

Potential Energy



Work vs. Potential Energy

Work

- **Any** force does work
- $\Delta K = W$ always
- Particle returns to start: $W \neq 0$ (perhaps)

$$W = \int_P^Q F_x dx$$

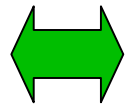
Potential Energy

- Potential energy = -work done by conservative force
- $\Delta K = -\Delta U$ **ONLY** when all forces are conservative forces
- Particle returns to start: $\Delta U = 0$

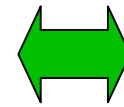
$$\Delta U = U(P) - U(Q)$$

Conservative forces

Work indep't
of path



No work over
closed path



No losses

gravity: $\vec{\mathbf{F}} = -mg \hat{\mathbf{j}}$

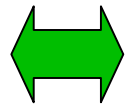
$$U = m g y$$

spring: $\vec{\mathbf{F}} = -kx \hat{\mathbf{i}}$

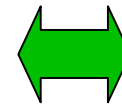
$$U = \frac{1}{2} k x^2$$

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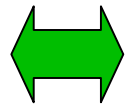
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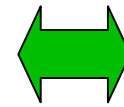
choose $U=0$ at level $y=0$

Conservative forces

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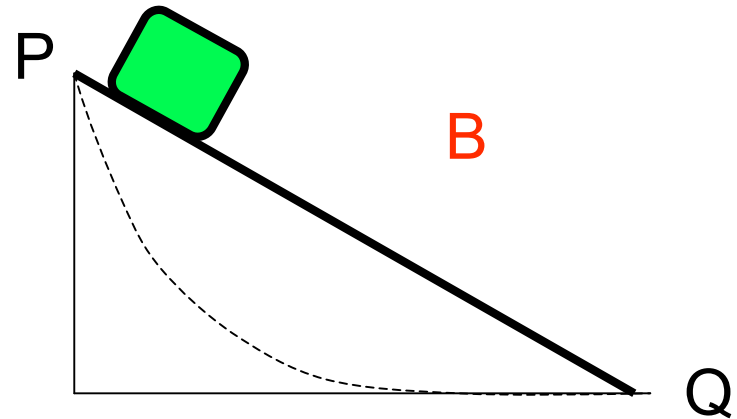
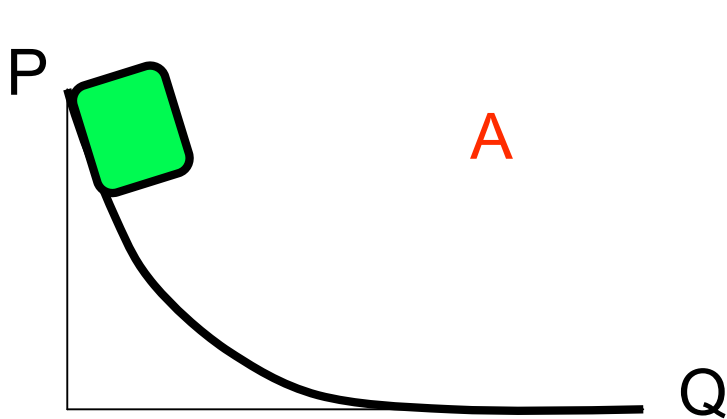
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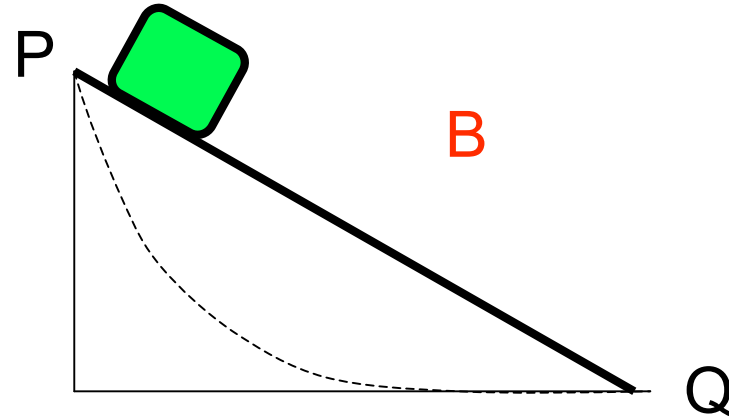
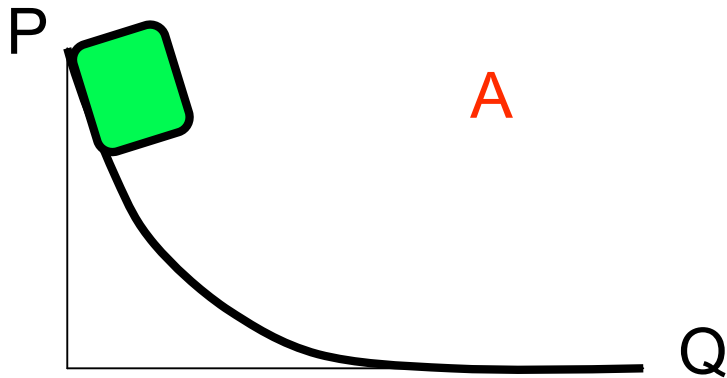
$$U = \frac{1}{2} k x^2$$

choose $U=0$ at rest position ($x=0$)



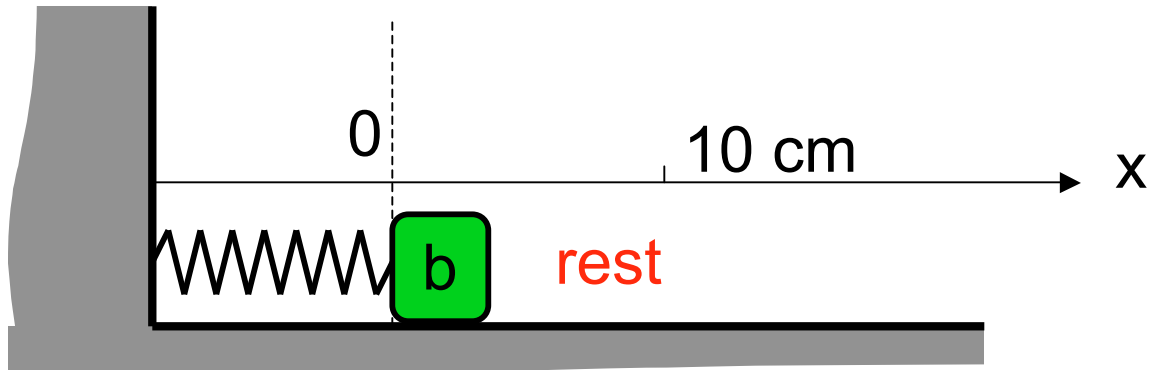
Block slides from P \rightarrow Q along two different frictionless slides. On which slide does gravity do more work?

