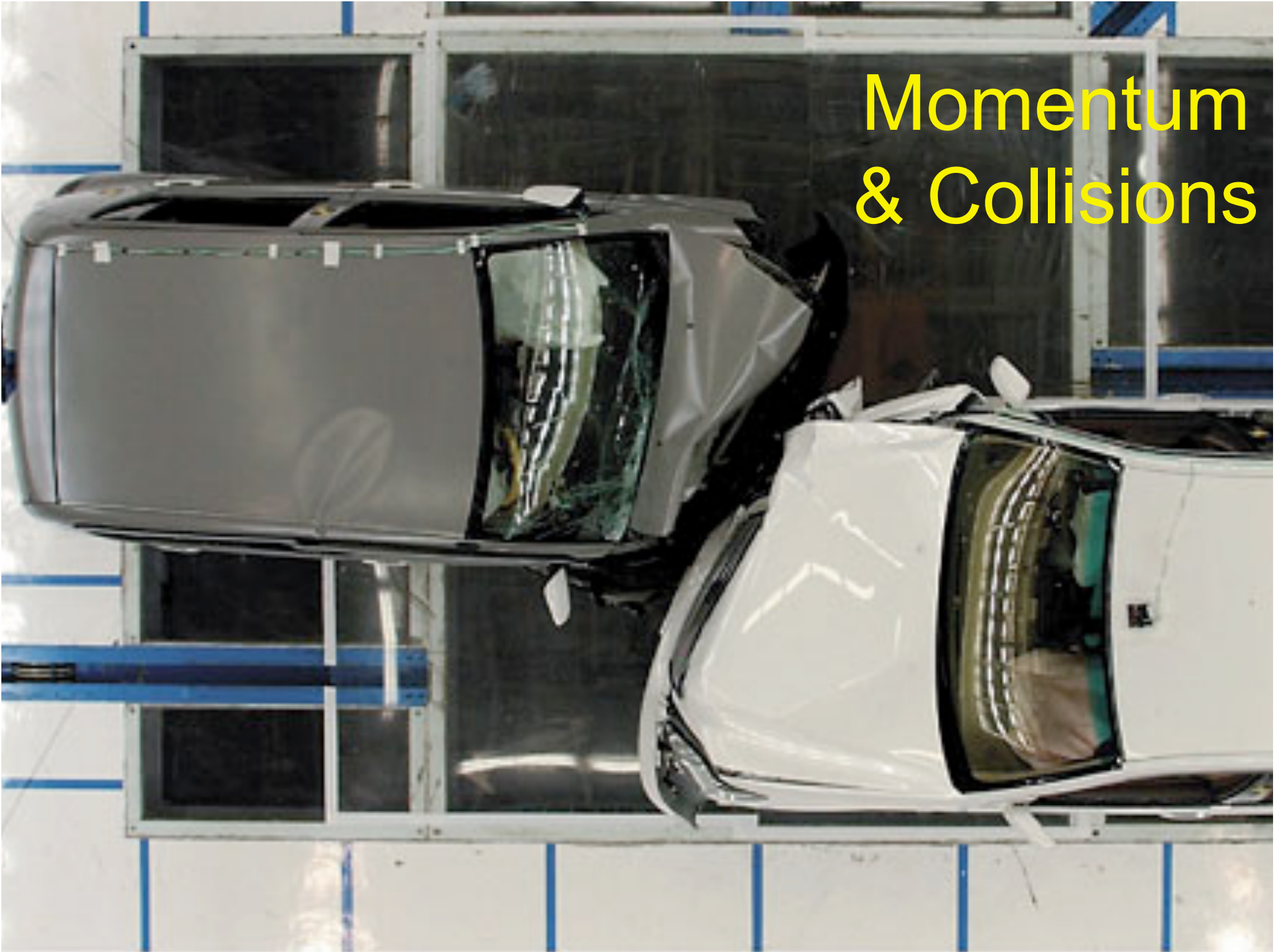


# Momentum & Collisions



# Reading Question

In physics the term **impulse** means:

- A The integral of momentum over time
- B The integral of momentum over distance
- C The integral of force over time
- D The integral of force over distance
- E A sudden urge that prompts an unpremeditated action

# Reading Question

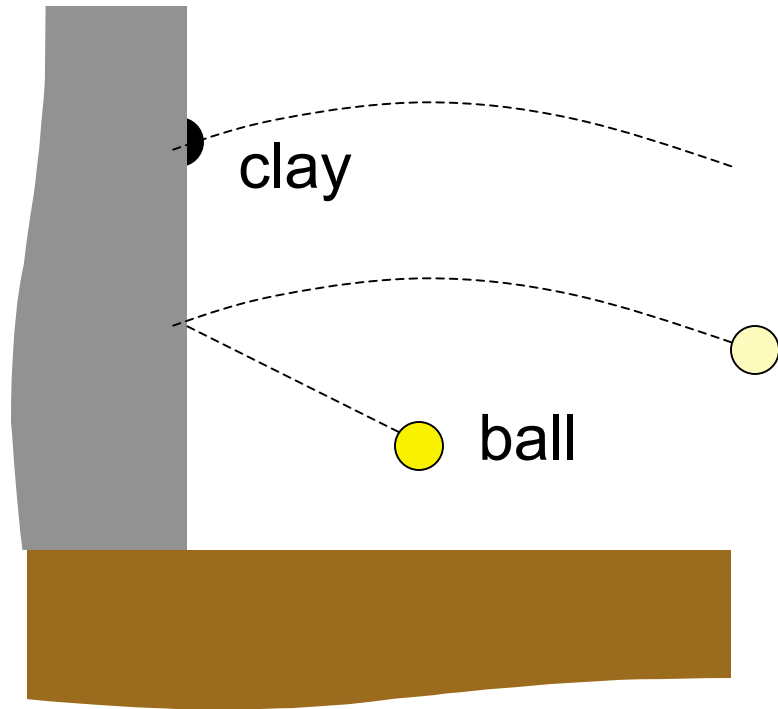
In physics the term **impulse** means:

- A The integral of momentum over time
- B The integral of momentum over distance
- C The integral of force over time
- D The integral of force over distance
- E A sudden urge that prompts an unpremeditated action

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$$

$$\vec{\mathbf{J}} = \int_{t_i}^{t_f} \vec{\mathbf{F}} dt$$

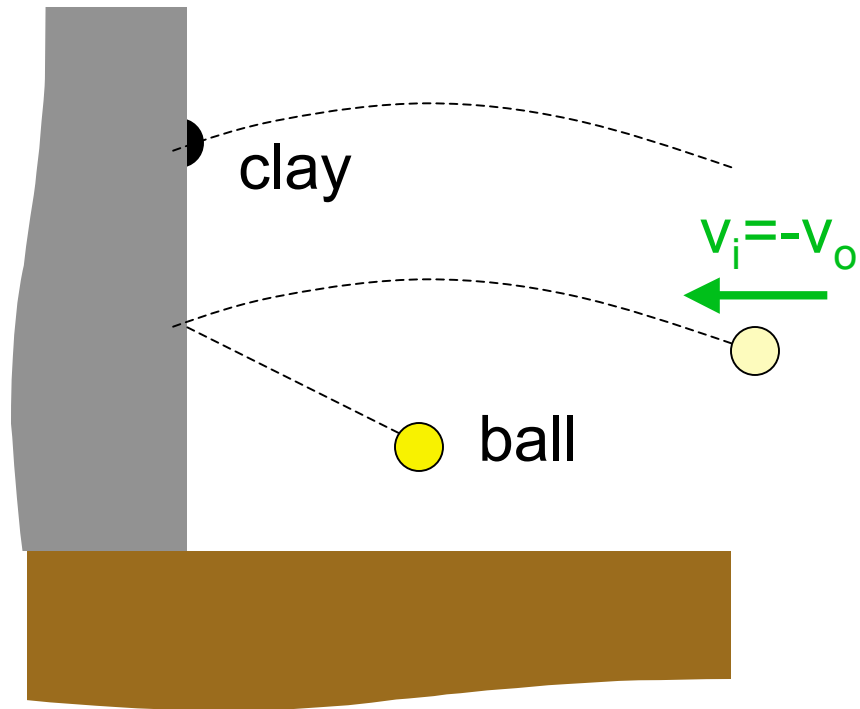
$$\vec{\mathbf{J}} = \int_{t_i}^{t_f} \frac{d\vec{\mathbf{p}}}{dt} dt = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \Delta\vec{\mathbf{p}}$$



# Impulse

Impulsively, I throw a ball against the wall and it bounces off. I then throw an equal mass of clay at the same speed. It sticks to the wall. In which case is the **impulse** of wall greater?

- A The ball
- B The clay
- C They are equal

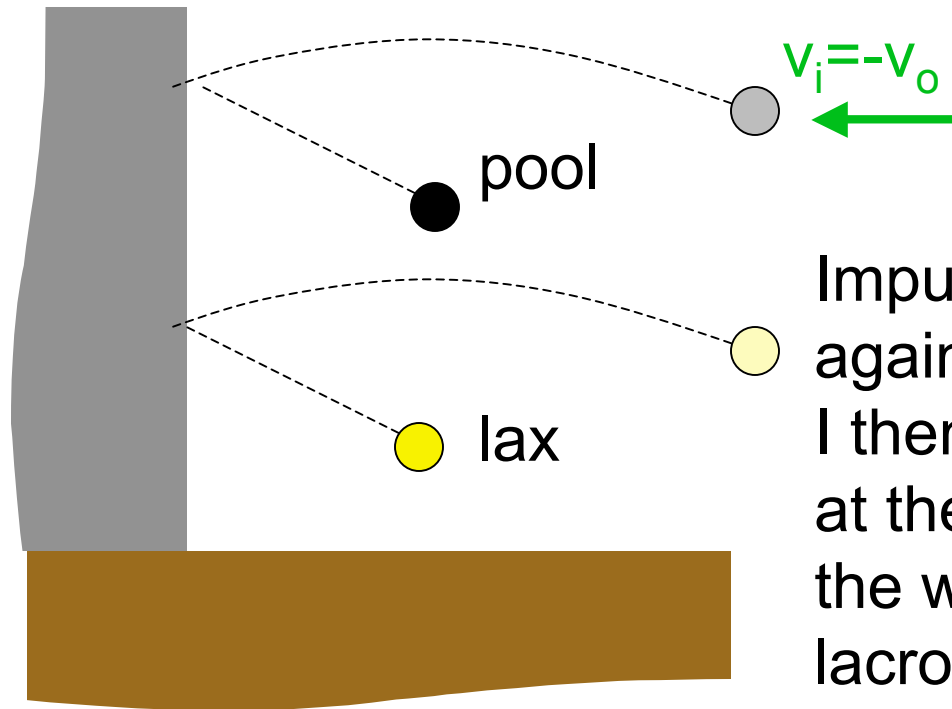


# Impulse

Impulsively, I throw a ball against the wall and it bounces off. I then throw an equal mass of clay at the same speed. It sticks to the wall. In which case is the **impulse** of wall greater?

- A** The ball
- B The clay
- C They are equal

	initial	final	$\Delta p$
ball	$-mv_o$	$+mv_f$	$m(v_f + v_o)$
clay	$-mv_o$	0	$mv_o$



# Impulse

Impulsively, I throw a lacrosse ball against the wall and it bounces off. I then throw an equal mass pool ball at the same speed. It bounces from the wall at the same speed that the lacrosse ball did BUT is in contact with the wall for much less time. In which case is the **impulse** of wall greater?

