

Torque



How vs. Why

kinematics: How linear variables change in time*

$$v = v_0 + at$$

$$x - x_0 = v_0t + \frac{1}{2}at^2$$

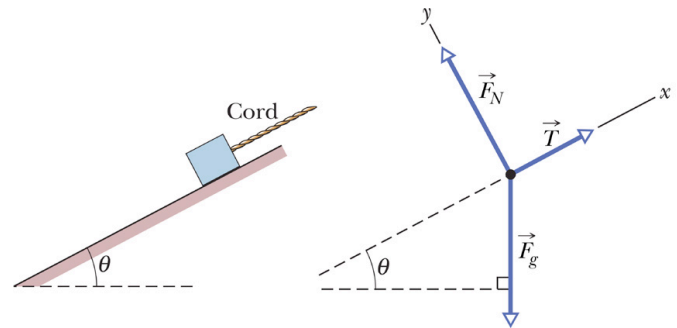
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t$$

$$x - x_0 = vt - \frac{1}{2}at^2$$

* provided **a** is constant

BUT why does a body undergo acceleration **a**?



A: Net force \leftrightarrow acceleration

$$\vec{F}_{\text{net}} = \sum \vec{F} = m \vec{a}_{\text{cm}}$$

Add all forces regardless of where they act on the body (FBD)

How vs. Why

kinematics: How angular variables change in time*

$$\omega = \omega_0 + \alpha t$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$$

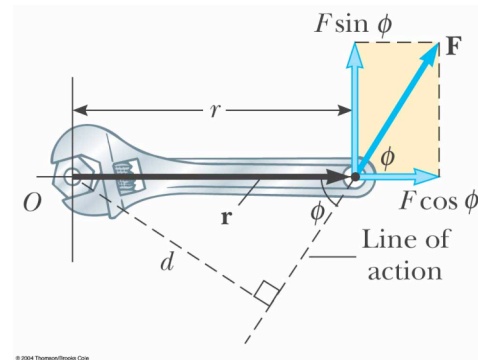
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$$

$$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$$

* provided α is constant

BUT **why** does a body undergo acceleration α about a given axis?



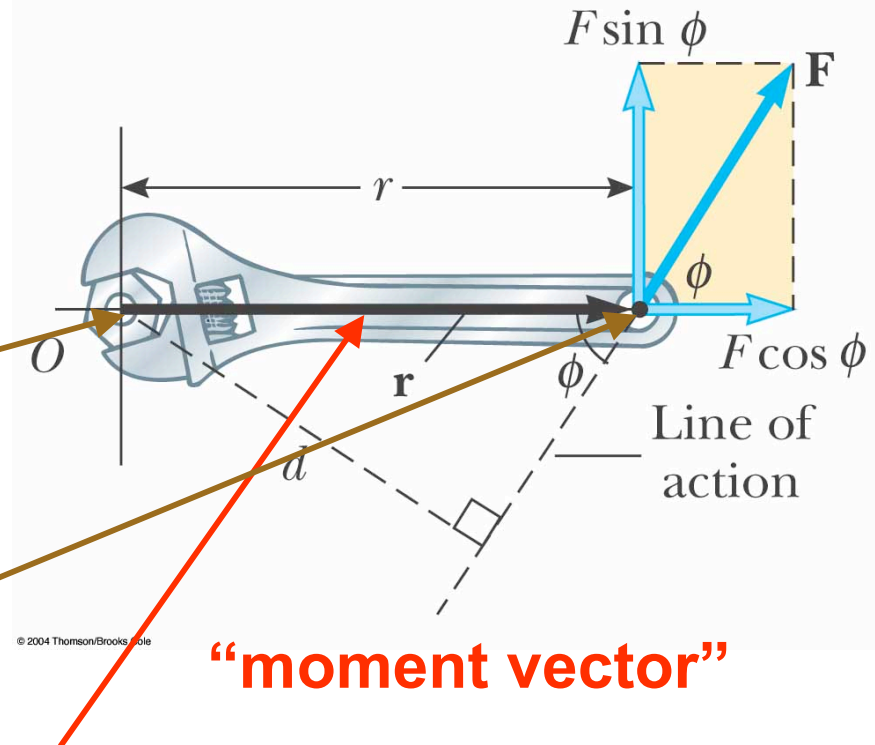
A: Net torque \leftrightarrow angular acceleration

$$\tau_{\text{net}} = \sum \tau = I\alpha$$

Computing torque

Ways torque differs from force:

1. Torque is computed about a given axis¹
2. One must know **where** force is applied²



Torque depends on vector $\vec{\mathbf{r}}$ between these points

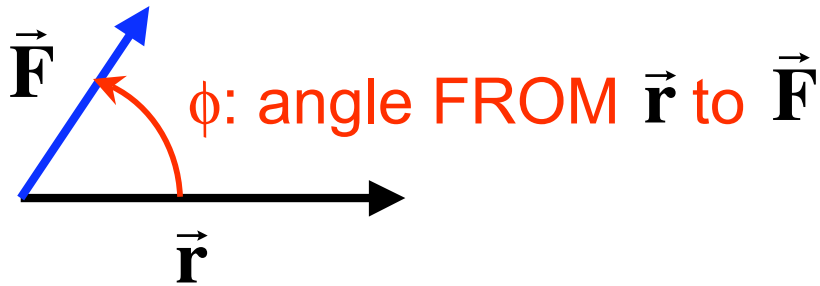
¹ To be precise we should always say “torque about O is...”

² Cannot use FBDs for torque

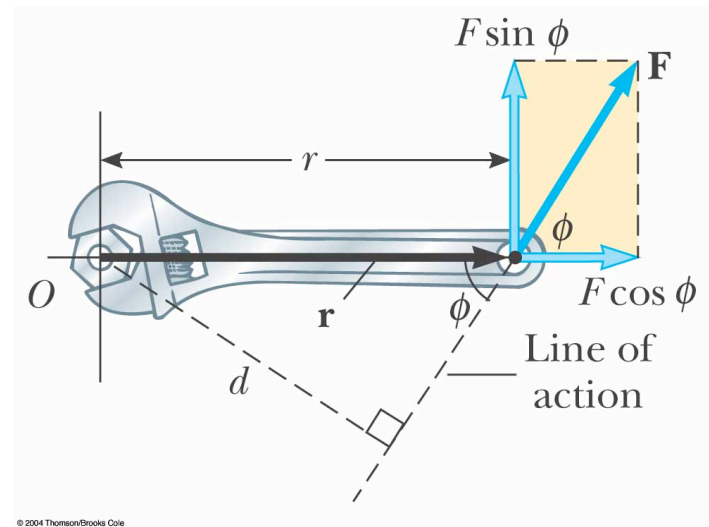
Ways to compute torque

1. With vectors and angles

place vectors tail-to-tail:



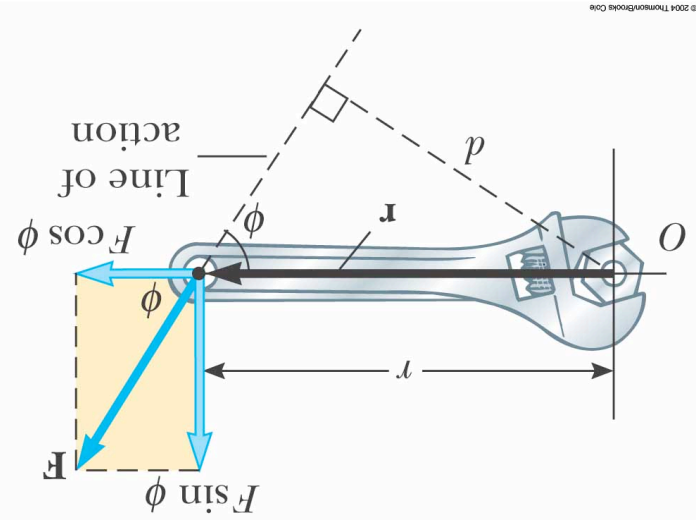
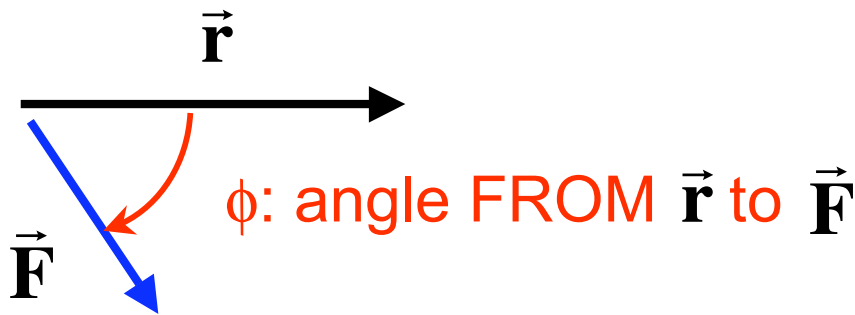
$$\tau = |\vec{\mathbf{r}}| |\vec{\mathbf{F}}| \sin \phi$$



