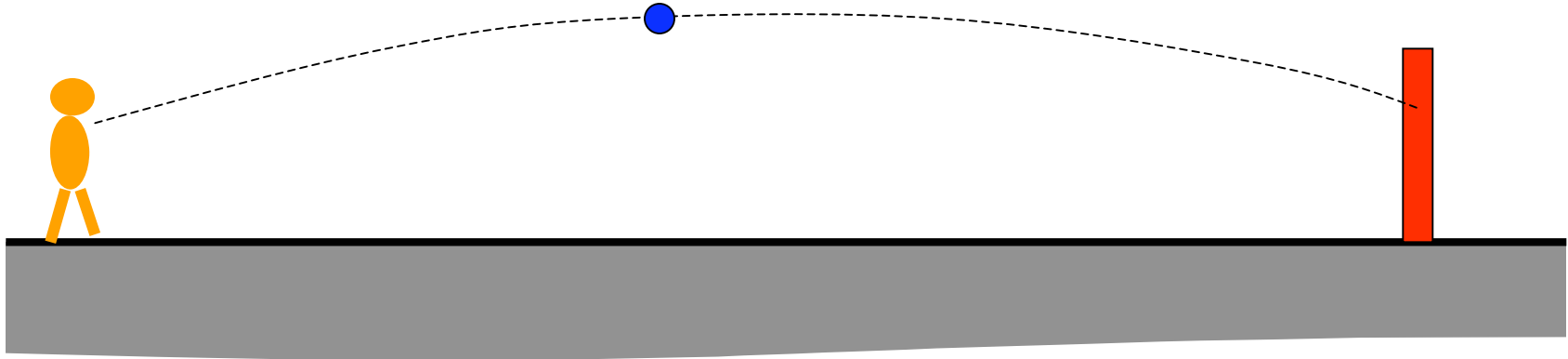


Collisions in 2d &



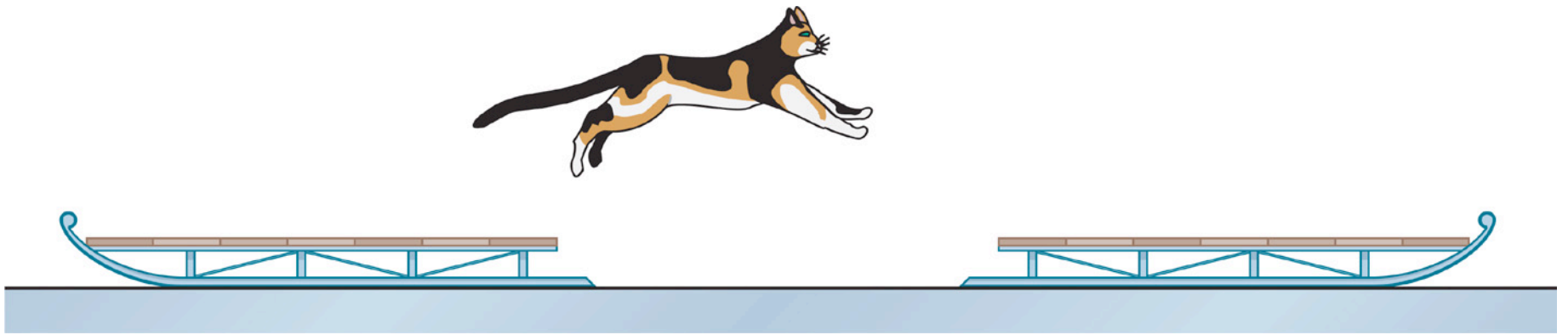
Craig wants to knock over a stone pillar by throwing a ball at it. He has two balls of identical mass, which he can throw with the same speed.

- A **stone** ball which will collide **perfectly elastically**
- A **clay** ball which will collide **perfectly inelastically**
(i.e. it will stick to the surface when it hits)

Which is more likely to knock it over (larger impulse)

- A** The stone ball
- B** The clay ball
- C** It won't matter

What is conserved?



