

Rolling

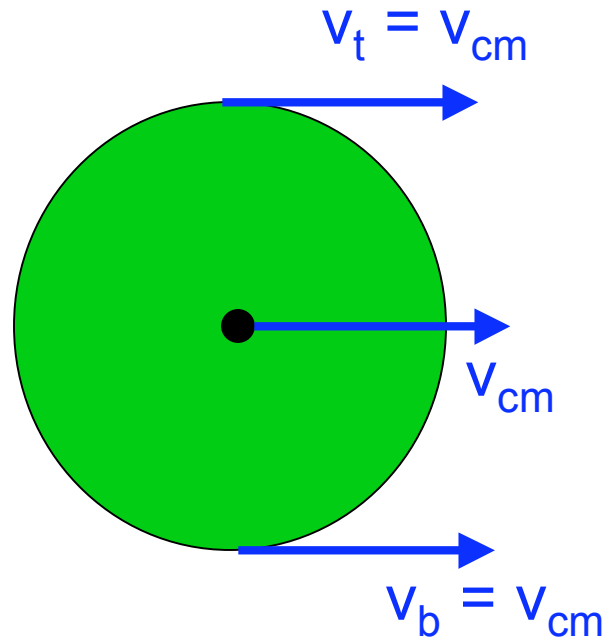


Motion of a disk

1. Translational motion (without rotation)

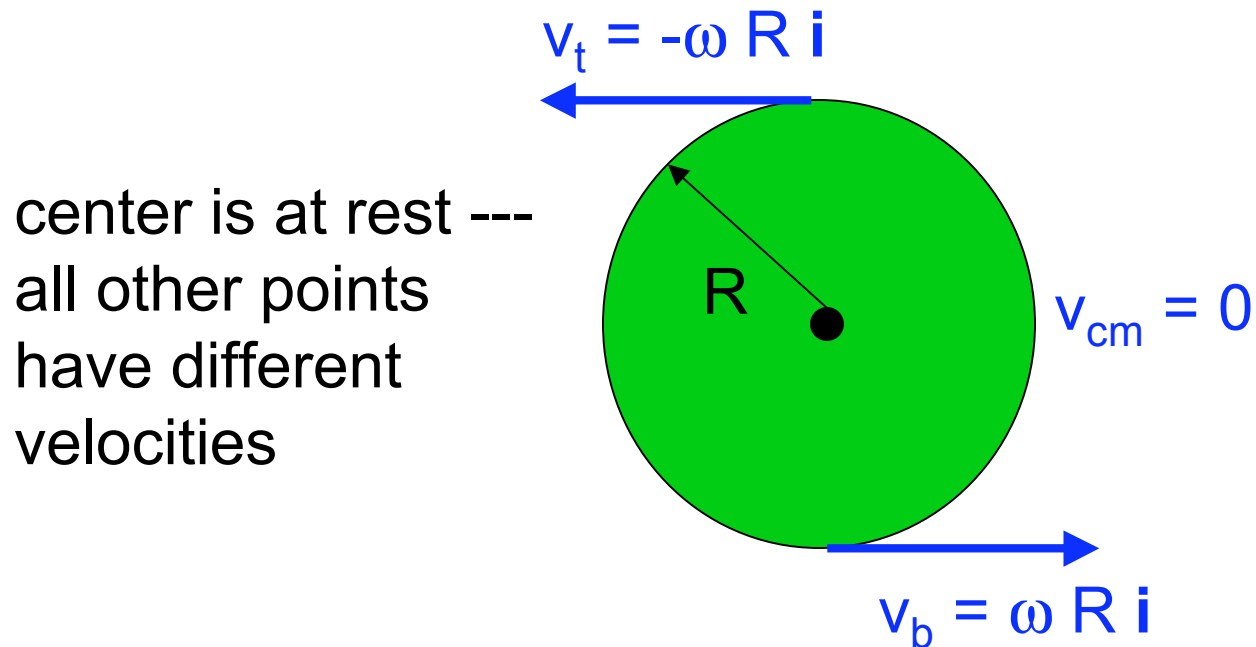
(imagine disk moving through empty space)

all points on disk
move with same
velocity.