Ways to compute torque

1. With vectors and angles

place vectors tail-to-tail:



$$\tau = |\vec{\mathbf{r}}| |\vec{\mathbf{F}}| \sin \phi$$

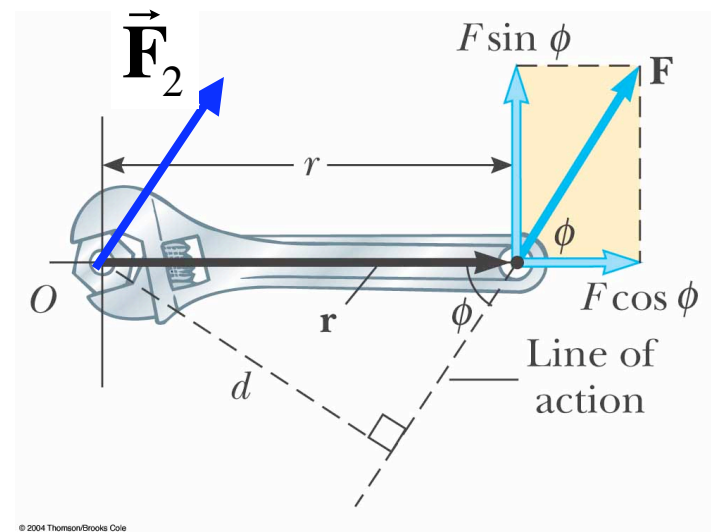
torque is negative
when ϕ is negative

Ways to compute torque

$\tau = 0$ when force is applied at axis

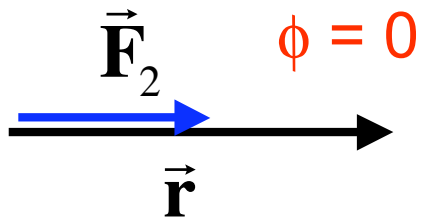
$$\vec{\mathbf{r}} = \mathbf{0}$$

$$\tau = |\vec{\mathbf{r}}| |\vec{\mathbf{F}}_2| \sin \phi = 0$$

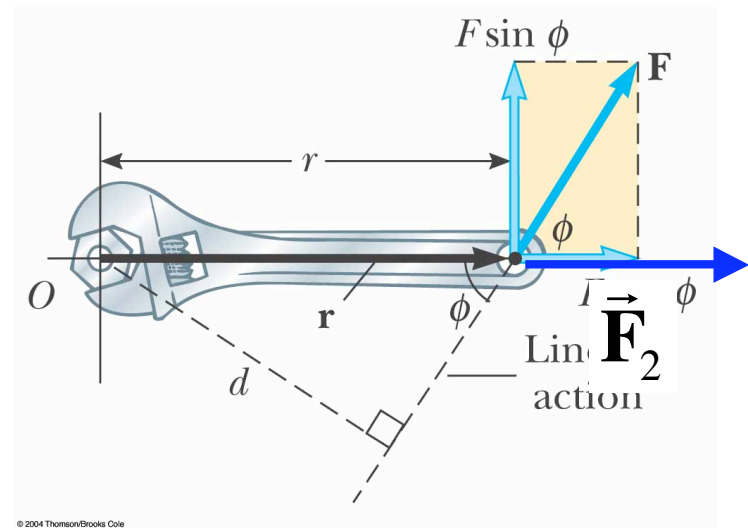


Ways to compute torque

$\tau = 0$ when force is \parallel to \vec{r}



$$\tau = |\vec{r}| |\vec{F}_2| \sin \phi = 0$$

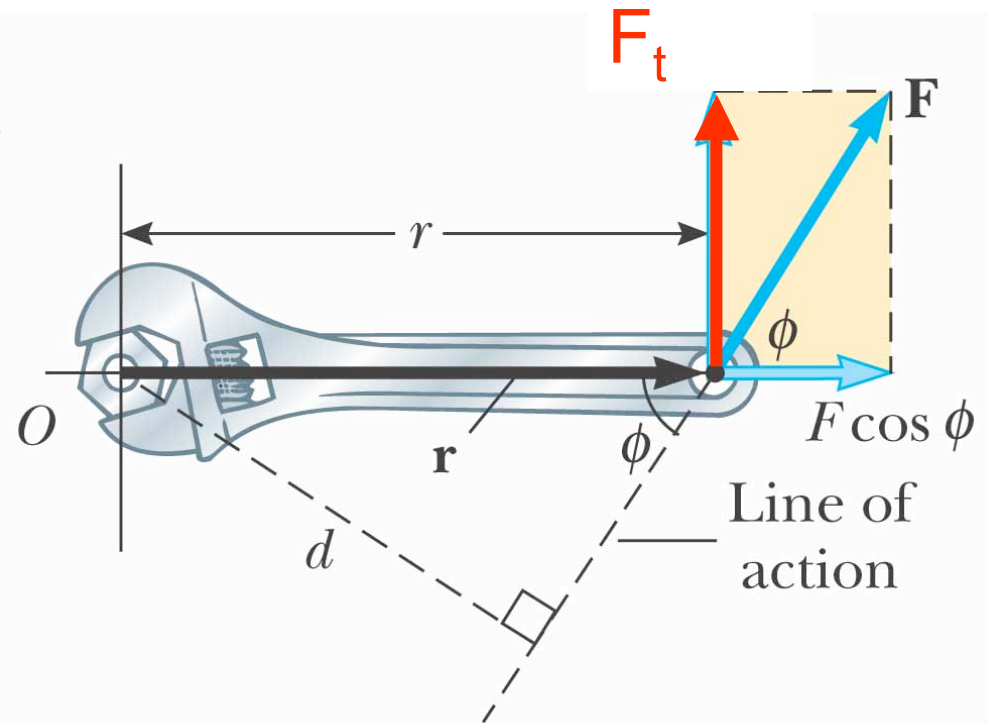


Ways to compute torque

2. Perpendicular component of Force

A. At the point of application find the component of \vec{F} which is perp. to \vec{r} F_t

$$|\tau| = |\vec{r}| F_t$$



