

# Angular Momentum

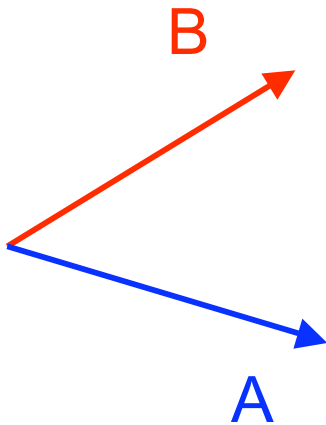
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# Quiz Next Class

On torque and cross products

# Cross products



A & B have equal magnitudes and are directed  $30^\circ$  apart. The angle between A and the cross product,  $A \times B$  is

- A  $0^\circ$
- B  $30^\circ$
- C  $90^\circ$
- D  $120^\circ$
- E need more information

# Torque

Here on Earth, the **weight force** is always directed downward. In cases where the weight force creates a **torque** about some point, that will be directed

- A Downward
- B Upward
- C Vertically (either up or down)
- D Horizontally**
- E It could be any combination of C & D depending on the situation

$$\begin{aligned}\vec{\tau} &= \vec{\mathbf{r}} \times \vec{\mathbf{F}} = (x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \times (-mg\hat{\mathbf{k}}) \\ &= -xmg(\hat{\mathbf{i}} \times \hat{\mathbf{k}}) - ymg(\hat{\mathbf{j}} \times \hat{\mathbf{k}}) = xmg\hat{\mathbf{j}} - ymg\hat{\mathbf{i}}\end{aligned}$$

# 30-sec RHR drills

The torque is:

- A Front
- B Left
- C Back
- D Right
- E zero

# Forms of Newton's 2<sup>nd</sup> Law

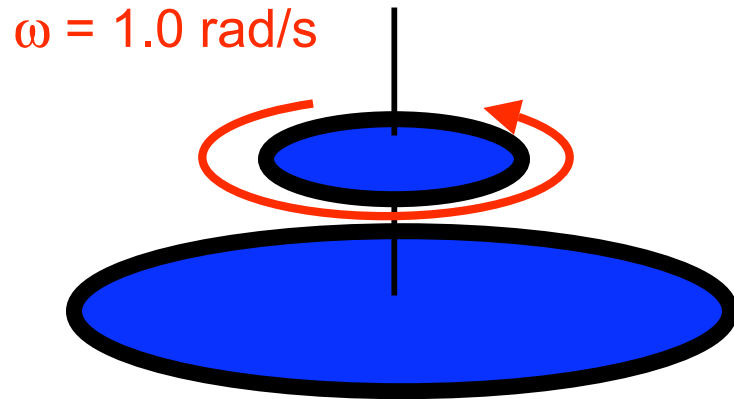
Linear	Rotational
$\sum \vec{\mathbf{F}} = m \vec{\mathbf{a}}_{\text{cm}}$	$\sum \vec{\tau} = I \vec{\alpha}$
$\sum \vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}$	$\sum \vec{\tau} = \frac{d\vec{\mathbf{L}}}{dt}$

for a rigid body:  $\vec{\mathbf{L}} = I\vec{\omega}$

$$\frac{d\vec{\mathbf{L}}}{dt} = \frac{d}{dt}(I\vec{\omega}) = I \frac{d\vec{\omega}}{dt} = I\vec{\alpha}$$

$$\vec{\tau} = 0 \quad \Rightarrow \quad \vec{\mathbf{L}} = \text{const.}$$

# Totally Inelastic collision



A solid disk of radius  $R$  spins at an initial rate of  $1.0 \text{ rad/s}$ . It falls onto a stationary disk of equal mass, but radius  $2R$ . The bottom disk lies on a frictionless surface. The friction between the two disks eventually cause them to spin together about their common axis. They spin at:

$$L_{z,i} = I_t(1 \text{ rad/s}) + I_b(0) = 0.5 m R^2$$

$$L_{z,f} = I_2 \omega_f$$

$$I_2 = 0.5 m R^2 + 0.5 m (2R)^2 = 2.5 m R^2$$

$$\omega_f = 0.5/2.5 = 0.2 \text{ rad/s}$$

**A** 0.2 rad/s

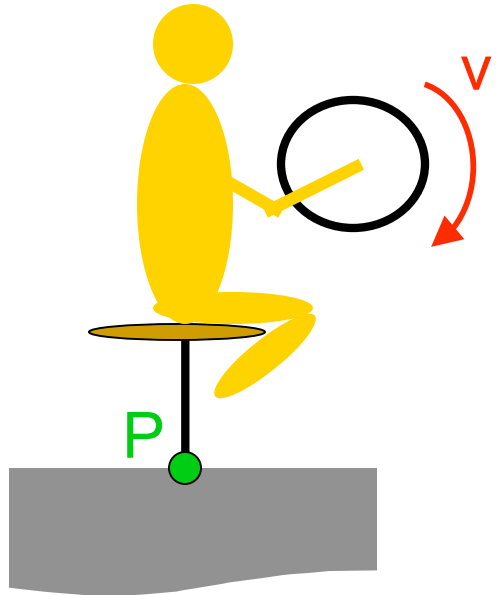
**B** 0.25 rad/s

**C** 0.33 rad/s

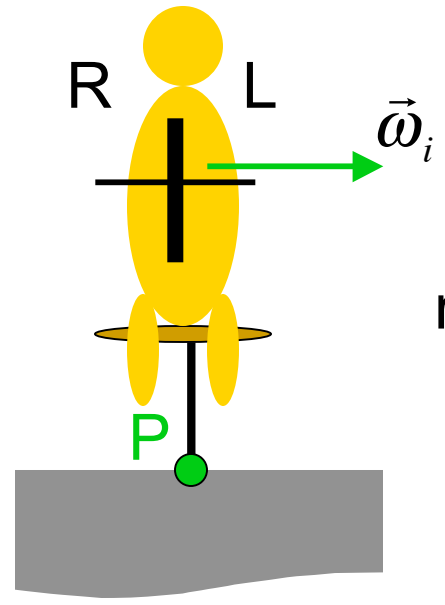
**D** 0.5 rad/s

**E** 1.0 rad/s

side view



front view view



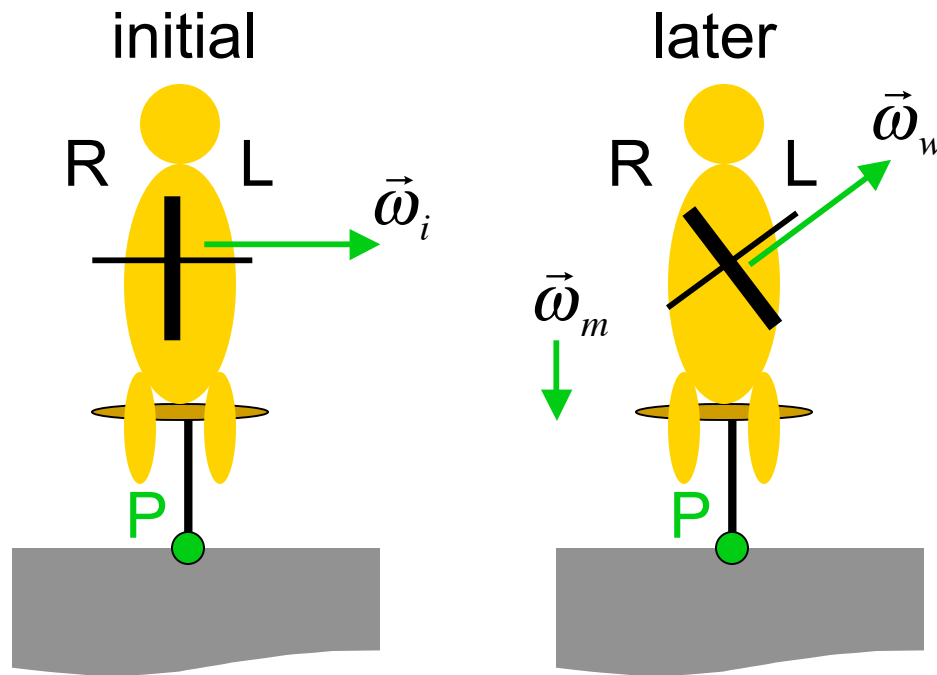
$$\tau_z = 0$$

from floor on  
me/wheel/stool system

$$L_z = \text{const.}$$

I sit on a stool holding a bicycle wheel. The stool pivots without friction about a vertical pivot. When the wheel's axis is horizontal the wheel is spun as shown. I now tip the wheel, moving **my left hand up**. In response the stool and I do what (viewed from above)?

- A Turn clockwise
- B Turn counter-clockwise
- C We do not move



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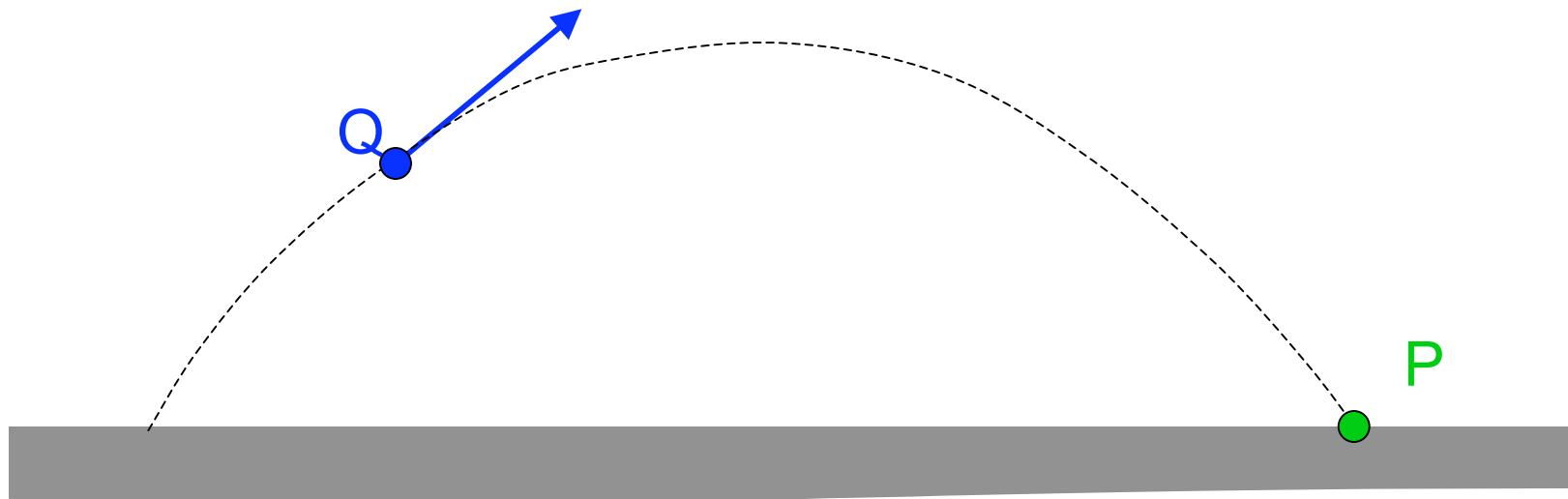
**C** We do not move