- A $W_A > W_B$
- B $W_A = W_B$
- C $W_A < W_B$

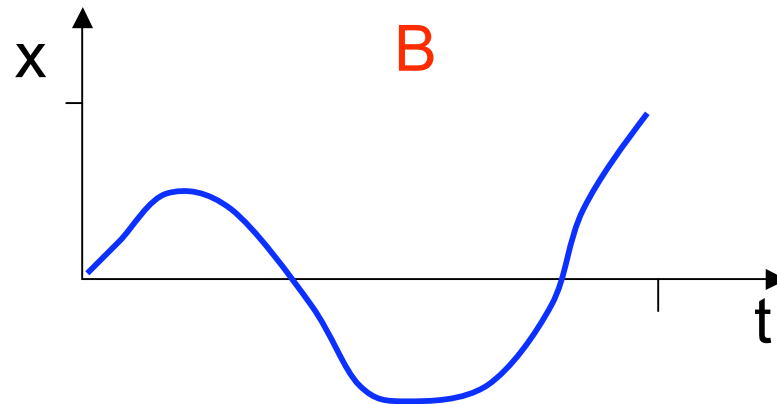
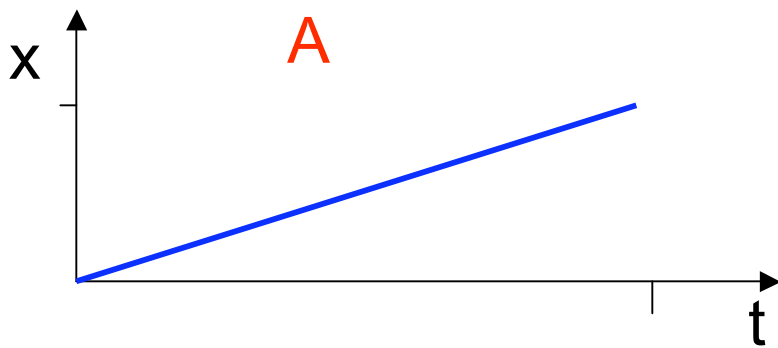


Block slides from P \rightarrow Q along two different frictionless slides. Which block reaches point Q quicker?

- A $\Delta t_A > \Delta t_B$
- B $\Delta t_A = \Delta t_B$
- C $\Delta t_A < \Delta t_B$

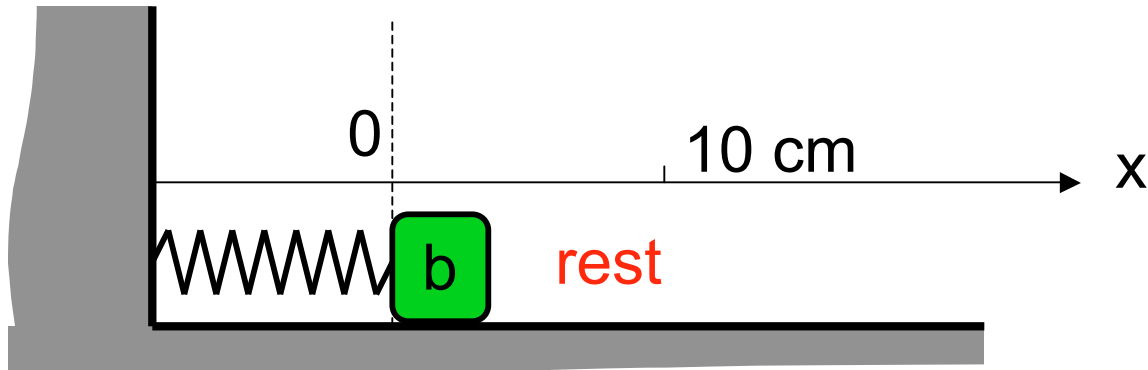


The spring is stretched to $x=10$ cm in two different ways by dragging the block across a table with friction.

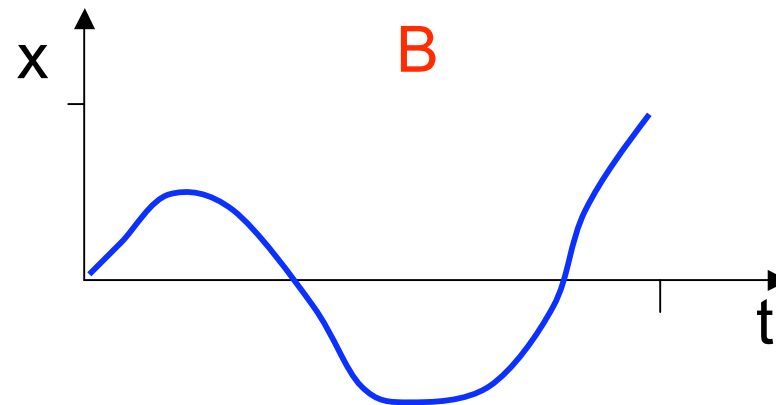
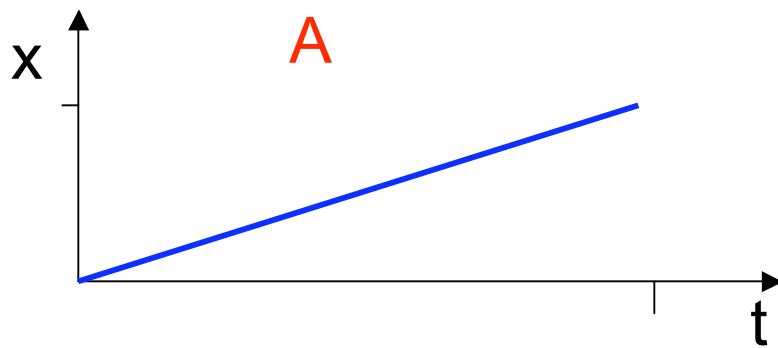


In which case is more work done **by the spring** on the block?

- A $W_A > W_B$
- B $W_A = W_B$
- C $W_A < W_B$



The spring is stretched to $x=10$ cm in two different ways by dragging the block across a table with friction.



In which case is greater work done **by friction** on the block?