© 2004 Thomson/Brooks Cole

B. determine sign from sense of rotation

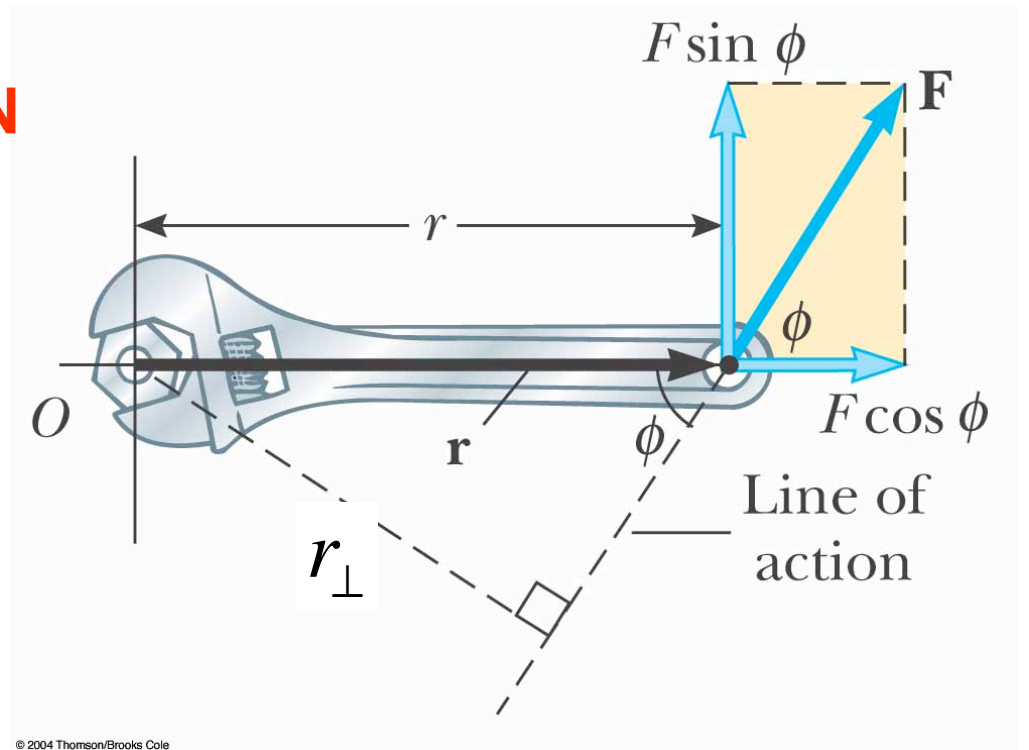
Ways to compute torque

3. Moment arm to line of action

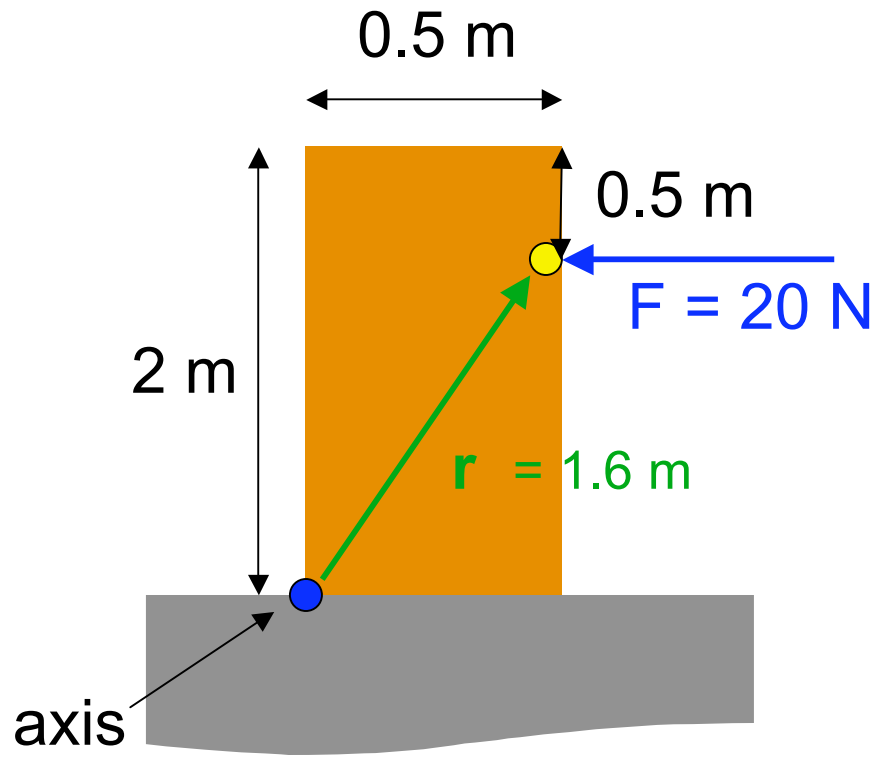
A. From the point of application run a line \parallel to \vec{F} **LINE OF ACTION**

B. From axis draw perp. to line of action:
MOMENT ARM r_{\perp}

$$|\tau| = r_{\perp} |\vec{F}|$$

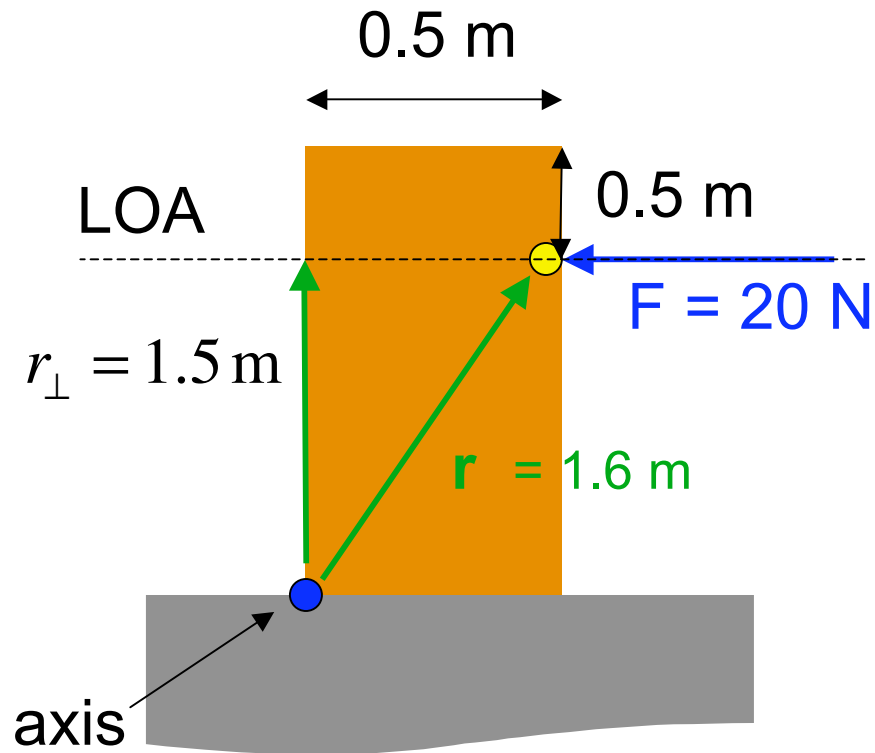


C. determine sign from sense of rotation



A crate of mass $m=10\text{ kg}$ is tipped by applying a 20 N force as shown. What is the torque about the far lower corner?

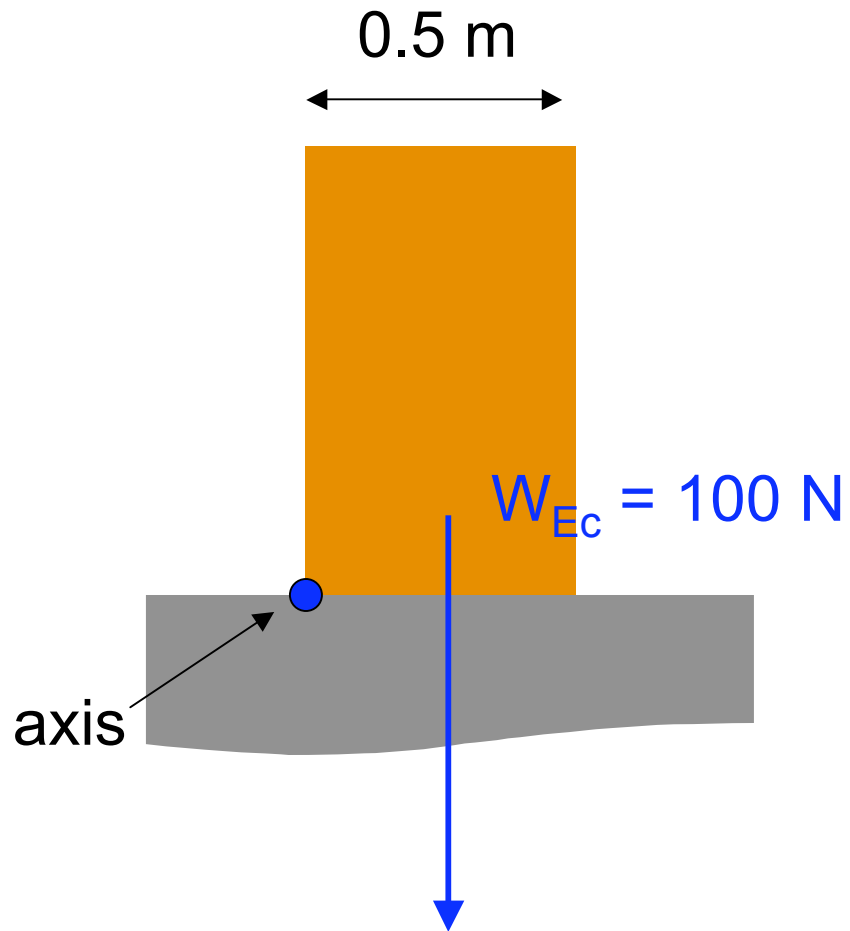
- A +32 N m
- B +30 N m
- C -30 N m
- D -32 N m



A crate of mass $m=10\text{ kg}$ is tipped by applying a 20 N force as shown. What is the torque about the far lower corner?