$$L_z = L_{z,i} = 0 = L_{z,w} + L_{z,m}$$

$$L_{z,w} = I_w \omega_{z,w} > 0$$

$$L_{z,m} = I_m \omega_{z,m} < 0$$

# Torque on a particle



A point particle ( $I_{cm}=0$ ) is launched along the parabolic trajectory shown. When it reaches point **Q**, the torque on the particle about point **P** is

**A** up & right

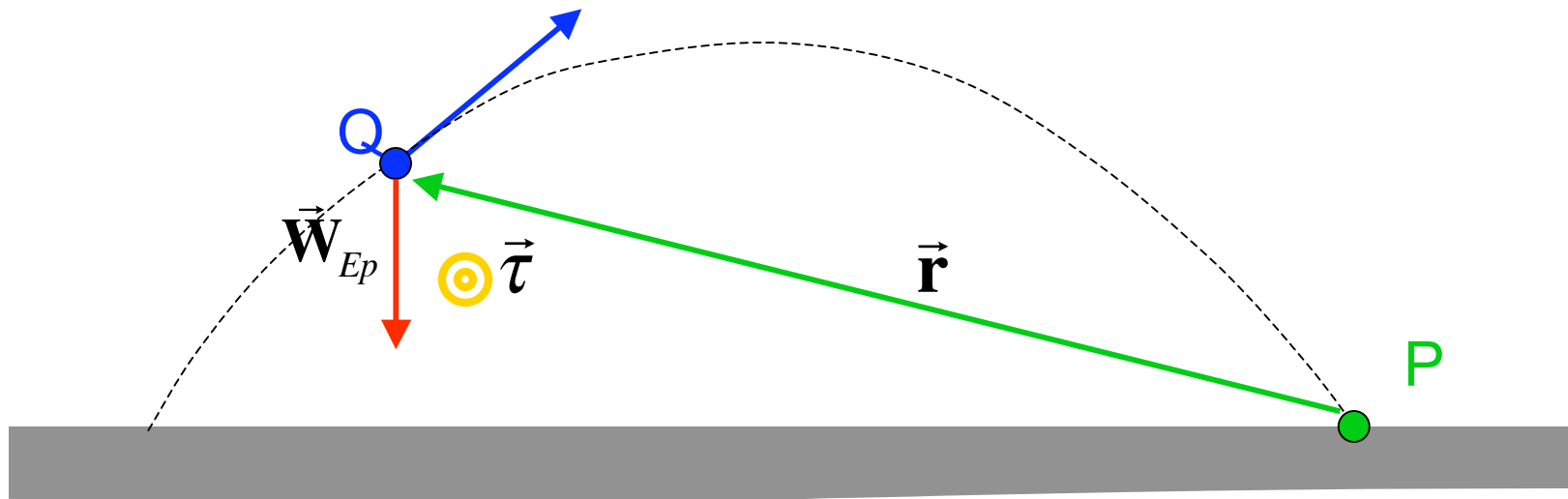
**B** downward

**E** zero

**C** into the board

**D** out of the board

# Torque on a particle

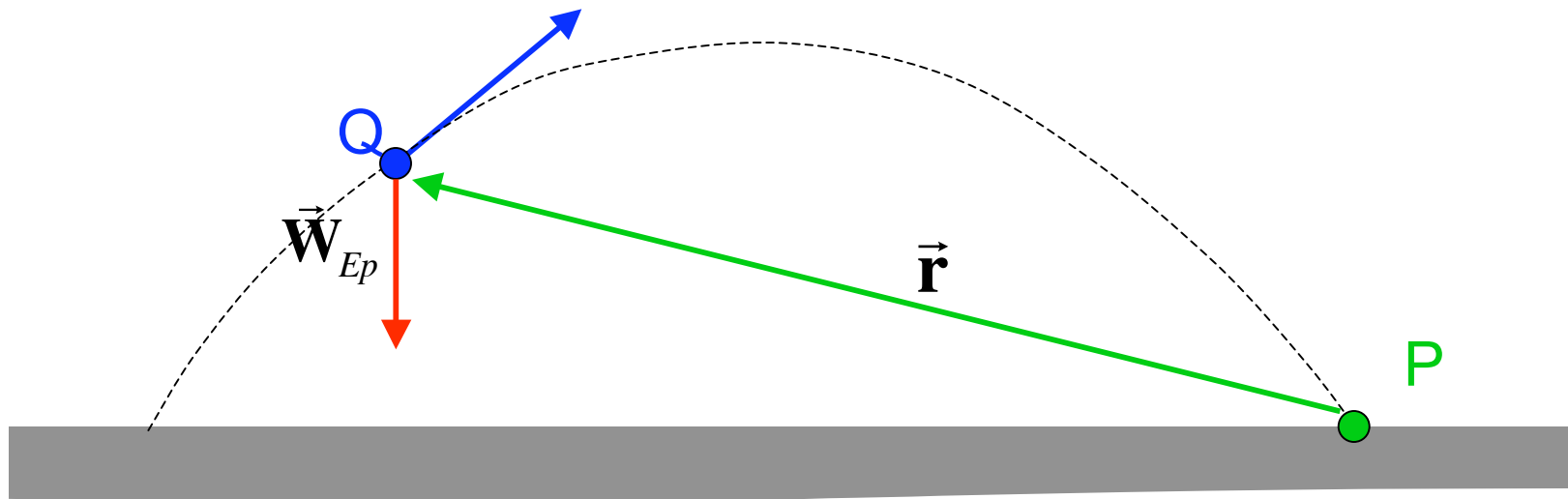