- A $W_A > W_B$
- B $W_A = W_B$
- C $W_A < W_B$

Work vs. Potential Energy

Work

- Work done **on particle A** by the force exerted by B.

$$W_{BA} = \int_P^Q F_{BA} dx$$

Potential Energy

- Potential energy is “stored” **within the AB system**

$$\Delta U = -W_{BA} = -\int_P^Q F_{BA} dx$$

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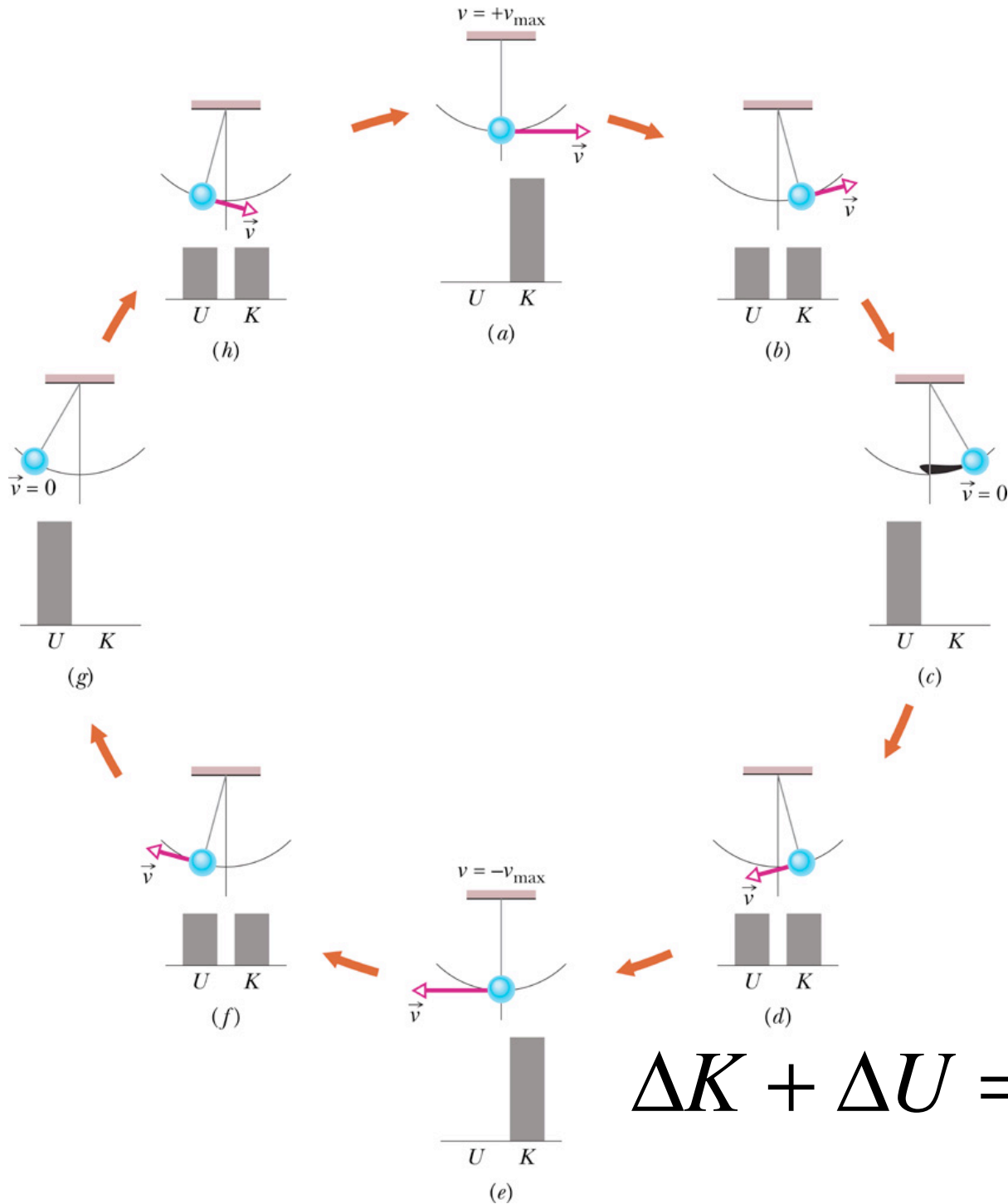
- Potential energy is “stored” **within the AB system**