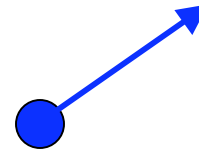
- A The lacrosse ball
- B The pool ball
- C They are equal

	initial	final	$\Delta p$
lacrosse	$-mv_o$	$+mv_f$	$m(v_o + v_f)$
pool	$-mv_o$	$+mv_f$	$m(v_o + v_f)$

# momentum of a system

one particle

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

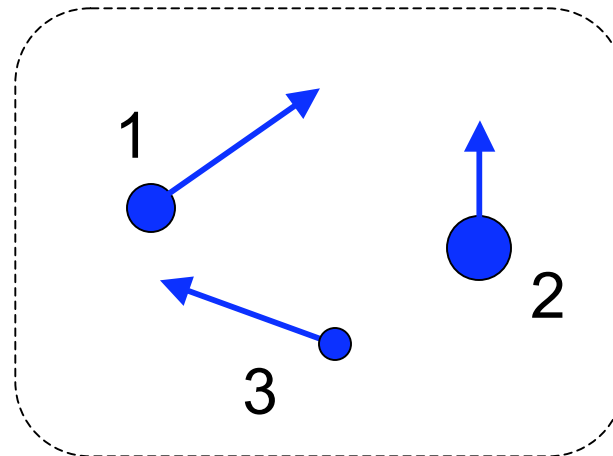


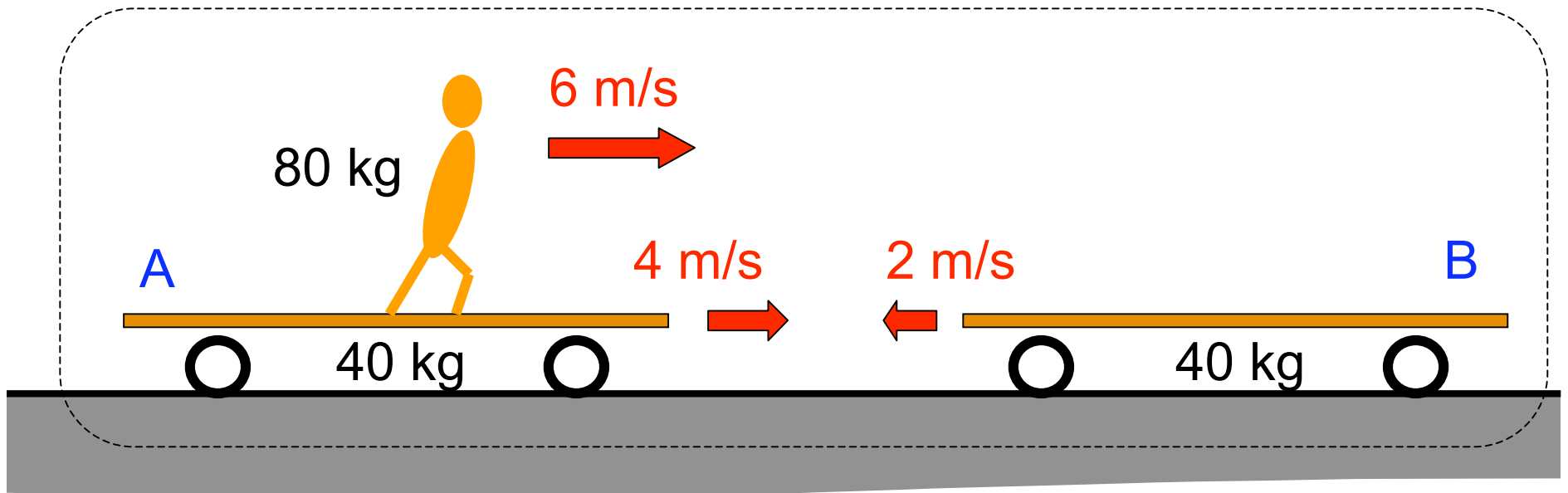
systems of particles

$$\vec{\mathbf{p}}_{\text{tot}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + m_3\vec{\mathbf{v}}_3$$

$$\frac{d\vec{\mathbf{p}}_{\text{tot}}}{dt} = m_1\vec{\mathbf{a}}_1 + m_2\vec{\mathbf{a}}_2 + m_3\vec{\mathbf{a}}_3$$

$$\frac{d\vec{\mathbf{p}}_{\text{tot}}}{dt} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_{\text{tot}}$$





2 identical carts (@ 40 kg) are moving towards one another. Carl (80 kg) runs atop the left cart a 2 m/s (relative to the cart). What is the momentum of the entire system.

A 160 kg m/s

B 480 kg m/s

C 560 kg m/s

D 640 kg m/s

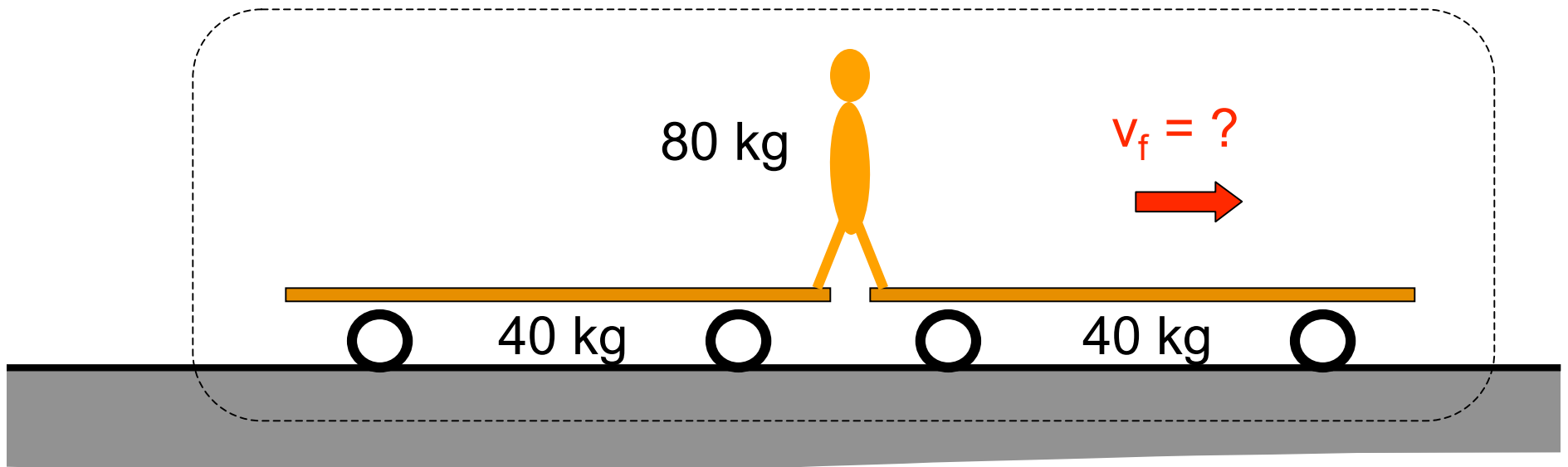
$$p_A = (40 \text{ kg})(4 \text{ m/s}) = 160 \text{ kg m/s}$$

$$p_B = (40 \text{ kg})(-2 \text{ m/s}) = -80 \text{ kg m/s}$$

$$p_C = (80 \text{ kg})(6 \text{ m/s}) = 480 \text{ kg m/s}$$

---


$$p_A + p_B + p_C = 560 \text{ kg m/s}$$



Carl manages to straddle the carts and coast along with the pair together. How fast are they going?

A 2.5 m/s

B 3 m/s

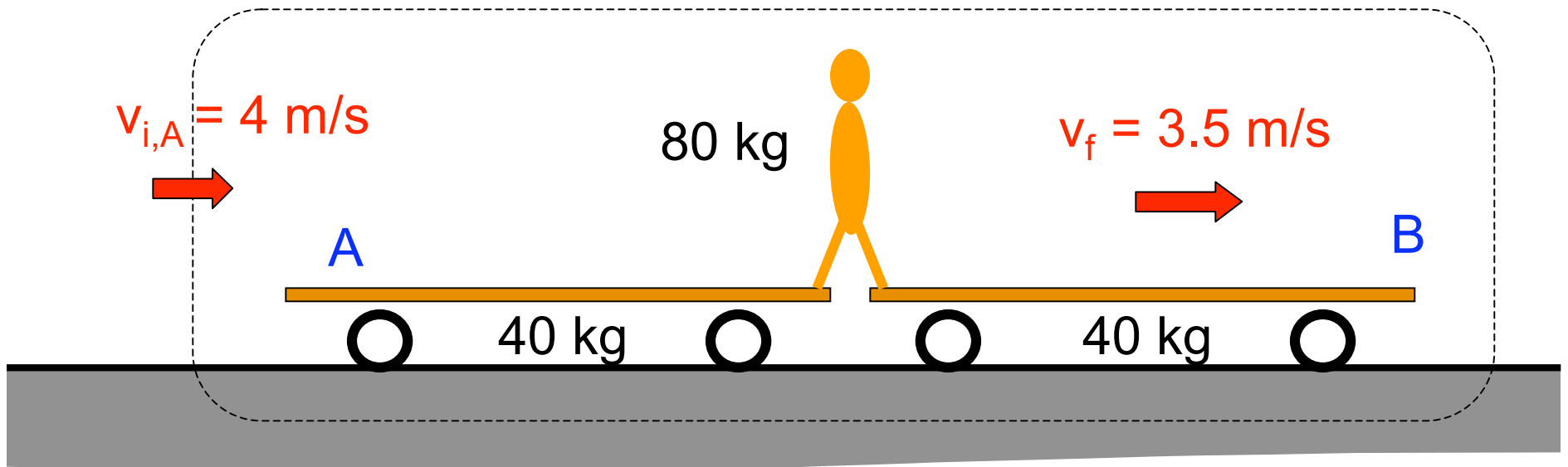
C 3.5 m/s

D 4 m/s

$$p_i = p_A + p_B + p_C = 560 \text{ kg m/s}$$

$$p_f = (160 \text{ kg}) v_f = 560 \text{ kg m/s}$$

$$v_f = \frac{560 \text{ kg m/s}}{160 \text{ kg}} = 3.5 \text{ m/s}$$



Carl manages to straddle the carts and coast along with the pair together. What impulse did he impart to car A?

A -140 kg m/s

B - 20 kg m/s

C 0 kg m/s

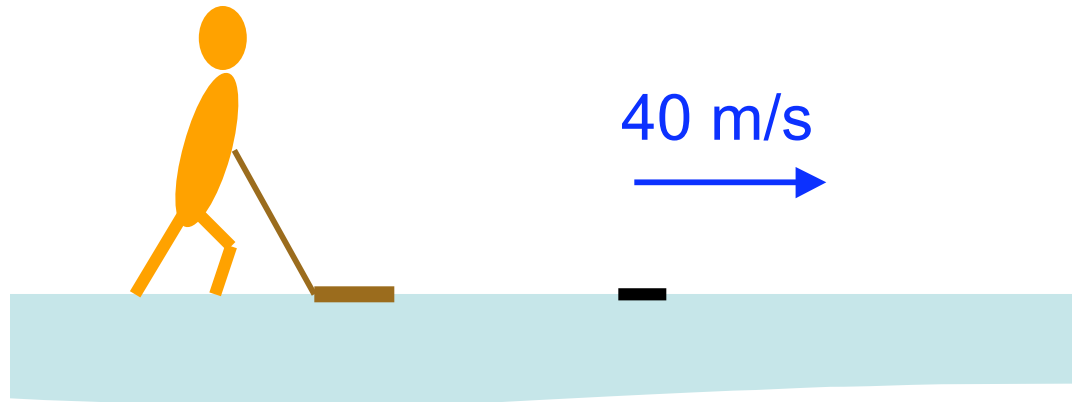
D + 20 kg m/s

E +140 kg m/s

$$p_{i,A} = (40 \text{ kg})(4 \text{ m/s}) = 160 \text{ kg m/s}$$

$$p_{fA} = (40 \text{ kg})(3.5 \text{ m/s}) = 140 \text{ kg m/s}$$

$$p_{fA} - p_{iA} = -20 \text{ kg m/s}$$

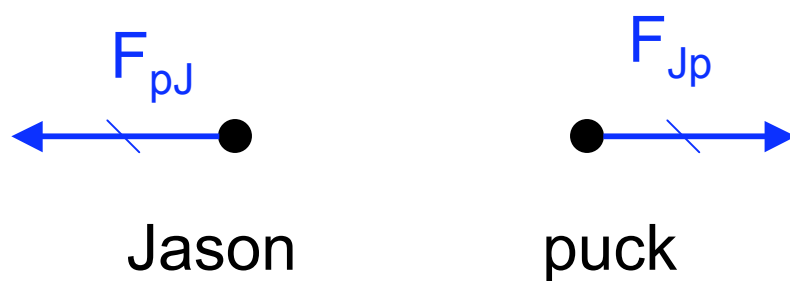


Jason (80 kg) is standing in street shoes on frictionless ice. From rest he shoots a hockey puck (0.25 kg) at 40 m/s. What velocity does Jason have after he shoots?