A W_{mech}

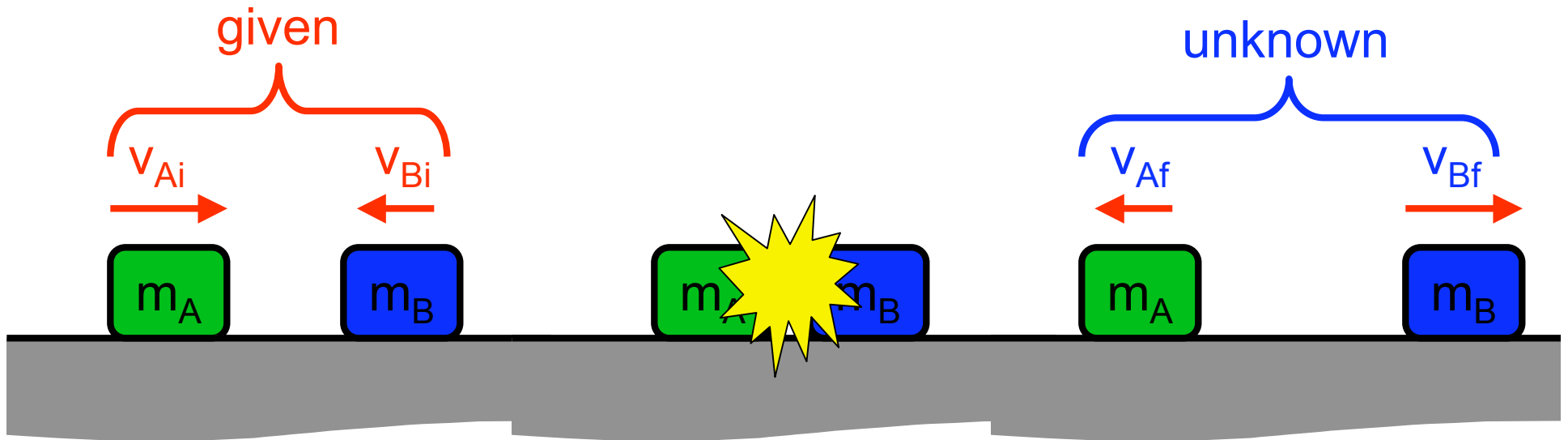
B p_{cat}

C $p_{\text{sys}, x}$

D W_{mech} & $p_{\text{sys}, x}$

E neither is conserved

Elastic Collisions in 1d

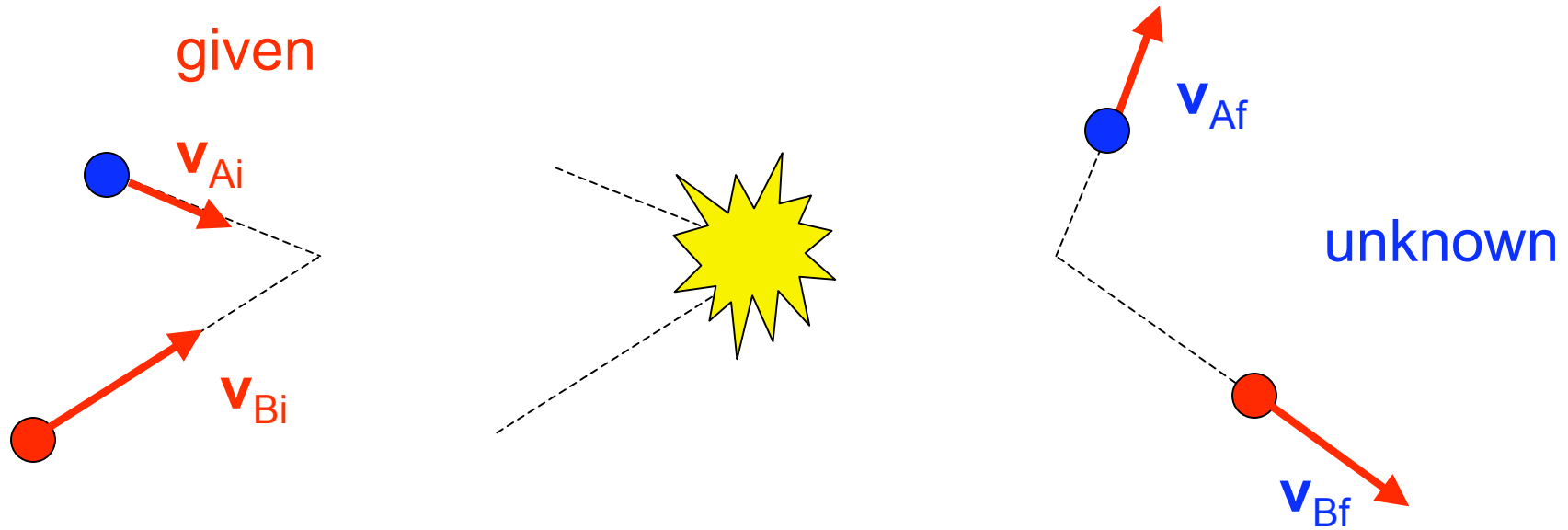


momentum: $p_i = m_A v_{Ai} + m_B v_{Bi} = m_A v_{Af} + m_B v_{Bf} = p_f$

energy: $K_i = 0.5 m_A v_{Ai}^2 + 0.5 m_B v_{Bi}^2 = 0.5 m_A v_{Af}^2 + 0.5 m_B v_{Bf}^2 = K_f$

✓ 2 equations for 2 unknowns

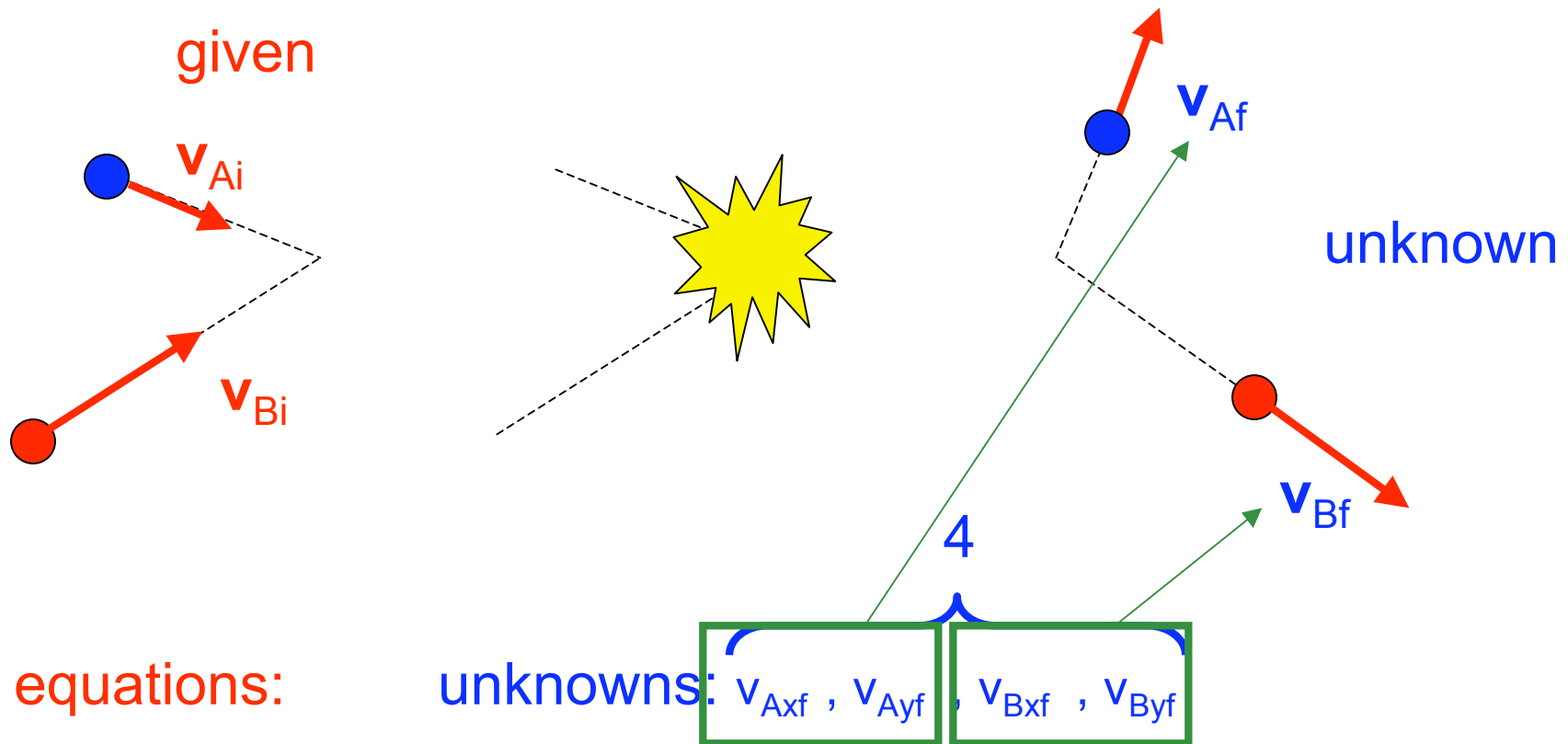
Elastic Collisions in 2d



Q: can we find the final velocities from conservation laws alone in 2d (as we could in 1d)?

