of the lower coronal null A7 (its fan comprising the more horizontal lines, and its spine the more vertical ones) while the thick dashed lines are the separatrix surface of photospheric null A5.

It is, however, important to recognise the limitations of our model. Our depiction of a delta spot as a small number of point sources is a rather crude one; it seems likely, though, that more realistic models would behave similarly. In the future, we plan to analyse MDI data from a flaring active regions in a similar way.

This model also ignores the energetics of the situation. In particular, a potential field is incapable of storing magnetic energy to be released in an explosive event, so it is impossible at this stage to state that this is the mechanism which allows a rapid, dramatic breakout.

We have also detailed a new, coronal local separator bifurcation. This bifurcation allows two null points to be created or destroyed well above the photosphere. It appears to be a critical stage towards magnetic breakout.

This model provides a step towards a more complete topological understanding of the magnetic breakout model. In time, these steps will hopefully lead to new insights into some of the processes involved in dynamic coronal magnetic events.

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Chapter 5

Elemental Flux Loops

Loney was watching the river in his mind, the loops and bends as gracefully etched in the winter cover as a blue racer snake frozen in the grass. Loney always wondered how that river knew where to bend, why it wandered with such feckless purpose. He wondered if it always sought the lowest ground, or was his mind such a shambles that he assumed there was a reason behind its constant shifting?

James Welch, The Death of Jim Loney.

Abstract

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The photosphere possesses many small, intense patches of magnetic flux. Each of these patches (or *sources*) is connected magnetically through the corona to several sources of opposite polarity. An *elemental flux loop* consists of all of the flux joining one such source to another. We find that each source is connected to twenty other sources, on average, and that the typical flux and diameter of elemental loops in the corona are 10^{16} Mx and 200 km; there are approximately 17 separators for each source. We also model a typical large-scale coronal loop consisting of many elemental loops and determine its complex internal topology. Each upright null lies at the end of about 22 separatrices, which tend to be clustered together in trunk-like structures, analogous to river-valleys in a geographical contour map. Prone nulls correspond to saddle points, while their spines are analogous to

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watersheds.

This work has been accepted by Solar Physics (Beveridge et al., 2003).

5.1 Introduction

Since magnetic loops are the fundamental building blocks of the corona (Rosner *et al.*, 1978), it is natural to seek to determine their nature, including their size distribution and topology.

Coronal loops may be observed in soft x-rays and in EUV. Their size and discrete nature have been attributed to their being anchored in small-scale photospheric elements (Litwin and Rosner, 1993). An *elemental flux loop* can then be defined as comprising all the field lines anchored to a given pair of source elements.

Our aim here is to develop a model for elemental flux loops, from which we might understand their magnetic properties. A key question then is: How many photospheric elements of opposite polarity are interconnected through the corona? Since it is possible that separator reconnection (Priest and Titov, 1996; Galsgaard and Nordlund, 1997; Longcope, 1996; Galsgaard *et al.*, 2000) contributes to coronal heating, it is important to find in particular the density and distribution of separators.

We model a large-scale coronal loop (a *super-loop*) anchored in unipolar areas of an active region. Each unipolar area consists of a large number of elemental sources, with flux $\Phi \sim 10^{17}$ Mx (Stenflo, 1994). Following Longcope and van Ballegooijen (2002), we identify the pattern of interconnections between such sources.

In the first part of our analysis, we examine the topology of a source plane containing a large number of point sources, and how it varies with the distribution of flux and fraction of flux with a particular sign. In particular, we find an approximation for the number of upright nulls in each case, from which the number of prone nulls can be calculated.

In the second part, we examine the number and distribution of separators when a strictly positive region is connected to a strictly negative region. We find a large number of magnetic separators within the super-loop (roughly 17 for each source of a given polarity). These have a tendency to be clustered closely together.

Finally, we deduce that each source is connected to, on average, twenty opposing sources; this leads immediately to the conclusion that an average elemental loop will have a flux $\Phi_L \sim 10^{16} Mx$, corresponding to a diameter of around 200 km in a coronal field of typical strength. A similar estimate is obtained by Priest *et al.* (2002).

In Section 5.2, we outline our model; in Section 5.3 we discuss the topology of the source plane. Section 5.4 examines our results on separators and with the loops themselves. We conclude in Section 5.5 with a discussion of our results.