- A $+32\text{ N m}$
- B $+30\text{ N m}$**
- C -30 N m
- D -32 N m

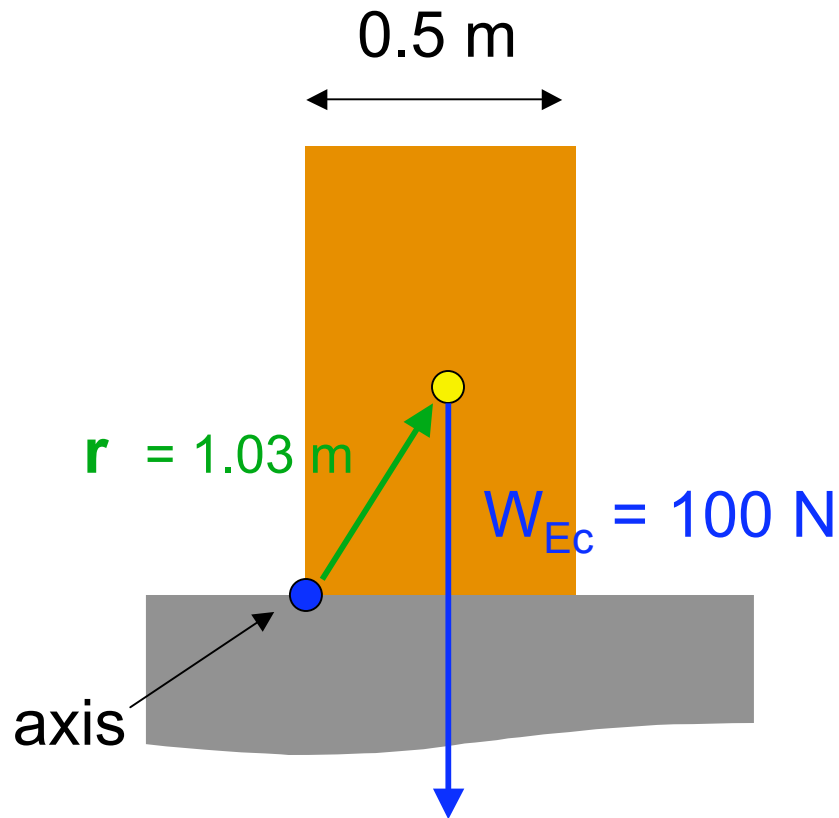
$$|\tau| = r_{\perp} |\vec{F}| = (1.5\text{ m})(20\text{ N})$$



The weight force

$$W=mg$$

Q: But where is it applied to the body?



What torque does gravity create about the axis?

The weight force

$$W=mg$$

Q: But where is it applied to the body?

A: It acts at the **center of gravity** which is exactly the same as the **center of mass**

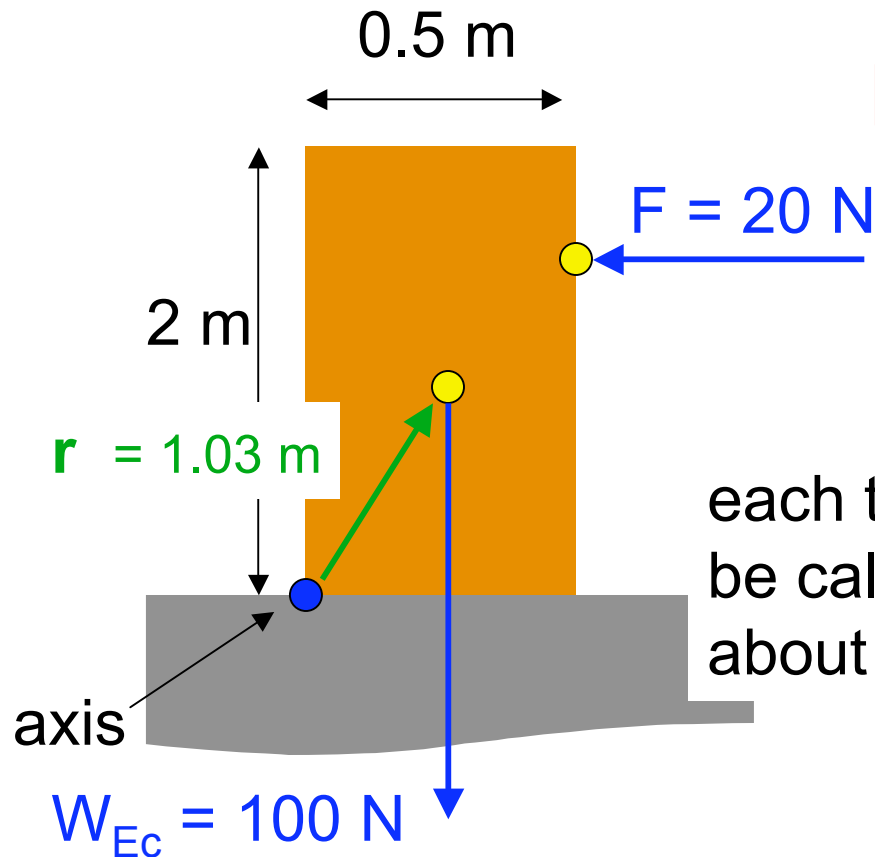
A +103 N m

B +25 N m

C -25 N m

D -103 N m

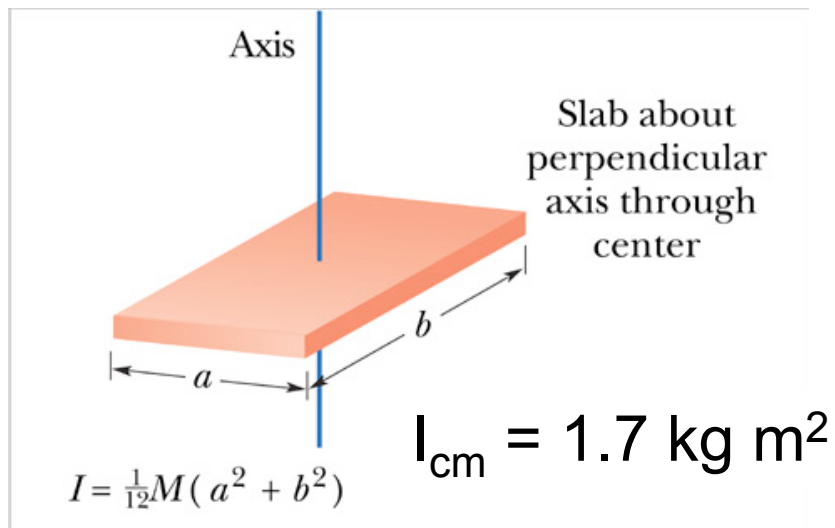
N2R (Newton's 2nd Law for rotation)



$$\tau_{\text{net}} = \sum \tau = I\alpha$$

each torque must be calculated about same axis

rotational inertia must be calculated about same axis as torques

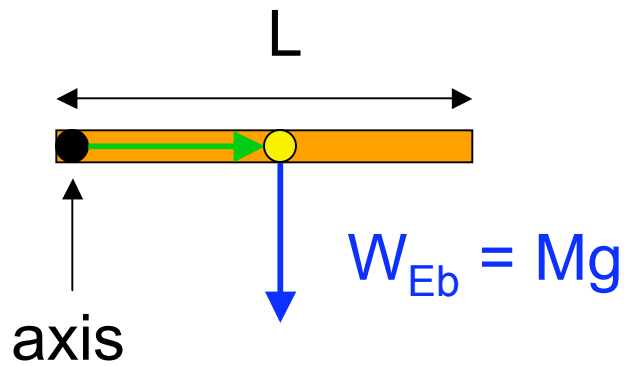


$$\tau_{\text{net}} = 30 - 25 = +5 \text{ N m}$$

What values of I do we divide $+5 \text{ N m}$ by?