- A Yes
- B No
- C I don't understand the question

Elastic Collisions in 2d



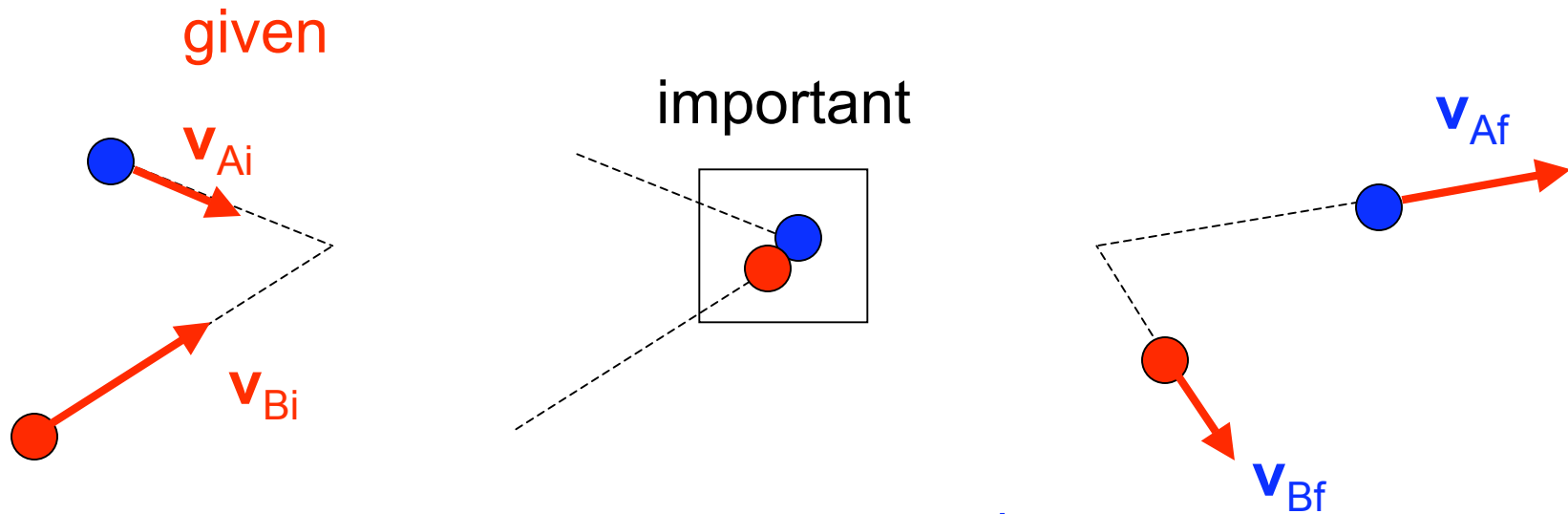
equations:

unknowns: v_{Axf} , v_{Ayf} , v_{Bxf} , v_{Byf}

$$\begin{cases}
 p_{xi} = m_A v_{Axi} + m_B v_{Bxi} = m_A v_{Axf} + m_B v_{Bxf} = p_{xf} \\
 p_{yi} = m_A v_{Ayi} + m_B v_{Byi} = m_A v_{Ayf} + m_B v_{Byf} = p_{yf} \\
 K_i = 0.5 m_A |\mathbf{v}_{Ai}|^2 + 0.5 m_B |\mathbf{v}_{Bi}|^2 = 0.5 m_A |\mathbf{v}_{Af}|^2 + 0.5 m_B |\mathbf{v}_{Bf}|^2 = K_f
 \end{cases}$$

Not enough information

Elastic Collisions in 2d



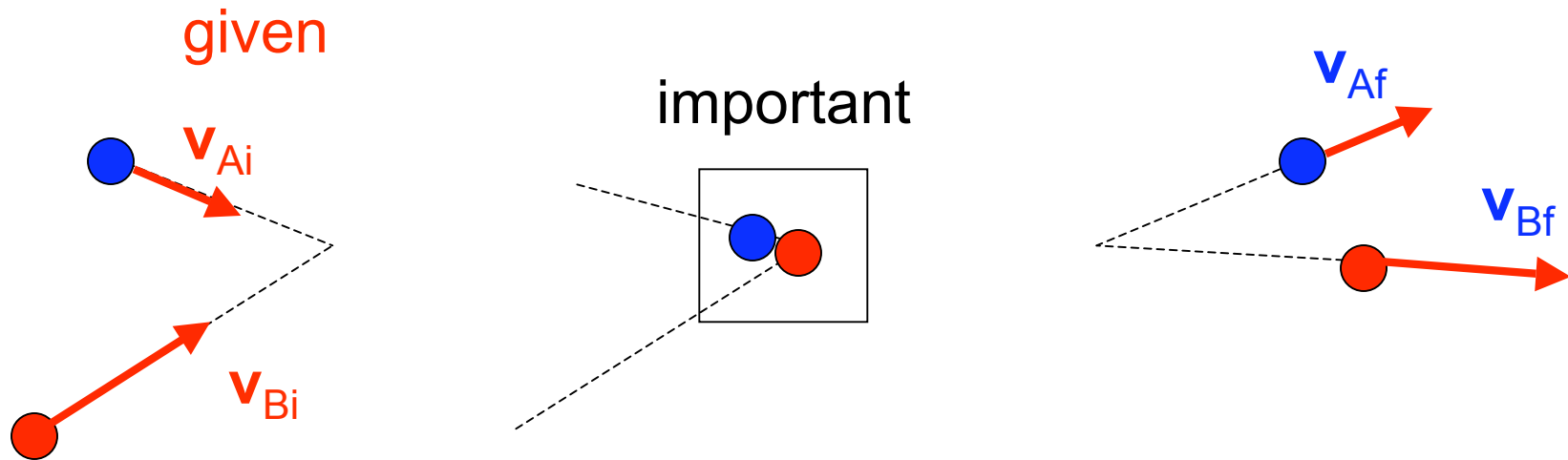
equations:

unknowns: v_{Axf} , v_{Ayf} , v_{Bxf} , v_{Byf}

3

$$\left\{ \begin{array}{l} p_{xi} = m_A v_{Axi} + m_B v_{Bxi} = m_A v_{Axf} + m_B v_{Bxf} = p_{xf} \\ p_{yi} = m_A v_{Ayi} + m_B v_{Byi} = m_A v_{Ayf} + m_B v_{Byf} = p_{yf} \\ K_i = 0.5 m_A |\mathbf{v}_{Ai}|^2 + 0.5 m_B |\mathbf{v}_{Bi}|^2 = 0.5 m_A |\mathbf{v}_{Af}|^2 + 0.5 m_B |\mathbf{v}_{Bf}|^2 = K_f \end{array} \right.$$

Elastic Collisions in 2d



equations:

unknowns: v_{Axf} , v_{Ayf} , v_{Bxf} , v_{Byf}

$$\begin{cases}
 p_{xi} = m_A v_{Axi} + m_B v_{Bxi} = m_A v_{Axf} + m_B v_{Bxf} = p_{xf} \\
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 \end{cases}$$