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# Torque on a particle



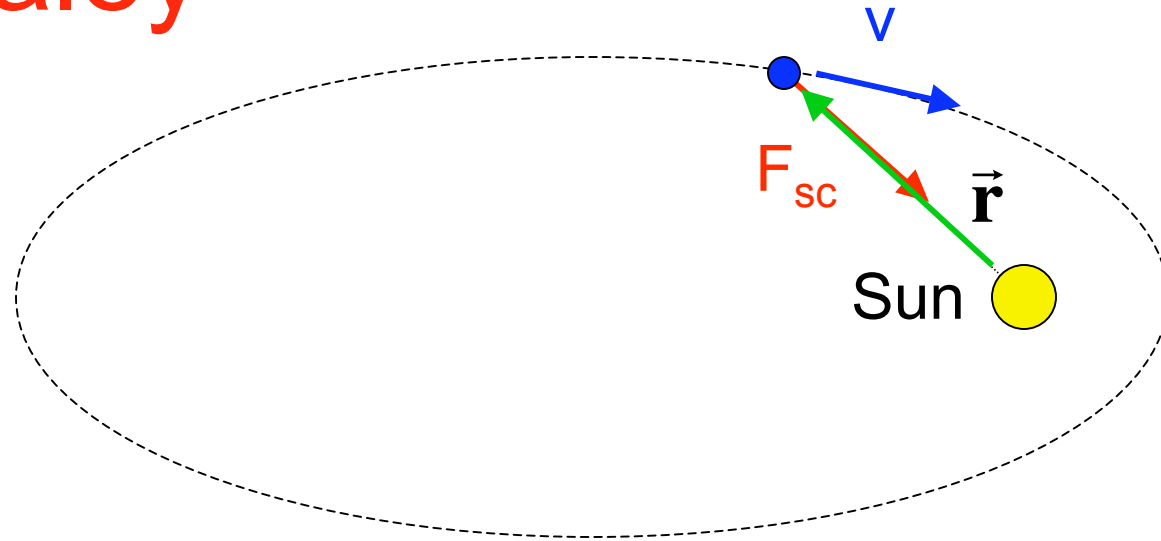
What is the **angular momentum** of point particle ( $I_{cm}=0$ )?

A: about center-of-mass:  $\vec{L} = 0$

A: about point P:  $\vec{L} = \vec{r} \times \vec{p}$

$$\frac{d\vec{L}}{dt} = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \vec{v} \times \vec{p} + \vec{r} \times \vec{F} = \vec{\tau}$$

# Comet Haley



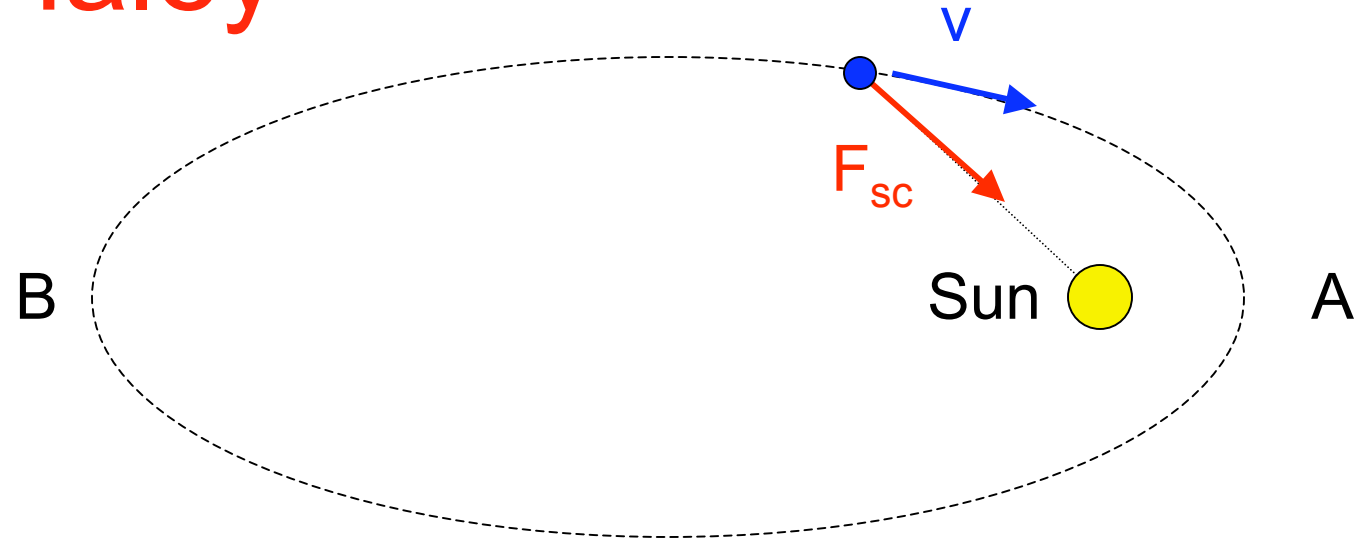
The torque **about the Sun**, due to the gravitational force of attraction from the Sun is