$$\Delta U = -W_{BA} = -\int_P^Q F_{BA} dx$$

Energy is transferred **from** system **to** do work on A

$$\Delta K + \Delta U = 0 = \Delta(\underbrace{K + U}_{E_{\text{mech}}})$$

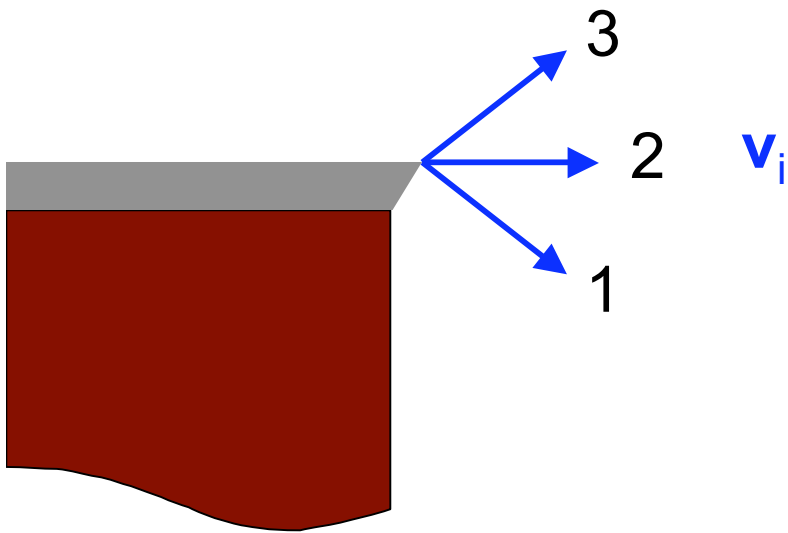
when all forces are conservative



conservation of
mechanical
energy

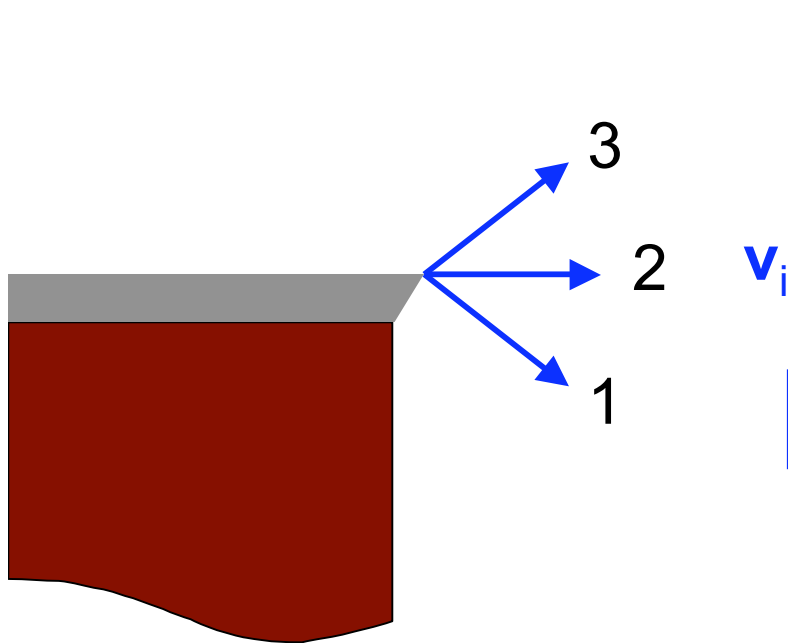
$$\Delta K + \Delta U = 0 = \Delta(K + U)$$

Three identical balls are thrown from the top of a building, with the same initial speeds, but at different angles. Air resistance may be neglected. Rank the speeds with which they strike the ground.



- A $|\mathbf{v}_{f1}| > |\mathbf{v}_{f2}| > |\mathbf{v}_{f3}|$
- B $|\mathbf{v}_{f1}| < |\mathbf{v}_{f2}| < |\mathbf{v}_{f3}|$
- C $|\mathbf{v}_{f1}| = |\mathbf{v}_{f3}| > |\mathbf{v}_{f2}|$
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D $|\mathbf{v}_{f1}| = |\mathbf{v}_{f2}| = |\mathbf{v}_{f3}|$

$$\Delta K + \Delta U = W_{\text{mech}} = 0$$

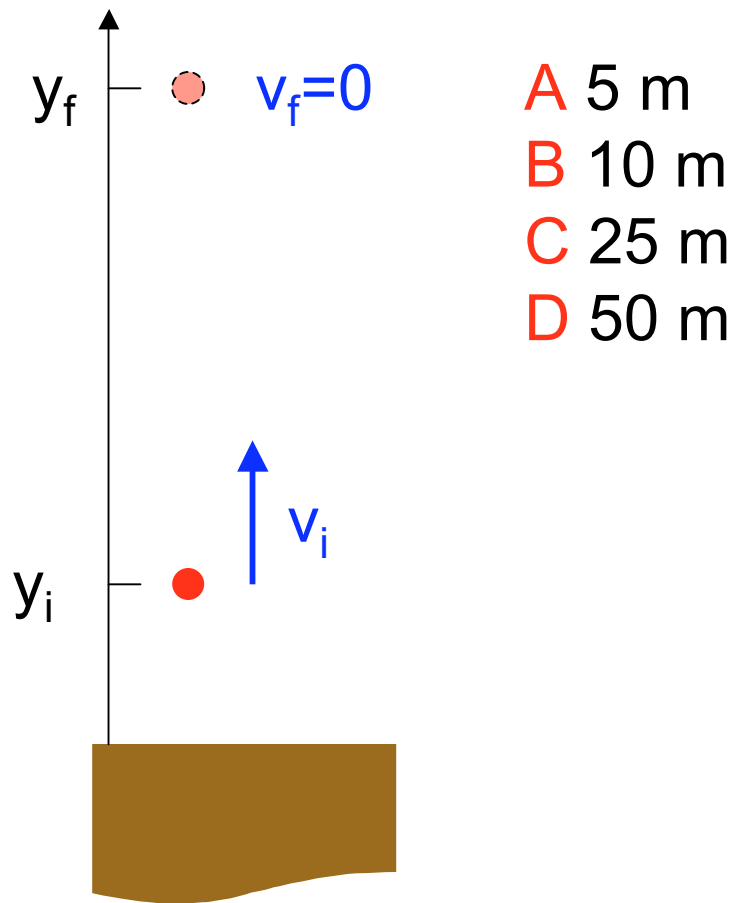
$$\Delta U = Mg(y_f - y_i) = \text{same}$$

$$K_i = M|\mathbf{v}_i|^2/2 = \text{same}$$

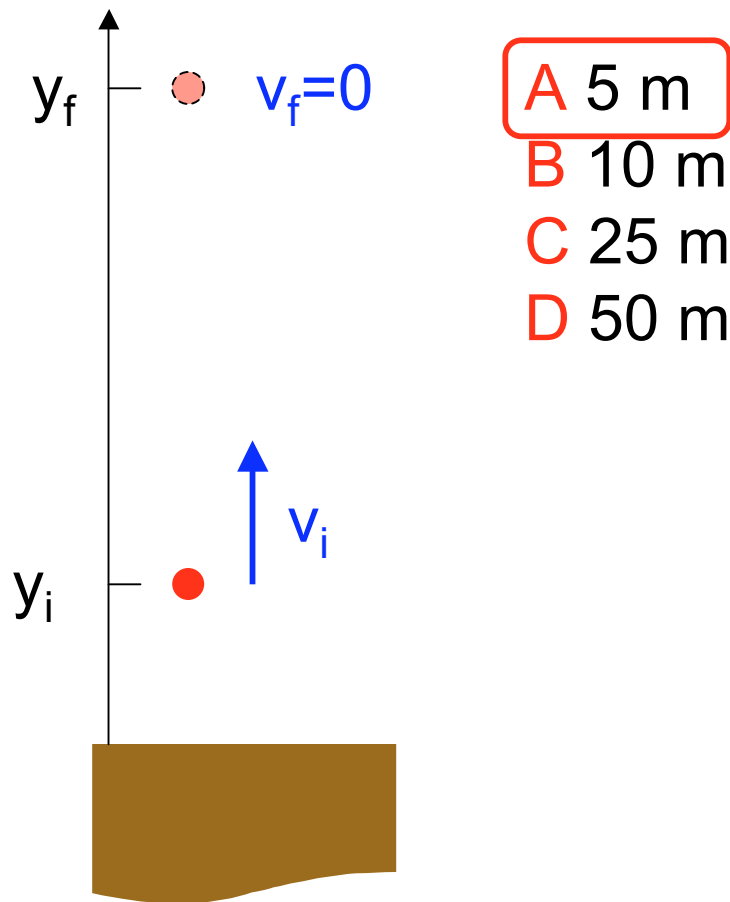
$$K_f = K_i - \Delta U = \text{same}$$

$$|\mathbf{v}_f|^2 = 2K_f/M = \text{same}$$

A ball is thrown upward with an initial velocity $\mathbf{v}_i = 10 \text{ m/s } \mathbf{j}$. How high does it go?