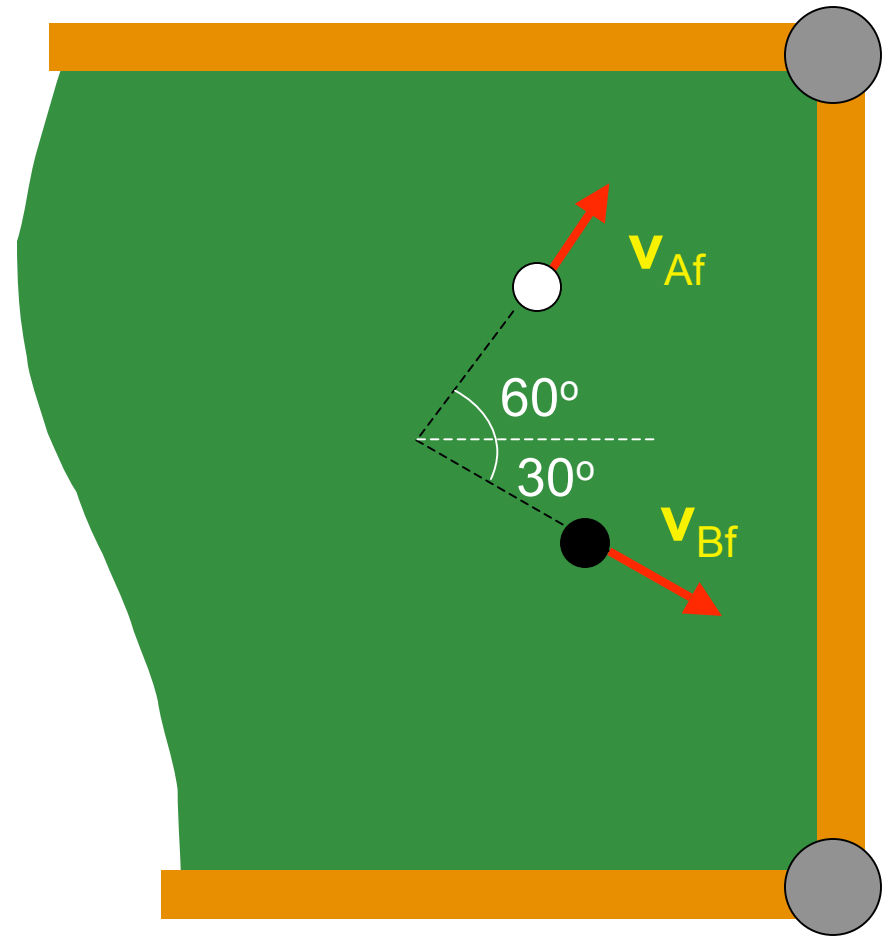
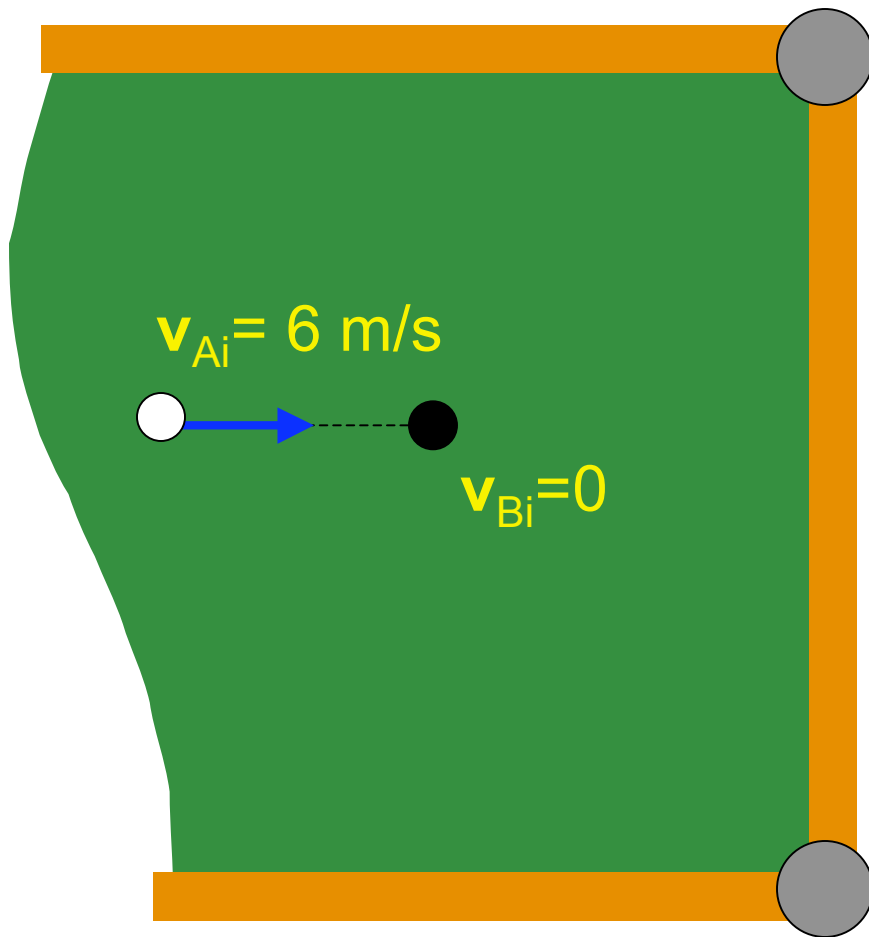
Cue ball is sent @ 6 m/s at the stationary 8-ball. The cue ball deflects to 60° . How fast is each ball moving after the collision?

A equal speeds

B Cue ball moves faster

C 8-ball moves faster

D not enough information

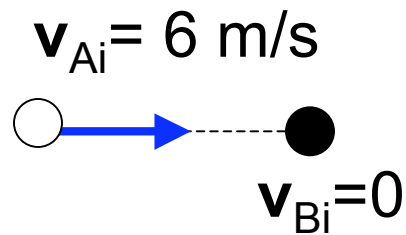


Cue ball is sent @ 6 m/s at the stationary 8-ball. The cue ball deflects to 60°. How fast is each ball moving after the collision?

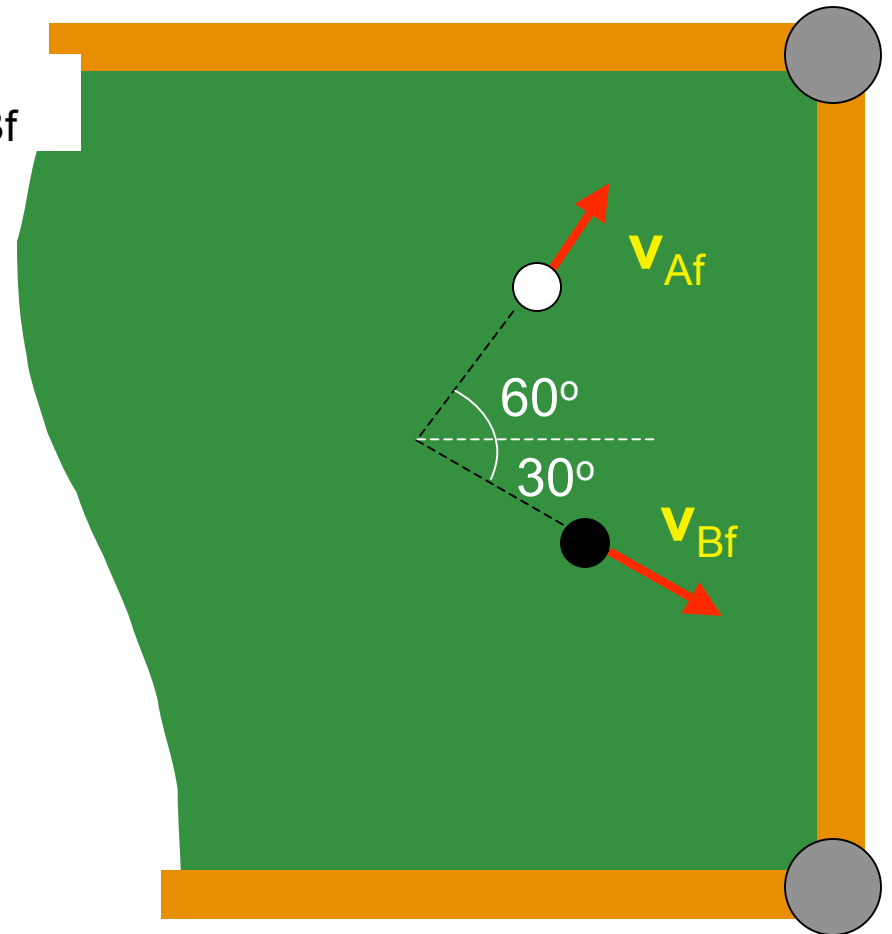
$$p_{fx} = [v_{Af} \cos(60^\circ) + v_{Bf} \cos(30^\circ)] m_A = 6 m_A$$

$$p_{fy} = [v_{Af} \sin(60^\circ) - v_{Bf} \sin(30^\circ)] m_A = 0$$

$$v_{Af} = v_{Bf} \sin(30^\circ) / \sin(60^\circ) = 0.58 v_{Bf}$$



$$\mathbf{p}_i = m_A 6 \hat{\mathbf{i}}$$



Cue ball is sent @ 6 m/s at the stationary 8-ball. The cue ball deflects to 60°. How fast is each ball moving after the collision?