- A inward (toward the Sun)
- B outward (away from the Sun)
- C into the page
- D out of the page

**E zero**

$$\vec{\tau} = \vec{r} \times \vec{F} = 0$$

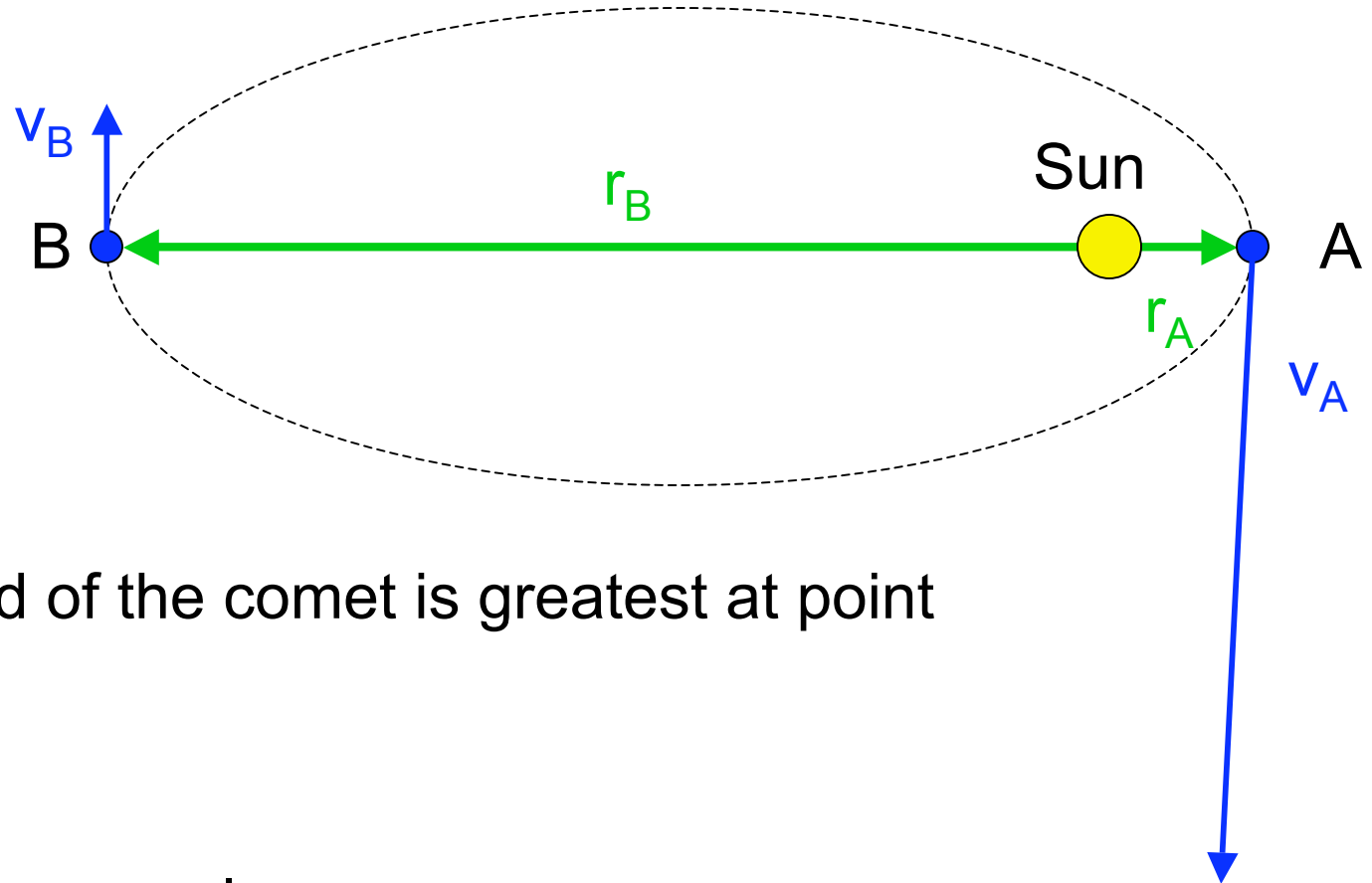
# Comet Haley



The speed of the comet is greatest at point

- A A
- B B
- C both are equal

# Comet Haley



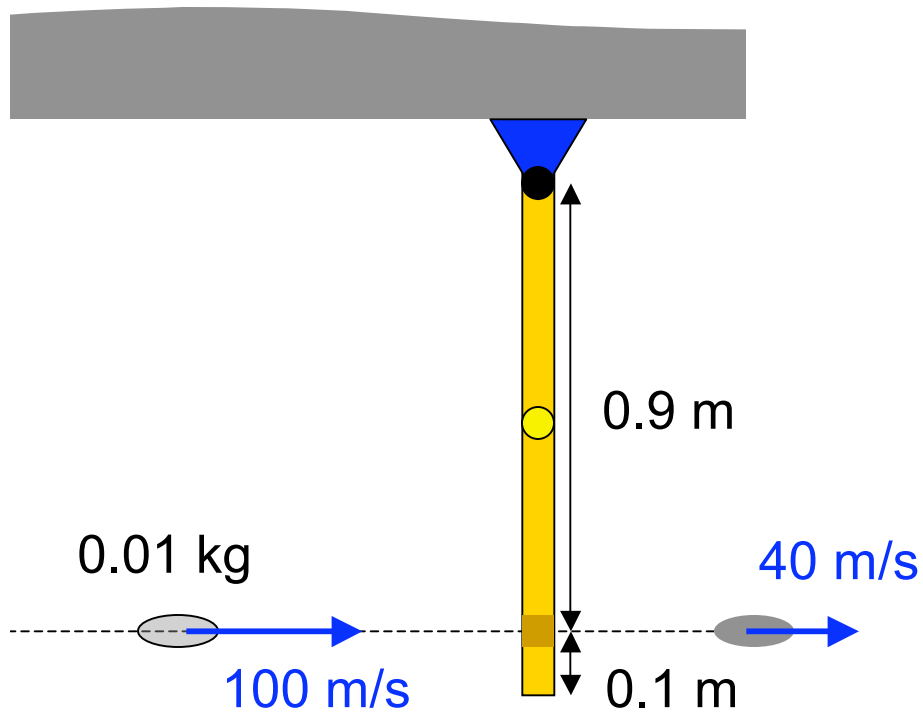
The speed of the comet is greatest at point

**A** A

B B

C both are equal