5.2 Model

Following Longcope and van Ballegooijen (2002), we consider a magnetic super-loop with a large aspect ratio geometry, as depicted in Figure 5.2.1: the loop length is much greater than its radius, and for simplicity, we treat it as approximately straight. At each end is a boundary representing a section of the photosphere and containing small, discrete flux sources. One end represents the positive polarity of an active region and the majority of its photospheric sources are positive. Unlike Longcope and van Ballegooijen, we do not restrict ourselves to strictly unipolar source planes. The result is a *magnetic carpet* layer below the merging height (Schrijver *et al.*, 1997; Parnell, 2000; Priest *et al.*, 2002). In our analysis, we assume the distance between these sources is large enough in comparison to their size that we can treat them as point sources.

We perform two numerical experiments, detailed below: the first examines the topology of the magnetic field within the source plane, which includes a determination of the proportion of upright nulls due to a given source configuration. The second examines how this topology relates to the coronal field, its separators and elemental coronal loops.

Each calculation in the two experiments is performed by a Monte Carlo method using numerous realisations of source distributions. A single realisation consists of 1000 sources whose positions are randomly generated to produce a planar Poisson point process of unit mean density within a disc centred at the origin (Kendall and Moran, 1963; Longcope *et al.*, 2003). The mean separation of two sources is:

$$\bar{r} = \int_0^\infty r p(r) dr$$

where the point distribution function for unit density is:

$$p(r) = 2\pi r e^{-\pi r^2}$$

and r represents the distance between nearest neighbours in units of length such that the mean density N = 1. This results in: $\bar{r} = 0.5$.

We perform two types of calculation, by making two different assumptions about the source magnitudes. In one type all magnitudes are equal, while in the other type the magnitudes are drawn randomly from an exponential distribution. We consider these two cases in recognition of the fact that the true size distribution of fundamental flux elements is unknown. Larger magnetic elements, such as those found in the quiet Sun (Schrijver *et al.*, 1997), show an exponential distribution of fluxes whose mean is $\Phi \simeq 10^{19}$ Mx. Each of these elements is, however, composed of numerous fundamental elements of typical size 10^{17} Mx.

The signs of the flux elements within a plane are chosen to produce a region with a prescribed level of *flux imbalance*, by making a fraction f of the sources positive. In the positive end of the cylinder f > 0.5 while in the negative end f < 0.5. Since the topological properties of a magnetic field do not depend on sign, we need only consider cases with $0.5 < f \le 1.0$

The first series of experiments characterises the magnetic field in the source plane. A unipolar region interspersed with opposing flux elements will contain a magnetic carpet within which their field lines close. The remaining field maps through the carpet to the merging layer (and thereafter through the corona to a region of opposing polarity). To clarify this, we determine how the density of both types of photospheric null point, upright and prone, vary with flux imbalance f.

The second of our experiments will apply the topology of the photospheric field to the corona. In the model of Longcope and van Ballegooijen, the topology of the coronal field is determined entirely by the topology of the field at each merging layer, rather than at the photosphere (the coronal field is by definition unidirectional, so it maps the field lines without changing their topology). The only cases where we can approximate the merging layer topology by the photospheric topology are those without a magnetic carpet (f = 0 and f = 1). We therefore restrict our consideration to the case f = 1 and furthermore assume that the coronal field is unstressed, and therefore maps between merging heights without distortion. Under these assumptions, separators occur along lines parallel to the

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cylinder's axis. These coincide with the intersections of separatrix curves when the two photospheric layers are superposed.

5.3 Topology of the source plane

In this section, we find the densities of prone and upright nulls as functions of flux imbalance f and the nature of source magnitudes. We do this by performing Monte Carlo simulations of the source planes, repeating the experiment 1000 times for each set of parameters: firstly, the strength distribution (uniform strengths or exponentially distributed) and secondly, the fraction of positively signed flux f, which we examine for f between 0.5 and 1.0 in steps of 0.1.

Each critical point of the field - each source and null point - corresponds to an extremum in the source plane of the field's scalar potential. A positive source or upright null corresponds to a maximum of the potential; a negative upright null or source corresponds to a minimum. A prone null, meanwhile, corresponds to a saddle point irrespective of its sign.

We determine the number of upright nulls by finding all the extrema of the potential. We do this using two gradient masks, one of which associates with each point in a grid the co-ordinates of the adjacent point with the highest value, while the other does so with the lowest value. From this information, using a sufficiently fine grid, it is easy to find all of the extrema reliably.

We then use a Newton solver to find the nulls near such extrema - this also removes from our list of nulls any maxima or minima corresponding to sources - and lastly we ensure that each of the nulls is indeed an upright null by examining the alignment of its spine field line.

We average the number of upright nulls over the 1000 experiments. The results are given in Table 5.1 and Figure 5.3; from these, we determine the number of prone nulls from the Euler characteristic equation (Equation 1.10). We find the following relationship between the number of upright nulls (n_u) , the flux (f) and the number of sources (S):

$$n_u = 4.0 \times e^{-8.04j}$$

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in the uniform case and

$$n_u = 3.2 \times e^{-8.04f}$$

in the exponential case.