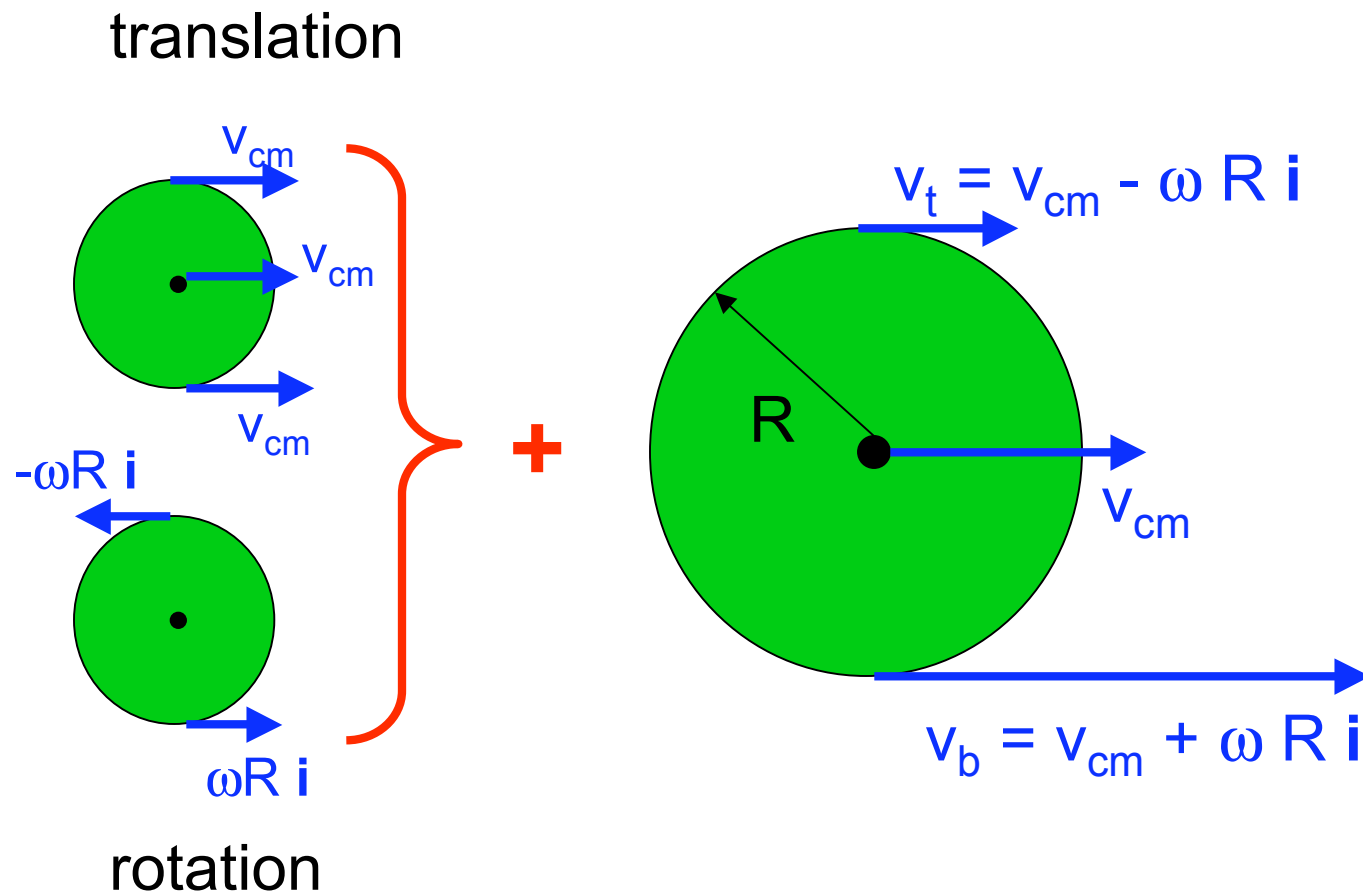
Motion of a disk

2. Rotation about fixed axis: the center (consider case $\omega > 0$)



Motion of a disk

Combination of translation & rotation

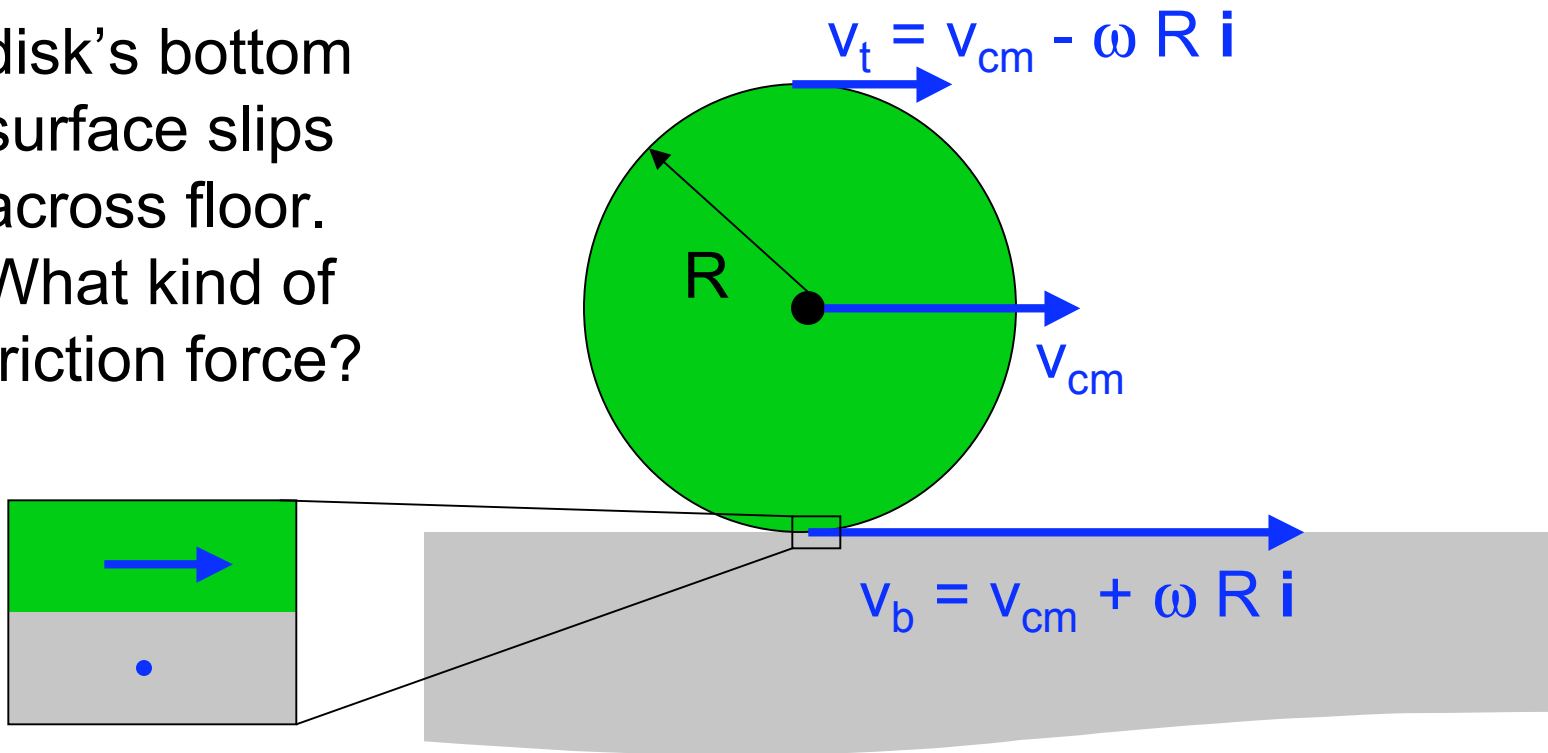


Motion of a disk

Combination of translation & rotation: **Slipping**

v_{cm} and ω can be anything at all (here $v_{cm} > 0$ and $\omega > 0$)

disk's bottom surface slips across floor.
What kind of friction force?

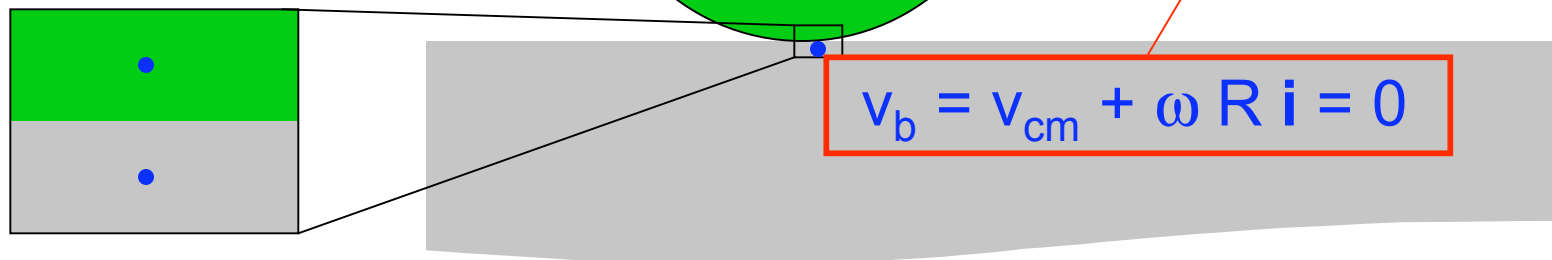


Motion of a disk

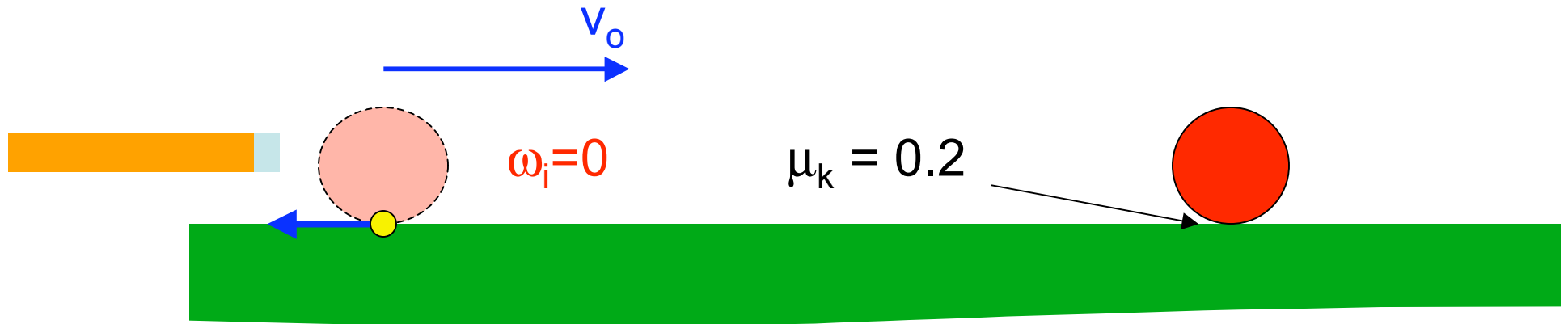
Special combination: **Rolling without slipping**

v_{cm} and ω are related to make $v_b = 0$

disk's bottom surface **DOES NOT** slip across floor. What kind of friction force?



a pool ball is hit at an initial speed $v_0 = 2.0 \text{ m/s}$, but is **not spinning**. How much time does it skid along the felt?