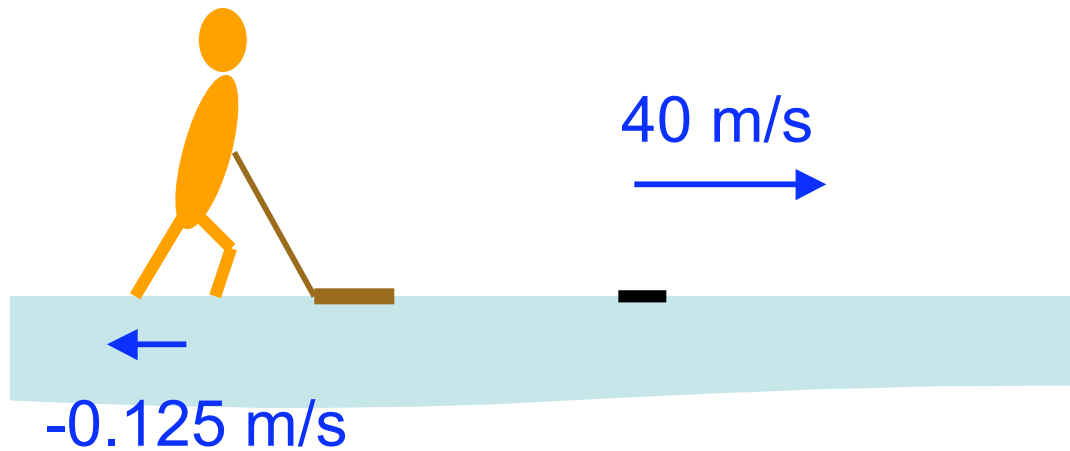
$$p_{p,f} = (+0.25 \text{ kg})(40 \text{ m/s}) = 10 \text{ kg m/s}$$

$$p_{J,f} = (80 \text{ kg}) v_{J,f} = -10 \text{ kg m/s}$$

$$v_{J,f} = -0.125 \text{ m/s}$$



- A -40 m/s
- B -1.2 m/s
- C -0.12 m/s
- D 0 m/s
- E +0.12 m/s



Which quantity has greater magnitude?

- A Work done by Jason on puck
- B Work done by puck on Jason
- C A & B are equal

$$\Delta E_{\text{mec}} = 200.625 \text{ J}$$

Jason (80 kg) is standing in street shoes on frictionless ice. From rest he shoots a hockey puck (0.25 kg) at 40 m/s. What velocity does Jason have after he shoots?

$$W = \Delta K$$

$$\begin{aligned} \Delta K_p &= 0.5 (0.25)(40)^2 \\ &= 200 \text{ J} \end{aligned}$$

$$\begin{aligned} \Delta K_J &= 0.5 (80)(0.125)^2 \\ &= 0.625 \text{ J} \end{aligned}$$

# What is conserved when?

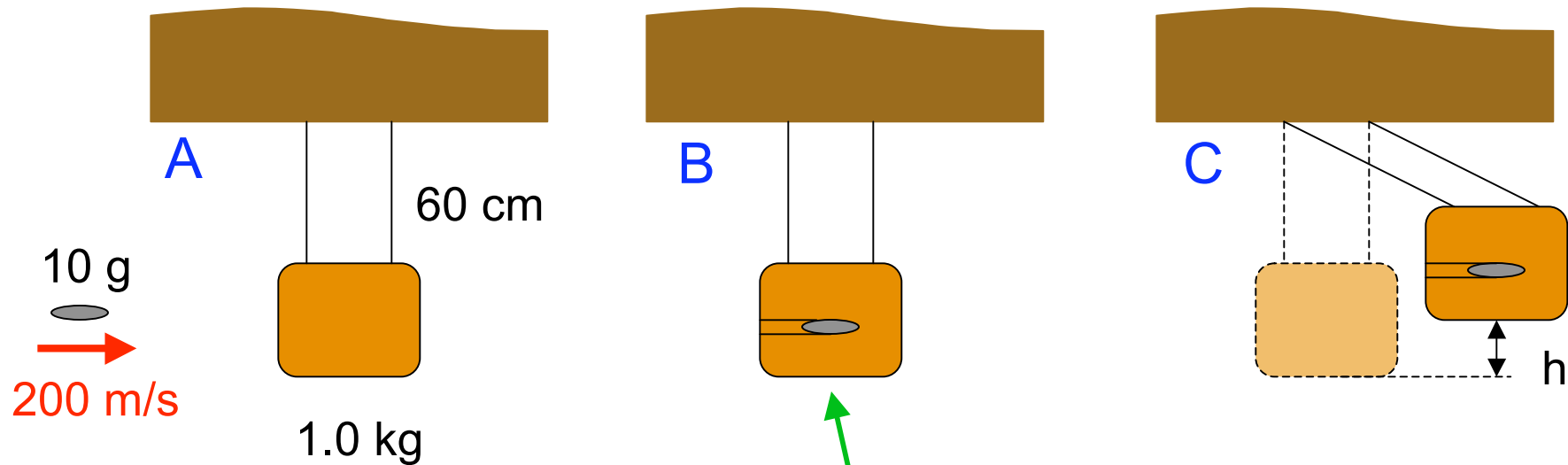
$W_{\text{mech}}$

- External forces acting on system do no work
- AND
- No internal energy in system

$p$

- No external forces acting on system
- OR
- A collision (very short duration)

so short that external forces have no effect



A 10 g bullet is fired at 200 m/s into a 1.0 kg wooden block, hung from the ceiling by light 60 cm cords.

Q: What property of the block/bullet system is conserved throughout stages A-C?

~~A  $W_{\text{mech}}$~~

~~C  $W_{\text{mech}}$  &  $p$~~

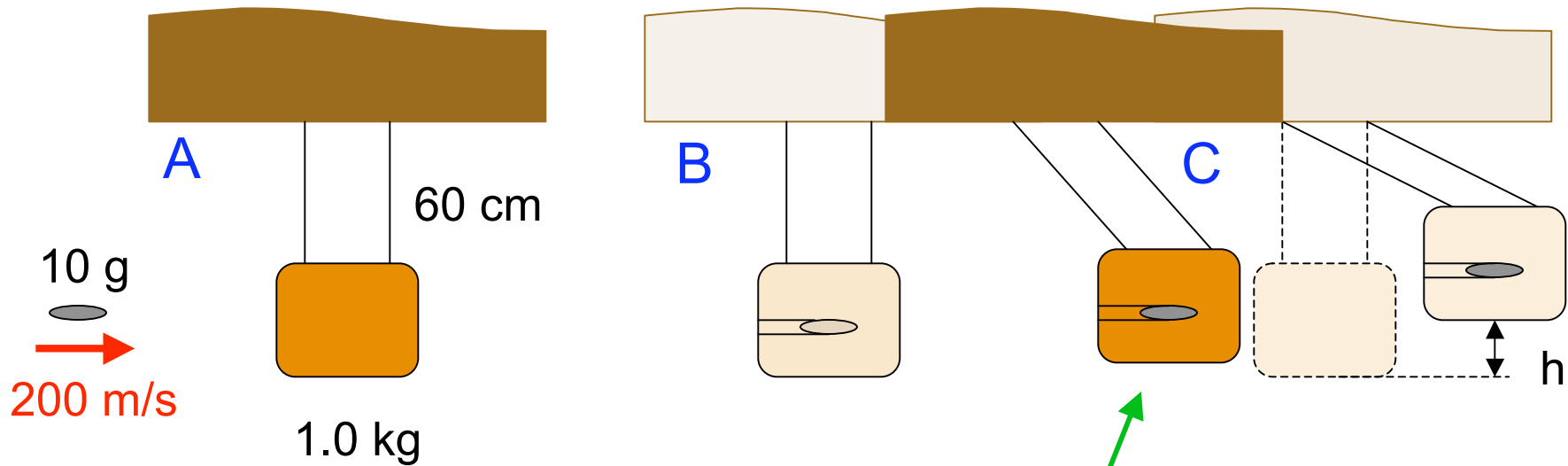
B  $p$

D neither is conserved

changes to system:

- deformed block
- heat

$W_{\text{mech}}$  lost @ stage B



A 10 g bullet is fired at 200 m/s into a 1.0 kg wooden block, hung from the ceiling by light 60 cm cords.

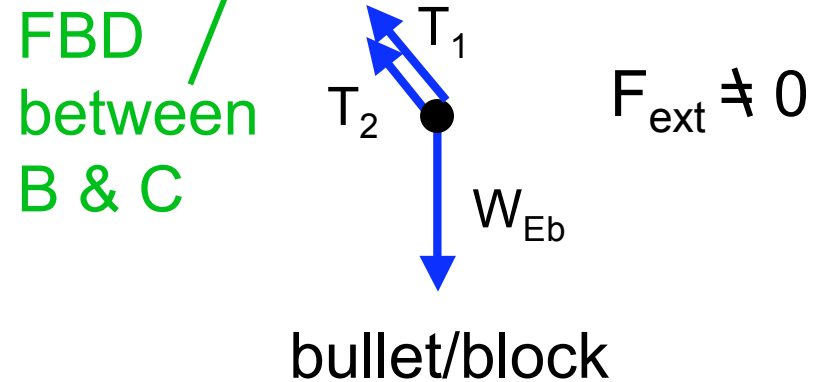
Q: What property of the block/bullet system is conserved throughout stages A-C?

~~A  $W_{\text{mech}}$~~

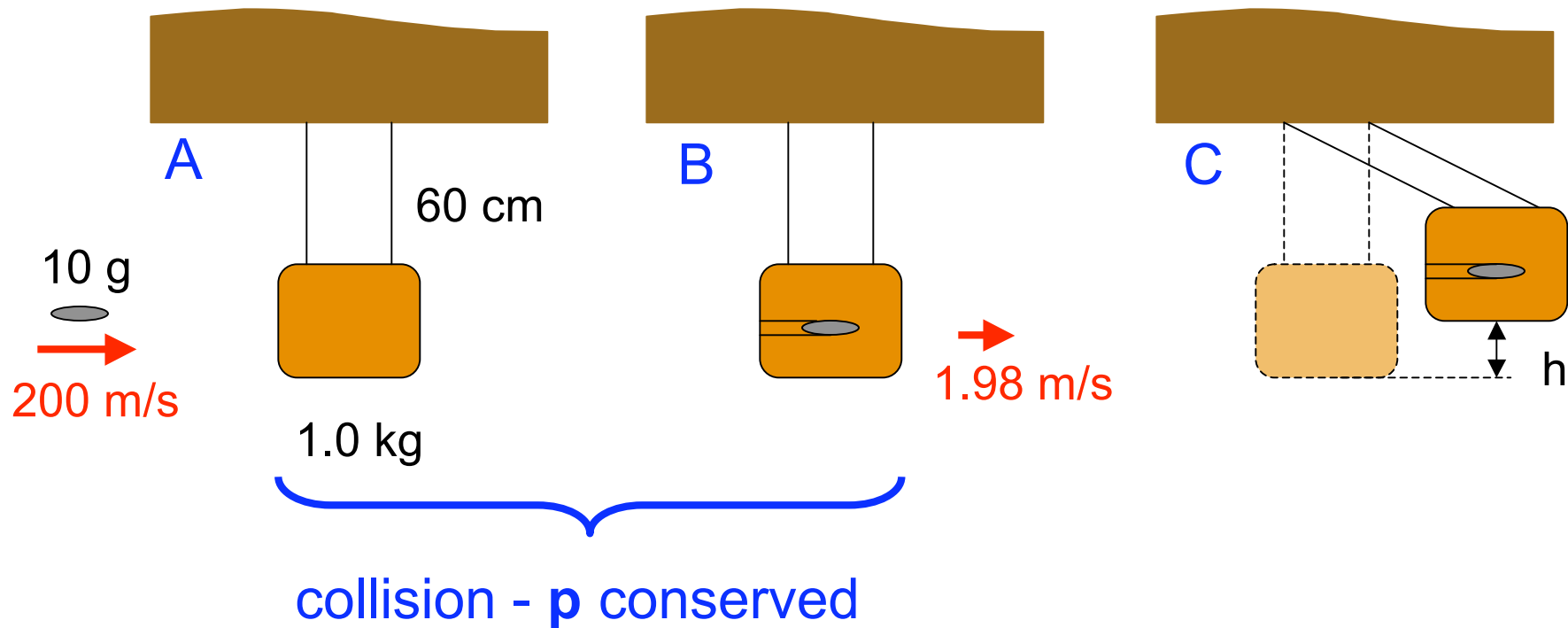
~~C  $W_{\text{mech}}$  &  $p$~~

~~B  $p$~~

D neither is conserved



A 10 g bullet is fired at 200 m/s into a 1.0 kg wooden block, hung from the ceiling by light 60 cm cords.