These results show that the less mixed a region, the more numerous the upright nulls; also the uniform distribution of source strengths produces slightly more upright nulls than the exponential distribution, approximately in the proportion 5:4.

5.4 Separators and flux loops

In this section, we find the density and distribution of separators in a super-loop and from this calculate the average value for the size of an elemental loop. We deduce the number of separators from the number of points where the projections of positive and negative separatrices cross. As mentioned previously, this assumes a straight coronal field so that the mapping is a simple overlay of the two topologies.

In the case where f = 1, none of the field lines close back into the source region. This implies there is no magnetic carpet, and we expect the topology at the merging height to be quite similar to that at the photosphere: we expect trunks to form at the merging height, but we do not expect the mapping to preserve relative areas. Since we are interested only in the topology, we assume the merging height to have the same configuration as the photosphere.

Section 5.3 found that the density of upright nulls in the exponential case with f = 1 is $d_u = 0.10$ nulls per unit area. Since this density assumed a distribution of sources with unit density, d_u also denotes the number of upright nulls per photospheric source.

From the Euler characteristic (Equation 1.10), we deduce the density of prone nulls to be $d_p = 1 + d_u = 1.1$. This implies that there are approximately 11 prone nulls for every upright null. Every prone null is the origin of exactly two separatrix curves within the photosphere, each of which must end at an upright null since there are no negative sources in the plane. This means that an average of about 22 separatrix field lines from surrounding prone nulls lead to each upright null.

Figure 5.4.3 shows an example of the separatrix curves connecting prone to upright nulls.



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Figure 5.2.1: A straight flux loop, after Longcope and van Ballegooijen (2002). The relative dimensions have been exaggerated for clarity, whereas the model assumes a long, thin loop with a small chromosphere so that $a \ll h \ll L$.

f	d_u	
	uniform	exponential
1.0	0.1198 ± 0.0055	0.0968 ± 0.0049
0.9	0.0588 ± 0.0038	0.0468 ± 0.0034
0.8	0.0243 ± 0.0025	0.0218 ± 0.0023
0.7	0.0115 ± 0.0017	0.0095 ± 0.0015
0.6	0.0058 ± 0.0012	0.0038 ± 0.0010
0.5	0.0020 ± 0.0007	0.0018 ± 0.0007

Table 5.1: Upright null density d_u (nulls per unit area) as a function of the fraction of positive flux f, when the source strength distribution is (left) uniform and (right) exponential.

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91 0.100 Upright null density d_u 0.010 0.001 0.5 0.6 0.9 1.0 0.7 0.8 Flux imbalance f (a) 0.100 Upright null density d_u 0.010 0.001 14 0.5 0.6 0.7 0.8 0.9 1.0 Flux imbalance f (b)

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Figure 5.3.2: Plots of the flux imbalance f against the upright null density d_u when the source strength distribution is (a) uniform and (b) exponential. The solid lines represent our fitted curves of the form (a) $n_u = 4.0 \times e^{-8.04f}$ and (b) $n_u = 3.2 \times e^{-8.04f}$, where S is the number of sources.

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Figure 5.4.3: A part of a simulated photosphere, about 5 units by 4 units, where one unit is twice the mean separation of sources. The sources (asterisks) determine the positions of the prone and upright null points (diamonds and triangles, respectively). The separatrices of prone nulls (solid lines) divide the plane into domains associated with a particular source. It can be seen that the separators tend to run closely together in trunks (a prime example runs down the centre of the diagram and ends at the upright null near (0.5, 0.5)). The spines (dashed lines) divide the plane into domains associated with a particular upright null. Also plotted are contours of the potential Fwhere $\nabla F = \mathbf{B}$. It can be seen that path of a trunk is analogous to a river valley, while the spines are analogous to watersheds.

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Figure 5.4.4: A part of two overlaid topologies, about 5 units by 4 units. The solid lines represent the separatrices of one photosphere, the dotted lines those of the other. The crosses, where these meet, mark separators. Where two trunks cross, a large number of separators occur in a tiny area.

It is evident from this typical view that the separatrices tend to form *trunks*, where several field lines run extremely close together - a clear example runs down the centre of the figure and ends at the upright null near (0.5, 0.5).

This tendency has important consequences for the density and distribution of separators (and hence for the density of elemental loops), since if one trunk crosses the projection of another, many separators can form in a very small area.

We next determine the locations of separators by superposing two realisations of the photospheric field, each of which is generated as described in the previous section. One of these realisations represents the positive end of a loop, the other the negative boundary. This superposition represents a mapping by way of straight coronal field lines.