$$I = I_{\text{cm}} + M (1.03)^2 = 12.3 \text{ kg m}^2$$

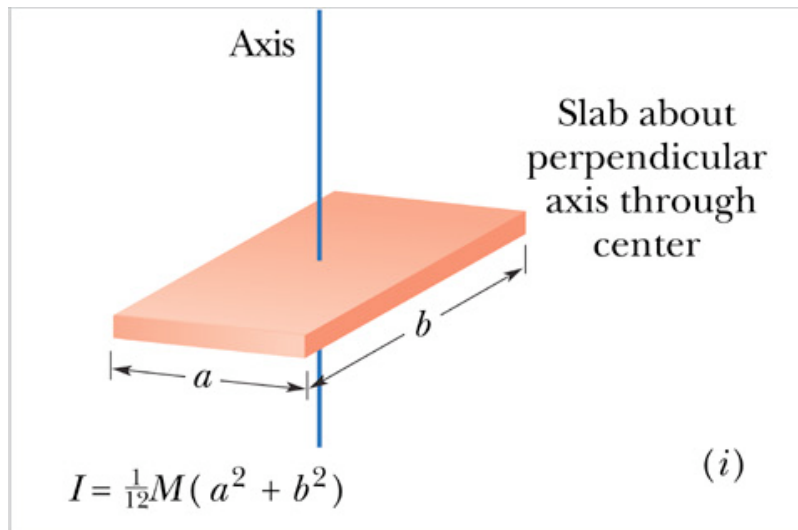
$$\alpha = (5 \text{ N m}) / (12.3 \text{ kg m}^2) = 0.41 \text{ rad/s}^2$$



A uniform bar swings freely about a frictionless axis at one end. It is released from rest when it is horizontal.

$$\tau_g = -(Mg)L/2$$

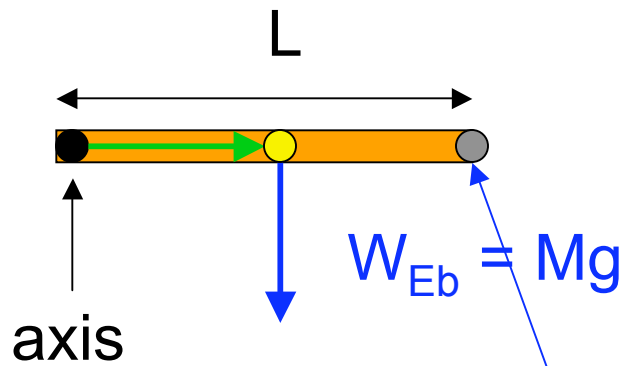
What is the rotational moment about the axis?



$$a \rightarrow 0$$

- A $M L^2/12$
- B $M L^2/4$
- C $M L^2/3$**
- D $M L^2$
- E $(13/12) M L^2$

$$I = I_{cm} + M (L/2)^2 = M L^2 (1/12 + 1/4)$$



What is the linear acceleration of the rod's tip?

- A 0
- B 0.333 g
- C 0.75 g
- D g
- E 1.5 g**

A uniform bar swings freely about a frictionless axis at one end. It is released from rest when it is horizontal.

$$\tau_g = -(Mg)L/2$$

$$I = M L^2 / 3$$

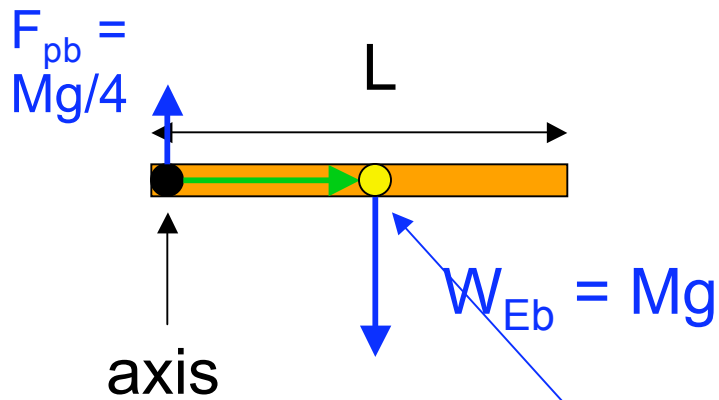
$$\tau_{\text{net}} = \sum \tau = I\alpha$$

$$-(Mg)L / 2 = (M L^2 / 3) \alpha$$

$$\alpha = -(3/2) g/L$$

$$a_t = r \alpha = -(3/2) g$$

what about a_c ?



What is the linear acceleration of the COM?

- A 0
- B 0.333 g
- C 0.75 g**
- D g
- E 1.5 g

A uniform bar swings freely about a frictionless axis at one end. It is released from rest when it is horizontal.

$$\tau_g = -(Mg)L/2$$

$$I = M L^2 / 3$$

$$\alpha = -(3/2) g/L$$

$$a_t = r \alpha = (L/2)(-3/2 g/L)$$

Doesn't $a_{cm} = -g$ because

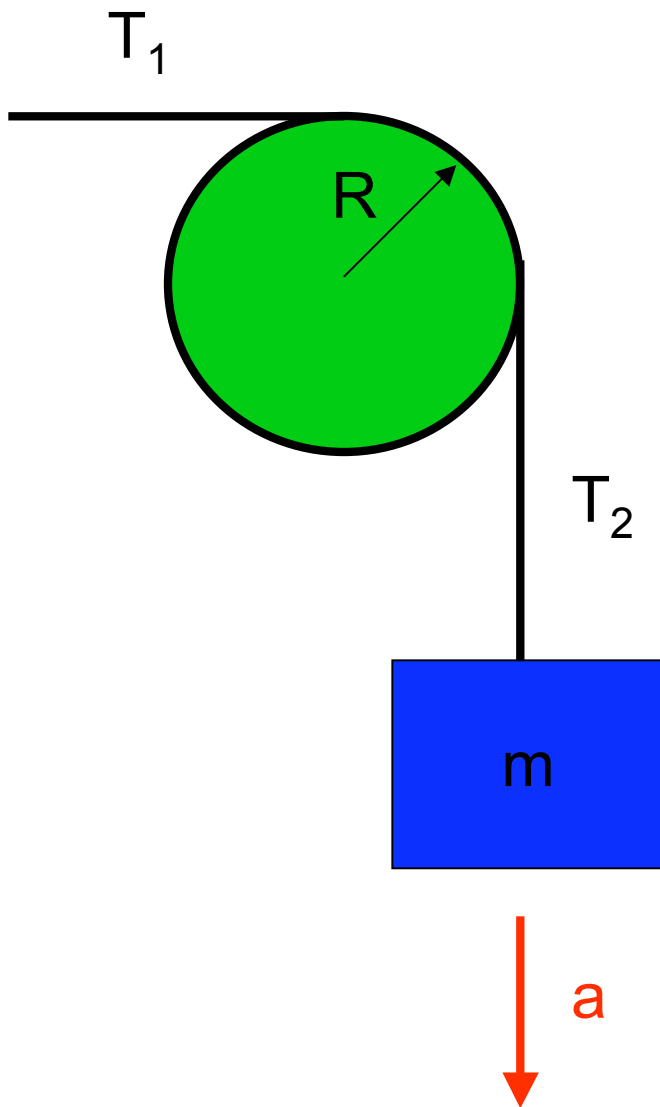
$$M \vec{a}_{cm} = \vec{F}_{net} = -Mg \hat{j}$$