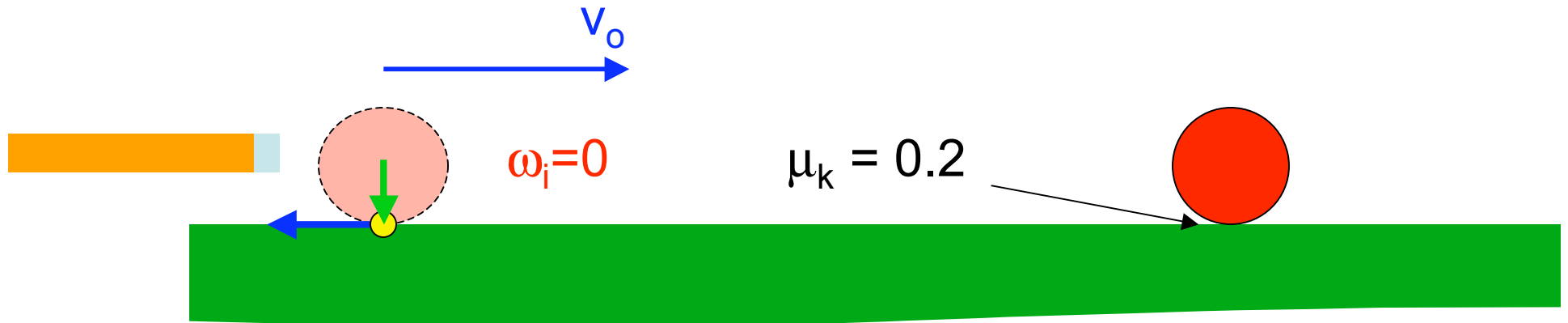


What is strategy for solving this?

1. Kinetic friction exerts **force** on the ball.

What sign? negative: $f_{tb,x} < 0$

a pool ball is hit at an initial speed $v_0 = 2.0 \text{ m/s}$, but is **not spinning**. How much time does it skid along the felt?

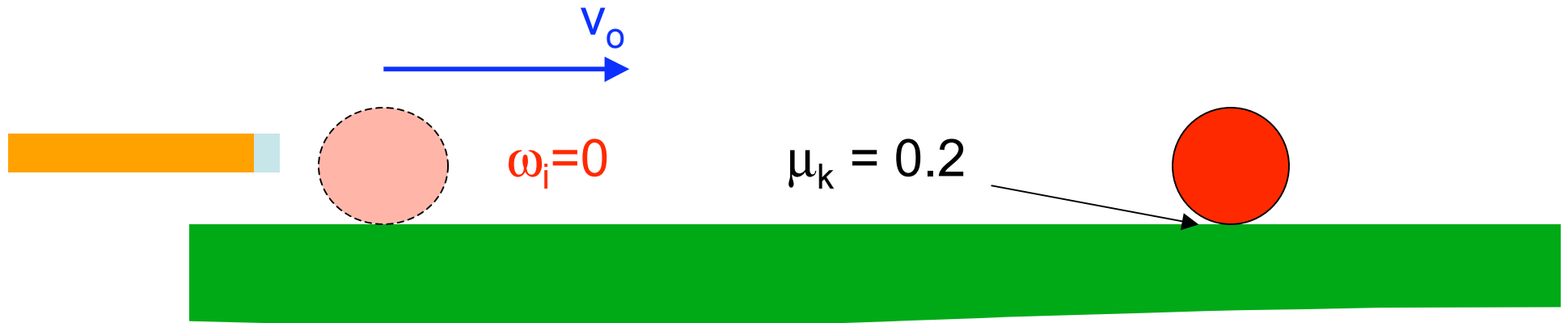


What is strategy for solving this?

1. Kinetic friction exerts (negative) **force** on the ball.
 $v_{cm}(t)$ decreases over time.
2. Kinetic friction exerts **torque** about ball's C.O.M.

What sign? **negative** ($\tau < 0$)

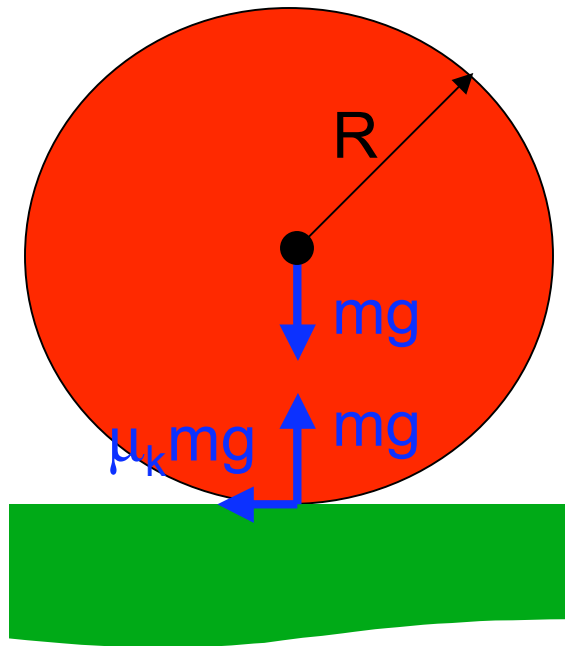
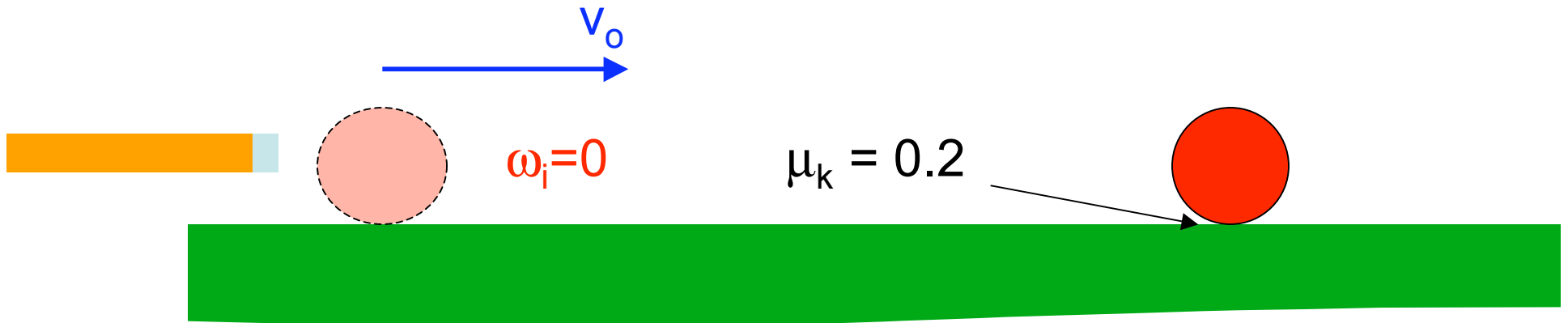
a pool ball is hit at an initial speed $v_o = 2.0 \text{ m/s}$, but is **not spinning**. How much time does it skid along the felt?



What is strategy for solving this?

1. Kinetic friction exerts (negative) **force** on the ball.
 $v_{cm}(t)$ decreases over time.
2. Kinetic friction exerts (negative) **torque** about ball's C.O.M. $\rightarrow \omega(t)$ becomes more negative from $\omega_i = 0$
3. At some time these satisfy the relation $\omega = -v_{cm}/R$
--- the ball stops slipping.

a pool ball is hit at an initial speed $v_o = 2.0 \text{ m/s}$, but is **not spinning**. How much time does it skid along the felt?