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$$\Delta K + \Delta U = \Delta E_{\text{mech}} = 0$$

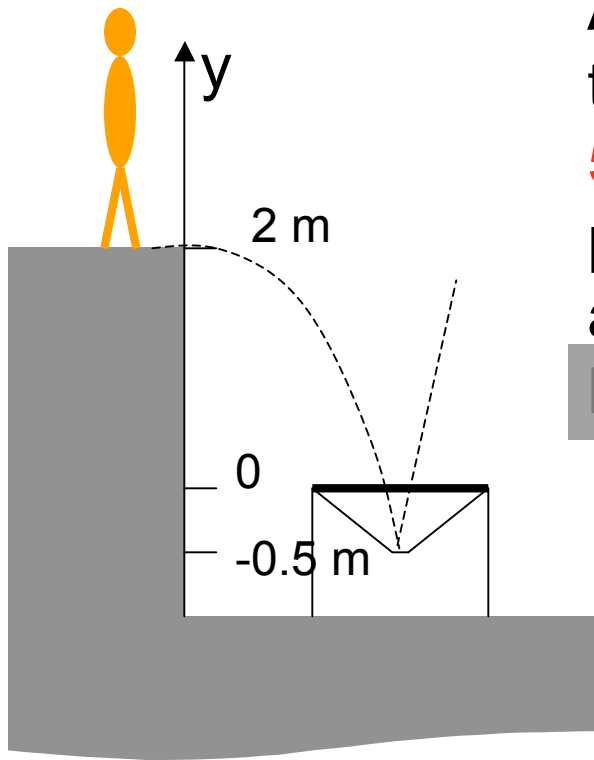
$$K_i = Mv_i^2/2 = 50 \text{ m}^2/\text{s}^2 M$$

$$K_f = Mv_f^2/2 = 0$$

$$\Delta K = K_f - K_i = -50 \text{ m}^2/\text{s}^2 M$$

$$\Delta U = Mg(y_f - y_i) = -\Delta K = 50 M$$

$$y_f - y_i = (50 \text{ m}^2/\text{s}^2)/g = 5 \text{ m}$$



An acrobat of mass **60 kg** jumps **2.0 m** onto a trampoline. She depresses the trampoline **50 cm** as she bounces off it. Treat her as a particle and the trampoline as a spring.

a. What is its spring constant?

b. What is her maximum acceleration?

from rest @ $y=2.0$ m to rest @ $y=-0.5$ m

$$\Delta K = 0 \implies \Delta U = 0 = \Delta U_g + \Delta U_s$$

$$\Delta U_g = mg(-0.5 - 2.0) = -1500 \text{ J}$$

$$\Delta U_s = 0.5 k [(0.5 \text{ m})^2 - 0] = k/8$$

$$k = 12,000 \text{ N/m}$$

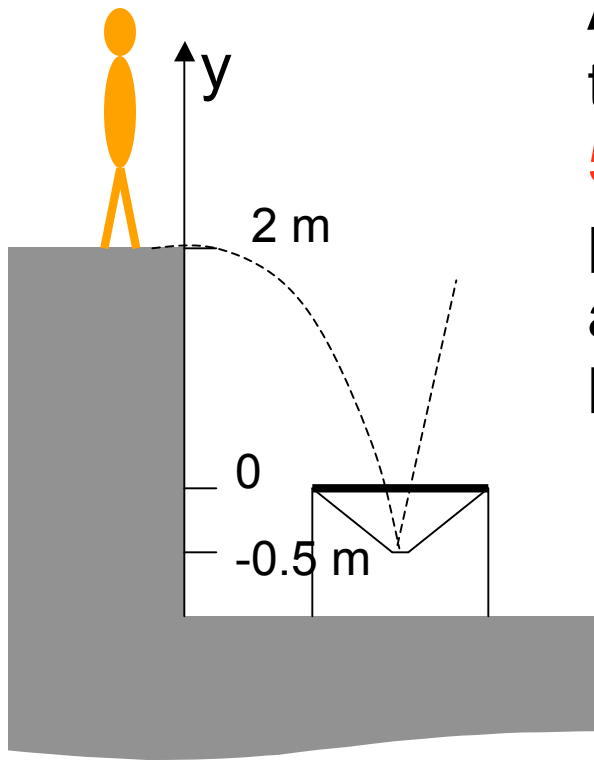
A 240 N/m

B 480 N/m

C 1200 N/m

D 6000 N/m

E 12000 N/m

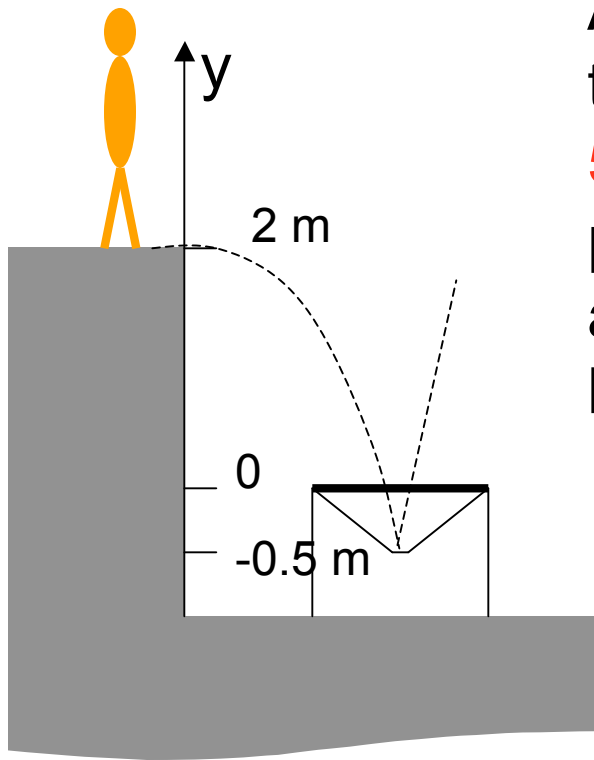


An acrobat of mass 60 kg jumps 2.0 m onto a trampoline. She depresses the trampoline 50 cm as she bounces off it. Treat her as a particle and the trampoline as a spring.