Tension & Pulleys

Problems *used to say*

- light* string
- light* pulley
- on frictionless axel

$$\rightarrow T_1 = T_2$$



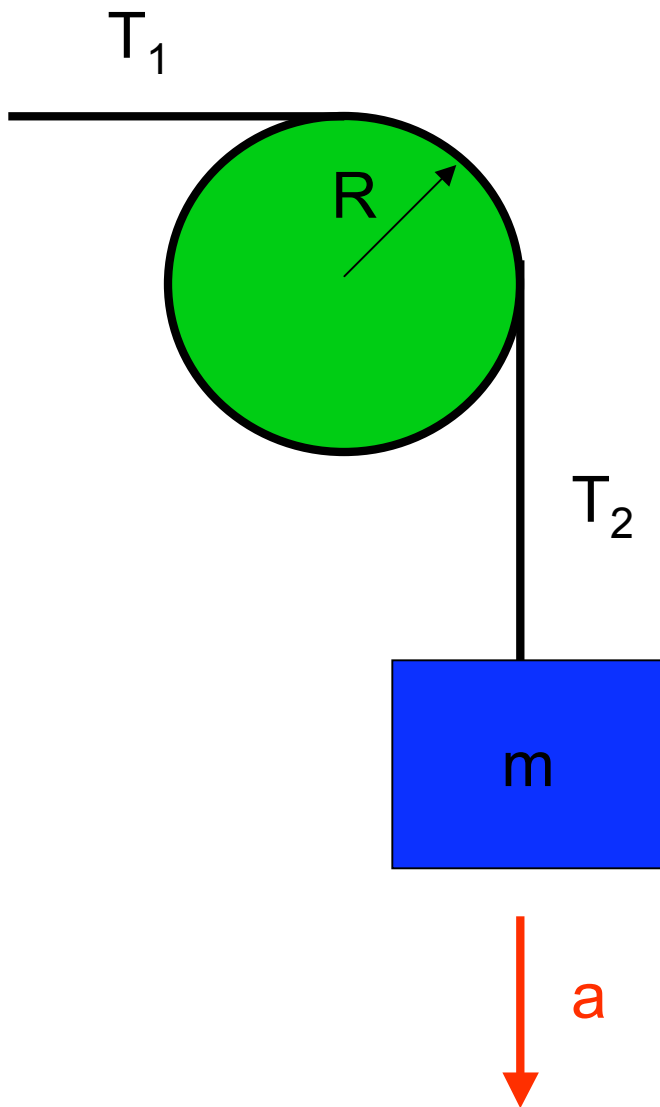
* i.e. massless

Tension & Pulleys

Now they say

- light* string
- ~~light* pulley~~
- on frictionless axel

→ $I \neq 0$ for pulley (about axis)



* i.e. massless

Tension & Pulleys

Now they say

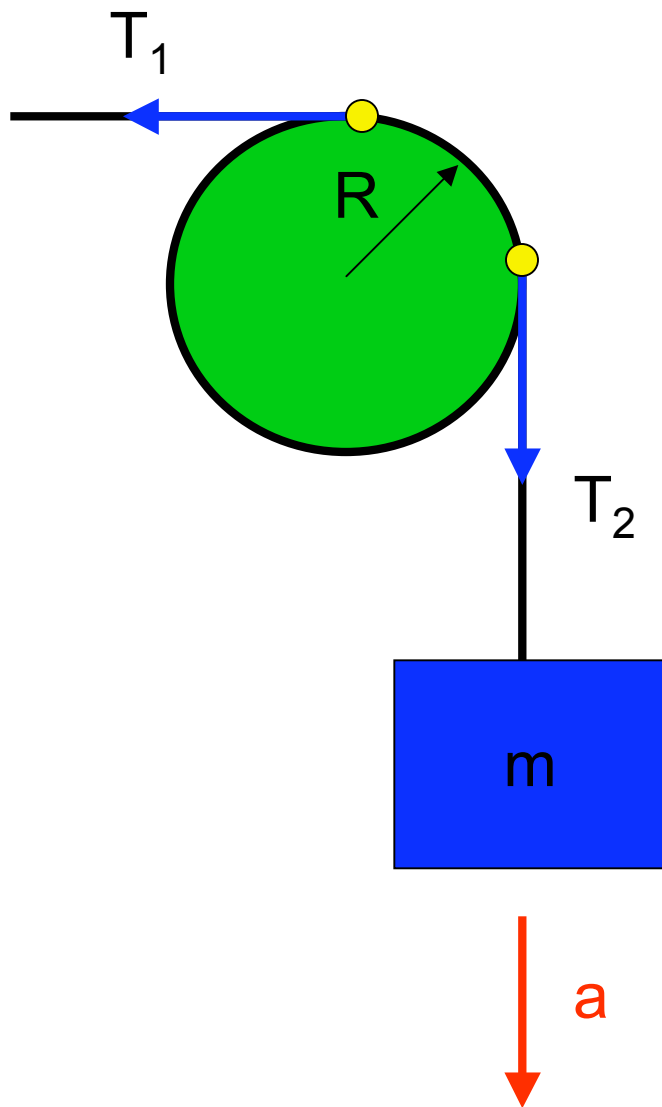
- light* string
- ~~light* pulley~~
- on frictionless axel
- string does not slip

→ $I \neq 0$ for pulley (about axis)

→ Tension acts on pulley at point where they 1st touch it.

→ $\tau = TR$

→ $\alpha = a/R$



* i.e. massless

Tension & Pulleys

Now they say

- light* string
- ~~light* pulley~~
- on frictionless axel
- string does not slip

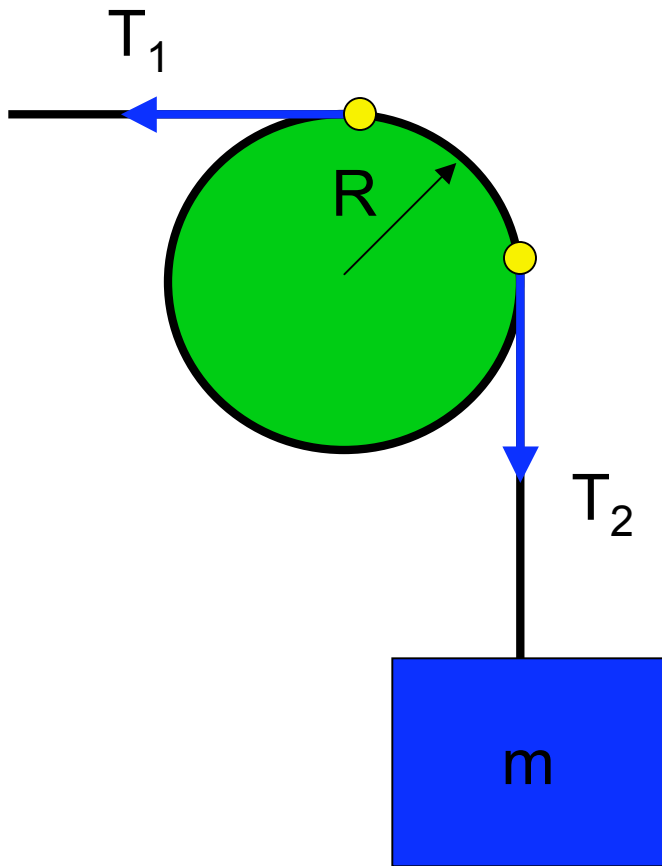
In the picture shown here:

A $T_1 < T_2$

B $T_1 = T_2$

C $T_1 > T_2$

D need more information



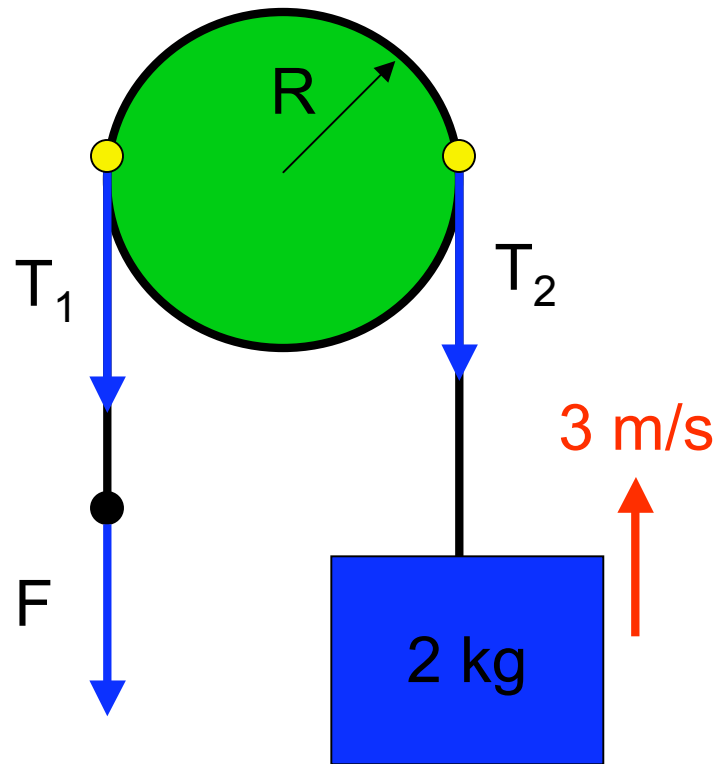
$$\alpha = a/R < 0$$

$$\tau_{\text{net}} = \alpha/I < 0$$

$$\tau_{\text{net}} = R(T_1 - T_2) < 0$$

* i.e. massless

Tension & Pulleys



$$\alpha = a/R = 0$$

$$\tau_{\text{net}} = \alpha/I = 0$$

$$\tau_{\text{net}} = R(T_1 - T_2) = 0$$

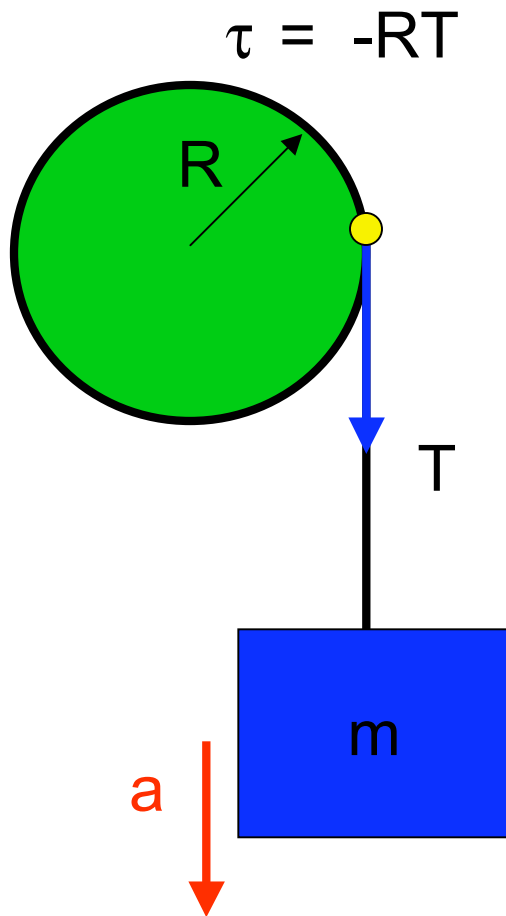
A downward force F is applied in order to pull the block upward at constant velocity of 3 m/s . The light string does not slip over the pulley, which is on a frictionless axle.

A $T_1 < T_2$

B $T_1 = T_2$

C $T_1 > T_2$

D need more information



The pulley and block each have mass m . A light string is wrapped about the pulley and does not slip. The block falls freely. What is the tension T ?

N2 for block:

$$a_y = T/m - g$$

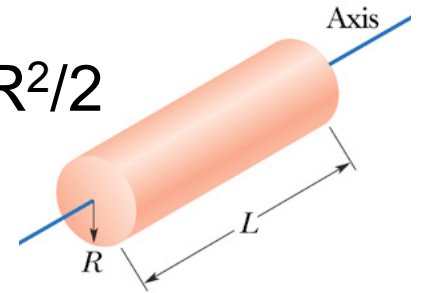
N2R for pulley*:

$$\alpha = \tau/I = -2T/mR$$

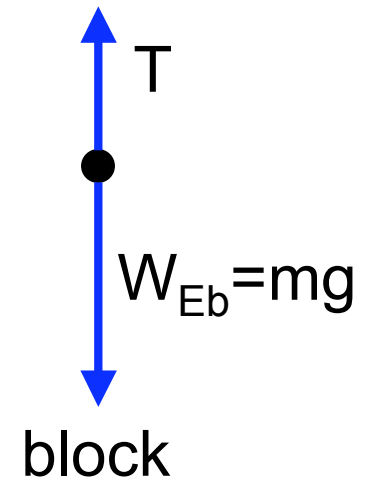
linear/angular relation

$$a_y = R\alpha = -2T/mR$$

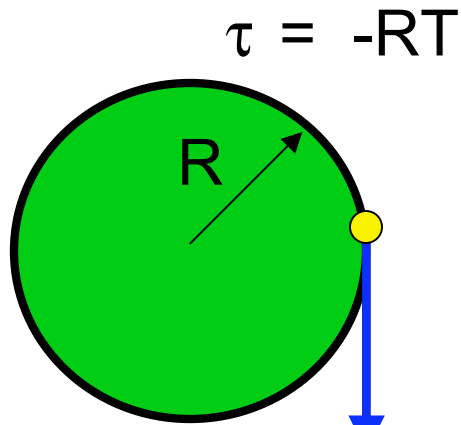
$$I = mR^2/2$$



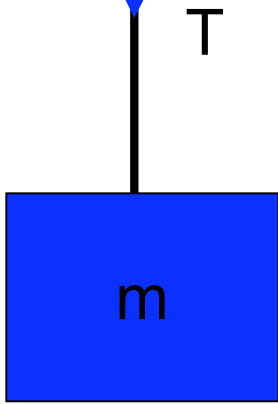
$$I = \frac{1}{2}MR^2$$



* about axle



The pulley and block each have mass m . A light string is wrapped about the pulley and does not slip. The block falls freely. What is the tension T ?



N2 for block:

$$a_y = T/m - g$$

N2R for pulley:

$$\alpha = \tau/I = -2T/mR$$

linear/angular relation

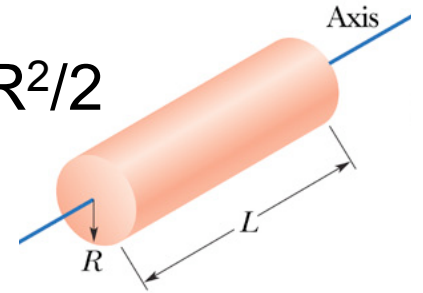
$$a_y = R\alpha = -2T/m$$

$$T/m - g = -2T/m$$

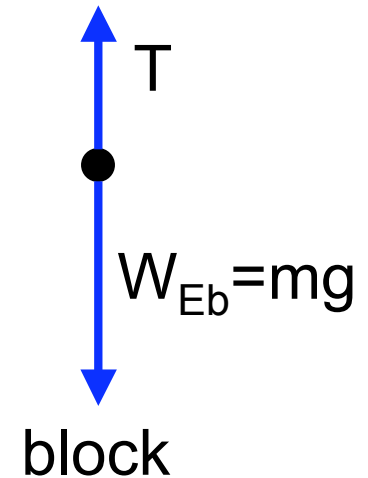
$$T = mg/3$$

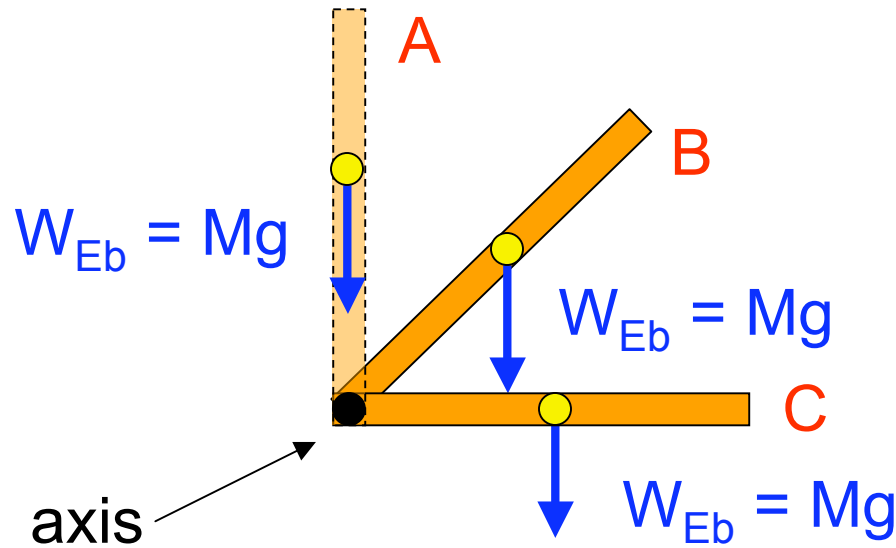
$$a_y = -(2/3)g$$

$$I = mR^2/2$$



$$I = \frac{1}{2}MR^2$$





A uniform bar swings freely about a frictionless axis at one end. It is perched vertically when it is nudged to fall.