Each crossing of the superposed photospheric separatrix curves is a *separator*. Figure 5.4.4 shows a piece of a superposition where the solid (respectively, dashed) curves are the separatrix curves on the positive (respectively, negative) boundary. The separators, where these lines cross, are marked by asterisks.

We then count the number of separators in a central subregion measuring 10 units by 10 units, and repeat this experiment 30 times. Were the points uniformly distributed so that their counts obeyed Poisson statistics, we would expect to find $N_{sep} = d_{sep}(10^2) = 1740 \pm 42$ in our 10 × 10 box. Instead we count 1665 ± 461, which varies more than an order of magnitude beyond expectation. This indicates that separators are not uniformly distributed, but tend to form in clusters.

We explore the effect of clustering through a simple model in which all separators occur in clusters of size C. These clusters are themselves distributed uniformly with density $d_C = d_{sep}/C$, where d_{sep} is the inferred separator density. A region of area A will therefore contain $\langle N_C \rangle = d_C A = d_{sep} A/C$ clusters. Trials will yield N_C counts centred on this mean with Poisson deviation $\sigma = \sqrt{\langle N_C \rangle}$. Thus the number of separators found within area A will be $N_{sep} = \langle N_{sep} \rangle \pm \sqrt{\langle N_{sep} \rangle} \sqrt{C}$. Our results are consistent with a cluster size $C \simeq 128$, which would occur at the intersection of two trunks with eleven separators each. Figure 5.4.5 shows this prediction against the actual results: in the lowest 70%, the prediction matches the results very well, although at the high end of the scale we consistently find more separators than we predict.

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Given that there are around 1.1 prone nulls per unit area and each prone null has exactly two separatrix field lines in the plane, each separatrix surface contains an average of around $17.4/2.2 \simeq 8$ separators.

As for the flux domains, we know there are d_{sep} separators per unit area, and hence in an area with S sources, a total of $n_{sep} = d_{sep}S$. In an approach similar to Longcope and Klapper (2002), we therefore apply Euler's theorem:

$$f = e - v + 2, (5.1)$$

where f represents faces, e edges and v vertices, to the superposition of two boundaries.

In the superposition, every face represents a flux domain, so $f = n_d$. We count as a vertex all photospheric nulls and separators, so we have $v = 2(n_p + n_u) + n_{sep}$ vertices. Edges are constituted by the parts of field lines joining vertices: that is, connecting two separators, two nulls, or a null and a separator. There are $e = 4n_p + 2n_{sep}$ edges.

Equation 5.1 then becomes

$$n_d = 2n_p - 2n_u + n_{sep} + 2,$$

where n_d is the number of domains. However, $n_p - n_u = S$; moreover, in a large area, we can ignore the 2. So,

$$n_d \sim (2 + d_{sep})S.$$

With our value of $d_{sep} = 18.1$, we find $n_d = 20.1 S$; on average, then, each source connects to about twenty opposing sources.

Stenflo-sized elements have a flux of $\Phi \sim 10^{17}$ Mx (Stenflo, 1994), so our loops have a flux $\Phi_L \sim 10^{16}$ Mx. Taking a typical coronal magnetic field strength of 10 gauss, we find an average diameter for each flux loop of around 200 km, corresponding to 0.25 arcsec. This is a little finer than the limit of TRACE resolution, and similar to the estimate of

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Figure 5.4.5: Cumulative histogram of separator number n_{sep} (solid line) against prediction (dashed curve). There is very good agreement for the lowest 70%, although we find more separators than predicted at higher values.

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Priest *et al.* (2002). It must be noted, however, that due to the propensity of separators to form trunks, many of these flux loops are expected to be very thin indeed and to have very little flux.

5.5 Conclusions

The *magnetic carpet* - the arrangement of connections between flux sources in the solar photosphere - is an incredibly complicated, and forever-changing, structure (e.g., Schrijver *et al.*, 1997; Parnell, 2000; Simon *et al.*, 2001). New sources are constantly emerging as old sources disappear; others coalesce to form larger sources, while others divide into smaller ones. Yet others come together and cancel each other out.

An important aspect of studying the phenomena due to the carpet is to determine in detail the topology due to its sources. Much progress has been made, both in the examination of small numbers of sources (for instance, the three-source case has been completely classified (Brown and Priest, 1999a) and a start has been made on four sources, both in a balanced scenario (Beveridge *et al.*, 2002) and in unbalanced cases (Brown and Priest, 1999b, 2001)) as well as in discussion of general concepts (Longcope, 2001).

Our analysis approaches the problem from a different angle, with a direct physical application. It is one of the first studies to predict flux loop sizes from a theoretical standpoint.