- What is its spring constant?
- What is her maximum acceleration?

$$k = 12,000 \text{ N/m}$$

- A 45 m/s^2
- B 50 m/s^2
- C 90 m/s^2
- D 100 m/s^2



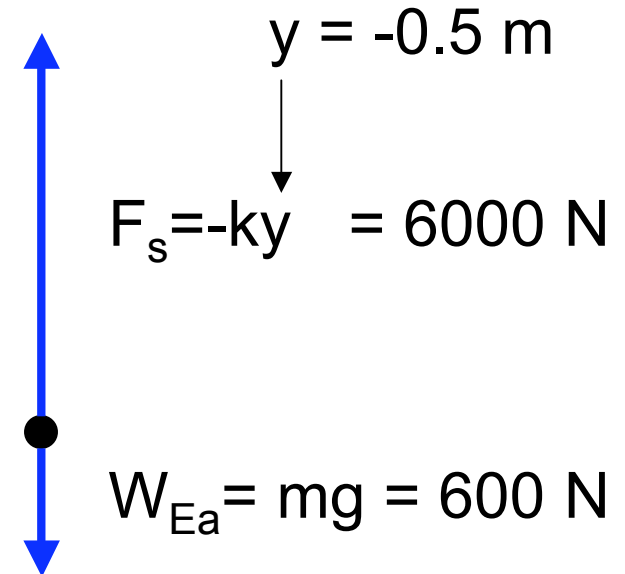
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$$F_{\text{net}} = 5400 \text{ N} = ma$$

$$a = 90 \text{ m/s}^2$$

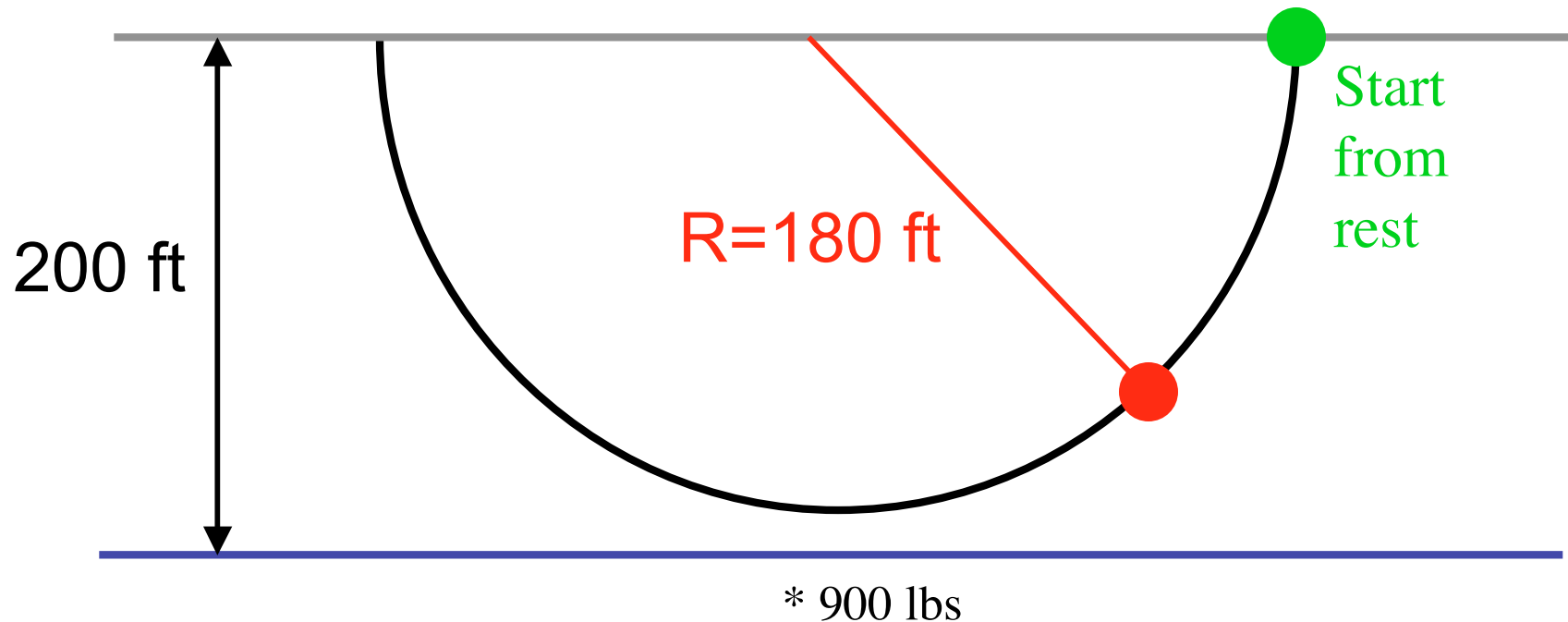


- 45 m/s²
- 50 m/s²
- 90 m/s²
- 100 m/s²

Do Not Try This...