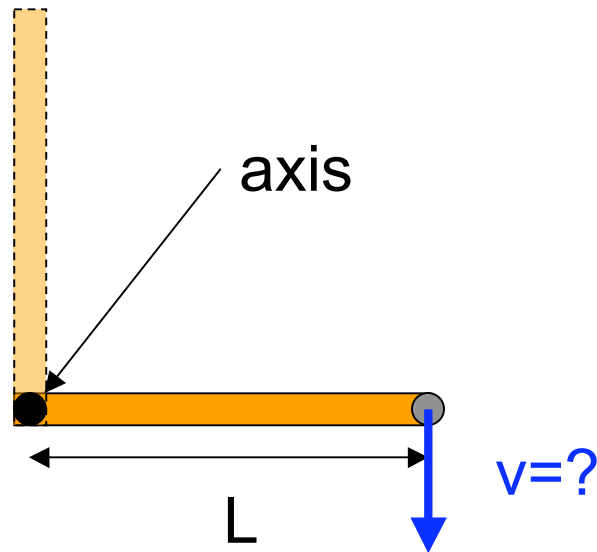
of the three times show, when is the torque from gravity the greatest:

A (vertical)

B (45°)

C (horizontal)

D all three are the same



A uniform bar swings freely about a frictionless axis at one end. It is perched vertically when it is nudged to fall. What is the linear speed of its tip as it passes the horizontal?

The kinematic equations:

- A** should be used here
- B** could be used, but will be hard

C cannot be used here

$$\omega = \omega_0 + \alpha t$$

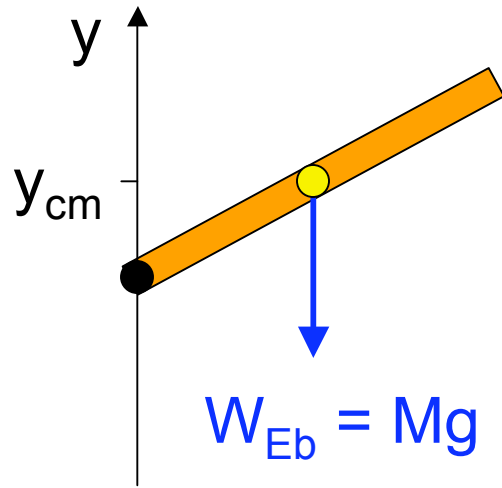
$$\theta - \theta_0 = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$$

$$\theta - \theta_0 = \omega t - \frac{1}{2} \alpha t^2$$

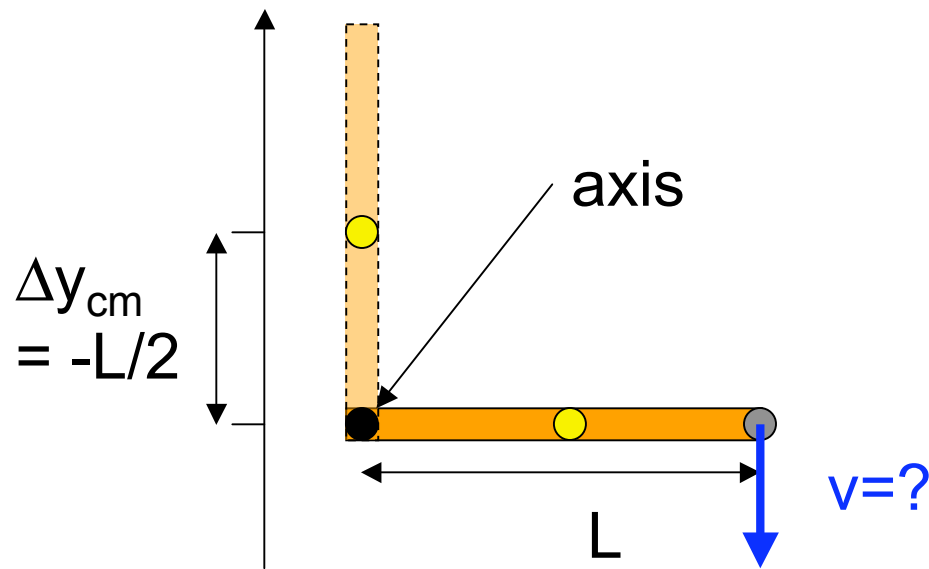
τ changes \rightarrow α changes



Gravitational Potential Energy

Weight force acts @ C.O.M.
→ potential energy
of body/Earth system depends
on height of C.O.M.

$$U_g = Mg y_{cm}$$



A uniform bar swings freely about a frictionless axis at one end. It is perched vertically when it is nudged to fall. What is the linear speed of its tip as it passes the horizontal?

Conservation of mechanical energy:

$$\Delta E = \Delta K + \Delta U = W = 0$$

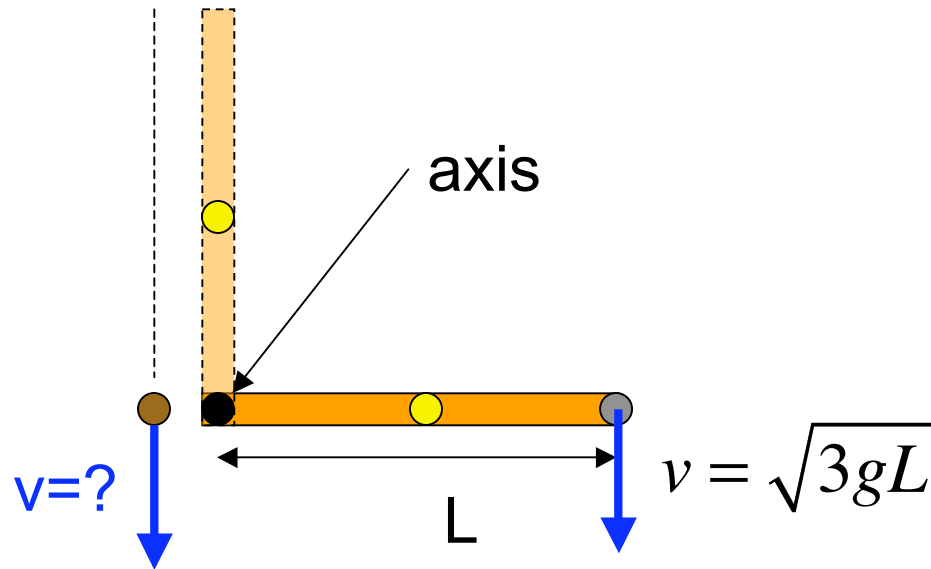
$$\Delta U = mg \Delta y_{\text{cm}} = -mgL/2$$

$$K_i = 0 \quad K_f = 0.5 I \omega^2 = mL^2 \omega^2/6$$

$$\Delta K = mL^2 \omega^2/6 = -\Delta U = mgL/2$$

$$\omega = \sqrt{\frac{3g}{L}}$$

$$v = \omega r = \sqrt{3gL}$$



A uniform bar swings freely about a frictionless axis at one end. It is perched vertically when it is nudged to fall. A penny falls from the tip when the bar is nudged. As it passes the axis, how does its speed compare to that of the tip when it was horizontal?

$$\Delta E = \Delta K + \Delta U = W = 0$$

$$\Delta U = mg \Delta y_p = -mgL$$

$$K_i = 0 \quad K_f = m v_p^2 / 2$$

$$\Delta K = m v_p^2 / 2 = -\Delta U = mgL$$

$$v_p = \sqrt{2gL}$$

- A the penny is faster
- B the penny is slower**
- C the two are the same
- D they cannot be compared