We find that the separatrix surfaces in the photospheric planes have a tendency to form *trunks* where many separators lie close together. One way to understand this is to consider the magnetic potential F, where

$\mathbf{B}=\nabla F.$

Supposing all of the sources to be positive, F has a local maximum at each source. At each prone null in the source plane, the potential has a saddle point, and at each upright null, F has a local minimum. By analogy to a geographical contour map, these trunks can be seen to follow valleys of the potential. This analogy can be extended to include the spines, which trace the watersheds of the potential landscape, as can be seen in Figure 5.4.3. One explanation for the valleys could be that, at a general potential null point, the fan field lines are not equally spaced but aligned preferentially towards one of the fan eigenvectors.

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We superpose two photospheric planes as an approximation to a straight super-loop. The intersections correspond to separators, while the the spaces in the network are the elemental flux loops.

We find that many of these loops will carry very little flux - where trunks intersect, a large number of loops and separators will occur in a small area. We expect this phenomenon to be replicated at the merging height, and suspect that direct methods may find fewer connections - it is unlikely that tracing field lines from sources would find all of the flux loops that we do.

We now plan to extend this analysis of model fields to more realistic ones by extrapolating MDI data. There remain, however, many questions even in this simplified model: is separator heating more or less efficient than the separatrix heating proposed in the Coronal Tectonics Model (Priest *et al.*, 2002)? Why exactly do the trunks form, and what do they represent? What happens physically when a large number of separators are clustered together in a tiny space? Does reconnection in such clusters contribute significantly to coronal heating?

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Chapter 6

Discussion and future work

The last word has golden rays. Méret Oppenheim

6.1 Discussion

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Magnetic charge topology is a relatively new means of investigating the properties of the solar corona. While in some respects the subject is well-developed, in others it is in its infancy.

In this thesis, we have attempted to develop it a little further, both in terms of our understanding of how magnetic field regions interact and in terms of physical applications for the model.

In Chapter 2, we catalogued all of the possible topologies for a balanced potential scenario with four magnetic point charges, finding that seven topologically distinct cases exist. When there are three positive and one negative sources, an upright null state (U), a separate state (S) and an enclosed state (E) are possible; when there are two sources of each sign, a nested state (N), a detached state (D), an intersecting state (I) and a coronal null state (CN) can exist.

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Figure 6.1.1: Interactions between topologies (circles) and bifurcations (arrows).

The field configuration can change between these by means of several bifurcations: two global bifurcations proper (the global separator (GSB) and the global spine-fan bifurcations (GSF)), two global quasi-bifurcations (the global separatrix (GSxQB) and the global spine (GSpQB) quasi-bifurcations), and two local bifurcations (the local separator (LSB) and the local double-separator bifurcations (LDSB)).

The interactions between the topologies and bifurcations are shown in Figure 6.1.1.

In Chapter 3, we lay the foundations for a topological cartoon of the Magnetic Breakout Model, by considering local bifurcations which take place above the source plane. These allow flux initially enclosed by overlying structures to 'break out' and connect to distant flux systems, as developed in Chapter 4.

Chapter 5 represents something of a change of direction, dealing with large numbers of point sources, which we use to model elemental flux loops. We consider these to be the building blocks for large-scale 'superloops' which might represent the loop structures seen in TRACE images. We find, in configurations of mixed polarity, results for the density of upright nulls as a function of the flux imbalance; by superposing two regions (each containing many sources of one polarity) we calculate characteristics of the separators that ensue, and find an average diameter for elemental loops of around 200km.

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6.2 Future work

Two of the critical areas for the development of MCT, and indeed the whole of solar physics, are flares and coronal heating, both of which we have touched upon in this thesis (in Chapters 3, 4 and 5, respectively).

To provide realistic, physical models, though, MCT in this form is at best limited because of its insistence on potential fields. This insistence is based largely on ease of computing - a potential field has a simple, analytical expression from which it is easy and quick to calculate field lines. It has many physical disadvantages, including the fact that magnetic energy cannot be stored by a potential field, which is automatically in the lowest energy state.

Linear force-free fields have (relatively complicated) analytical expressions, and some thought has gone into force-free topology

A topological model for flares has frequently been considered in the past: Gorbachev and Somov (1988) and Antiochos *et al.* (1999) were by no means the first. However, the improved resolution of space-based telescopes such as TRACE allow us to examine the corona more closely and infer more accurately coronal magnetic structures, and hence to improve our models.

We intend to extend our potential analysis of a δ -spot region to include magnetic shear near the photosphere.

In this way, we hope to determine the three-dimensional magnetic skeleton of a 'breakout' situation, and how the topology changes between the two. We expect the bifurcation to resemble those outlined in Chapters 3 and 4. By developing a force-free model, we may also be able to analyse the energy available for release in such a situation.