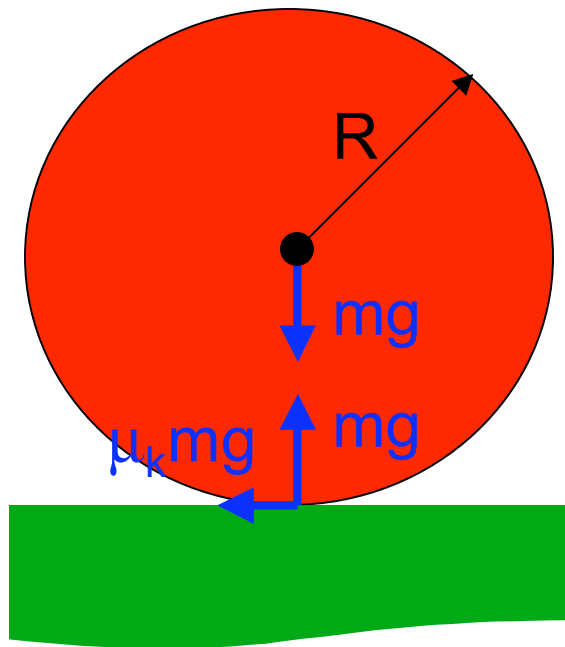
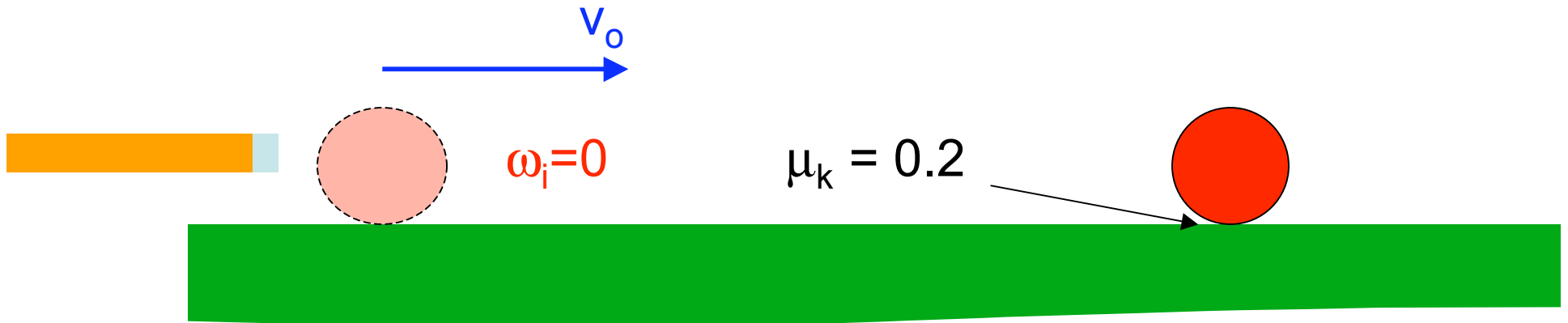


1. Force: $\mathbf{F} = -\mu_k mg \mathbf{i}$

$$\mathbf{a}_{\text{com}} = \mathbf{F}/m = -\mu_k g \mathbf{i}$$

$$v_{x,\text{com}} = v_o - \mu_k g t$$

a pool ball is hit at an initial speed $v_0 = 2.0 \text{ m/s}$, but is **not spinning**. How much time does it skid along the felt?



2. Torque: $\tau = -\mu_k mg R$

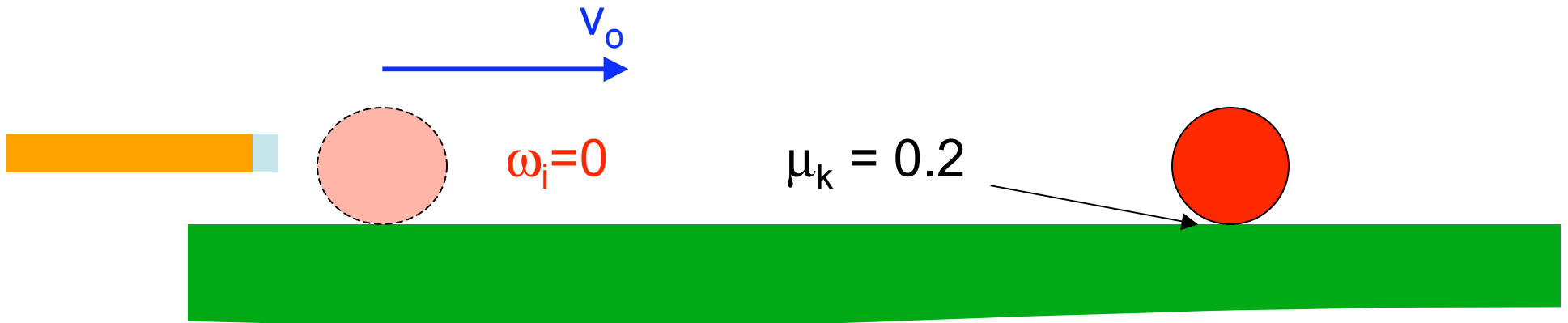
$$I_{cm} = (2/5) mR^2$$

$$\alpha = \tau/I = -(5/2)\mu_k g/R$$

constant α & $\omega_i = 0$

$$\omega = \alpha t = -(5/2)\mu_k g t/R$$

a pool ball is hit at an initial speed $v_o = 2.0 \text{ m/s}$, but is **not spinning**. How much time does it skid along the felt?

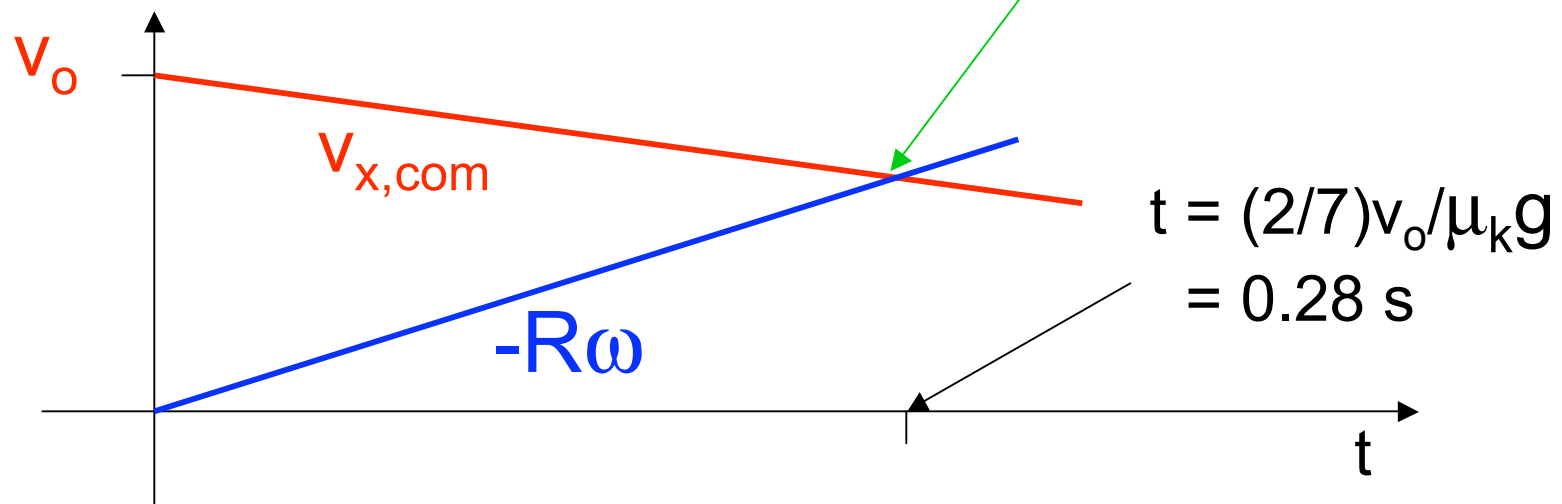


$$\omega = -(5/2)\mu_k g t / R$$

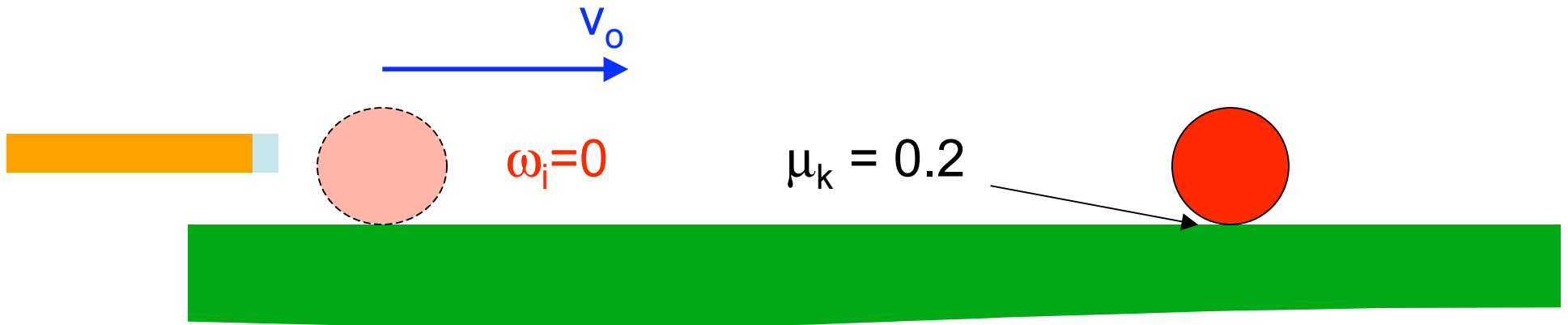
$$v_{x,\text{com}} = v_o - \mu_k g t$$

$$-R\omega = (5/2)\mu_k g t$$

$$-R\omega = v_{x,\text{com}}$$



a pool ball is hit at an initial speed $v_0 = 2.0 \text{ m/s}$, but is **not spinning**. How much time does it skid along the felt?



