

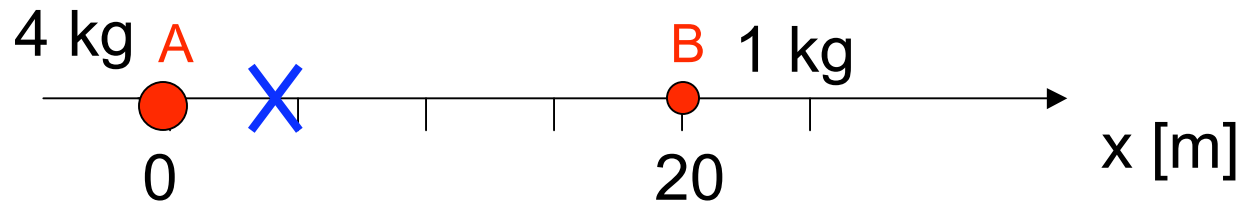
# Center of Mass



## Q from reading

When applying Newton's 2<sup>nd</sup> law  $\sum \vec{F} = m \vec{a}$  to a finite object (i.e. a system of particles) the acceleration  $\vec{a}$  is that of

- A All the particles
- B One particular particle
- C The center of mass of the particle system
- D Trick question: Newton's 2<sup>nd</sup> law applies only to single particles



Where is the CM?

**A**  $x_{\text{CM}} = 4 \text{ m}$

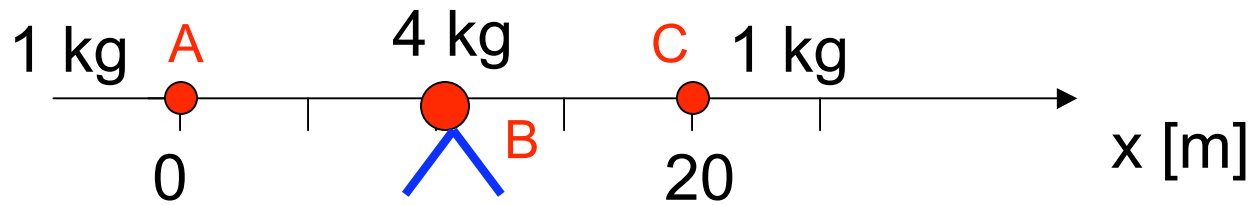
**B**  $x_{\text{CM}} = 5 \text{ m}$

**C**  $x_{\text{CM}} = 10 \text{ m}$

**D**  $x_{\text{CM}} = 15 \text{ m}$

**E**  $x_{\text{CM}} = 20 \text{ m}$

$$x_{\text{CM}} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{(4)(0) + (1)(20)}{4 + 1} = \frac{20 \text{ kg m}}{5 \text{ kg}} = 4 \text{ m}$$



Where is the CM?

A  $x_{CM} = 4$  m

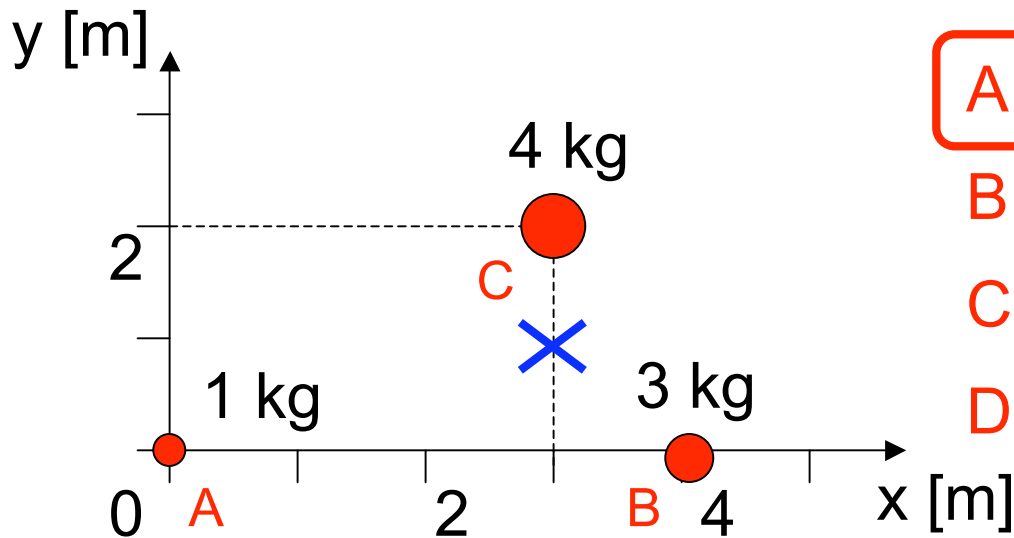
B  $x_{CM} = 5$  m

C  $x_{CM} = 10$  m

D  $x_{CM} = 15$  m

E  $x_{CM} = 20$  m

$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(1)(0) + (4)(10) + (1)(20)}{4 + 1 + 1} = \frac{60 \text{ kg m}}{6 \text{ kg}} = 10 \text{ m}$$



Where is the CM?

**A**  $\mathbf{r}_{CM} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}})\text{m}$

**B**  $\mathbf{r}_{CM} = (2.33\hat{\mathbf{i}} + 0.67\hat{\mathbf{j}})\text{m}$

**C**  $\mathbf{r}_{CM} = (2\hat{\mathbf{i}} + 0.67\hat{\mathbf{j}})\text{m}$

**D**  $\mathbf{r}_{CM} = (2.33\hat{\mathbf{i}} + \hat{\mathbf{j}})\text{m}$

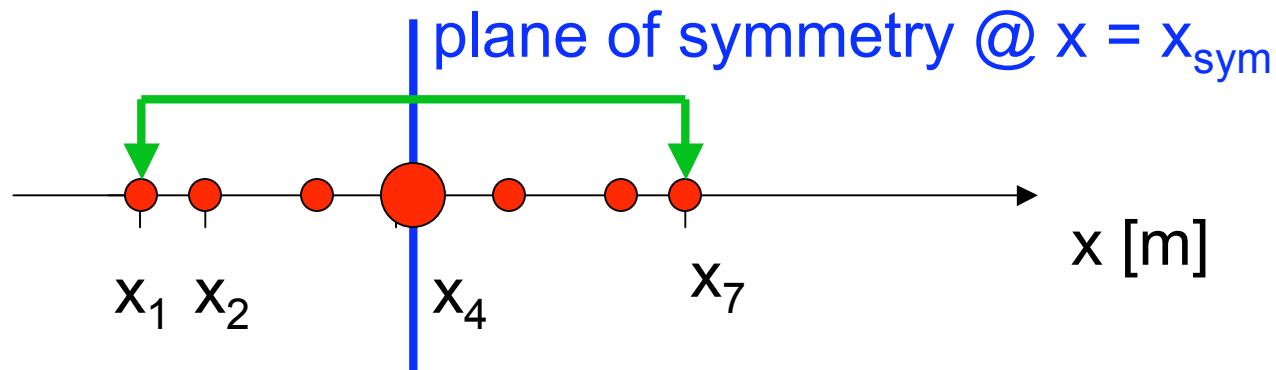
$$x_{CM} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{0 + 12 + 12}{1 + 3 + 4} = 3\text{m}$$

$$y_{CM} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C} = \frac{0 + 0 + 8}{1 + 3 + 4} = 1\text{m}$$

$$\mathbf{r}_{CM} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}})\text{m}$$

$$x_{com} = \frac{1}{M} \sum_{i=1}^7 m_i x_i$$

## Using symmetry



### plane of symmetry:

For every particle to the left of the plane there is an identical particle located an equal distance to the right... like a reflection.