As for coronal heating, we expect the interior structure of loops to play a significant role. We plan to develop our simulations of straight loops to include dynamic and magnetic carpet effects.

We also wish to investigate what significance the close clustering of separators has. A suggestion for this work is to extend the Longcope and van Ballegooijen (2002) model of a dynamic super-loop to more complicated end-planes. We suspect that with even five or six sources at either end, trunk-like behaviour, and hence separator clustering, will be

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observed.

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To this end, we plan to work with the Coronal Tectonics Model of Priest *et al.* (2002). In this model, coronal heating is effected by the movement of flux tubes against each other, in analogy to geophysical plate tectonics. Numerical simulations so far undertaken have failed to take any topological effects into account.

Lastly, our understanding of the uses of MCT is constantly changing. A vital part of any future work is the development of more sophisticated tools for determining the topological nature of solar magnetic fields, and a wider understanding of their implications.

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Glossary

A glossary of some terms used in this thesis. The reference given is generally to the first or fullest definition of a term in the body of the thesis.

Bifurcation: A topological change from one state to another (Section 1.4).

Connectivity: The existence of flux between two source regions. All field lines beginning at one source and end at another form a region of connectivity (Section 1.3.2).

Corona: In reality, the outer layer of the solar atmosphere (Chapter 1). In MCT, it is represented by the half space above the photosphere (Section 1.3).

Coronal null: A null which doesn't lie in the photosphere (Section 1.4.1).

Delta spot: A sunspot group in which a substantial amount of flux of one polarity lies withing the penumbra of the main sunspot of opposite polarity (Section 4.2).

Domain graph: A graph representing all of the flux domains in a given configuration of sources (Section 1.3.2).

Elemental loop: A small loop, consisting of all the flux connecting two sources, of which *superloops* are made up (Section 5.1).

Euler characteristic: Equations 1.10 and 1.11, which relate the numbers of positive and negative sources and nulls in an MCT configuration.

Fan: The plane of field lines near a magnetic null diverging from or converging on the null (Section 1.3.2).

Flux domain: All of the field lines connecting two source regions (Section 1.3.2).

Force-free: A field satisfying Equation 1.6, $\mathbf{j} \times \mathbf{B} = \mathbf{0}$, or $\nabla \times \mathbf{B} = \alpha(\mathbf{r})\mathbf{B}$

Global bifurcation: A bifurcation which changes the global structure of the field without creating or destroying a null (Section 1.4.2).

Half separator: A separator which lies along the intersection of two flux domains rather than four. This is a symptom of a homovertebraic null (Section 1.3.2).

Heterovertebraic null: A null with two spine sources. (Section 1.3.2).

Homovertebraic null: A null with only one spine source. (Section 1.3.2).

Linear force-free field: A *force-free* field where $\alpha(\mathbf{r}) = \alpha_0$ (Section 1.2).

Local bifurcation: A bifurcation in which a pair of nulls are created or destroyed (Section 1.4.1).

Magnetic Breakout: the opening of initially low-lying field due to shearing; reconnection at a null point high in the corona weakens the overlying field until the lower flux can escape explosively (Chapter 4).

Magnetic Charge Topology (MCT): A technique for using point sources to construct magnetic fields (Section 1.3).

Negative null: A null in which the spine sources are positive and the fan field lines diverge from the null (Section 1.3.2).

Null points: Locations at which the magnetic field vanishes (Section 1.3.2).

Photosphere: In reality, the thin layer of the solar atmosphere closest to the surface (Section 1.1); in MCT, the plane in which the flux sources lie (Section 1.3).

Photospheric domain: A flux domain which includes field lines lying in the photosphere (Section 1.3.2).

Photospheric null: A null lying in the same plane as the flux sources (Section 1.3.2).

Positive null: A null in which the spine sources are negative and the fan field lines converge on the null (Section 1.3.2).

Prone null: A photospheric null whose spine lies in the source plane (Section 1.3.2).

Proper separator: A separator which lies on the boundary of four flux domains (Section 1.3.2).

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Purely coronal domain: A flux domain with no photospheric field lines (Section 1.3.2).

Quasi-static equilibrium: The assumption that structures moving far slower than the Alfvén speed can be treated as stationary (Section 1.2

Reynolds number (magnetic): The ratio of the advection term to the diffusion term in the induction equation 1.4; it works out to be

$$R_m = \frac{v_0 L_0}{\eta}$$

(Equation 1.5)

Separator: A field line connecting two magnetic null points, or equivalently the intersection of two separatrix surfaces (Section 1.3.2). See also *proper* and *half* separators.