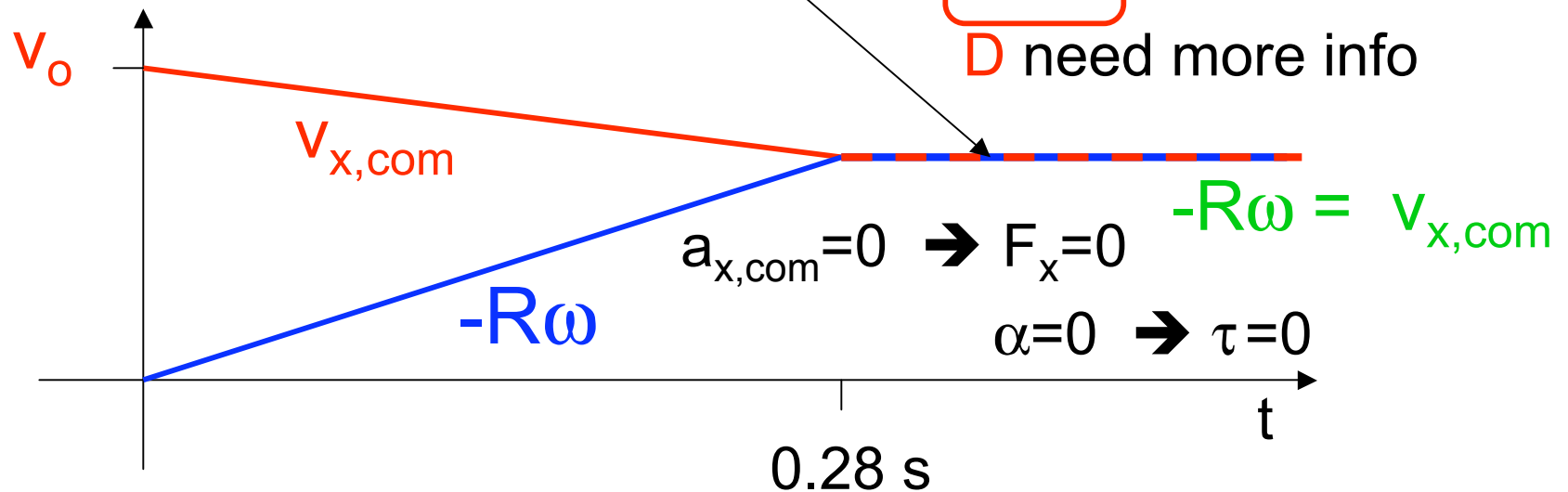
what is the friction force on the ball as it rolls without slipping?

A $\mu_k mg$

B $\mu_s mg$

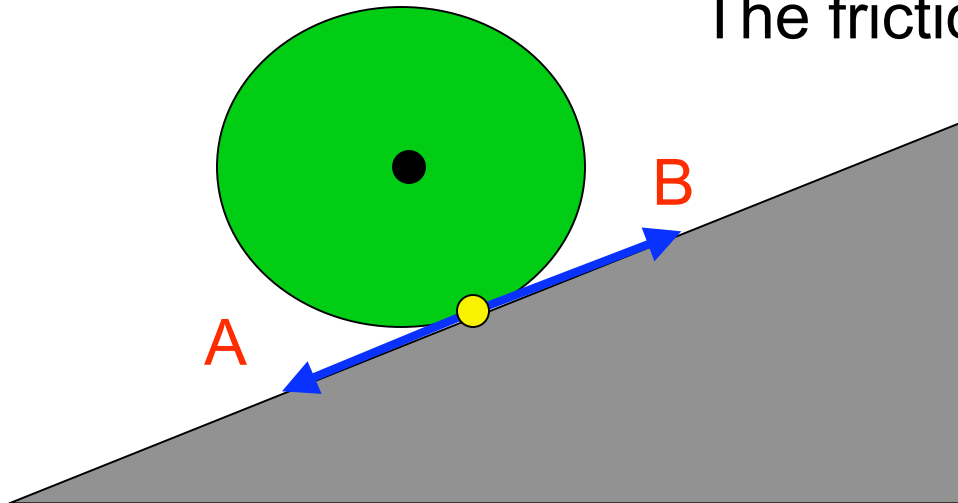
C 0

D need more info



Rolling down an incline

the disk rolls freely **without slipping**.
naturally it accelerates downward.
The friction force is directed



A downhill

B uphill

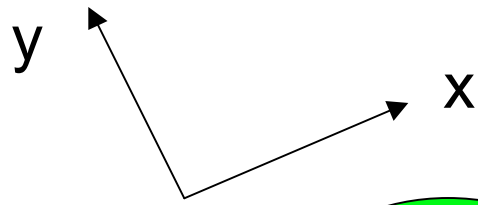
C 0 (no force)

D need more info

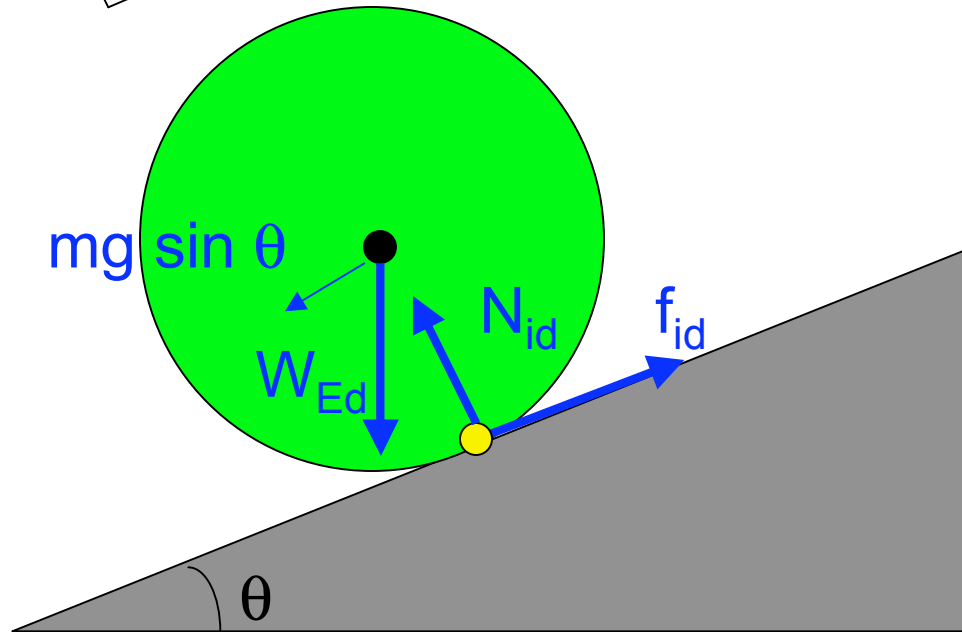
$\omega > 0$ (rolling down)

ω increases as v_{cm} increases

$\rightarrow \alpha > 0 \quad \rightarrow \tau > 0$

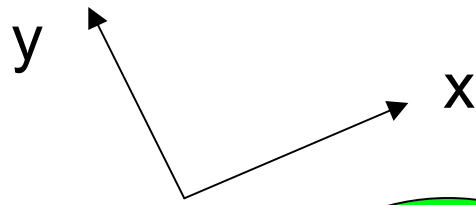


the disk rolls freely **without slipping**.
what is its acceleration?