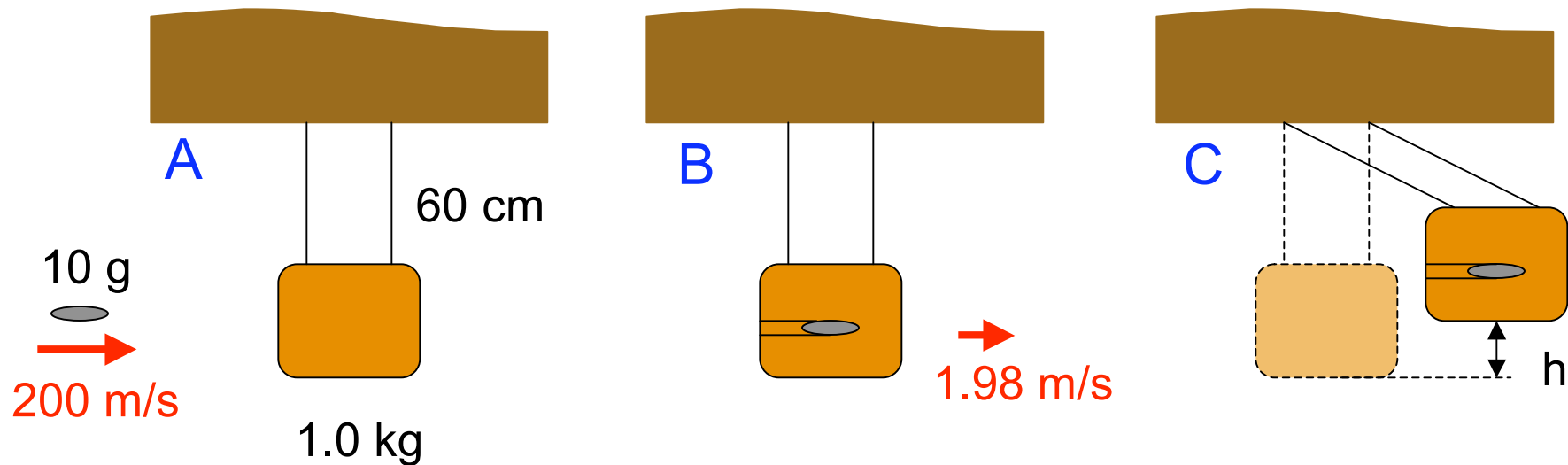


$$p_i = (0.01 \text{ kg})(200 \text{ m/s}) + (1 \text{ kg})(0 \text{ m/s})$$
$$= 2 \text{ kg m / s}$$

$$p_f = (1.01 \text{ kg}) v_f = 2 \text{ kg m/s}$$

$$v_f = 1.98 \text{ m/s}$$

A 10 g bullet is fired at 200 m/s into a 1.0 kg wooden block, hung from the ceiling by light 60 cm cords.



$$K_A = 0.5 (0.01 \text{ kg})(200 \text{ m/s})^2 = 200 \text{ J}$$

$$W_{\text{mech}} = K + U \text{ conserved}$$

$$K_i = 0.5 (1.01 \text{ kg})(1.98 \text{ m/s})^2 = 1.98 \text{ J}$$

$$K_f = 0$$

$$U_i = 0$$

$$U_f = mgh$$

---

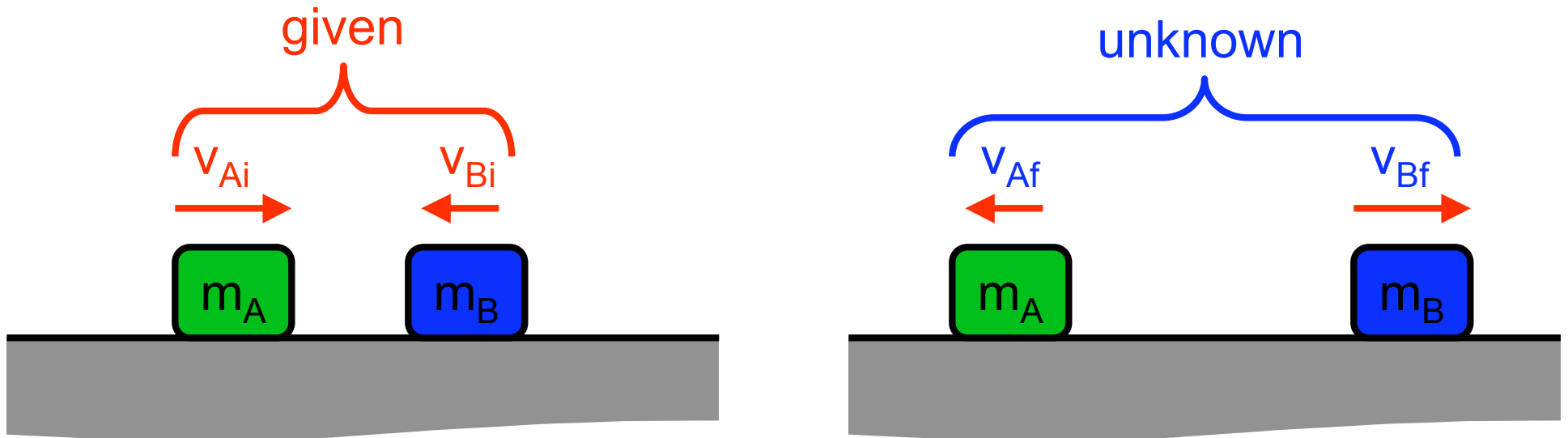

$$W_i = 1.98 \text{ J}$$

---


$$W_f = mgh = 1.98 \text{ J}$$

$$h = 20 \text{ cm}$$

# Elastic Collisions in 1d



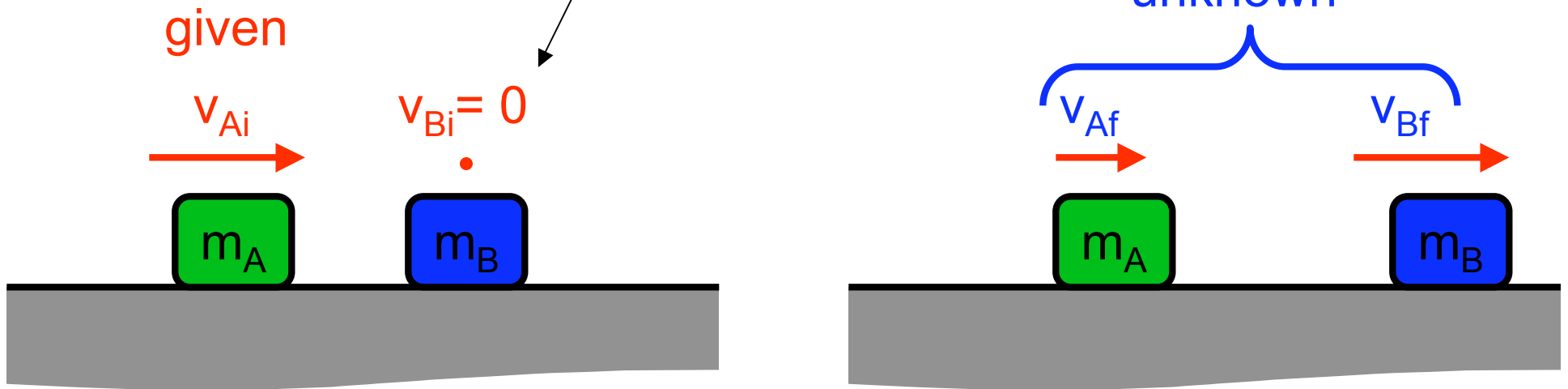
momentum:  $p_i = m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf} = p_f$

energy:  $K_i = 0.5 m_A v_{Ai}^2 + 0.5 m_B v_{Bi}^2 = 0.5 m_A v_{Af}^2 + 0.5 m_B v_{Bf}^2 = K_f$