Separatrix: A surface made up of fan field lines, dividing space into different flux domains (Section 1.3.2).

Skeleton: The basic structure of a magnetic field, consisting of any *null points*, *separatrix* surfaces, *spine* field lines and *separators* (Section 1.3.2).

Spine: An isolated field line leading into or away from a null point, perpendicular to the fan (Section 1.3.2).

Spine sources: The sources to which the spine field lines of a null connect. (Section 1.3.2).

Superloop: A theoretical construct representing a coronal loop. It connects two boundary layers, one of which contains many positive sources, and the other of which contains many negative sources (Section 5.1).

Trunk: A large number of separatrix surfaces very close together (Section 5.4).

Upright null: A photospheric null whose spine is orthogonal to the source plane (Section 1.3.2).

Zero curve: A curve on which two magnetic components (usually B_x and B_y) are equal to zero (Section 3.2).

Zero surface: A surface on which one of the magnetic components (e.g. B_z) is equal to zero (Section 3.2).
Appendix A: Some useful proofs

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Theorem 1 A separator connects two nulls if and only if the fan of one null is bounded in part by the spine of the other (Figure 6.2).

Proof: Let a separator connect two nulls, so that their fan planes intersect. Consider field lines beginning a small distance δ from one spine (without loss of generality, the red one). These field lines approach the red null and spread out close to the red fan. Because the red and blue fans intersect in the separator (which ends in the null), the blue fan must separate these field lines into different flux domains. One of the field lines must lie in the blue plane. Since δ can be reduced without limit, the blue fan must be bounded by the red spine. A similar argument can be used for the red fan and the blue spine.

For the other implication, let one fan (blue) be bounded by the spine of the other null (red). Consider field lines in the red fan plane. Because the red spine and red fan are non-coplanar (in the potential case, they are orthogonal), the red fan and blue fan must intersect, giving rise to a separator between the two.

Theorem 2 No stable coronal nulls exist in a three-source configuration.

Proof: Assume an isolated coronal null exists. The magnetic field is given by Equation 1.8:

$$\mathbf{B}(\mathbf{r}) = \sum_{i=1}^{n} \epsilon_i \frac{\mathbf{r} - \mathbf{r}_i}{|\mathbf{r} - \mathbf{r}_i|^3},\tag{A.1}$$

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Figure A.2: A separator joins two nulls if and only if the fan of one is bounded by the spine of the other.

with n = 3.

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There exists, by assumption, a point $\mathbf{r}_{\mathbf{N}} = (x_N, y_N, z_N)$ where $\mathbf{B}(\mathbf{r}_{\mathbf{N}}) = \mathbf{0}$ and $z_N > 0$. Without loss of generality, we can re-scale the geometry so that one source lies at the origin with strength 1, and a second source lies at the point (1, 0). This gives a simplified equation for the field:

$$\mathbf{B}(\mathbf{r}) = \frac{\mathbf{r}}{|\mathbf{r}|^3} + \epsilon_1 \frac{\mathbf{r} - \hat{\mathbf{x}}}{|\mathbf{r} - \hat{\mathbf{x}}|^3} + \epsilon_2 \frac{\mathbf{r} - \mathbf{r_2}}{|\mathbf{r} - \mathbf{r_2}|^3}.$$
(A.2)

For photospheric sources, the *z*-component of Equation A.2 is given by:

$$0 = \frac{z}{|\mathbf{r}|^3} + \epsilon_1 \frac{z}{|\mathbf{r} - \hat{\mathbf{x}}|^3} + \epsilon_2 \frac{z}{|\mathbf{r} - \mathbf{r_2}|^3}.$$
 (A.3)

Since, by assumption, z > 0, it follows that

$$0 = \frac{1}{|\mathbf{r}|^3} + \epsilon_1 \frac{1}{|\mathbf{r} - \hat{\mathbf{x}}|^3} + \epsilon_2 \frac{1}{|\mathbf{r} - \mathbf{r}_2|^3}.$$
 (A.4)

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The y-component of A.2 is given by

$$0 = \frac{y}{|\mathbf{r}|^3} + \epsilon_1 \frac{y}{|\mathbf{r} - \hat{\mathbf{x}}|^3} + \epsilon_2 \frac{y - y_2}{|\mathbf{r} - \mathbf{r}_2|^3},\tag{A.5}$$

or, using Equation A.4,

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$$0 = \epsilon_2 \frac{-y_2}{|\mathbf{r} - \mathbf{r_2}|^3} \tag{A.6}$$

This means that the third source must lie on the x-axis, and the three sources are colinear. Because of the symmetry of such a case, it is topologically unstable, contradicting our original assumption.