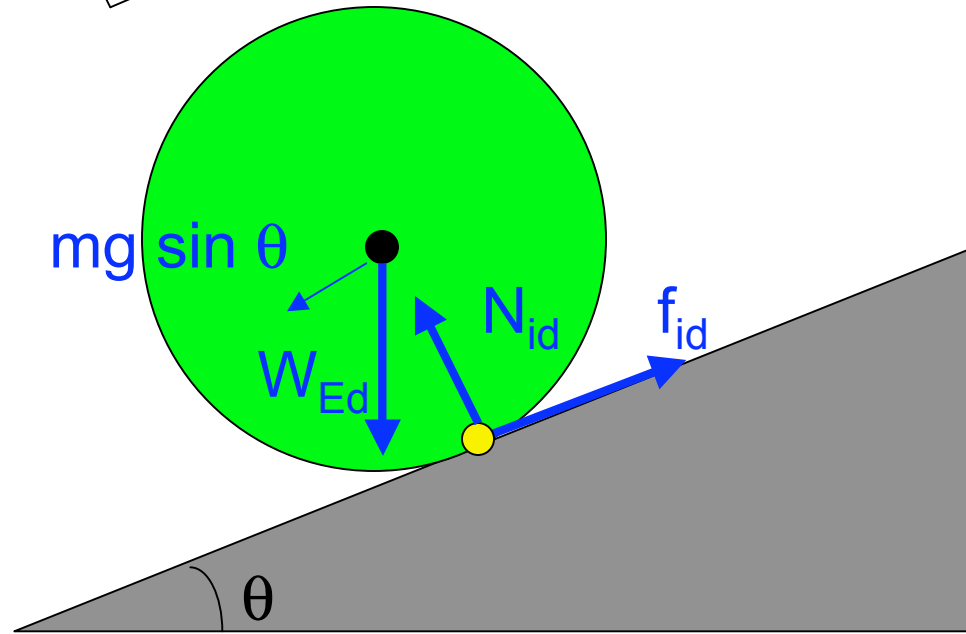


1. Forces along ramp (F_x):

$$m a_{x,cm} = f_{id} - mg \sin \theta$$



the disk rolls freely **without slipping**.
what is its acceleration?

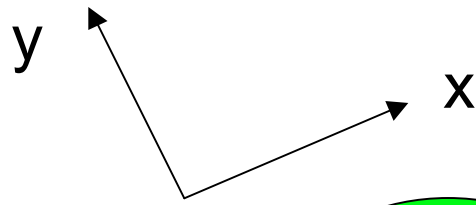


1. Forces along ramp (F_x):

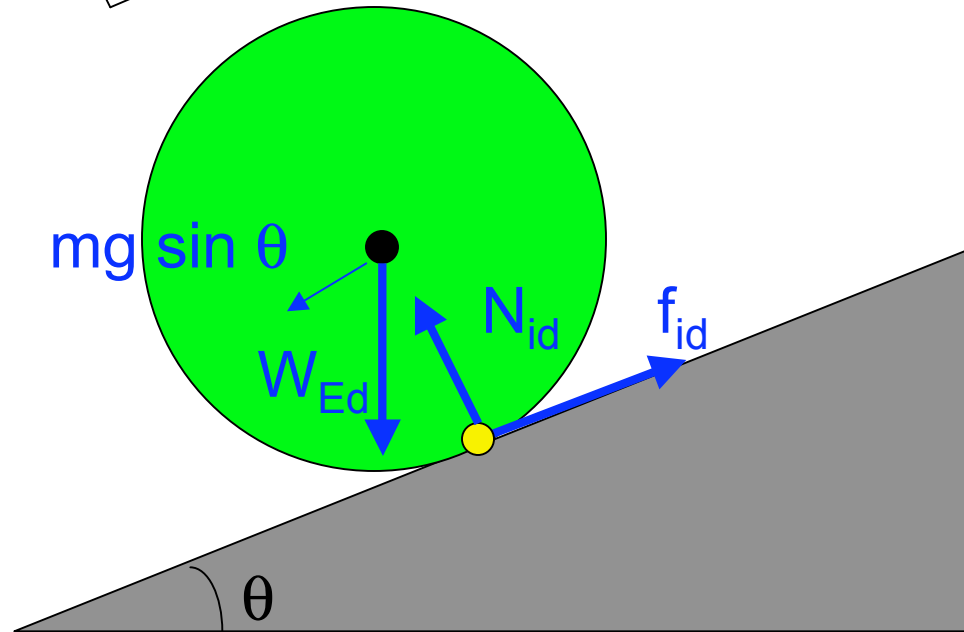
$$m a_{x,cm} = f_{id} - mg \sin \theta$$

2. Torques about C.O.M.

~~from weight?~~ why?



the disk rolls freely **without slipping**.
what is its acceleration?

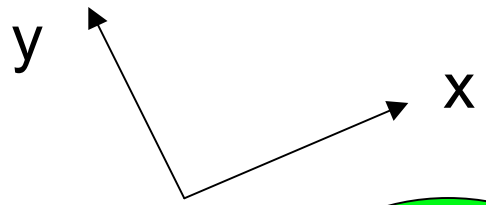


1. Forces along ramp (F_x):

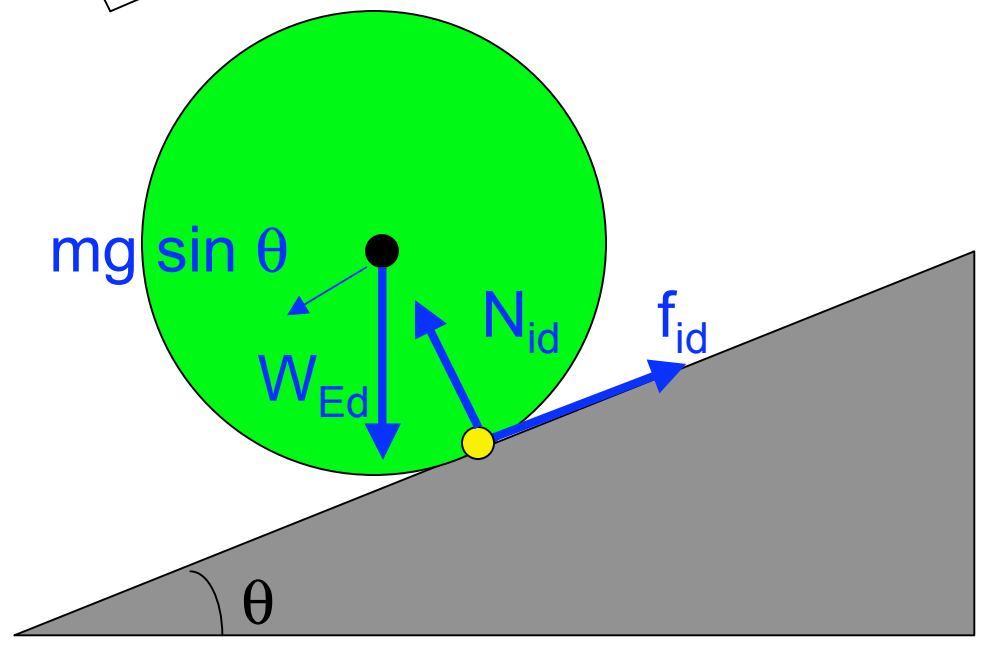
$$m a_{x,cm} = f_{id} - mg \sin \theta$$

2. Torques about C.O.M.

~~from normal force~~ why?



the disk rolls freely **without slipping**.
what is its acceleration?