2 equations for 2 unknowns

# Elastic Collisions in 1d

a special case



momentum:  $m_A v_{Ai} = m_A v_{Af} + m_B v_{Bf}$

energy:  $0.5 m_A v_{Ai}^2 = 0.5 m_A v_{Af}^2 + 0.5 m_B v_{Bf}^2$

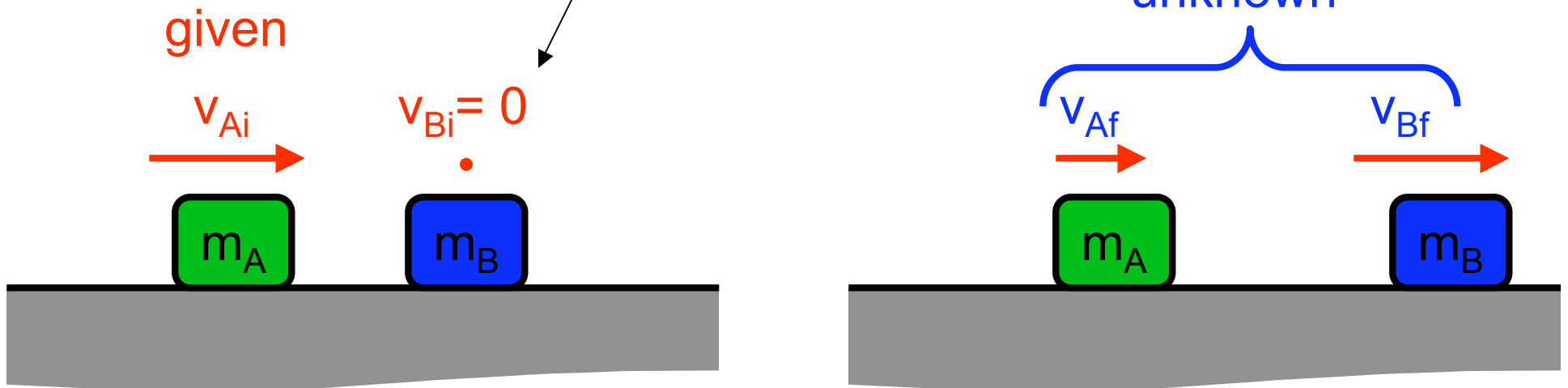
substitute in  
quadratic

$$v_{Af} = v_{Ai} - \frac{m_B}{m_A} v_{Bf}$$

solve linear eqn.  
for 1 unknown

# Elastic Collisions in 1d

a special case



momentum:  $m_A v_{Ai} = m_A v_{Af} + m_B v_{Bf}$

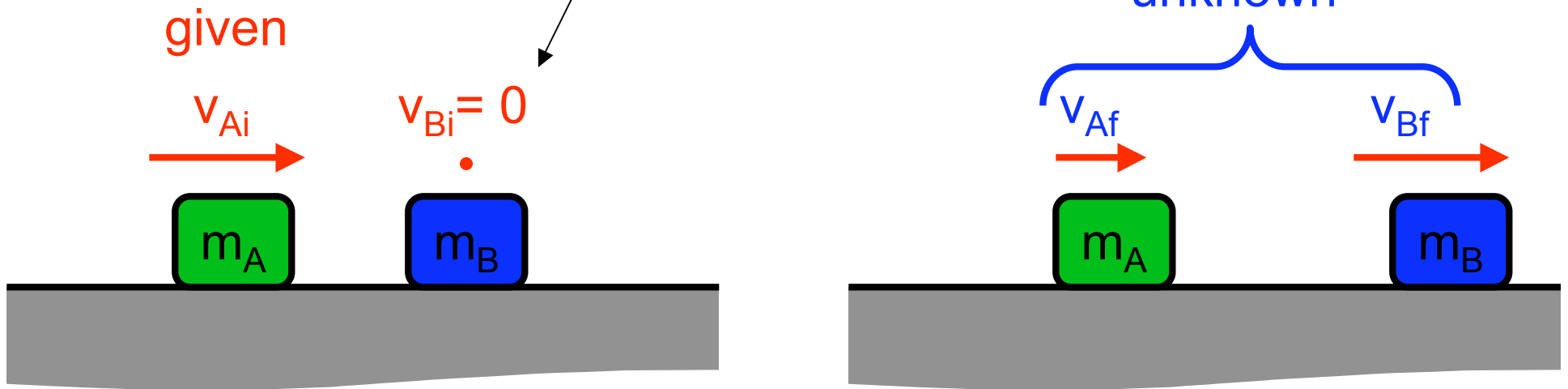
energy:  $0.5 m_A v_{Ai}^2 = 0.5 m_A v_{Af}^2 + 0.5 m_B v_{Bf}^2$

$$v_{Af} = v_{Ai} - \frac{m_B}{m_A} v_{Bf}$$

$$\frac{1}{2} m_A v_{Ai}^2 = \frac{1}{2} m_A \left( v_{Ai}^2 - 2 \frac{m_B}{m_A} v_{Ai} v_{Bf} + \frac{m_B^2}{m_A^2} v_{Bf}^2 \right) + \frac{1}{2} m_B v_{Bf}^2$$

# Elastic Collisions in 1d

a special case



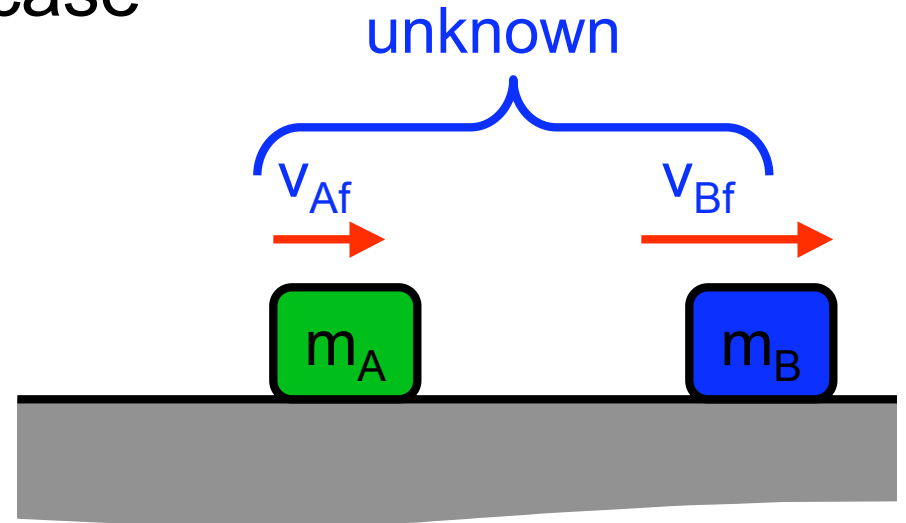
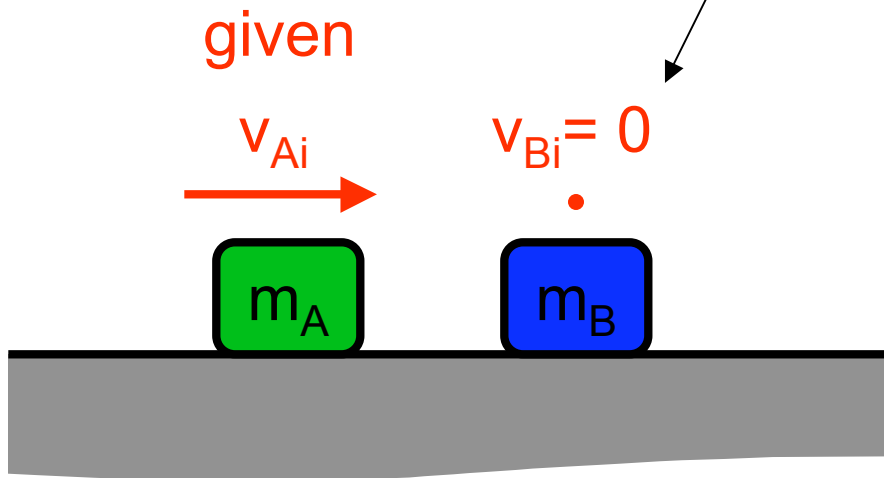
mc

$$\frac{1}{2} m_A v_{Ai}^2 = \frac{1}{2} m_A \left( v_{Ai}^2 - 2 \frac{m_B}{m_A} v_{Ai} v_{Bf} + \frac{m_B^2}{m_A^2} v_{Bf}^2 \right) + \frac{1}{2} m_B v_{Bf}^2 - \frac{m_B}{m_A} v_{Bf}^2$$

$$\cancel{\frac{1}{2} m_A v_{Ai}^2} = \cancel{\frac{1}{2} m_A v_{Ai}^2} - m_B v_{Ai} v_{Bf} + \frac{1}{2} \frac{m_B^2}{m_A} v_{Bf}^2 + \frac{1}{2} m_B v_{Bf}^2$$

# Elastic Collisions in 1d

a special case



momentum

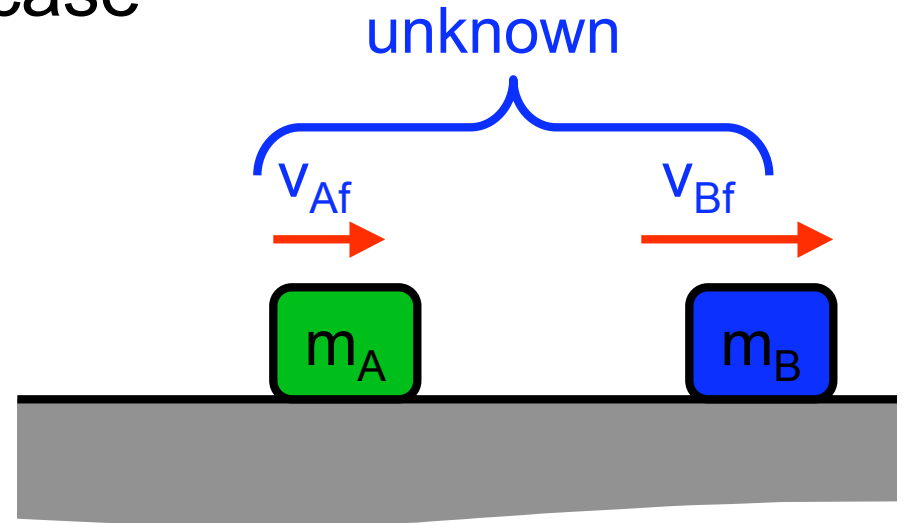
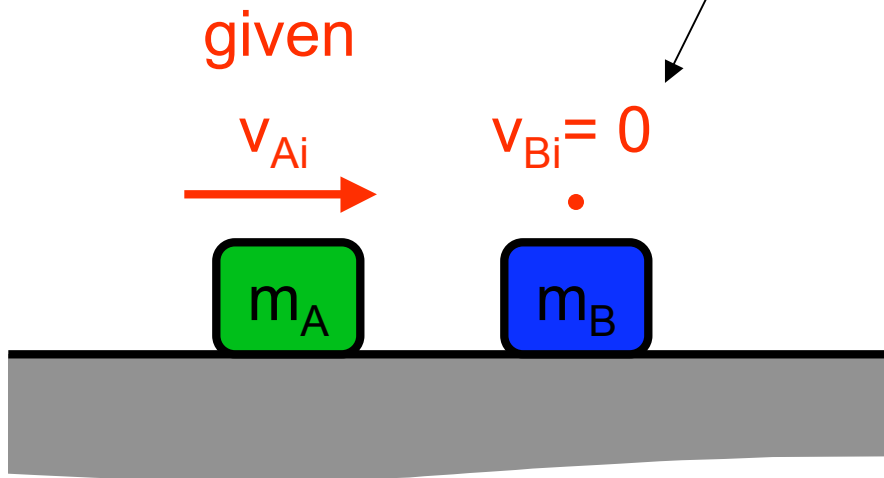
$$e \left[ \cancel{\frac{1}{2} m_A v_{Ai}^2} \right] = \cancel{\frac{1}{2} m_A v_{Ai}^2} - m_B v_{Ai} v_{Bf} + \frac{1}{2} \frac{m_B^2}{m_A} v_{Bf}^2 + \frac{1}{2} m_B v_{Bf}^2 - \left[ \frac{m_B}{m_A} v_{Bf}^2 \right]$$

$$0 = \underbrace{-m_B v_{Ai} v_{Bf}}_b + \frac{1}{2} \left( \frac{m_B^2}{m_A} + m_B \right) \underbrace{v_{Bf}^2}_a$$

quadratic w/ c=0

# Elastic Collisions in 1d

a special case



mom

$$0 = -m_B v_{Ai} v_{Bf} + \frac{1}{2} \left( \frac{m_B^2}{m_A} + m_B \right) v_{Bf}^2$$

$$v_{Af} = v_{Ai} - \frac{m_B}{m_A} v_{Bf}$$

factor

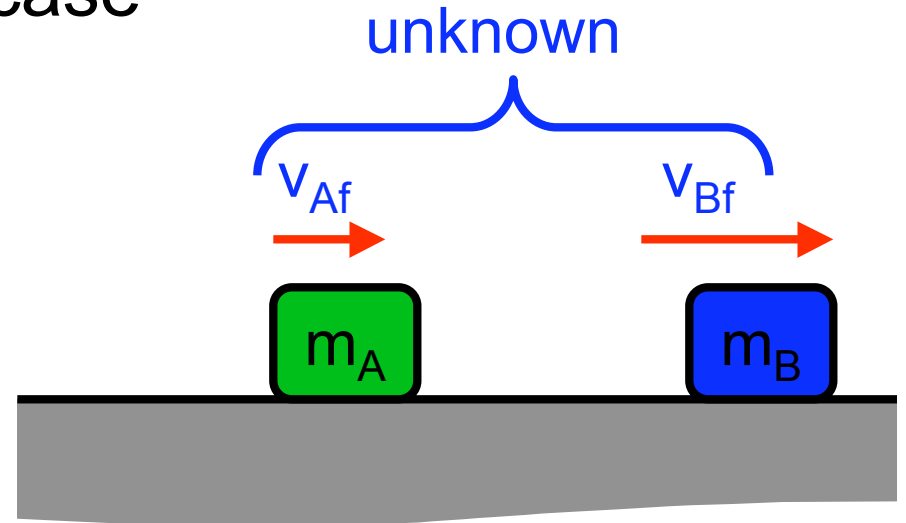
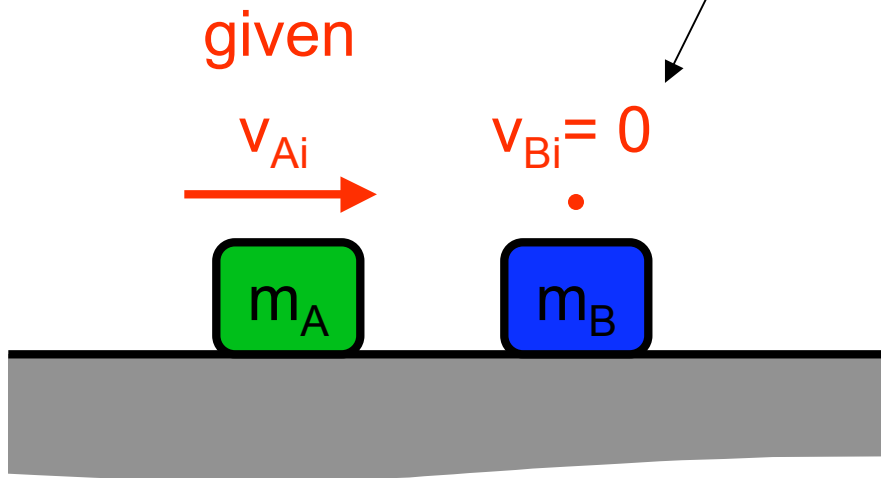
$$0 = -m_B v_{Bf} \left[ -v_{Ai} + \frac{1}{2} \left( \frac{m_B}{m_A} + 1 \right) v_{Bf} \right]$$