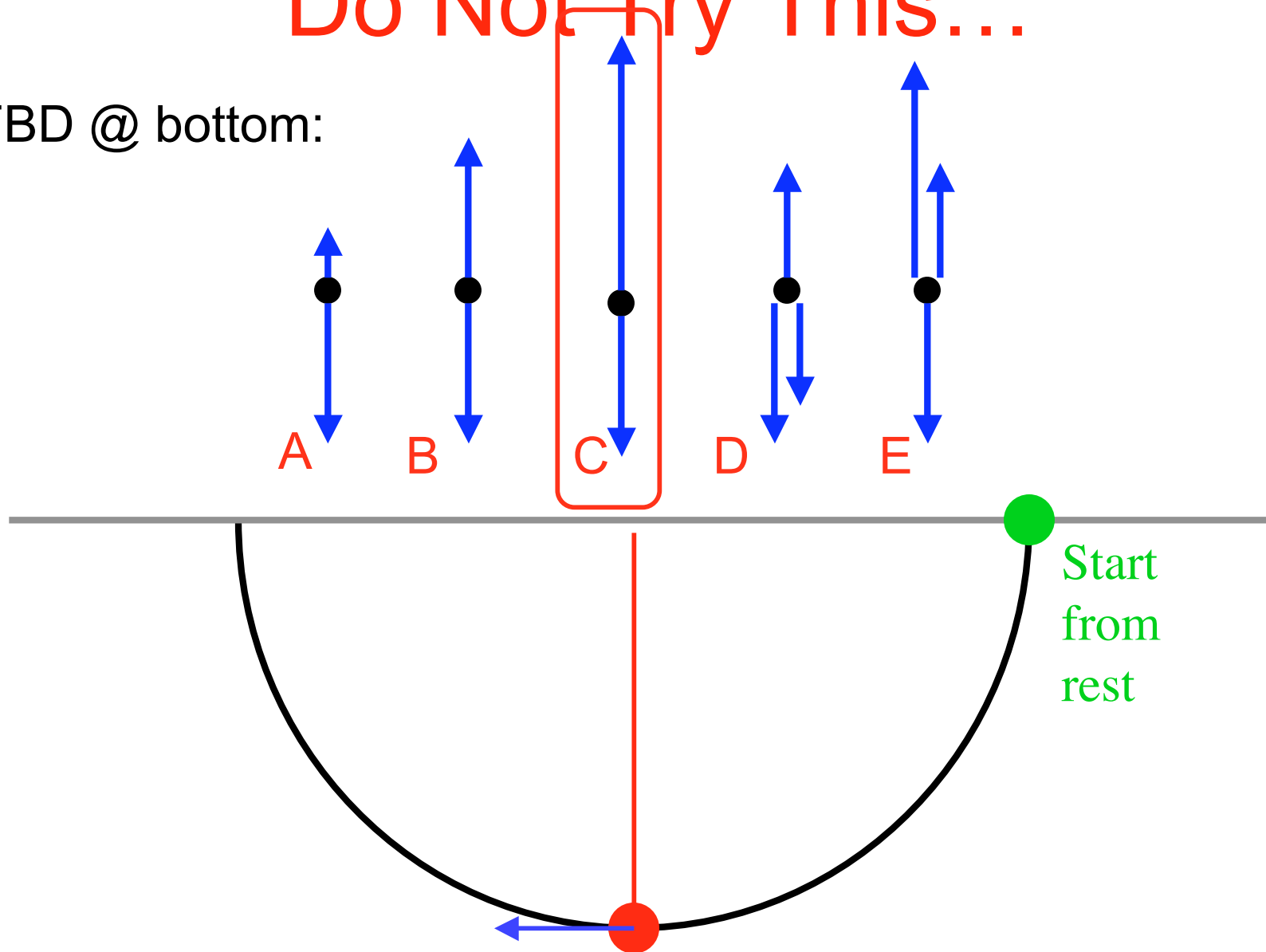


On April 27, 1999 Steve Trotter and 4 other thrill-seekers attempted to “pendulum swing” from the Tampa Bay Skyway Bridge. For added safety they used a steel cable capable of supporting **TWICE** their combined weight*. Was this cable strong enough?



Do Not Try This...

FBD @ bottom:



Do Not Try This...

$$\mathbf{a} = \frac{v^2}{R} \hat{\mathbf{j}} = \frac{2}{m} \frac{K}{R} \hat{\mathbf{j}} = 2g \hat{\mathbf{j}}$$