1. Forces along ramp (F_x):

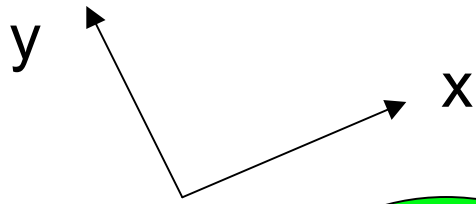
$$m a_{x,cm} = f_{id} + mg \sin \theta$$

2. Torques about C.O.M.

$$I_{cm} \alpha = \tau = f_{id} R$$

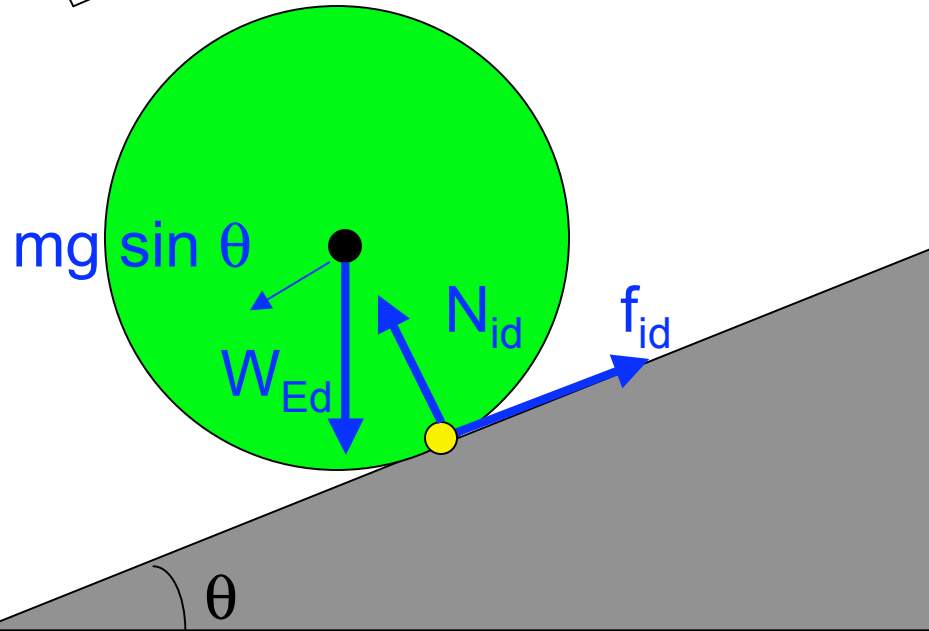
2 equations

3 unknowns



the disk rolls freely **without slipping**.
what is its acceleration?

whence a 3rd equation?



no-slip condition
relates translation
& rotation

$$\omega = -v_{x,cm}/R$$

→ $\alpha = -a_{x,cm}/R$

1. Forces along ramp (F_x):

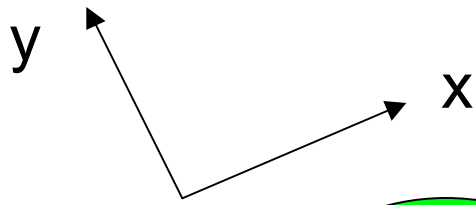
$$m a_{x,cm} = f_{id} + mg \sin \theta$$

2. Torques about C.O.M.

$$I_{cm} \alpha = \tau = f_{id} R$$

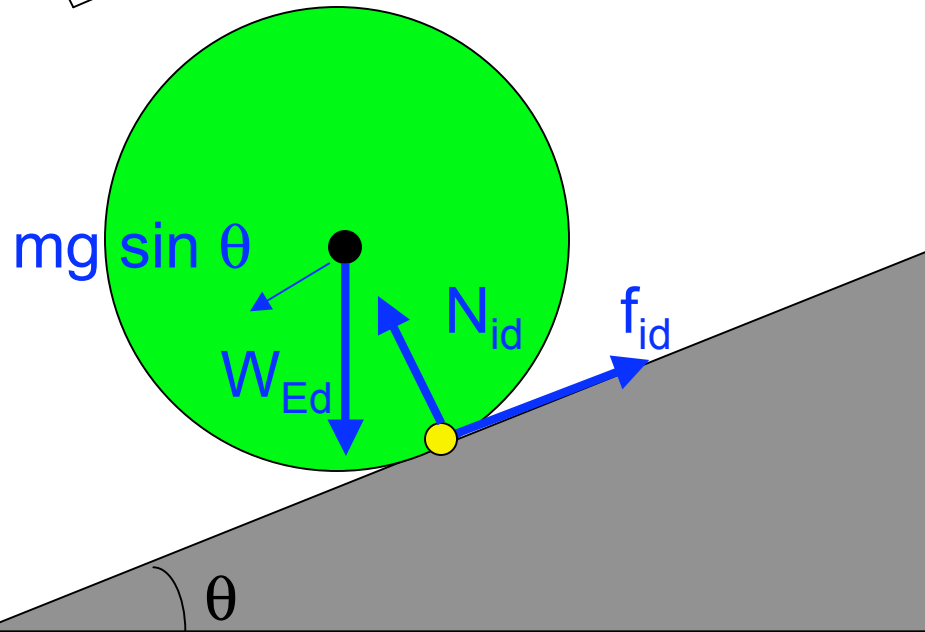
$$f_{id} = I_{cm} \alpha / R = -I_{cm} a_{x,cm} / R^2$$

$$m a_{x,cm} = -I_{cm} a_{x,cm} / R^2 - mg \sin \theta$$



the disk rolls freely **without slipping**.
what is its acceleration?

whence a 3rd equation?



no-slip condition
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$$\omega = -v_{x,cm}/R$$

→ $\alpha = -a_{x,cm}/R$

1. Forces along ramp (F_x):

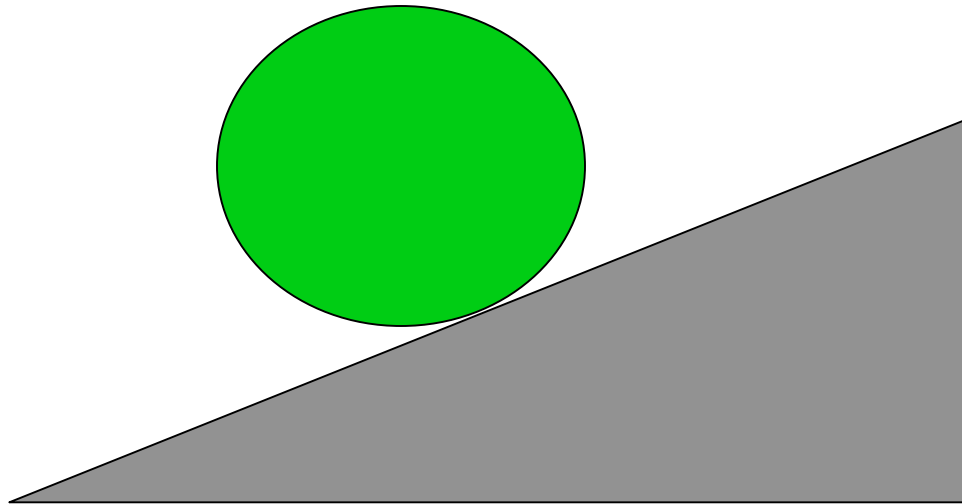
$$m a_{x,cm} = f_{id} - mg \sin \theta$$