In fact, the symmetry implies that the null is not isolated, but forms part of a null ring.

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Appendix B: Details of calculations and graphics

Throughout this thesis, nulls have been found, field lines calculated, topologies depicted and bifurcation diagrams drawn, without any explanation as to the methods used to do so. This Appendix aims to redress that imbalance. Many of the codes used were developed by Daniel Brown.

B.1 Null points

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Null points are found using a Newton-Raphson solver. Initial guesses are generated in several ways: in some cases, it is relatively clear where nulls ought to be and possible to make an explicit guess. In others, the zero surfaces method is automated for use in the source place. In still others, a three-dimensional network is used to find these surfaces in space.

B.2 Skeletons and field lines

The skeleton of the field is found by extrapolating certain field lines from the null points. As described in Section 1.3.2, these spine and fan field lines are determined by the eigenvectors of the null matrix of Equation 1.9 near a null point \mathbf{x}_N .

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One of the eigenvalues λ_1 is of opposite sign to the others; the eigenvector \mathbf{e}_1 corresponding to this eigenvalue determines the direction of the spine. We take the start point of the spine field lines to be $\mathbf{x}_N \pm \epsilon \mathbf{e}_1$.

The other two eigenvectors (\mathbf{e}_2 and \mathbf{e}_3) define a fan plane. We take the start points to be $\mathbf{x}_0(\theta_i) = \mathbf{x}_N + \epsilon [\cos(\theta_i)\mathbf{e}_2 + \sin(\theta_i)\mathbf{e}_3]$, where $0 < \theta_i < 2\pi$ and $i = 1 \dots n_{fan}$ where n_{fan} is the number of desired fan field lines.

Field lines $\mathbf{r}(s)$ are computed by integrating the field-line equation:

$$\frac{\mathrm{d}\mathbf{r}(s)}{\mathrm{d}s} = \frac{\mathbf{B}[\mathbf{r}(s)]}{|\mathbf{B}[\mathbf{r}(s)]|},\tag{B.1}$$

beginning at some initial point \mathbf{r}_0 , using a Runge-Kutta algorithm with variable step size.

B.3 Separators

Finding separators is not easy, since field lines tend to diverge away from null points. We find approximate separators by examining the connectivity of fan field lines from each null point. If the connectivity changes between two field lines beginning at $\mathbf{x}_0(\theta_i)$ and $\mathbf{x}_0(\theta_{i+1})$, a separator field line must begin at $\mathbf{x}_0(\theta)$ for some θ between θ_i and θ_{i+1} .

By gradually refining the value of θ , we can usually approximate a separator for a good part of its length. By applying the same approach at the null point to which the separator connects, we can find an approximation for another section of the separator.

B.4 Drawing topologies

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The topologies are plotted in three-dimensions by IDL; we then use the xfig package to trace, clarify and add colour to the diagrams by approximating certain field lines as splines.

Our approximate separators are treated as single field lines and turned into splines in the same way.

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B.5 Drawing bifurcation diagrams

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Bifurcation diagrams are automated as far as is possible - for instance, local doubleseparator bifurcations are found by varying parameters until the sign of a null point changes, and global separator bifurcations by tracking the connectivity of the separatrix field lines.

However, the quasi-bifurcations are not easy to find automatically since they involve changes at a large distance from the sources. These are simply found by inspection.

Once the data has been collected, we connect nearby bifurcation points using smooth curves. There is some scope for inaccuracy in this stage, but the overall picture will be at least approximately correct.

Appendix C: An unlikely-looking coronal null state

We have been unable to find a proof that a coronal null is impossible for a four-source case with three sources of the same sign (without loss of generality, assumed positive), although no such null has (so far as we know) ever been found.

In this Appendix, we show what the topology of such a state would have to look like and provisionally discount it.

We make the assumptions here that upright nulls cannot undergo a local double-separator bifurcation and that the four-source case lacks sufficient complexity to allow a coronal local separator bifurcation.

Then, the basic state for such a configuration consists of two positive prone nulls (i.e., either the separate or the enclosed state). We assume that one of these nulls undergoes a local double-separator bifurcation leaving the topology shown in Figure C.1.

The fan of the coronal null now forms a dome bounded in the photosphere by the spine field lines of the remaining, positive null, both of which connect to the negative source. These two, then, are connected by a half-separator (since the negative null is homovertebraic).

The reason this topology seems unlikely is that every local double-separator bifurcation observed (as far as we know) has taken place along an existing separator. In this case, no separator exists before the bifurcation.

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Figure C.1: Only possible topology for coronal null in four-source case with three sources of the same sign. The fan of the coronal null forms a dome bounded by the fan of the blue photospheric null. The separator between the two is a half-separator.

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