(particles lying within the plane of symmetry are their own reflection)

$$i=1 \leftrightarrow i=7$$

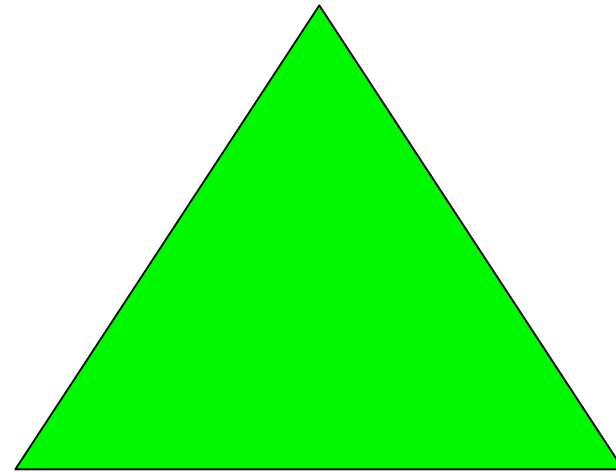
$$\frac{1}{2}(x_1 + x_7) = x_{sym}$$

The center of mass always lies within the plane of symmetry

# Using symmetry

How many different lines of symmetry does the equilateral triangle have?

- A 0
- B 1
- C 2
- D 3
- E 6



equilateral triangle

# Using symmetry

How many different lines of symmetry does the equilateral triangle have?

A 0

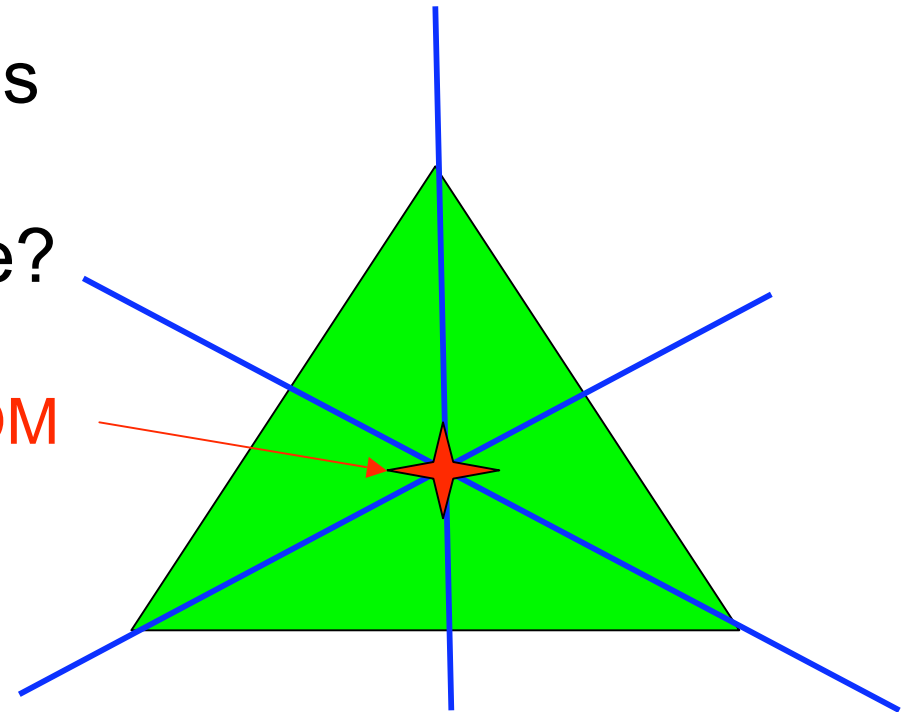
B 1

C 2

D 3

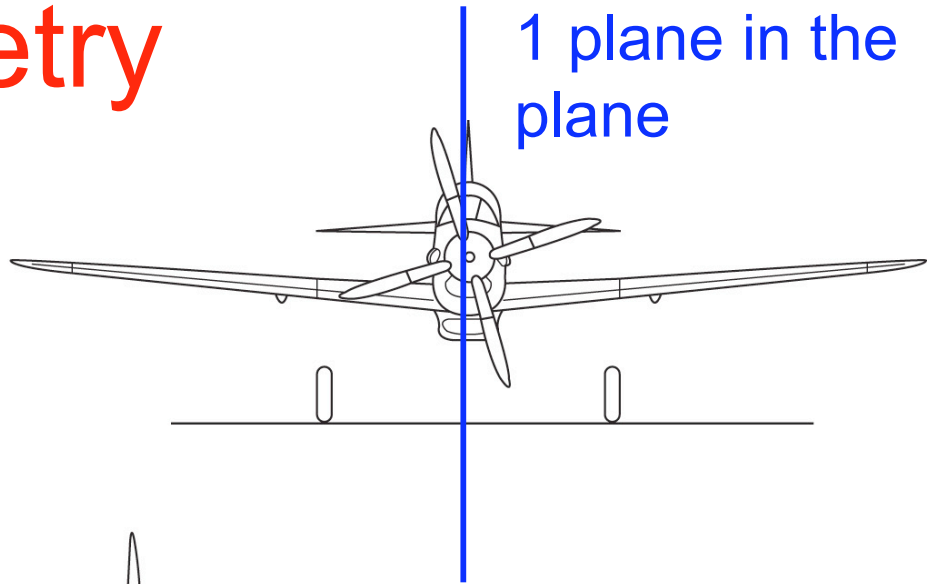
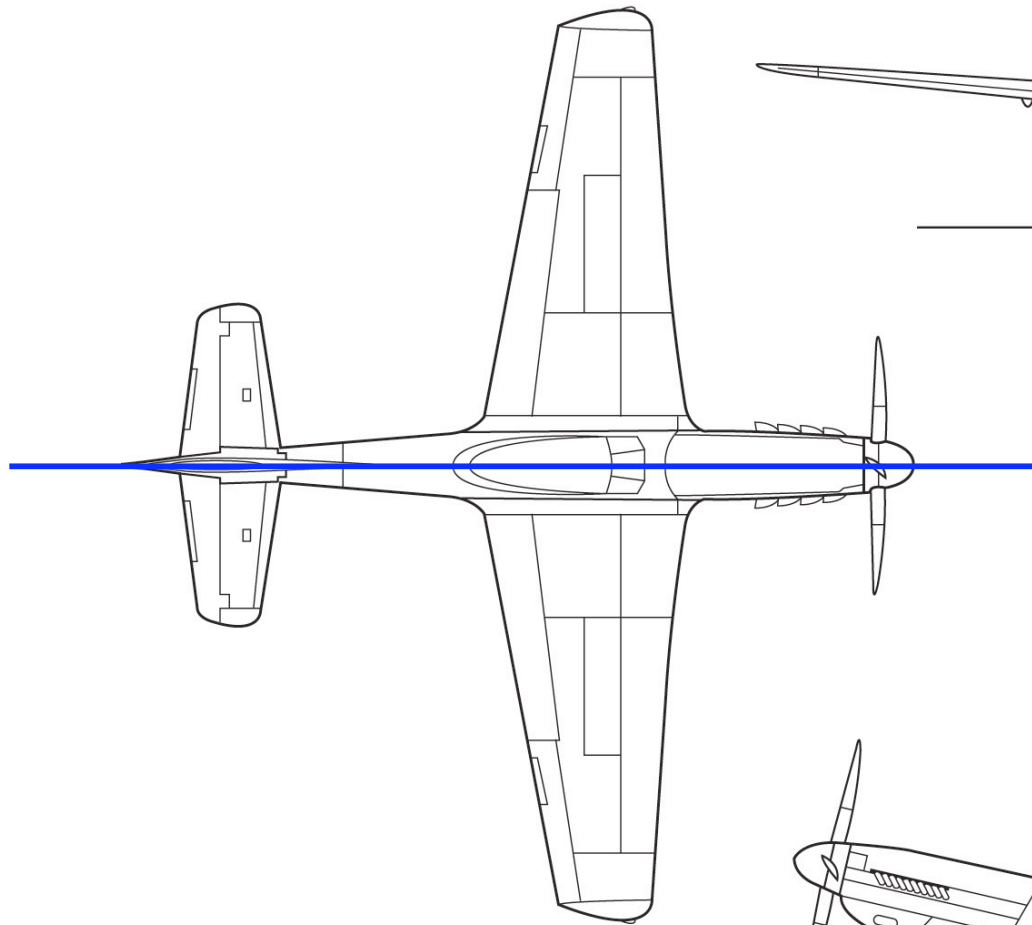
E 6