2. Torques about C.O.M.

$$I_{cm} \alpha = \tau = f_{id} R$$

$$m a_{x,cm} = - I_{cm} a_{x,cm} / R^2 - mg \sin \theta$$

$$a_{x,cm} = - \frac{g \sin \theta}{1 + I_{cm} / mR^2}$$



Rolling down an incline

$$a_{x,cm} = - \frac{g \sin \theta}{1 + I_{cm} / mR^2}$$

$$m a_{x,cm} = \cancel{f_{id}} - mg \sin \theta$$

$$I_{cm} \alpha = \tau = f_{id} R$$

If the disk slid down the incline without rolling ($f_{id}=0$) it would go

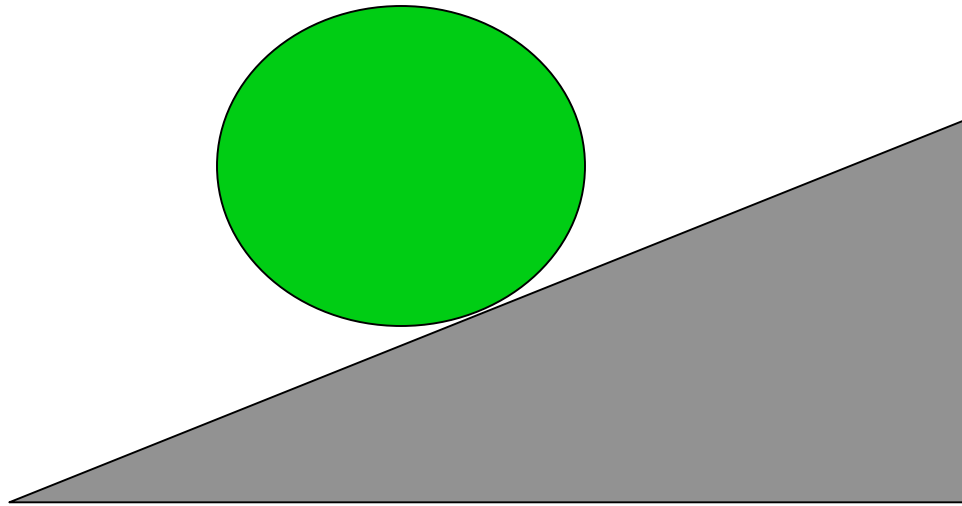
A faster

B slower

C the same speed

$$m a_{x,cm} = - mg \sin \theta$$

$$a_{x,cm} = -g \sin \theta$$



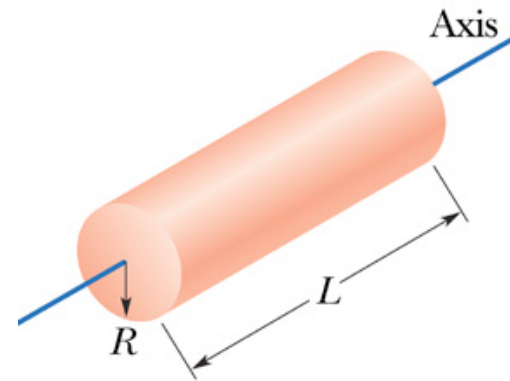
Rolling down an incline

$$a_{x,cm} = - \frac{g \sin \theta}{1 + I_{cm} / mR^2}$$

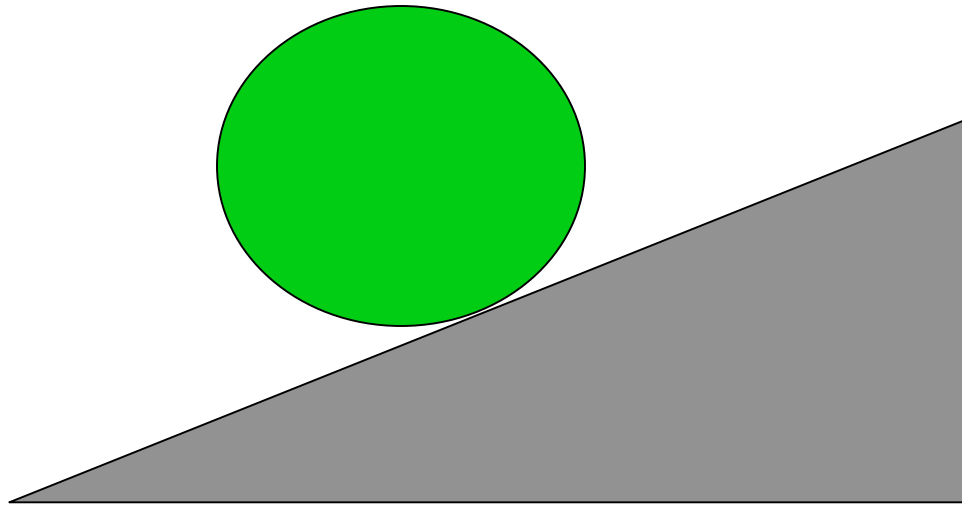
Compared to a smaller, lighter disk, a large, heavy disk will roll

- A faster
- B slower
- C the same speed**

$$a_{x,cm} = - \frac{g \sin \theta}{1 + \frac{1}{2}} = - \frac{2}{3} g \sin \theta$$



$$I = \frac{1}{2}MR^2$$

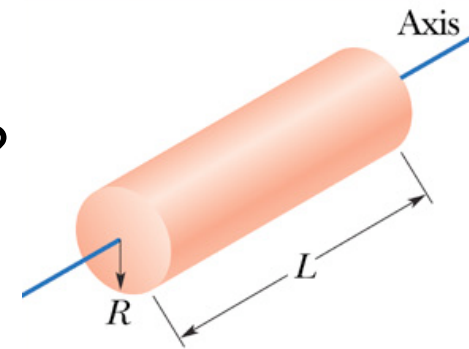


Rolling down an incline

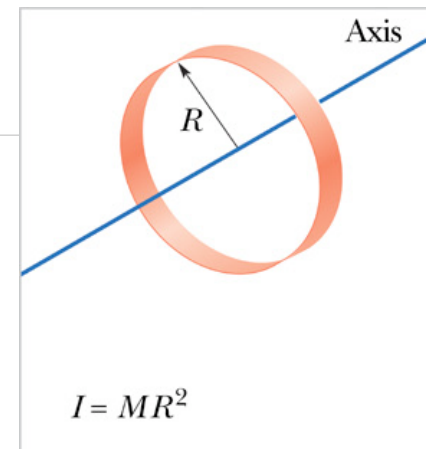
$$a_{x,cm} = - \frac{g \sin \theta}{1 + I_{cm} / mR^2}$$

A hoop and a solid disk are released together. Which reaches the bottom first?

- A** the disk
- B** the hoop
- C** its a tie



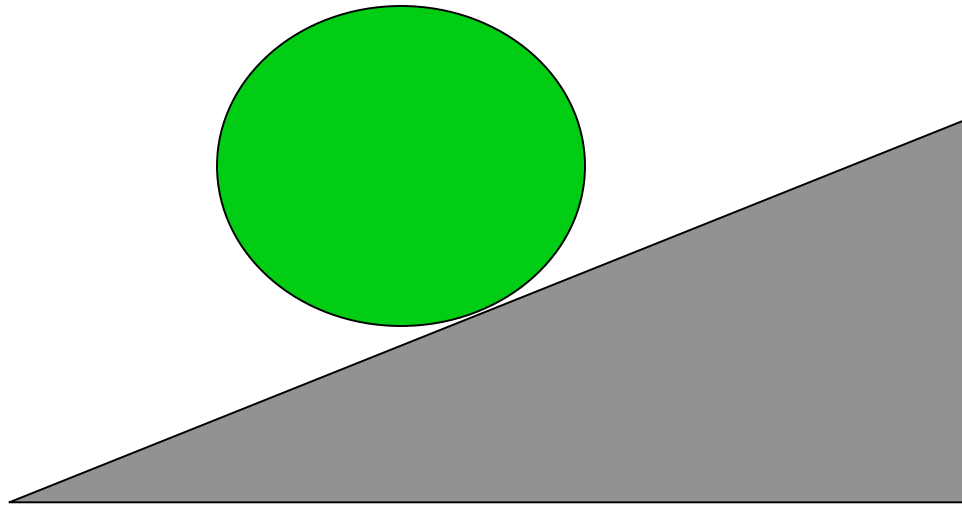
$$I = \frac{1}{2}MR^2$$



$$I = MR^2$$

$$a_{\text{disk}} = - \frac{g \sin \theta}{1 + \frac{1}{2}} = - \frac{2}{3} g \sin \theta$$

$$a_{\text{hoop}} = - \frac{g \sin \theta}{1 + 1} = - \frac{1}{2} g \sin \theta$$



Rolling down an incline

$$a_{x,cm} = - \frac{g \sin \theta}{1 + I_{cm} / mR^2}$$

A solid disk and a hollow sphere are released together. Which reaches the bottom first?

- A** the disk
- B** the sphere
- C** its a tie

$$a_{\text{disk}} = - \frac{g \sin \theta}{1 + \frac{1}{2}} = - \frac{2}{3} g \sin \theta \quad a_{\text{sphere}} = - \frac{g \sin \theta}{1 + \frac{2}{3}} = - \frac{3}{5} g \sin \theta$$