$$p_{fx} = [v_{Af} \cos(60^\circ) + v_{Bf} \cos(30^\circ)]m_A = 6 m_A$$

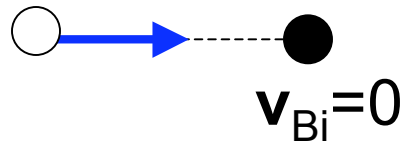
$$p_{fy} = [v_{Af} \sin(60^\circ) - v_{Bf} \sin(30^\circ)]m_A = 0$$

C 8-ball moves faster

$$v_{Af} = v_{Bf} \sin(30^\circ) / \sin(60^\circ) = 0.58 v_{Bf}$$

$$0.5 v_{Af} + 0.87 v_{Bf} = 6 = 1.15 v_{Bf} = 6$$

$$v_{Ai} = 6 \text{ m/s}$$



$$v_{Bf} = 5.2 \text{ m/s}$$

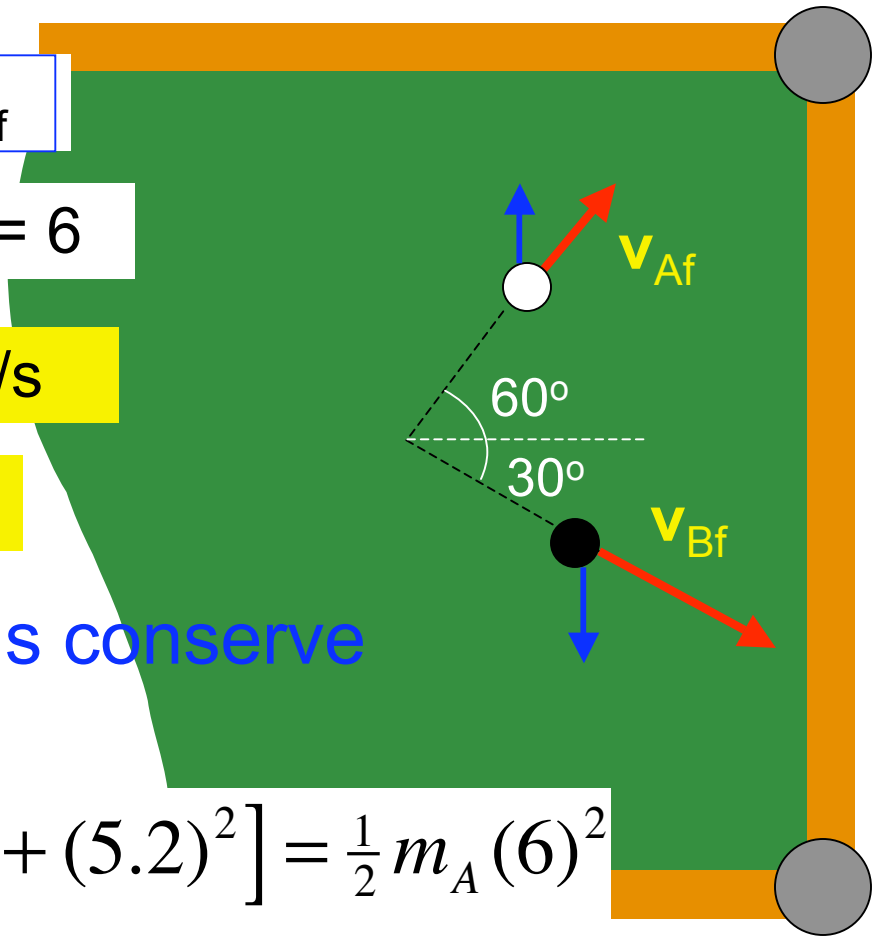
$$v_{Af} = 3 \text{ m/s}$$

$$p_i = m_A 6 \hat{i}$$

Q: Does this conserve energy?

$$\frac{1}{2} m_A v_{Af}^2 + \frac{1}{2} m_B v_{Bf}^2 = \frac{1}{2} m_A [(3)^2 + (5.2)^2] = \frac{1}{2} m_A (6)^2$$

9 + 27



Rotation

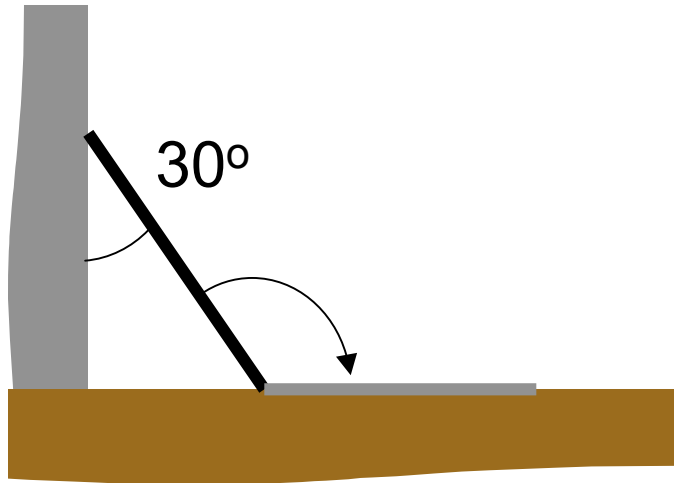


Reading Question

The units of angular acceleration α are

- A m/s^2
- B dimensionless
- C rad (radians)
- D rad/s^2

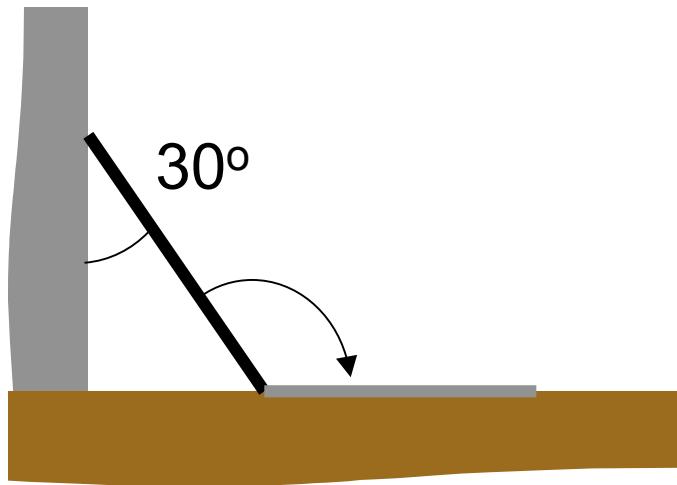
Rotation



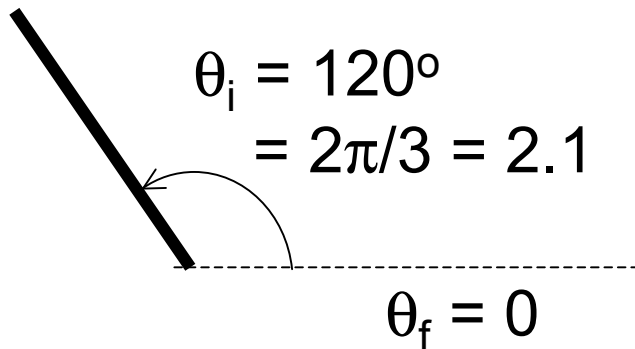
A ladder is leaning against a wall, as shown, when a sudden gust of wind knocks it over. It pivots about its end and takes **1.5 sec** to go from the wall to the ground. What is its average angular velocity?