COM



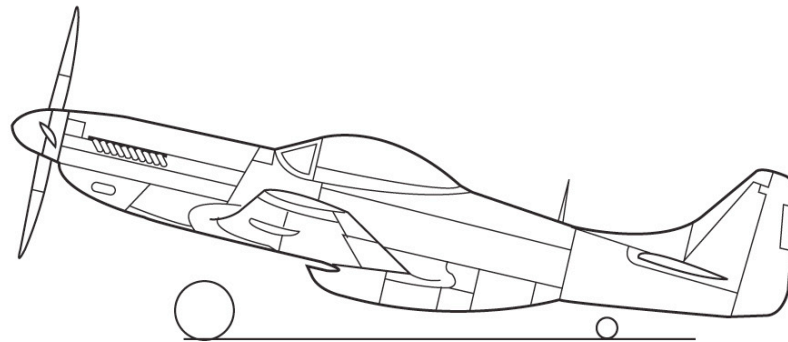
equilateral triangle

# Using symmetry



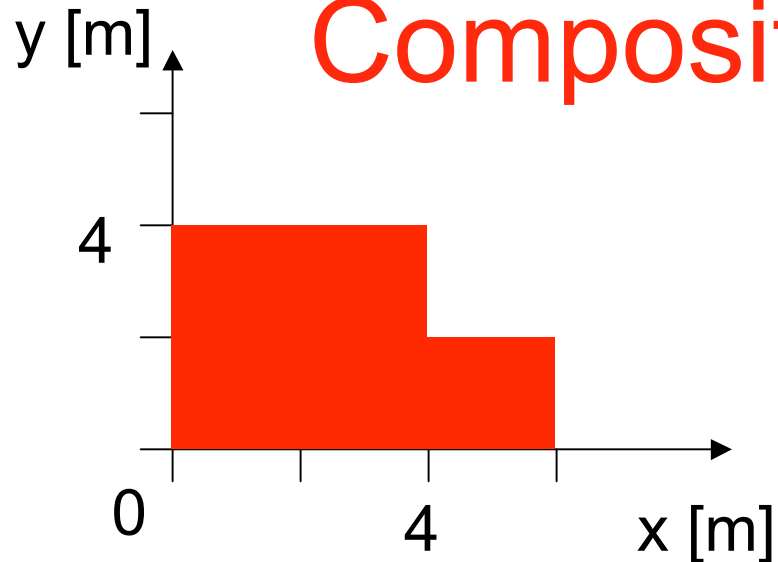
1 plane in the plane

$$x_{com} = \frac{1}{M} \int x \, dm$$



# Composition

A uniform plate is composed of two squares. Where is its CM?



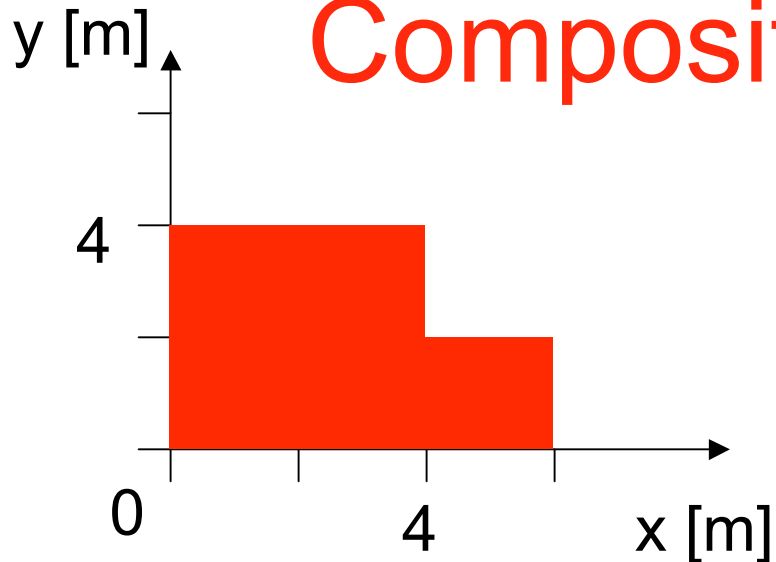
Trick: replace each piece by a particle:

mass: same

location: COM

# Composition

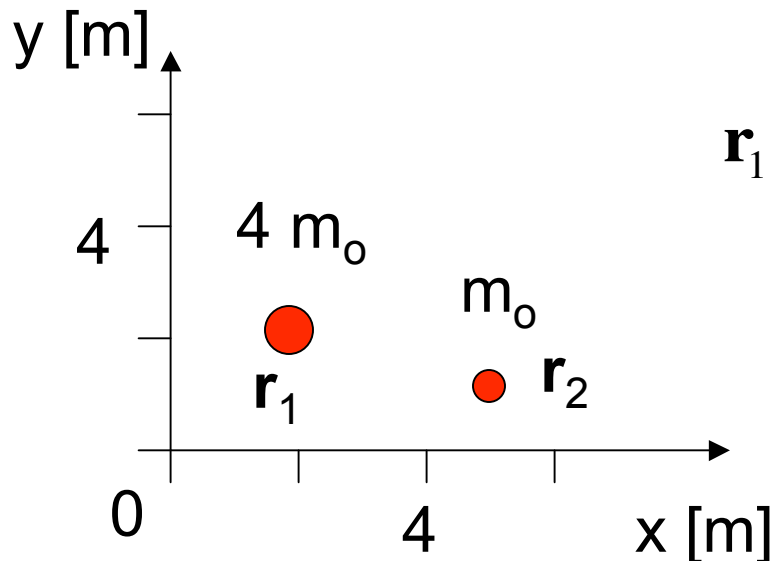
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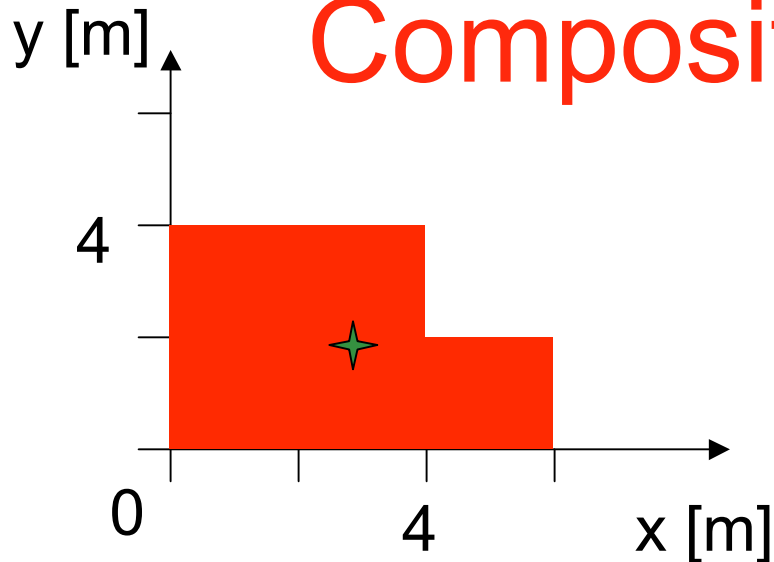
$$\mathbf{r}_1 = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}})m \quad \mathbf{r}_2 = (3\hat{\mathbf{i}} + 1\hat{\mathbf{j}})m$$