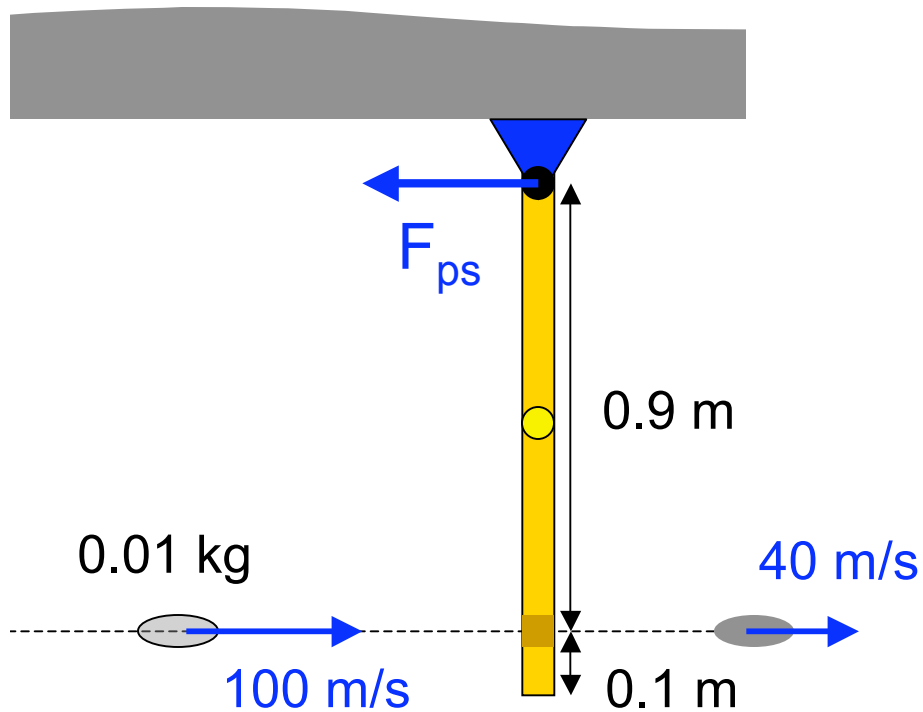
$$L_A = r_A v_A = L_B = r_B v_B$$



A 0.5kg sign hangs from a frictionless pivot. A ne'er-do-well fires a 10 g bullet through it. It passes through the sign 90 cm below the pivot, entering at 100 m/s and exiting at 40 m/s. The sign is a rigid body and experiences an immediate reaction from the pivot as the bullet passes through it.

During the (brief) collision what property of the **sign/bullet system** is conserved?