$$v_{Bf} = 0$$

$$v_{Bf} = \frac{2}{1 + m_B/m_A} v_{Ai}$$

# Elastic Collisions in 1d

a special case



momentum:  $m_A v_{Ai} = m_A v_{Af} + m_B v_{Bf}$

energy:  $0.5 m_A v_{Ai}^2 = 0.5 m_A v_{Af}^2 + 0.5 m_B v_{Bf}^2$

$$v_{Af} = v_{Ai} - \frac{m_B}{m_A} v_{Bf}$$

substitute

solution 1:  $v_{Af} = v_{Ai}$

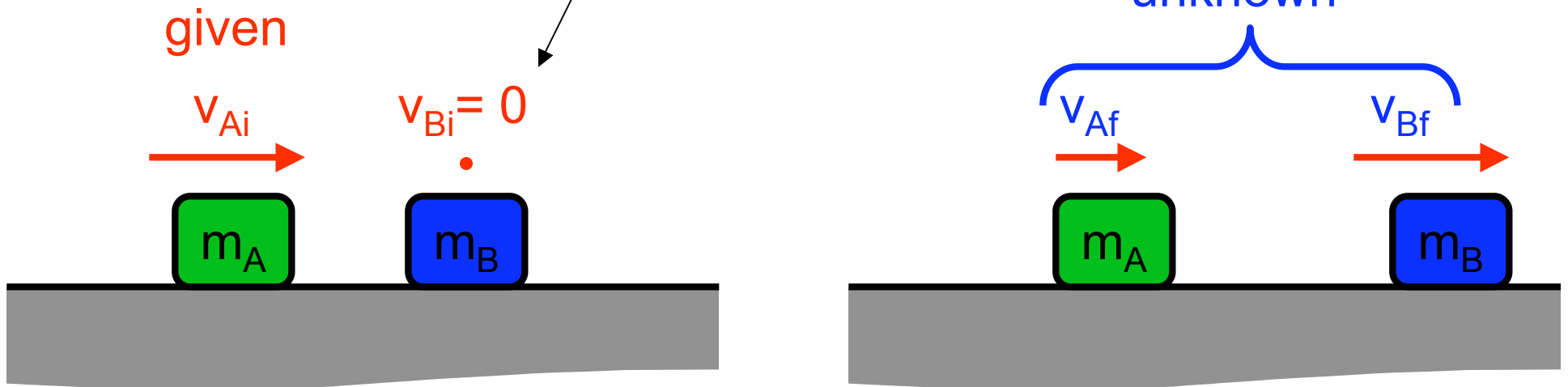
$$v_{Bf} = 0$$

solution 2:

$$v_{Bf} = \frac{2}{1 + m_B / m_A} v_{Ai}$$

# Elastic Collisions in 1d

a special case



momentum:  $m_A v_{Ai} = m_A v_{Af} + m_B v_{Bf}$

energy:  $0.5 m_A v_{Ai}^2 = 0.5 m_A v_{Af}^2 + 0.5 m_B v_{Bf}^2$

$$v_{Af} = v_{Ai} - \frac{m_B}{m_A} v_{Bf}$$

solution 1:  $v_{Af} = v_{Ai}$

$v_{Bf} = 0$

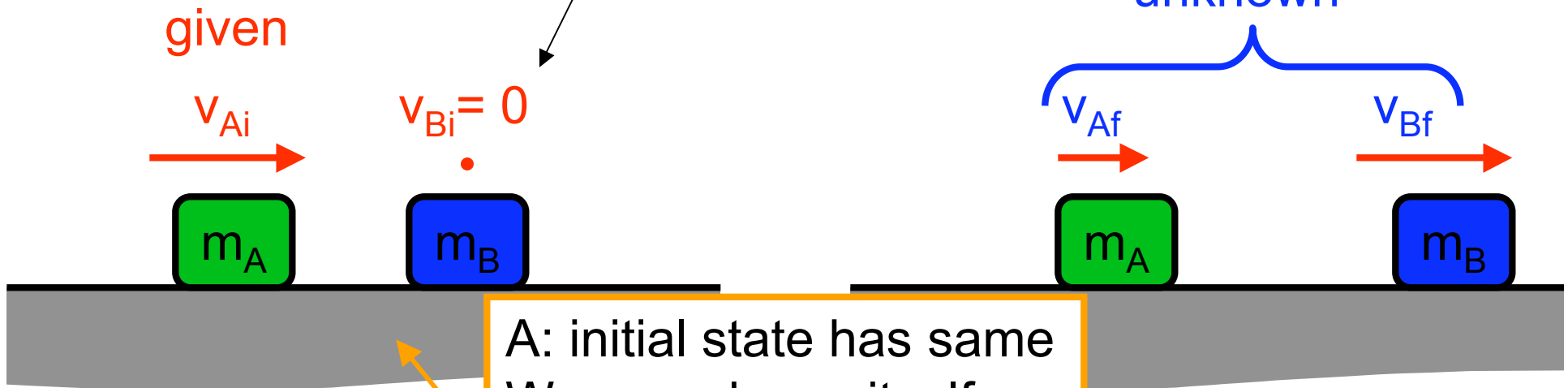
substitute

solution 2:  $v_{Af} = \frac{1 - m_B/m_A}{1 + m_B/m_A} v_{Ai}$

$$v_{Bf} = \frac{2}{1 + m_B/m_A} v_{Ai}$$

# Elastic Collisions in 1d

a special case



A: initial state has same  $W_{\text{mech}}$  and  $p$  as itself.

Q: What is the meaning of **solution 1**?

momentum:  $m_A v_{Ai} = m_A v_{Af} + m_B v_{Bf}$

energy:  $0.5 m_A v_{Ai}^2 = 0.5 m_A v_{Af}^2 + 0.5 m_B v_{Bf}^2$

**solution 1:**

$$v_{Af} = v_{Ai}$$

$$v_{Bf} = 0$$

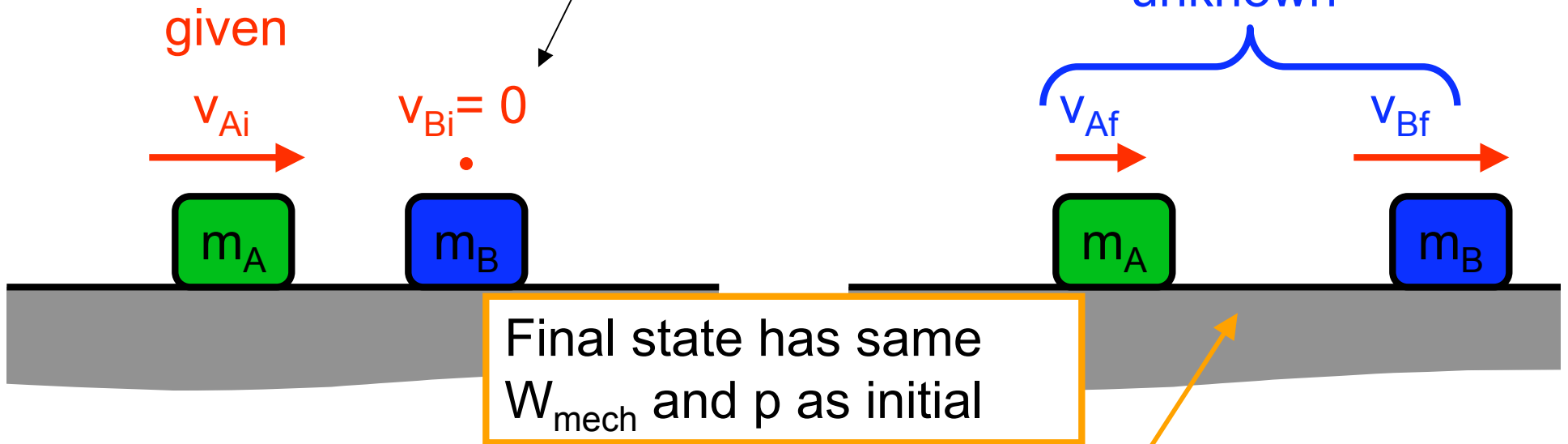
**solution 2:**

$$v_{Af} = \frac{1 - m_B / m_A}{1 + m_B / m_A} v_{Ai}$$

$$v_{Bf} = \frac{2}{1 + m_B / m_A} v_{Ai}$$

# Elastic Collisions in 1d

a special case



momentum:  $m_A v_{Ai} = m_A v_{Af} + m_B v_{Bf}$

energy:  $0.5 m_A v_{Ai}^2 = 0.5 m_A v_{Af}^2 + 0.5 m_B v_{Bf}^2$

solution 1:

$$v_{Af} = v_{Ai}$$

$$v_{Bf} = 0$$

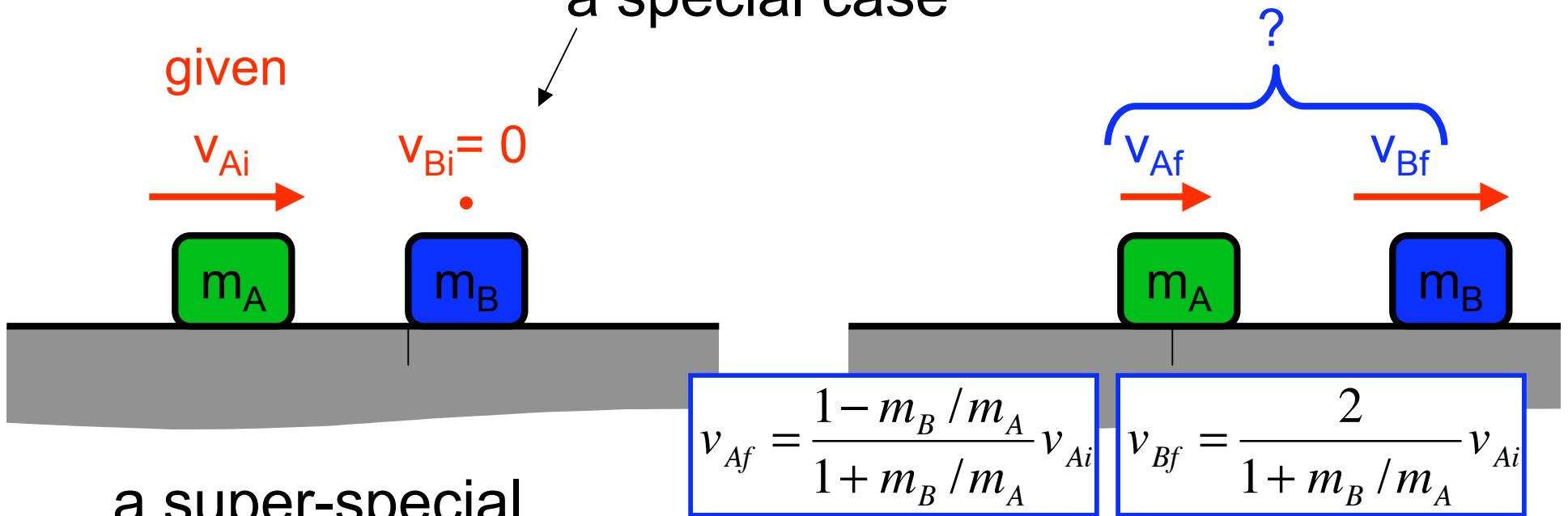
solution 2:

$$v_{Af} = \frac{1 - m_B / m_A}{1 + m_B / m_A} v_{Ai}$$

$$v_{Bf} = \frac{2}{1 + m_B / m_A} v_{Ai}$$

# Elastic Collisions in 1d

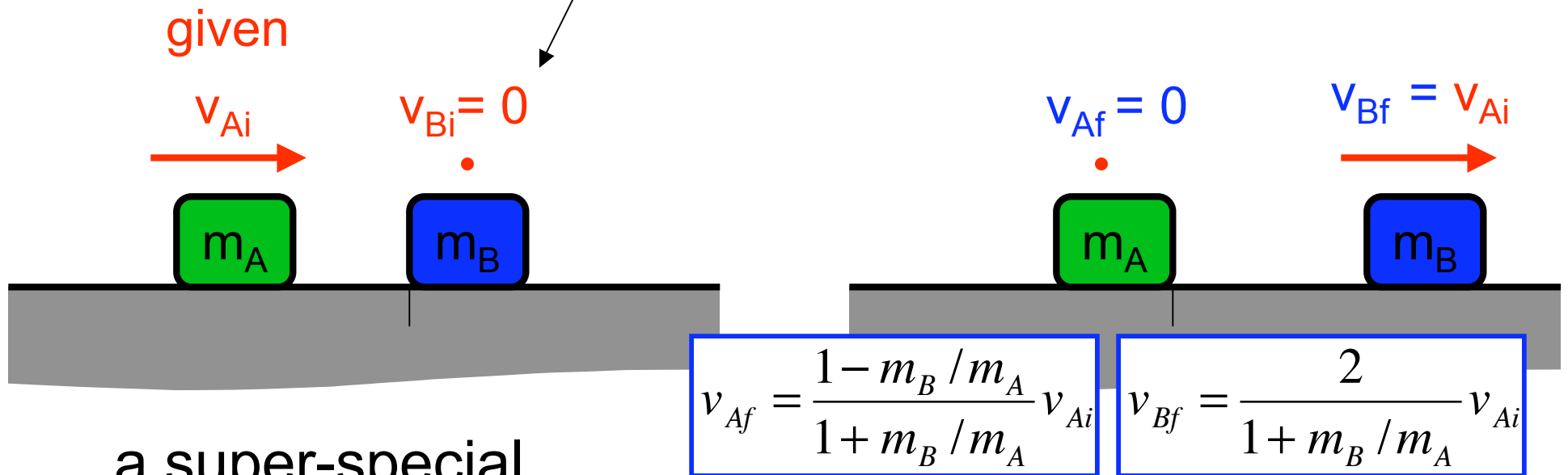
a special case



a super-special  
case:  $m_B = m_A$

# Elastic Collisions in 1d

a special case

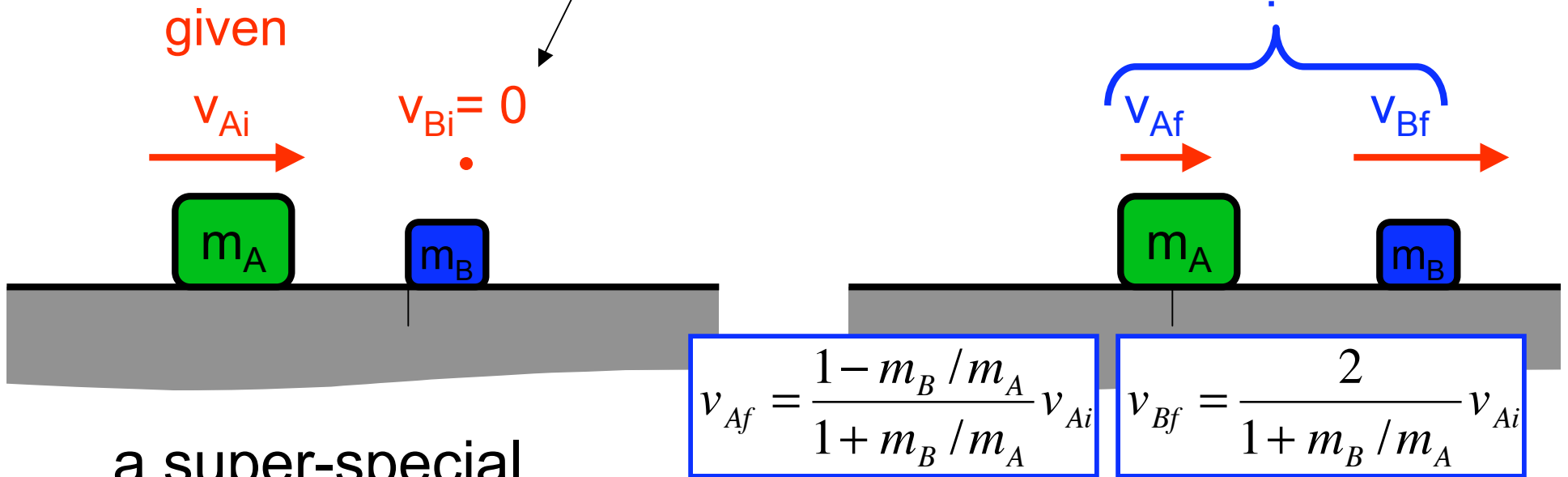


a super-special  
case:  $m_B = m_A$

“A transfers all its initial  
momentum to B”

# Elastic Collisions in 1d

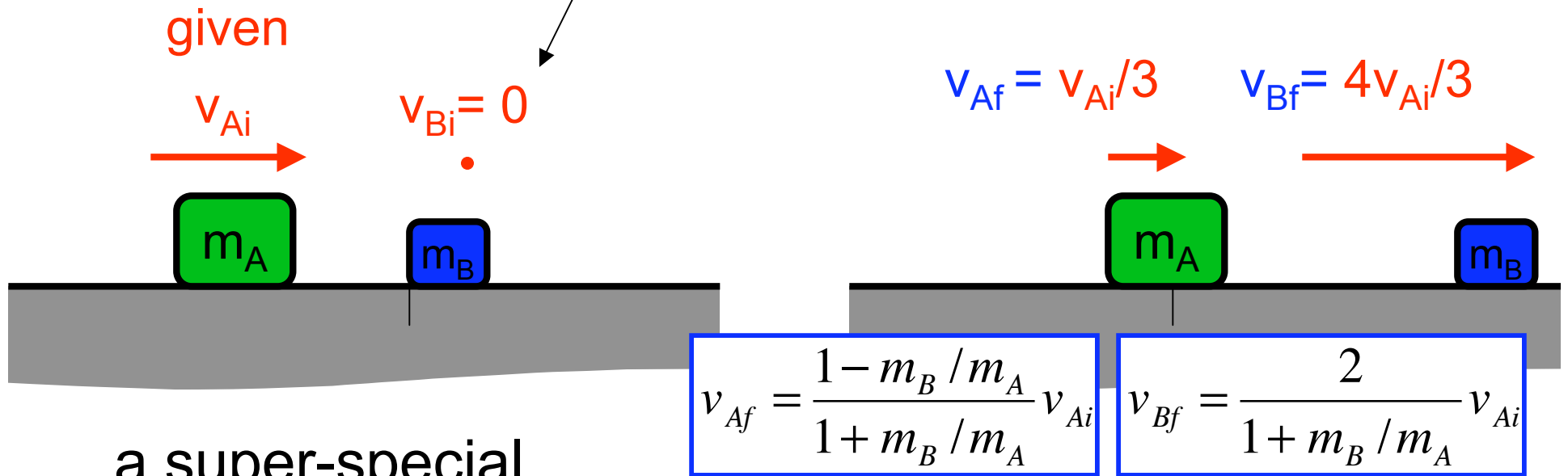
a special case



a super-special  
case:  $m_B = 0.5 m_A$

# Elastic Collisions in 1d

a special case



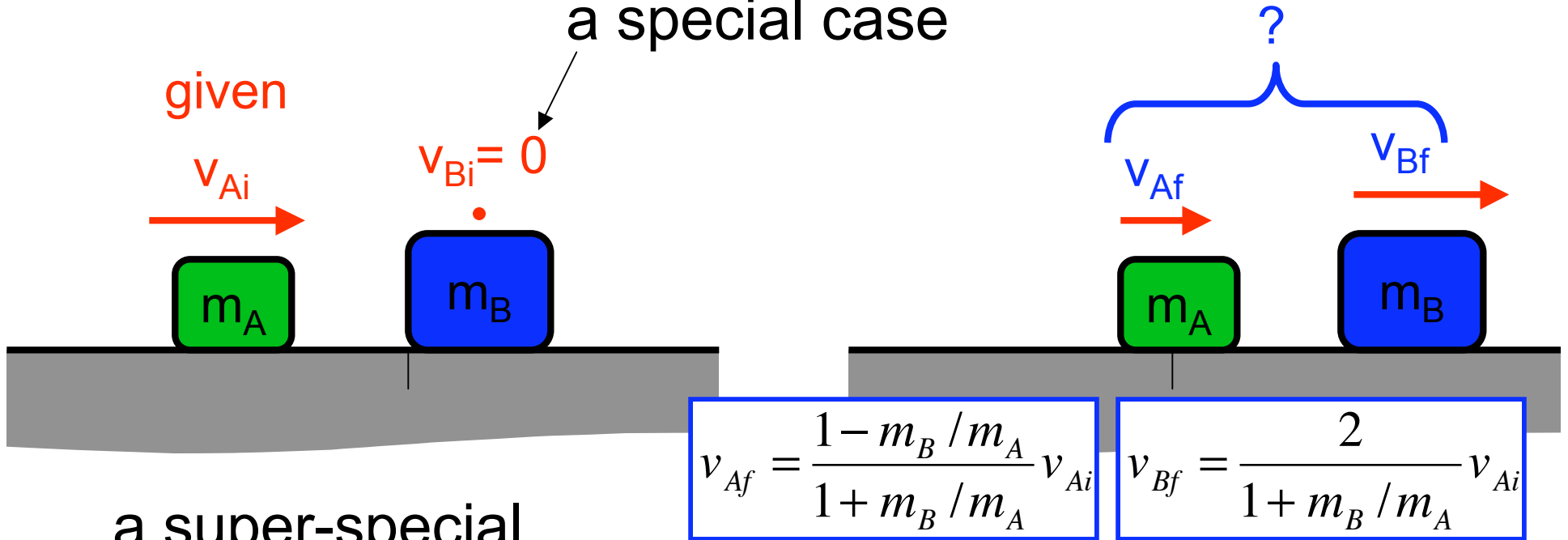
a super-special  
case:  $m_B = 0.5 m_A$

“A transfers 2/3 of its  
initial momentum to B”

“... which sends B away  
faster than A arrived”