$$x_{CM} \frac{2m \cdot 4m_o + 3m \cdot m_o}{4m_o + m_o} = \frac{11}{5}m$$

$$y_{CM} \frac{2m \cdot 4m_o + 1m \cdot m_o}{4m_o + m_o} = \frac{9}{5}m$$

# Composition

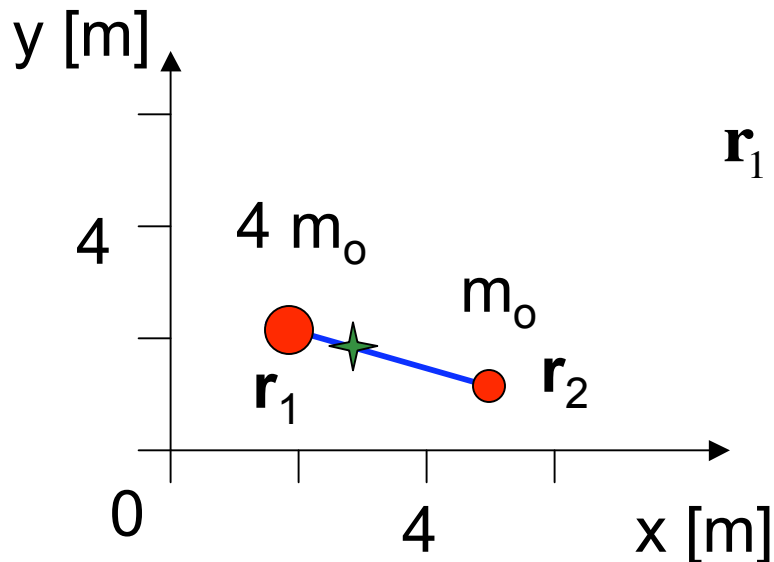
A uniform plate is composed of two squares. Where is its CM?



Trick: replace each piece by a particle:

mass: same

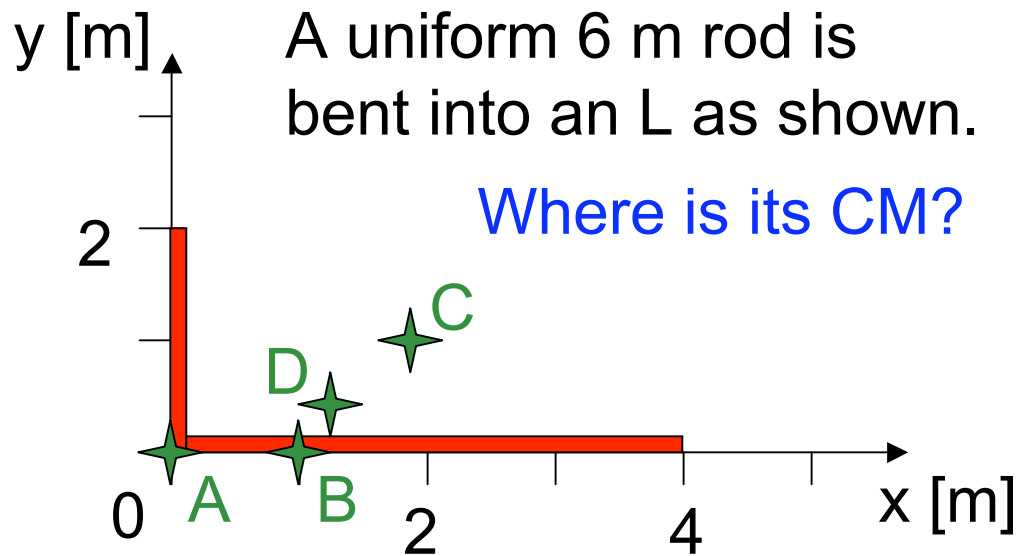
location: COM



$$\mathbf{r}_1 = (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}})m \quad \mathbf{r}_2 = (3\hat{\mathbf{i}} + 1\hat{\mathbf{j}})m$$

$$x_{CM} \frac{2m \cdot 4m_o + 3m \cdot m_o}{4m_o + m_o} = \frac{11}{5}m$$

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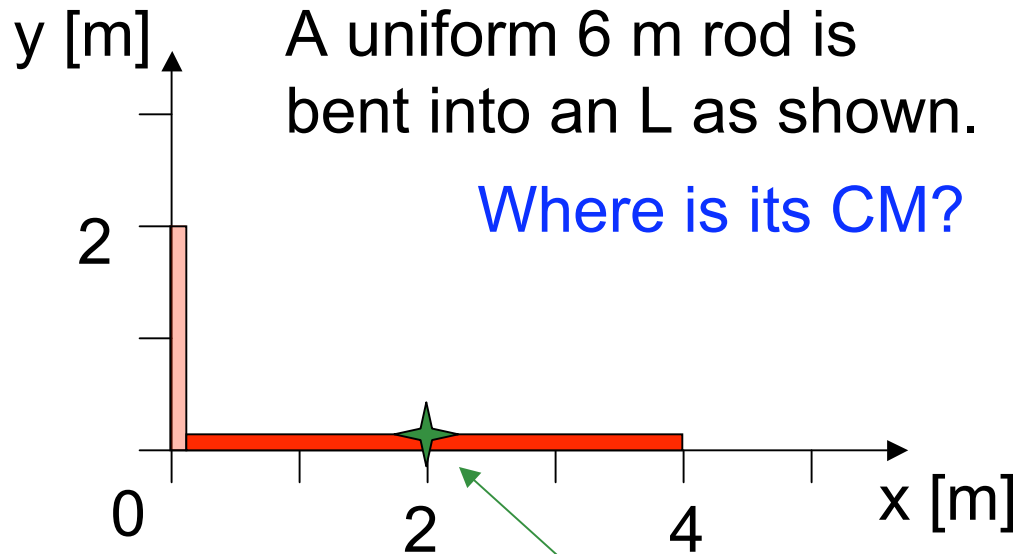


A  $\mathbf{r}_{CM} = (0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}) \text{ m}$

B  $\mathbf{r}_{CM} = \hat{\mathbf{i}} \text{ m}$

C  $\mathbf{r}_{CM} = (2\hat{\mathbf{i}} + 1\hat{\mathbf{j}}) \text{ m}$

D  $\mathbf{r}_{CM} = (1.33\hat{\mathbf{i}} + 0.33\hat{\mathbf{j}}) \text{ m}$



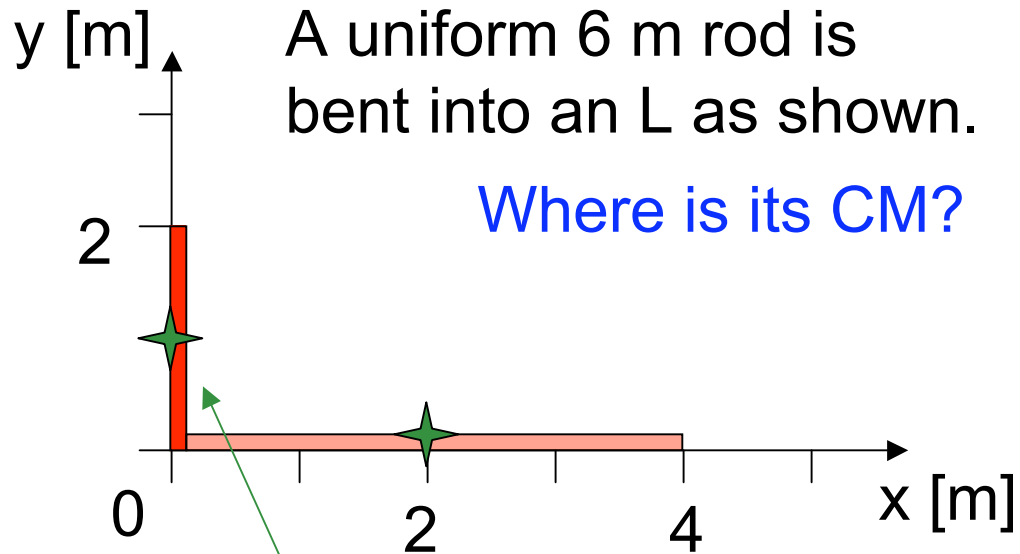
A  $\mathbf{r}_{CM} = (0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}) \text{ m}$

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C  $\mathbf{r}_{CM} = (2\hat{\mathbf{i}} + 1\hat{\mathbf{j}}) \text{ m}$