$$\sum F_y = ma_y = 2mg$$

$$T = 3mg$$

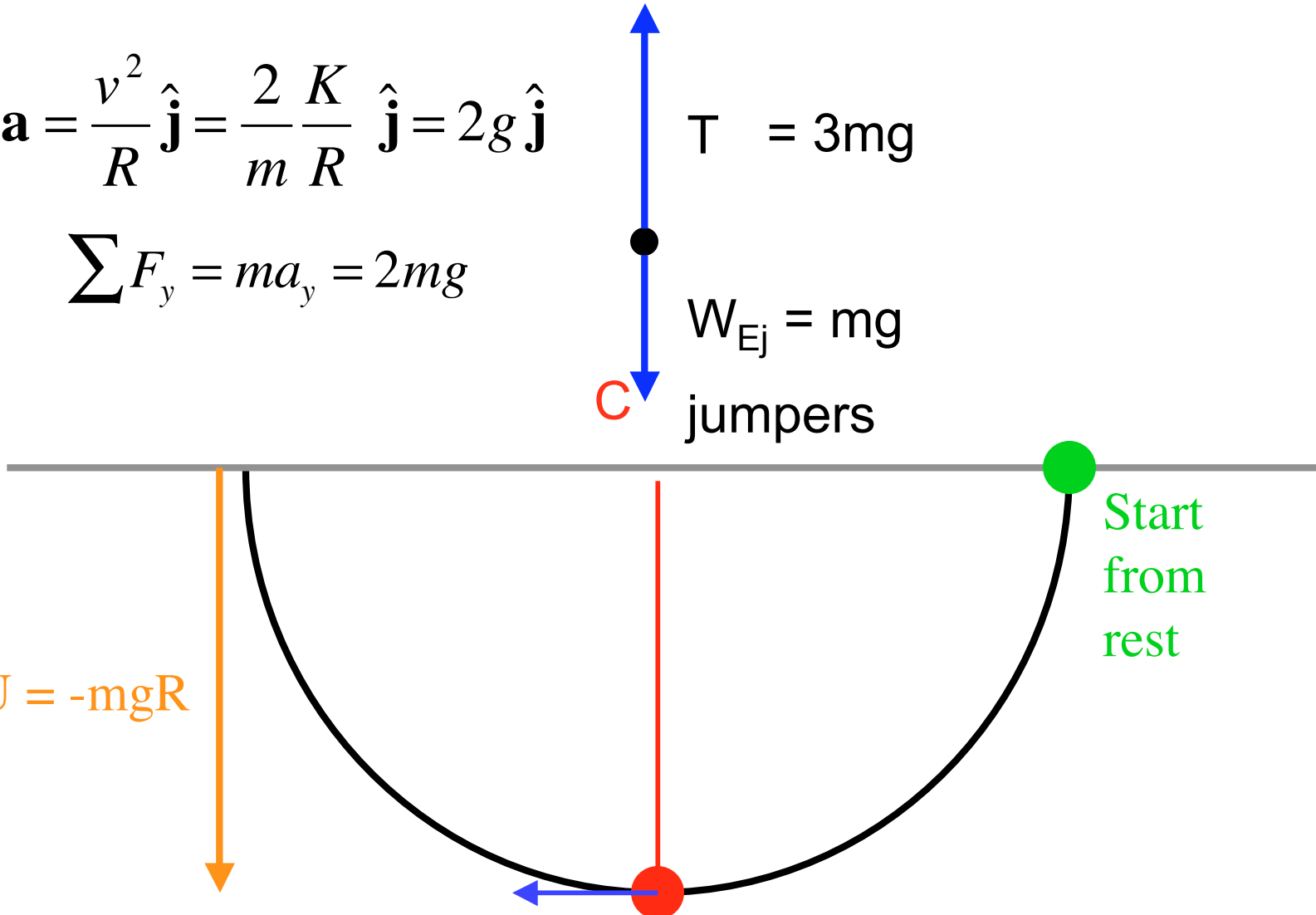
$$W_{Ej} = mg$$

C jumpers

$$\Delta U = -mgR$$

Start
from
rest

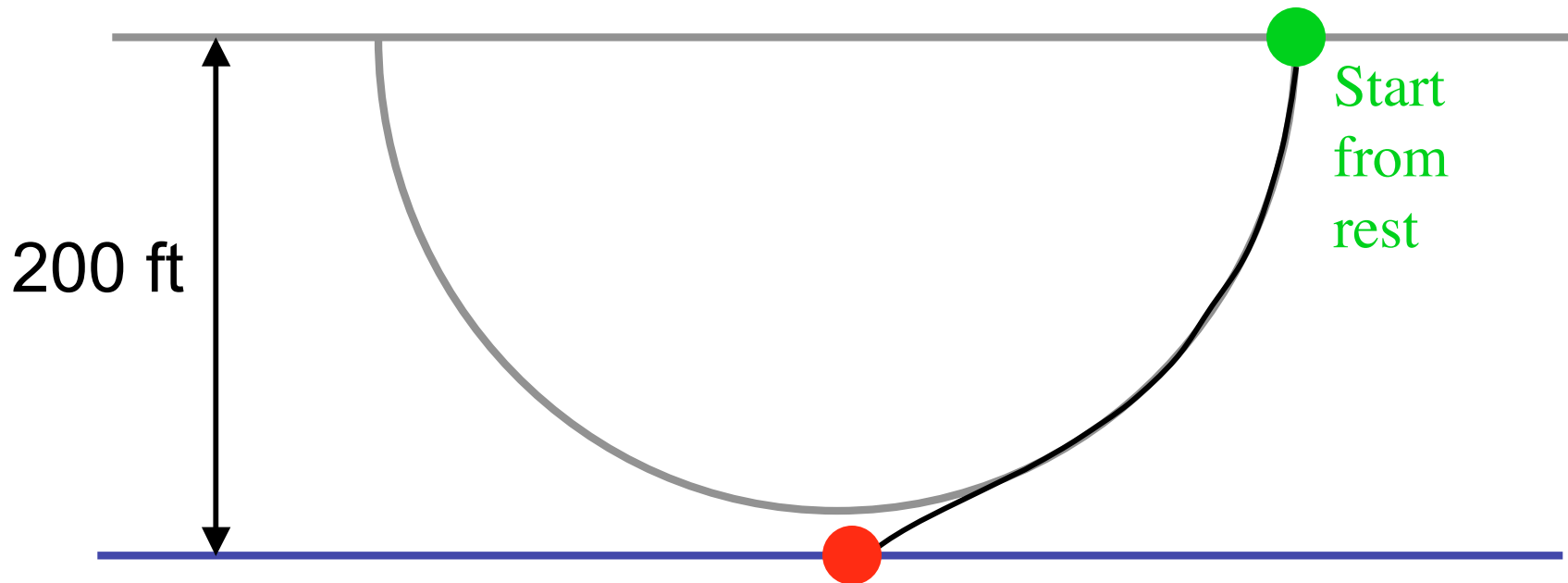
$$\Delta K = -\Delta U = mgR = mv^2/2$$



Do Not Try This...



To both support their weight AND provide centripetal acceleration the cable needed to be rated at least **THREE TIMES** their weight. It was not. The cable broke and four were hospitalized.* How fast were they traveling when they hit the water?

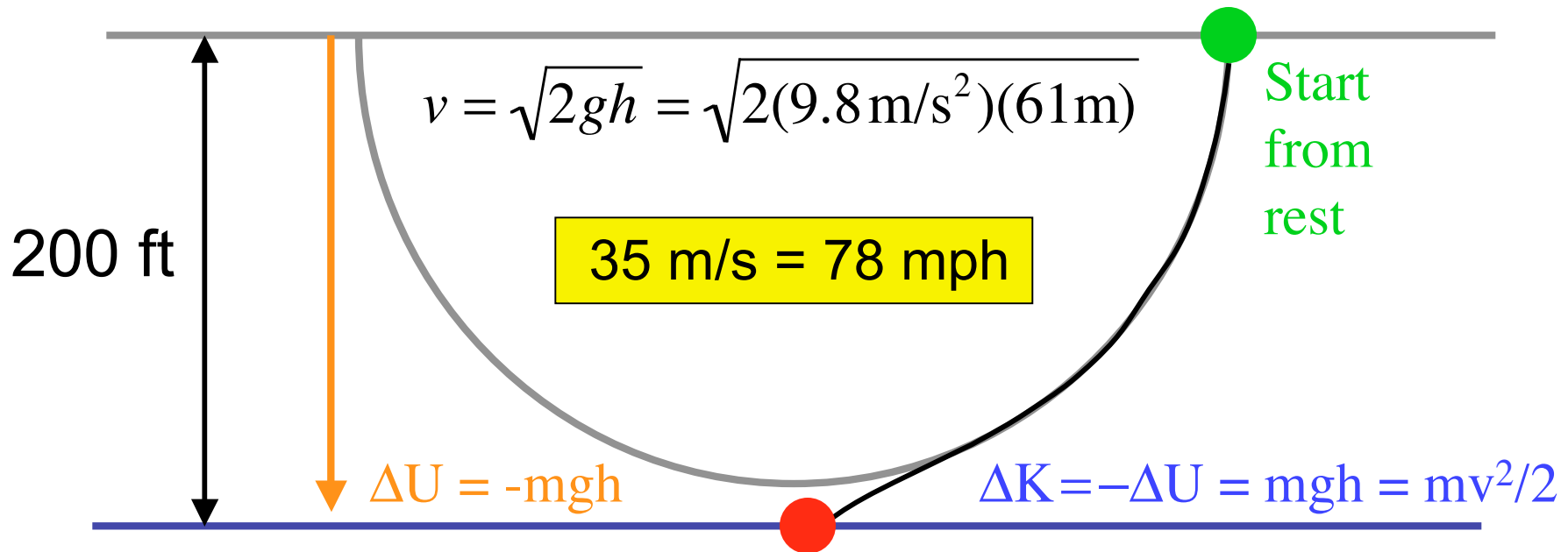


* Amazingly nobody was killed.

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