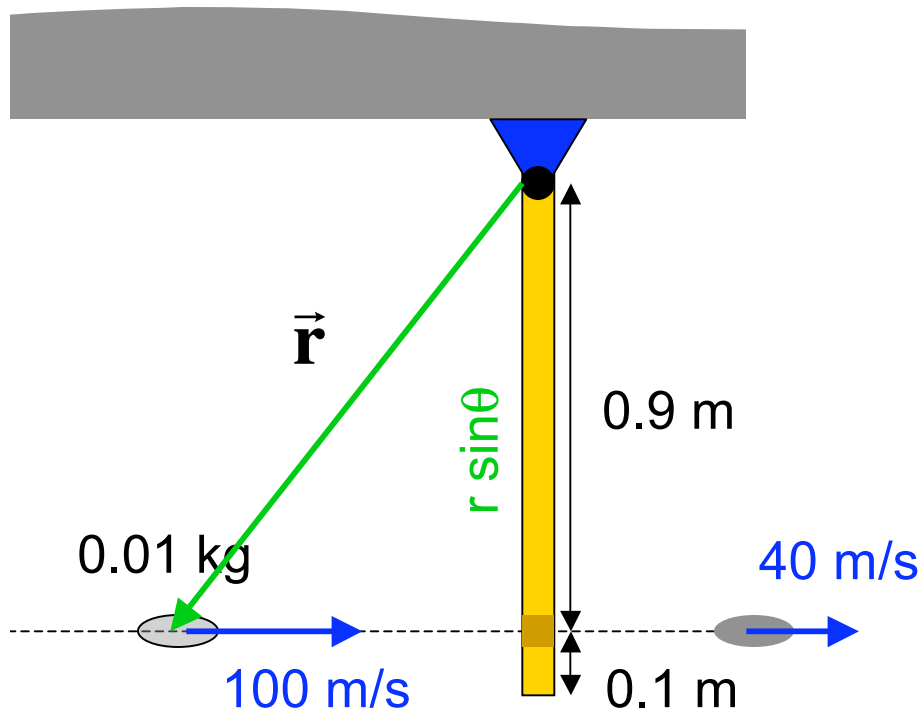
- A linear momentum
- B angular momentum about the COM
- C angular momentum about the pivot
- D A and B
- E A and C



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### Angular momentum about pivot:

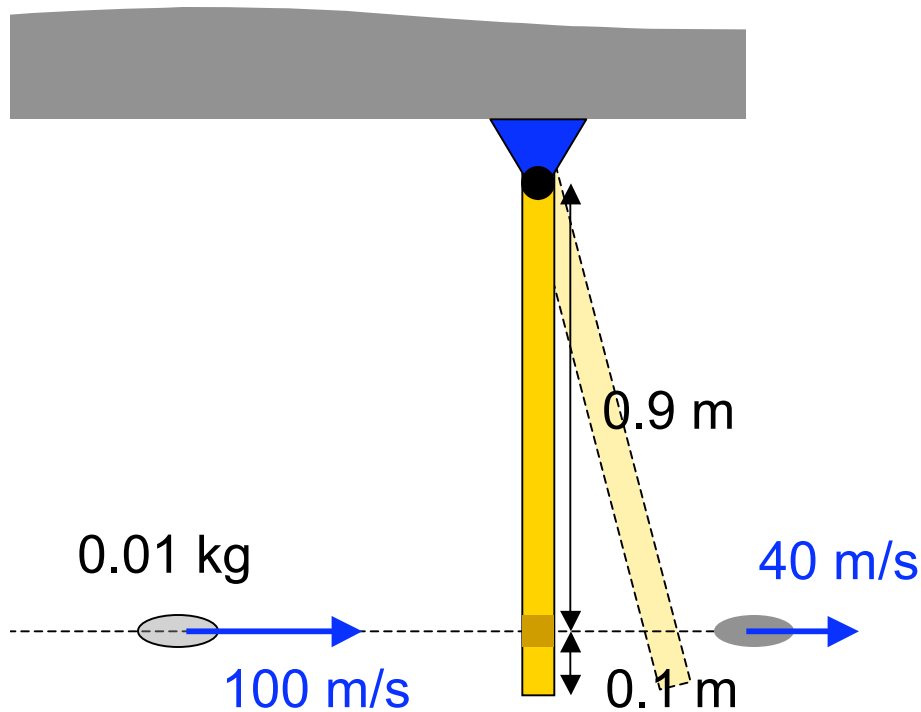
$$\text{Initial: } L_{b,i} = (0.01\text{kg})(100\text{ m/s})(0.9\text{ m}) = 0.9\text{ kg m}^2/\text{s}$$

$$\text{Final: } L_{b,f} = (0.01\text{ kg})(40\text{ m/s})(0.9\text{ m}) = 0.36\text{ kg m}^2/\text{s}$$

$$L_{s,f} = I_p \omega_f \qquad L_{s,f} = 0.54\text{ kg m}^2/\text{s}$$

$$I_p = (1/3) ML^2 = 0.167\text{ kg m}^2$$

$$\omega_f = 3.24\text{ rad/s}$$



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$$I_p = (1/3) ML^2 = 0.167 \text{ kg m}^2$$

Immediately after collision:

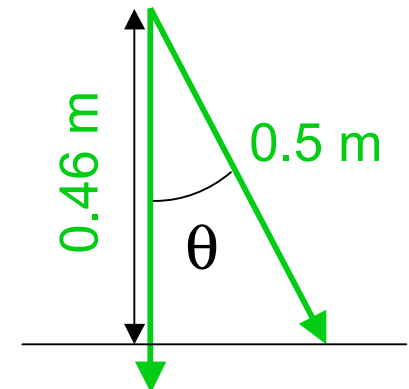
$$\omega_f = 3.24 \text{ rad/s} \quad K_s = 0.5 I_p \omega^2 = 0.87 \text{ J}$$