D  $\mathbf{r}_{CM} = (1.33\hat{\mathbf{i}} + 0.33\hat{\mathbf{j}}) \text{ m}$

CM of lower piece... by symmetry



A  $\mathbf{r}_{CM} = (0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}) \text{ m}$

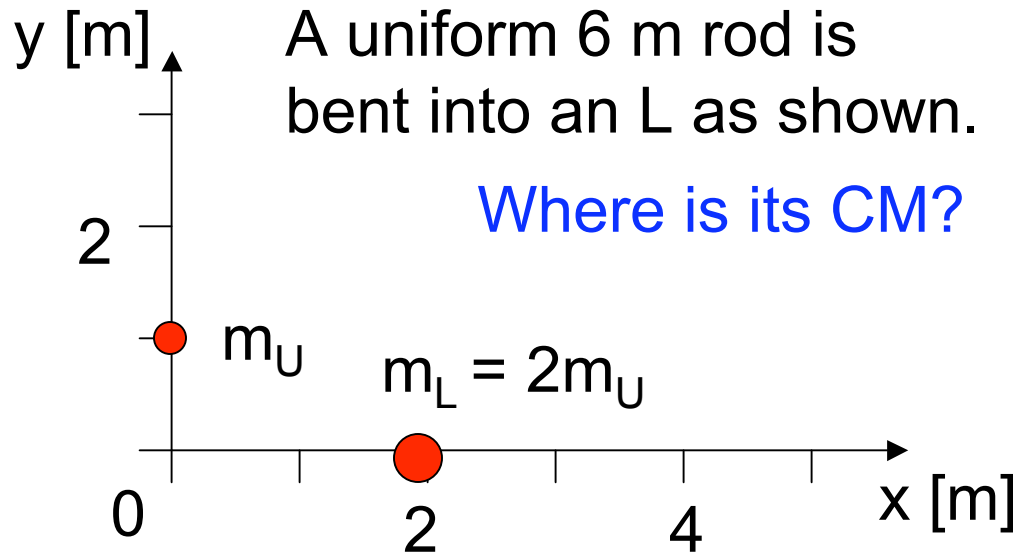
B  $\mathbf{r}_{CM} = \hat{\mathbf{i}} \text{ m}$

C  $\mathbf{r}_{CM} = (2\hat{\mathbf{i}} + 1\hat{\mathbf{j}}) \text{ m}$

D  $\mathbf{r}_{CM} = (1.33\hat{\mathbf{i}} + 0.33\hat{\mathbf{j}}) \text{ m}$

CM of lower piece... by symmetry

CM of upper piece... by symmetry

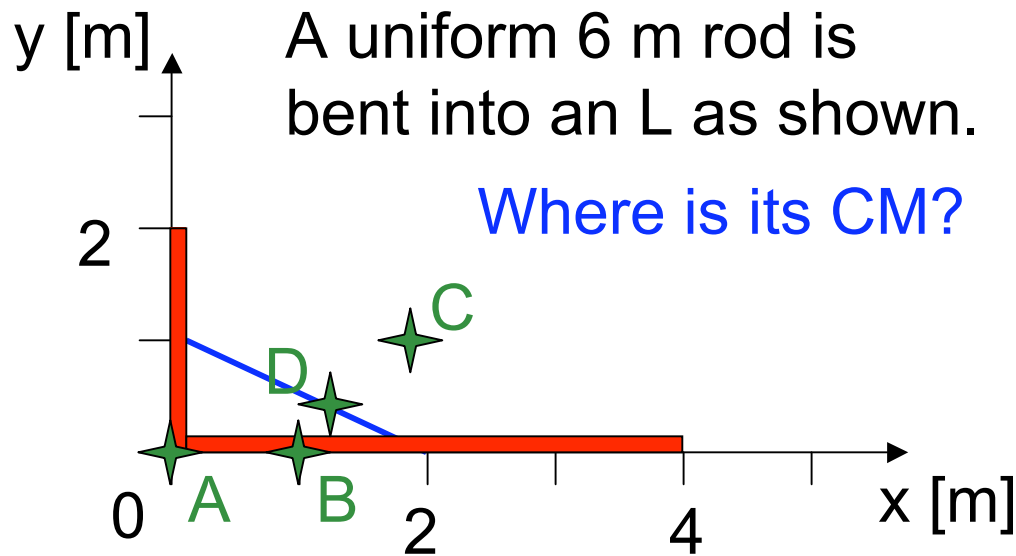


- A**  $\mathbf{r}_{CM} = (0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}) \text{ m}$
- B**  $\mathbf{r}_{CM} = \hat{\mathbf{i}} \text{ m}$
- C**  $\mathbf{r}_{CM} = (2\hat{\mathbf{i}} + 1\hat{\mathbf{j}}) \text{ m}$
- D**  $\mathbf{r}_{CM} = (1.33\hat{\mathbf{i}} + 0.33\hat{\mathbf{j}}) \text{ m}$

Replace CMs by points

$$x_{CM} = \frac{m_U x_U + m_L x_L}{m_U + m_L} = \frac{0 \cdot m_U + 2 \cdot 2m_U}{m_U + 2m_U} = 1.333 \text{ m}$$

$$y_{CM} = \frac{m_U y_U + m_L y_L}{m_U + m_L} = \frac{1 \cdot m_U + 0 \cdot 2m_U}{m_U + 2m_U} = 0.333 \text{ m}$$



A  $\mathbf{r}_{CM} = (0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}) \text{ m}$

B  $\mathbf{r}_{CM} = \hat{\mathbf{i}} \text{ m}$

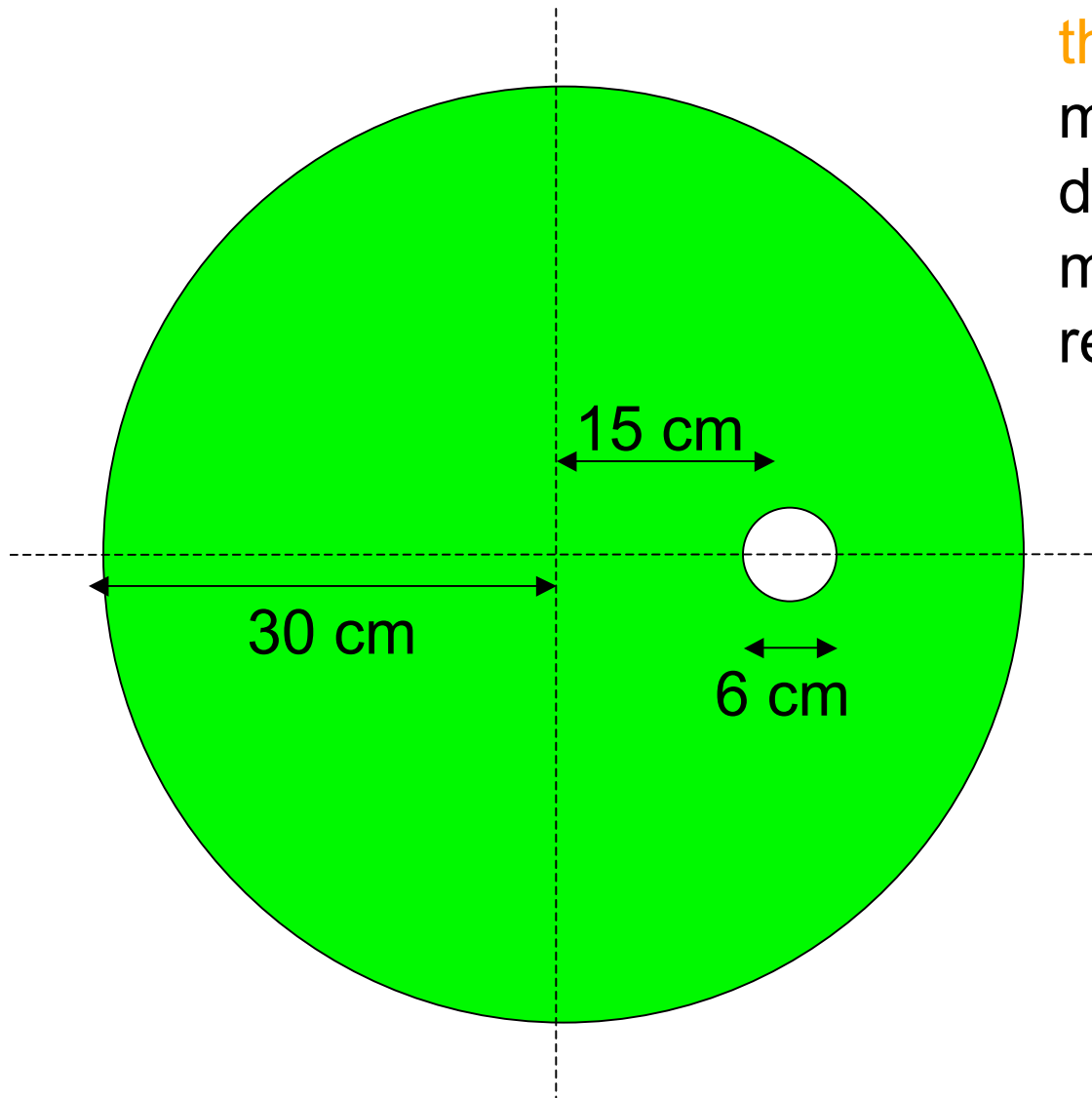
C  $\mathbf{r}_{CM} = (2\hat{\mathbf{i}} + 1\hat{\mathbf{j}}) \text{ m}$