- A -1.40 rad/sec
- B -0.35 rad/sec
- C +0.35 rad/sec
- D +1.40 rad/sec

Rotation

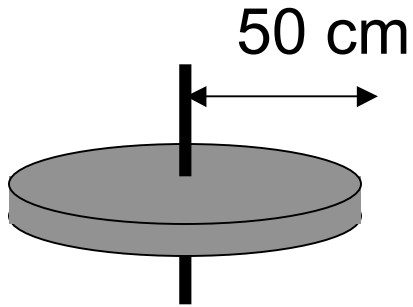


A ladder is leaning against a wall, as shown, when a sudden gust of wind knocks it over. It pivots about its end and takes **1.5 sec** to go from the wall to the ground. What is its average angular velocity?



$$\Delta\theta = \theta_f - \theta_i = -2.1 \text{ rad}$$

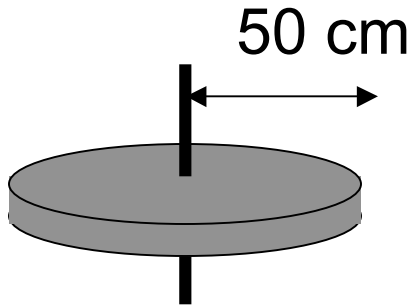
- A -1.40 rad/sec
- B -0.35 rad/sec
- C +0.35 rad/sec
- D +1.40 rad/sec



A 50 cm potter's wheel spins down at a **constant** angular acceleration $\alpha = 0.5 \text{ rad/s}^2$ from an initial ang. vel. of $\omega_0 = -16.0 \text{ rad/sec}$. How much time does it take to come to rest?

which equation would you use?

- A $\omega = \omega_0 + \alpha t$
- B $\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$
- C $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
- D $\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
- E $\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$



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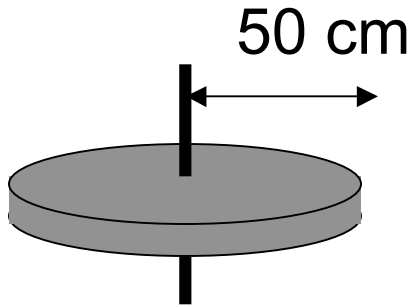
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$$\omega_i = -16$$

$$\omega_f = 0$$

$$t = -(\omega_f - \omega_i)/\alpha \\ = 32 \text{ sec}$$



A 50 cm potter's wheel spins down at a **constant** angular acceleration $\alpha = 0.5 \text{ rad/s}^2$ from an initial angular velocity of $\omega_0 = -16.0 \text{ rad/sec}$. How many rotations will it undergo?

$$\omega = \omega_0 + \alpha t$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

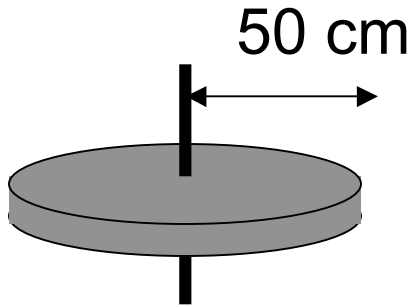
$$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$$

$$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$$

$$\omega_i = -16 \quad \omega_f = 0$$

$$t = 32 \text{ sec}$$

- A 512
- B 256
- C 81
- D 41



A 50 cm potter's wheel spins down at a **constant** angular acceleration $\alpha = 0.5 \text{ rad/s}^2$ from an initial ang. vel. of $\omega_0 = -16.0 \text{ rad/sec}$. How many rotations will it undergo?

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$$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$$

$$\omega_i = -16 \quad \omega_f = 0$$

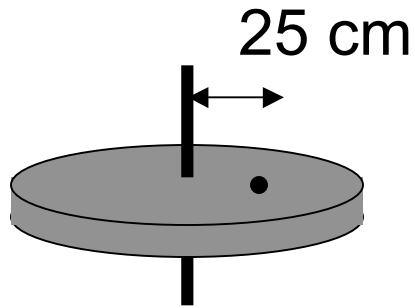
$$t = 32 \text{ sec}$$

$$0 = (-16)^2 + 2(0.5)(\theta - \theta_0)$$

$$\theta - \theta_0 = -256 \text{ rad}$$

$$(\theta - \theta_0)/2\pi = -40.74$$

- A 512
- B 256
- C 81
- D 41



A 50 cm potter's wheel spins down at a **constant** angular acceleration $\alpha = 0.5 \text{ rad/s}^2$ from an initial angular velocity of $\omega_0 = -16.0 \text{ rad/sec}$. A lump of clay sits at a point 25 cm from the center. What is the linear speed of the clay at $t=8 \text{ sec}$?

$$\omega = \omega_0 + \alpha t$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$$

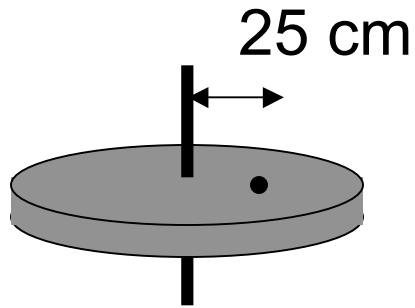
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$$

$$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$$

$$\omega_i = -16$$

- A 12 m/s
- B 6 m/s
- C 4 m/s
- D 3 m/s
- E 1.5 m/s



A 50 cm potter's wheel spins down at a **constant** angular acceleration $\alpha = 0.5 \text{ rad/s}^2$ from an initial angular velocity of $\omega_0 = -16.0 \text{ rad/sec}$. A lump of clay sits at a point 25 cm from the center. What is the linear speed of the clay at $t=8 \text{ sec}$?