# Elastic Collisions in 1d

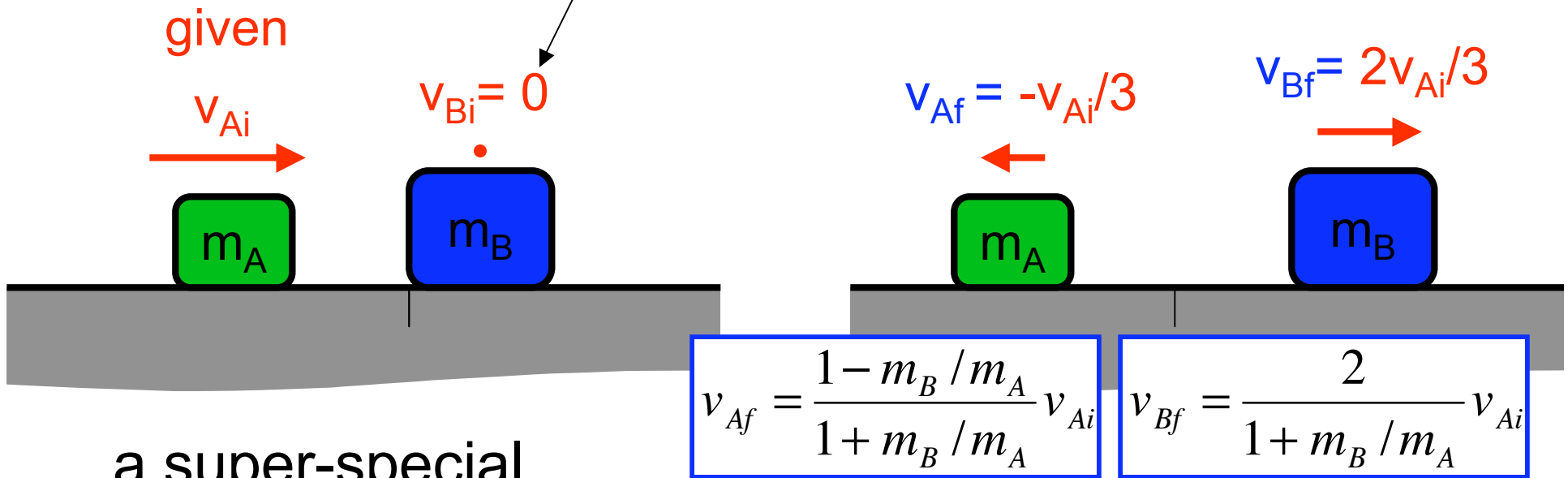
a special case



a super-special  
case:  $m_B = 2m_A$

# Elastic Collisions in 1d

a special case



a super-special case:  $m_B = 2m_A$

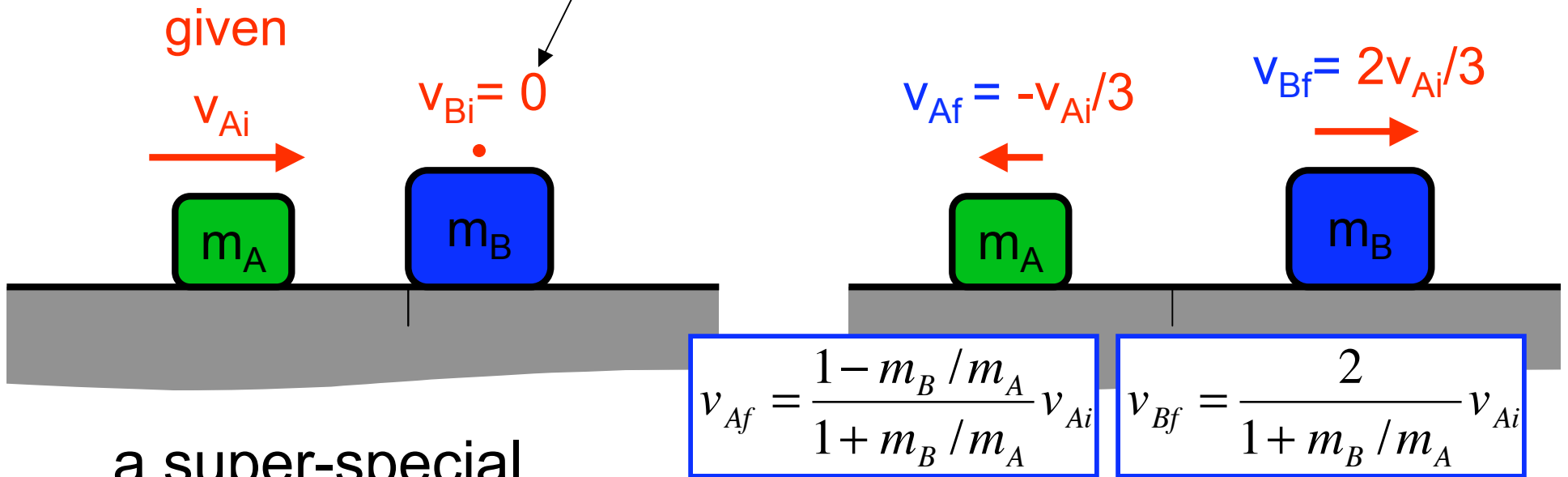
whenever  $m_B > m_A$   
block A will **reflect**  
i.e.  $v_{Af} < 0$

How much of its initial momentum does A transfer to B?

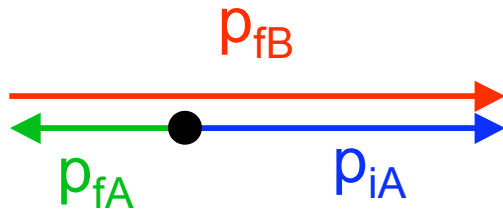
- A all:  $p_{fB} = p_{iA}$
- B less than all:  $p_{fB} < p_{iA}$
- C more than all:  $p_{fB} > p_{iA}$

# Elastic Collisions in 1d

a special case



a super-special case:  $m_B = 2m_A$



$$p_{iA} = p_{fA} + p_{fB}$$

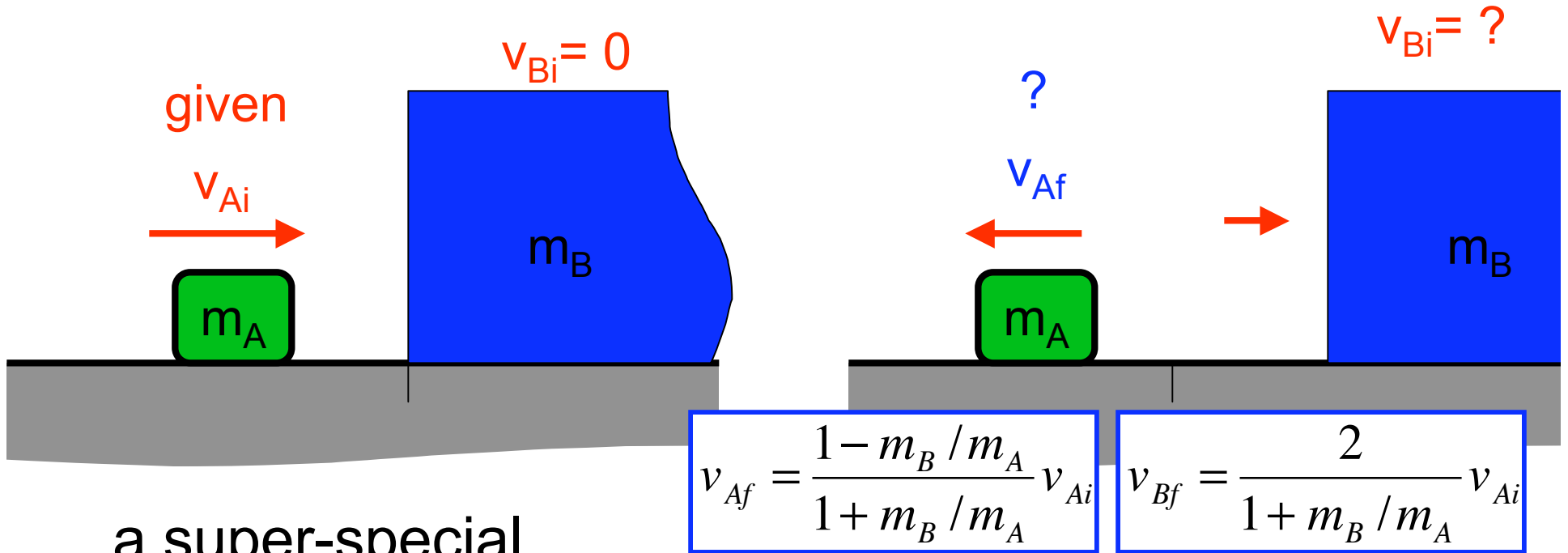
How much of its initial momentum does A transfer to B?

A all:  $p_{fB} = p_{iA}$

B less than all:  $p_{fB} < p_{iA}$

C more than all:  $p_{fB} > p_{iA}$

# Elastic Collisions in 1d



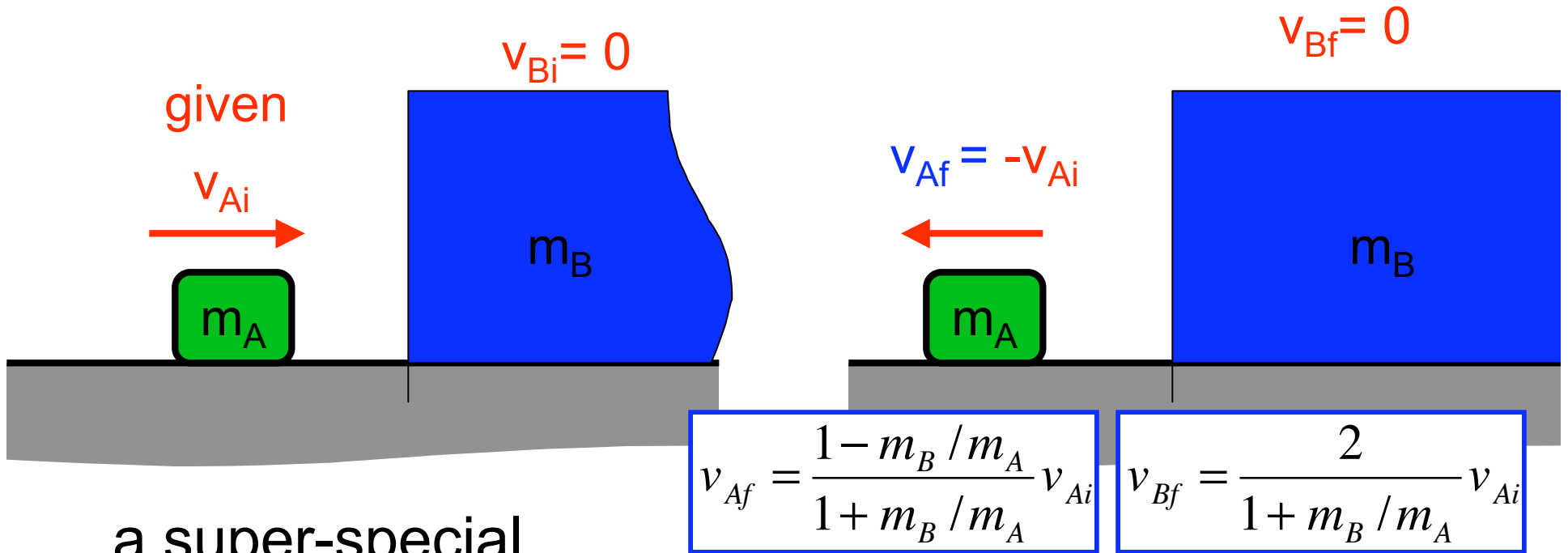
a super-special  
case:  $m_B \gg m_A$

$$\frac{m_B}{m_A} \gg 1$$

$$v_{Af} \approx \frac{-m_B/m_A}{+m_B/m_A} v_{Ai} = -v_{Ai}$$

$$v_{Bf} \approx \frac{2}{m_B/m_A} v_{Ai} = \frac{2m_A}{m_B} v_{Ai} \approx 0$$

# Elastic Collisions in 1d



a super-special  
case:  $m_B \gg m_A$

$$\frac{m_B}{m_A} \gg 1$$

$$p_{Bf} = m_B v_{Bf} = 2 v_{Ai}$$

$$v_{Af} \approx \frac{-m_B/m_A}{+m_B/m_A} v_{Ai} = -v_{Ai}$$

$$v_{Bf} \approx \frac{2}{m_B/m_A} v_{Ai} = \frac{2m_A}{m_B} v_{Ai} \approx 0$$