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$$x_{CM} = \frac{m_U x_U + m_L x_L}{m_U + m_L} = \frac{0 \cdot m_U + 2 \cdot 2m_u}{m_U + 2m_U} = 1.333 \text{ m}$$

$$y_{CM} = \frac{m_U y_U + m_L y_L}{m_U + m_L} = \frac{1 \cdot m_U + 0 \cdot 2m_u}{m_U + 2m_U} = 0.333 \text{ m}$$

# Holes



A uniform metal disk has radius 30 cm. A circular hole of radius 3 cm is drilled in it, centered 15 cm from the edge of the disk. If the mass of the original disk is designated  $m_o$  what is the mass of the material removed by drilling?

A  $m_o/10$

B  $m_o/31.4$

C  $m_o/100$

D  $m_o/314$

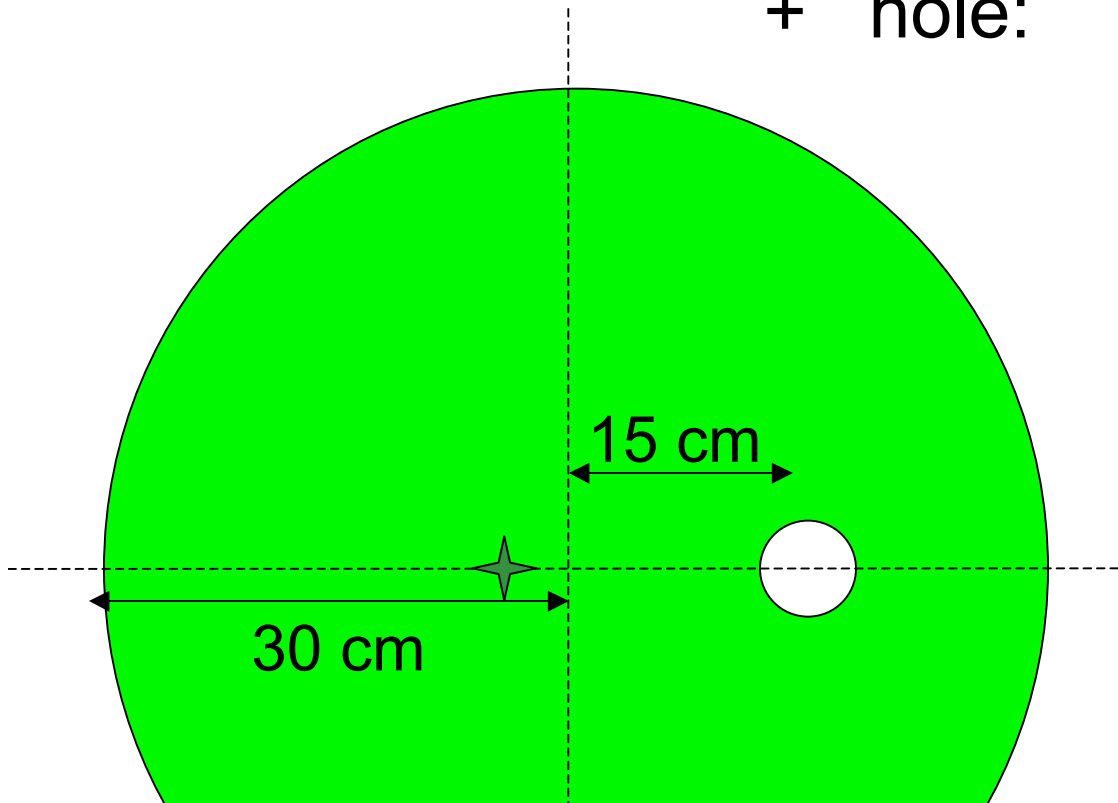
$$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left( \frac{r_1}{r_2} \right)^2$$

# Holes

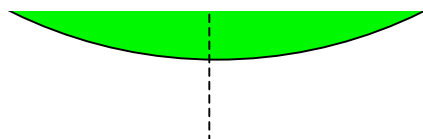
Drilled disk =

solid disk: mass  $m_d = m_o$

+ hole: mass  $m_h = -0.01 m_o$



$$x_{CM} = \frac{m_d x_d + m_h x_h}{m_d + m_h} = \frac{(1)(0) + (-0.01)(15)}{1 - 0.01} = \frac{-0.15 \text{ cm}}{0.99} = -0.15 \text{ cm}$$



# Why the CM?

$$\mathbf{r}_{CM} = \frac{m_A \mathbf{r}_A + m_B \mathbf{r}_B + m_C \mathbf{r}_C}{m_A + m_B + m_C}$$

$$\frac{d\mathbf{r}_{CM}}{dt} = \frac{m_A (d\mathbf{r}_A / dt) + m_B (d\mathbf{r}_B / dt) + m_C (d\mathbf{r}_C / dt)}{m_A + m_B + m_C}$$

# Why the CM?

$$\mathbf{r}_{CM} = \frac{m_A \mathbf{r}_A + m_B \mathbf{r}_B + m_C \mathbf{r}_C}{m_A + m_B + m_C}$$

$$\frac{d\mathbf{r}_{CM}}{dt} = \frac{m_A (d\mathbf{r}_A / dt) + m_B (d\mathbf{r}_B / dt) + m_C (d\mathbf{r}_C / dt)}{m_A + m_B + m_C}$$

$$\mathbf{v}_{CM} = \frac{d\mathbf{r}_{CM}}{dt} = \frac{m_A \mathbf{v}_A + m_B \mathbf{v}_B + m_C \mathbf{v}_C}{m_A + m_B + m_C}$$