$$\omega = \omega_0 + \alpha t$$

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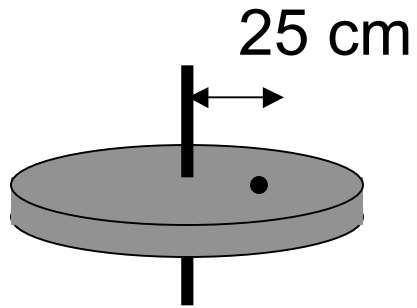
$$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$$

$$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$$

$$\omega_i = -16 \quad \omega = -16 + 0.5(8) = -12 \text{ rad/s}$$

$$v = \omega r = -12 (0.25) = 3 \text{ m/s}$$

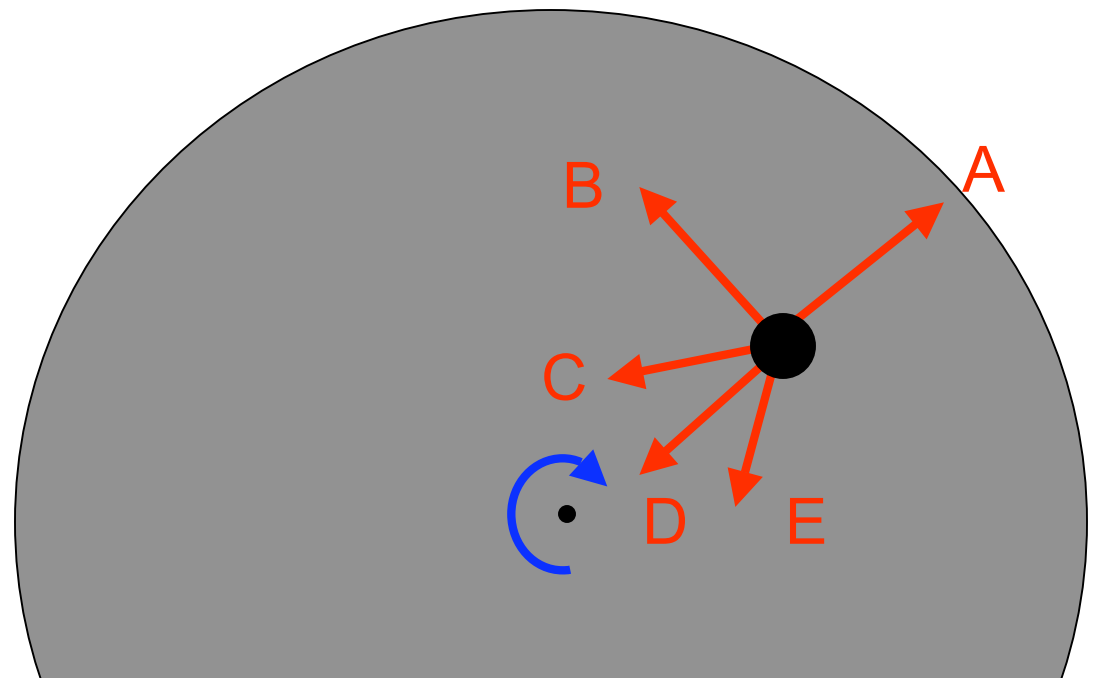
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- B 6 m/s
- C 4 m/s
- D 3 m/s
- E 1.5 m/s

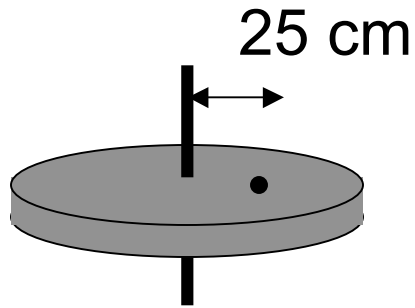


$$\omega = -1 \text{ rad/s}$$

$$v = 0.25 \text{ m/s}$$

A 50 cm potter's wheel spins down at a **constant** angular acceleration $\alpha = 0.5 \text{ rad/s}^2$ from an initial angular velocity of $\omega_0 = -16.0 \text{ rad/sec}$. A lump of clay sits at a point 25 cm from the center. Which could be the acceleration of the clay at 30 sec?





$$\omega = -1 \text{ rad/s}$$

$$v = 0.25 \text{ m/s}$$

$$a_c = \frac{v^2}{r} = \frac{(\omega r)^2}{r} = \omega^2 r$$

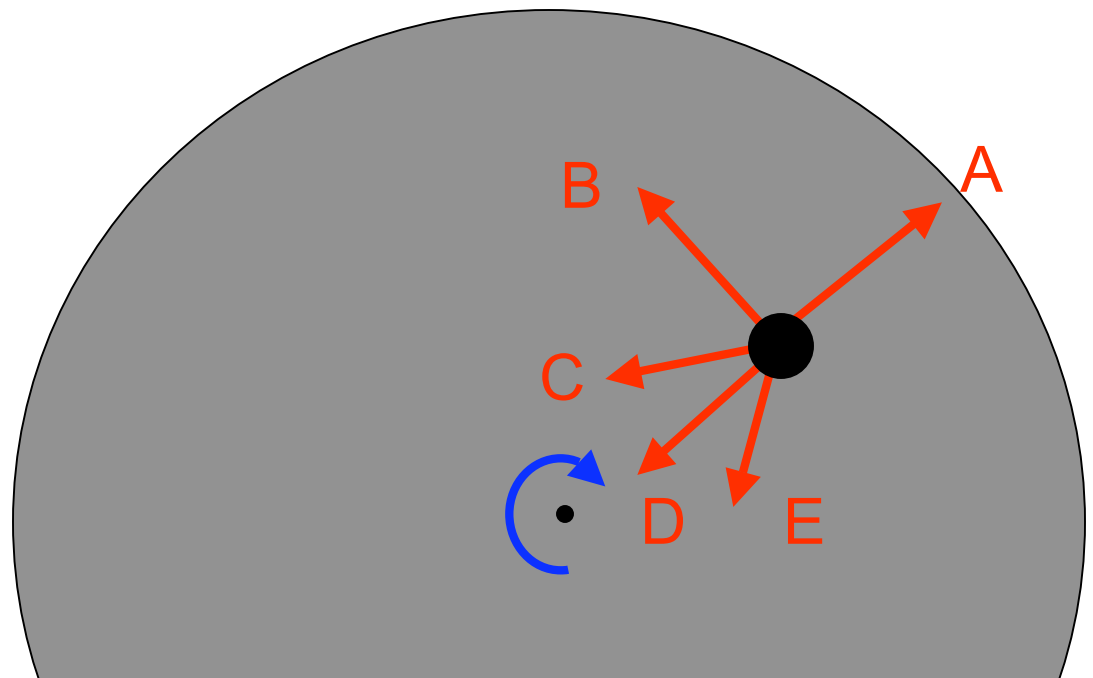
$$a_c = 0.25 \text{ m/s}^2$$

$$a_t = \alpha r$$

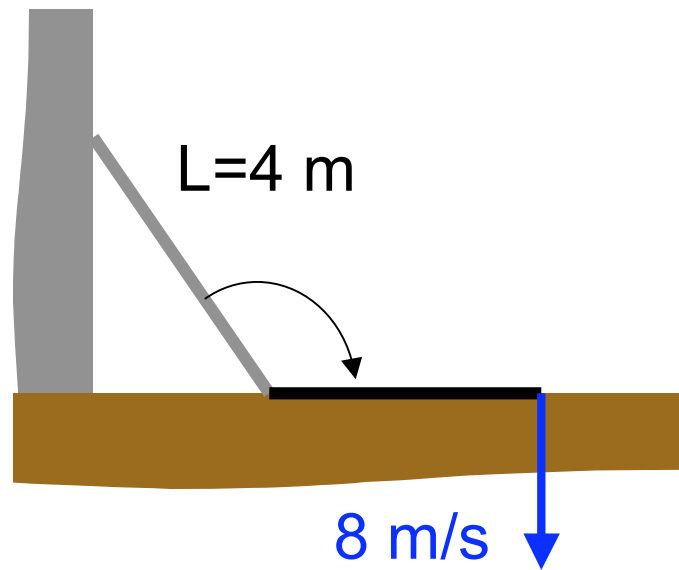
$$a_t = +0.125 \text{ m/s}^2$$

$$\vec{a} = \vec{a}_t + \vec{a}_c$$

A 50 cm potter's wheel spins down at a **constant** angular acceleration $\alpha = 0.5 \text{ rad/s}^2$ from an initial angular velocity of $\omega_0 = -16.0 \text{ rad/sec}$. A lump of clay sits at a point 25 cm from the center. Which could be the acceleration of the clay at 30 sec?