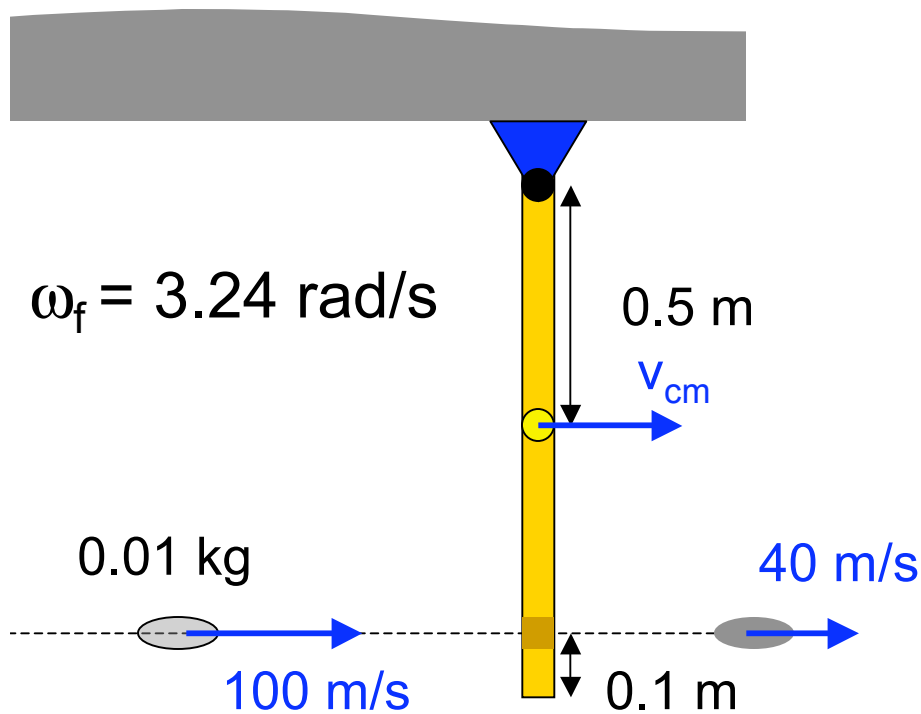
Conservation of mechanical energy (of sign)

$$\Delta K = -0.87 \text{ J} = -\Delta U = -mg \Delta y_{\text{cm}}$$

$$\Delta y_{\text{cm}} = 0.044 \text{ m}$$

$$\theta = \cos^{-1}(0.46/0.5) = 24^\circ$$





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## Linear Momentum

Immediately before collision:

$$p_{b,i} = (0.01 \text{ kg})(100 \text{ m/s}) = 1.0 \text{ kg m/s}$$

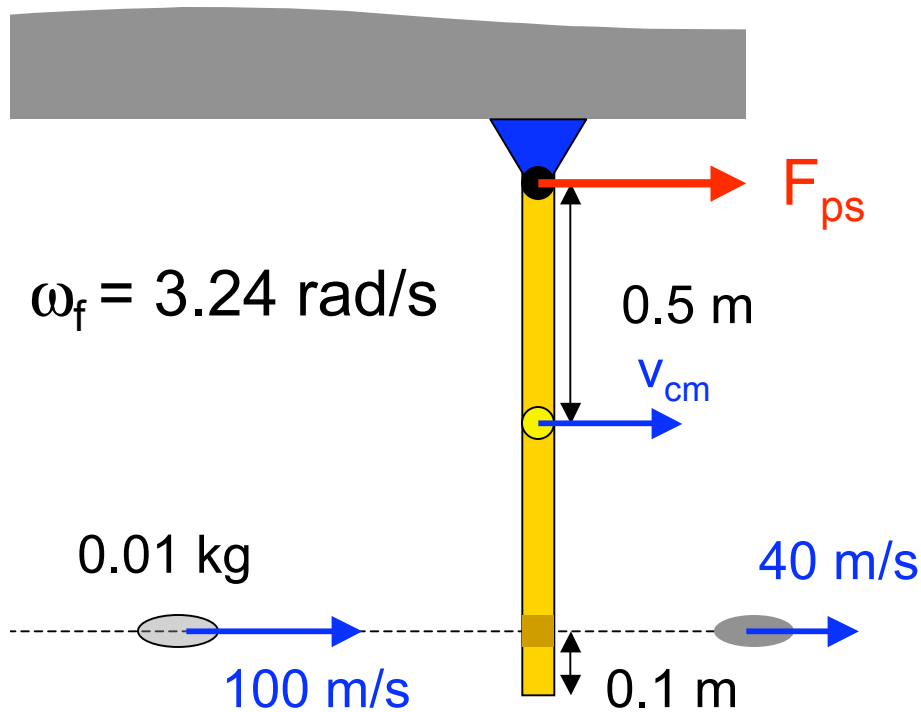
$$p_i = 1.0 \text{ kg m/s}$$

Immediately after collision:

$$p_{b,f} = (0.01 \text{ kg})(40 \text{ m/s}) = 0.4 \text{ kg m/s}$$

$$p_f = 1.21 \text{ kg m/s}$$

$$v_{cm} = (0.5 \text{ m}) \omega_f = 1.62 \text{ m/s} \quad p_{s,f} = 0.81 \text{ kg m/s}$$



A 0.5kg sign hangs from a frictionless pivot. A ne'er-do-well fires a 10 g bullet through it. It passes through the sign 90 cm below the pivot, entering at 100 m/s and exiting at 40 m/s. The sign is a rigid body and experiences an immediate reaction from the pivot as the bullet passes through it. What is the impulse of the pivot on the sign?

$$\Delta p = 1.21 - 1.0 = +0.21 \text{ kg m/s}$$

momentum **increases**

### Linear Momentum

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$$p_{b,i} = (0.01 \text{ kg})(100 \text{ m/s}) = 1.0 \text{ kg m/s}$$

$$p_i = 1.0 \text{ kg m/s}$$

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