# Why the CM?

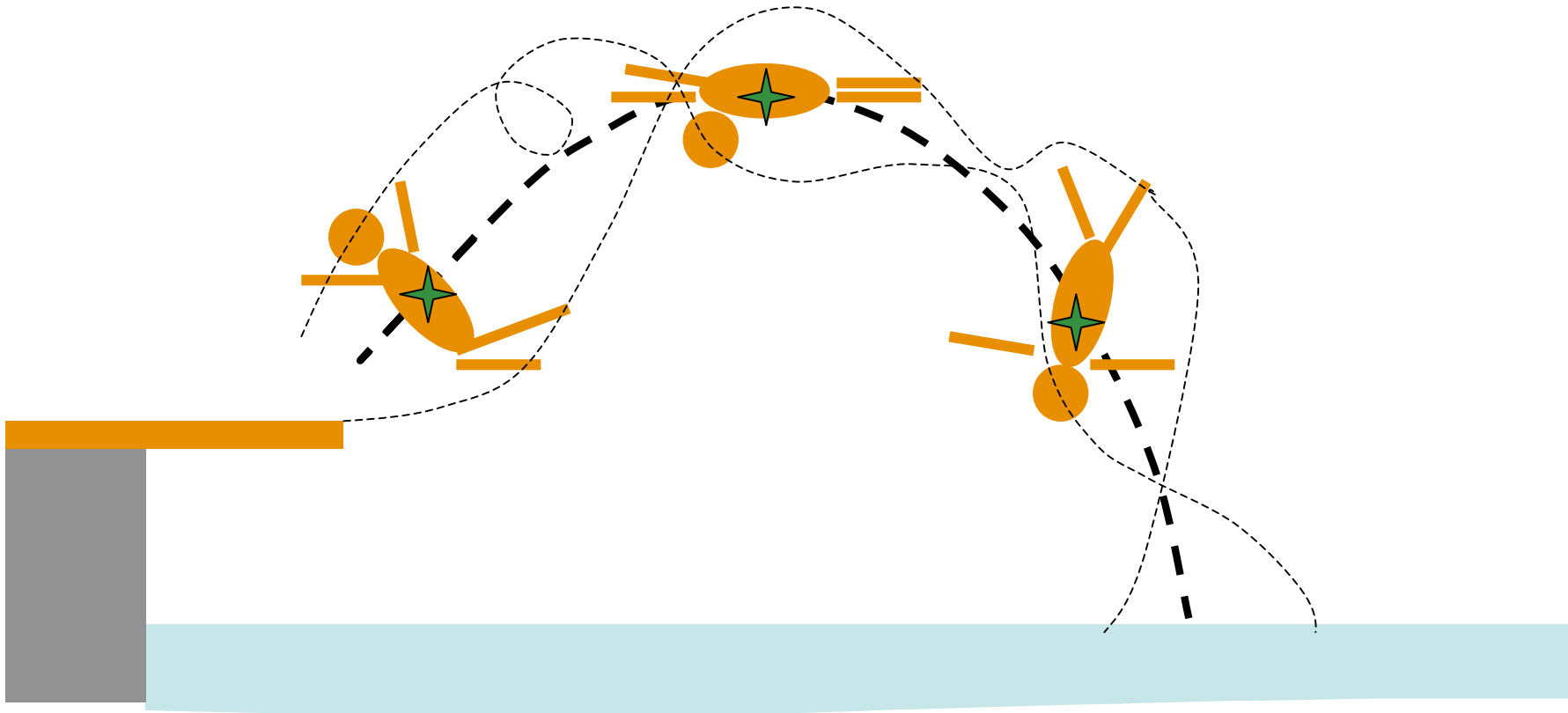
$$\mathbf{r}_{CM} = \frac{m_A \mathbf{r}_A + m_B \mathbf{r}_B + m_C \mathbf{r}_C}{m_A + m_B + m_C}$$

$$\frac{d\mathbf{r}_{CM}}{dt} = \frac{m_A (d\mathbf{r}_A / dt) + m_B (d\mathbf{r}_B / dt) + m_C (d\mathbf{r}_C / dt)}{m_A + m_B + m_C}$$

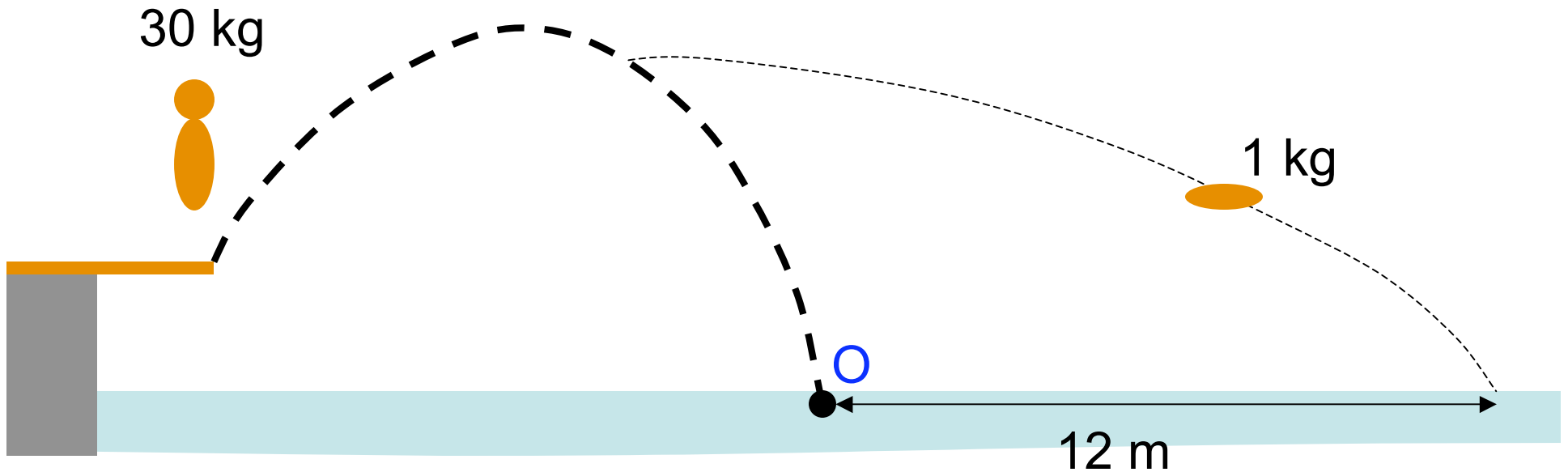
$$\mathbf{v}_{CM} = \frac{d\mathbf{r}_{CM}}{dt} = \frac{m_A \mathbf{v}_A + m_B \mathbf{v}_B + m_C \mathbf{v}_C}{m_A + m_B + m_C} = \frac{\mathbf{p}}{m_{tot}} \quad \text{of any system}$$

$$\mathbf{p} = m_{tot} \mathbf{v}_{CM} \quad \frac{d\mathbf{p}}{dt} = m_{tot} \frac{d\mathbf{v}_{CM}}{dt} = m_{tot} \mathbf{a}_{CM} = \mathbf{F}_{net}$$

N2



A diver's CM will follow a simple parabolic trajectory (i.e. free-fall motion) no matter what she does with her head, arms or legs



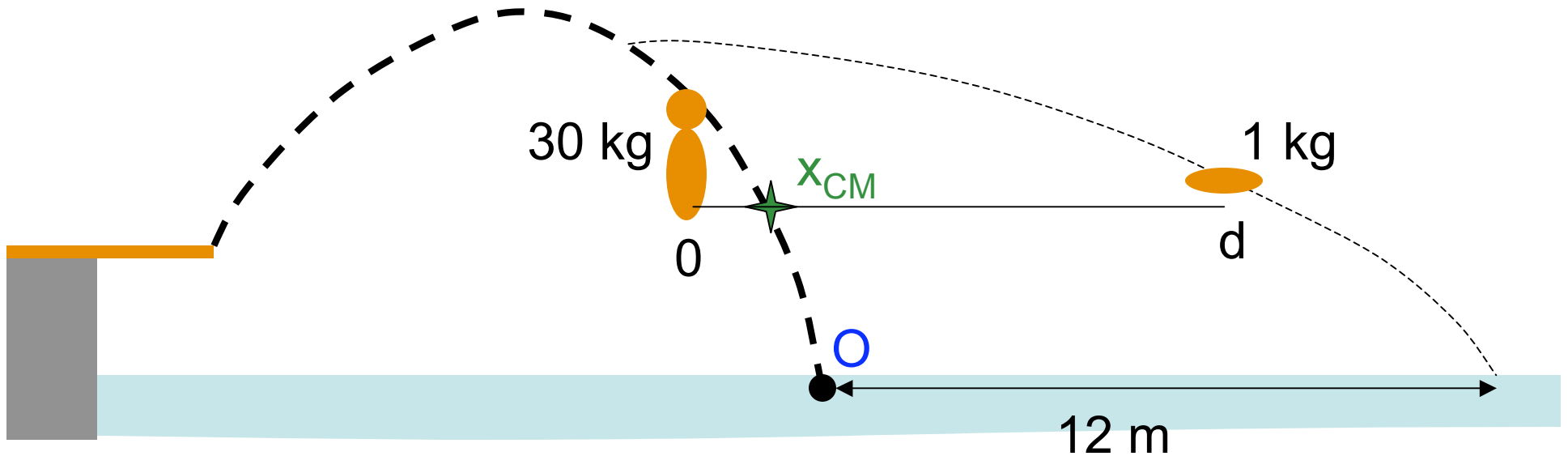
Billy (30 kg) jumps off a diving board w/ a 1 kg ball. While in mid air he throws the ball horizontally to a point 12 m beyond where he *would have landed* had he held on to it (point O). Where does he land relative to that point?