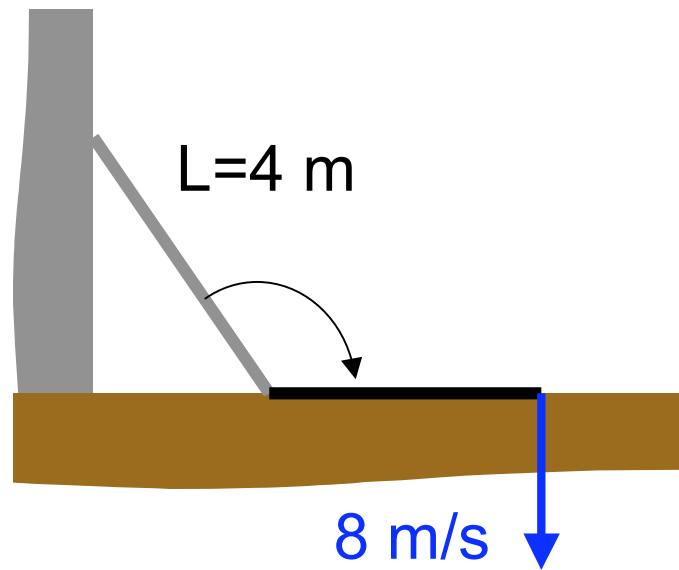
Rotation



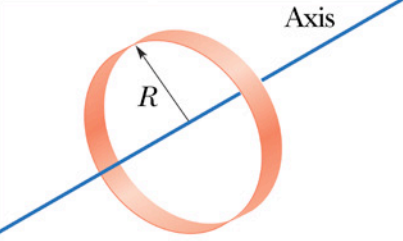
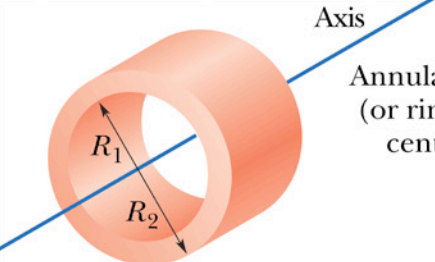
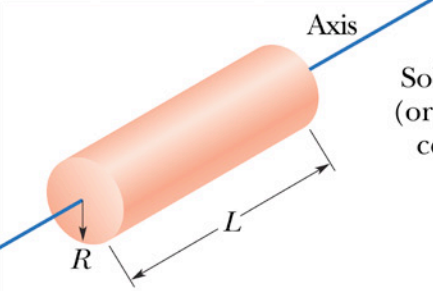
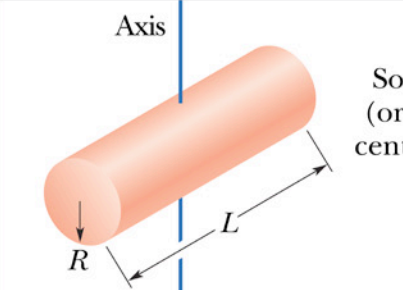
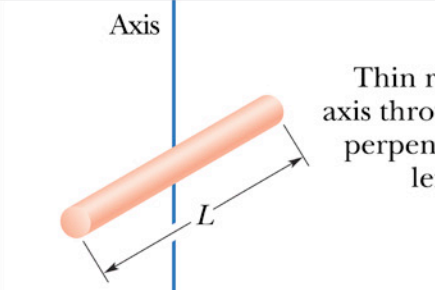
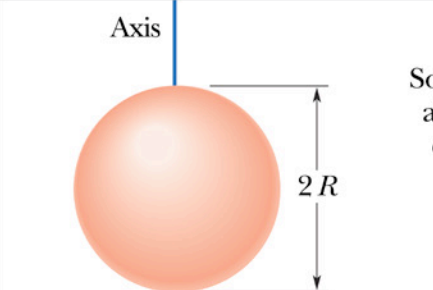
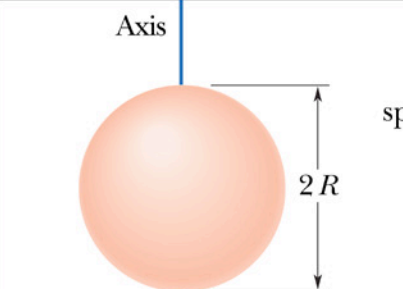
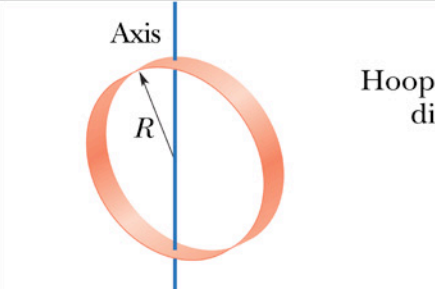
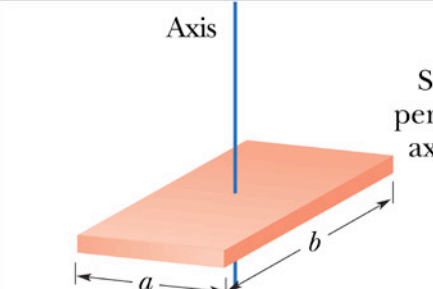
A 4m ladder ($m=10\text{ kg}$) is leaning against a wall, as shown, when a sudden gust of wind knocks it over. At the instant it hits the ground the ladder's tip is moving at 8 m/s . What is the ladder's angular velocity at that time?

- A -2 rad/s
- B -0.5 rad/s
- C $+0.5\text{ rad/s}$
- D $+2\text{ rad/s}$

Rotation



A 4m ladder ($m=10\text{ kg}$) is leaning against a wall, as shown, when a sudden gust of wind knocks it over. At the instant it hits the ground the ladder's tip is moving at 8 m/s . What is the kinetic energy of the ladder?

 <p>Hoop about central axis</p> <p>$I = MR^2$</p> <p>(a)</p>	 <p>Annular cylinder (or ring) about central axis</p> <p>$I = \frac{1}{2}M(R_1^2 + R_2^2)$</p> <p>(b)</p>	 <p>Solid cylinder (or disk) about central axis</p> <p>$I = \frac{1}{2}MR^2$</p> <p>(c)</p>
 <p>Solid cylinder (or disk) about central diameter</p> <p>$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$</p> <p>(d)</p>	 <p>Thin rod about axis through center perpendicular to length</p> <p>$I = \frac{1}{12}ML^2$</p> <p>(e)</p>	 <p>Solid sphere about any diameter</p> <p>$I = \frac{2}{5}MR^2$</p> <p>(f)</p>
 <p>Thin spherical shell about any diameter</p> <p>$I = \frac{2}{3}MR^2$</p> <p>(g)</p>	 <p>Hoop about any diameter</p> <p>$I = \frac{1}{2}MR^2$</p> <p>(h)</p>	 <p>Slab about perpendicular axis through center</p> <p>$I = \frac{1}{12}M(a^2 + b^2)$</p> <p>(i)</p>