A at point O

B short of point O

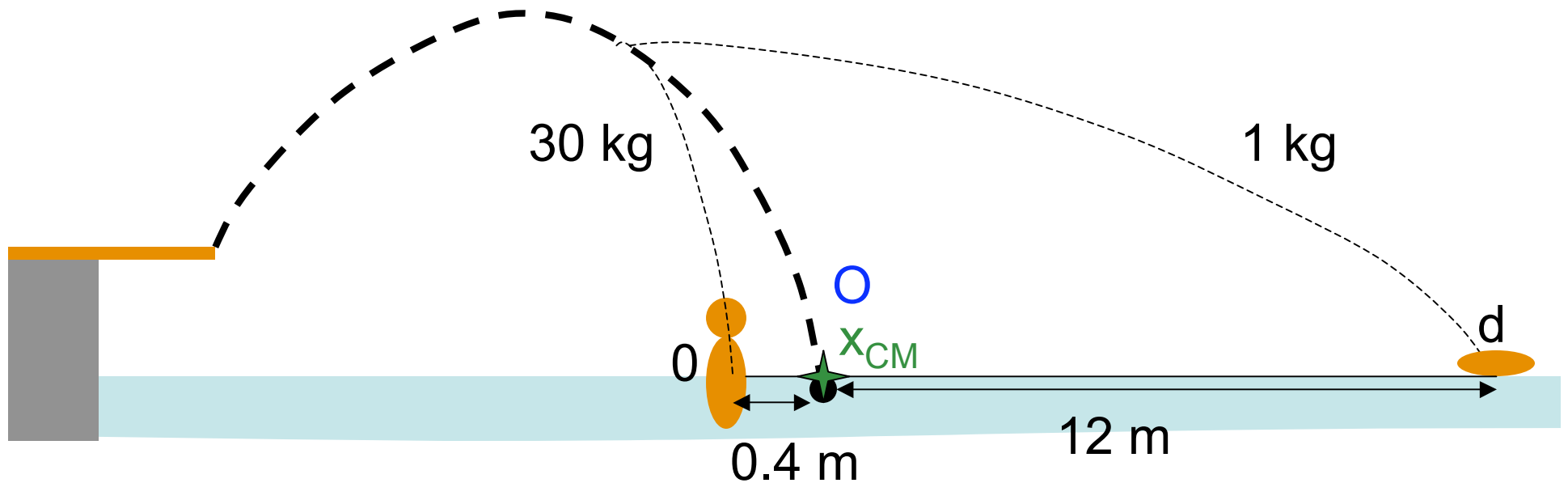
C long of point O

D he could land at any of A, B or C



Billy (30 kg) jumps off a diving board w/ a 1 kg ball. While in mid air he throws the ball horizontally to a point 12 m beyond where he *would have landed* had he held on to it (point  $\circ$ ). Where does he land relative to that point?

$$x_{CM} = \frac{1}{31} d$$



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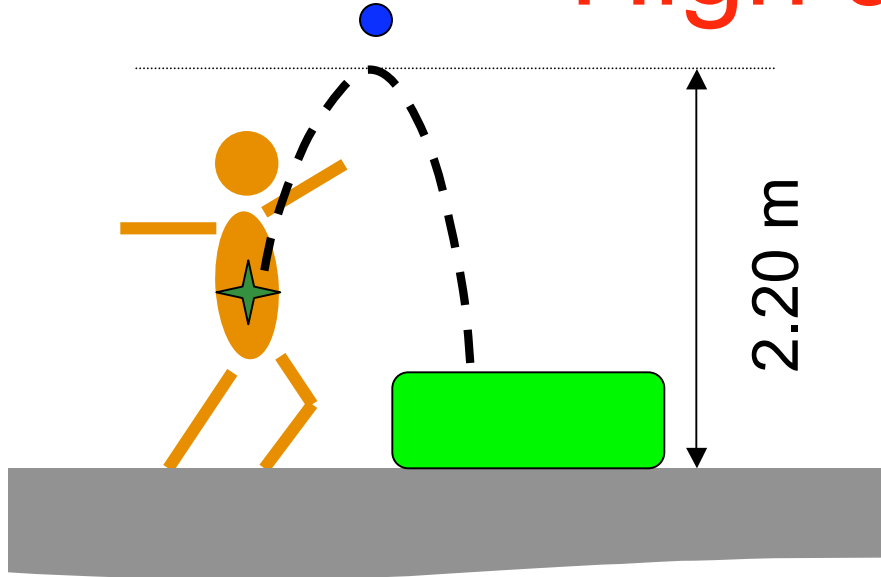
$$x_{CM} = \frac{1}{31} d$$

$$d = (31 \times 12 / 30) \text{ m}$$

$$d - x_{CM} = \frac{30}{31} d = 12 \text{ m}$$

$$x_{CM} = (12 / 30) \text{ m} = 0.4 \text{ m}$$

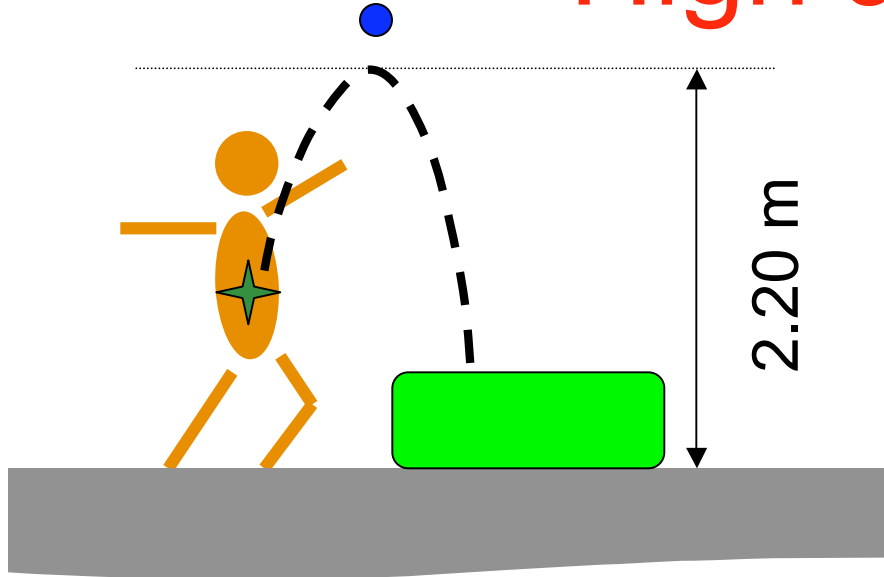
# High Jumping



- A **No way.** If his COM does not clear the bar, his body will not either
- B **Maybe.** It is possible for him to move in the air & affect the motion of his COM
- C **Yes.** It is possible for part of his body to be above his COM.

Richard is able to supply sufficient force to propel his COM to a height of 2.20 m with sufficient range to clear a high jump bar. Is it possible for him to use this jump to clear a 2.25 m high bar?

# High Jumping



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- B **Maybe.** It is possible for him to move in the air & affect the motion of his COM

C **Yes.** It is possible for part of his body to be above his COM.

