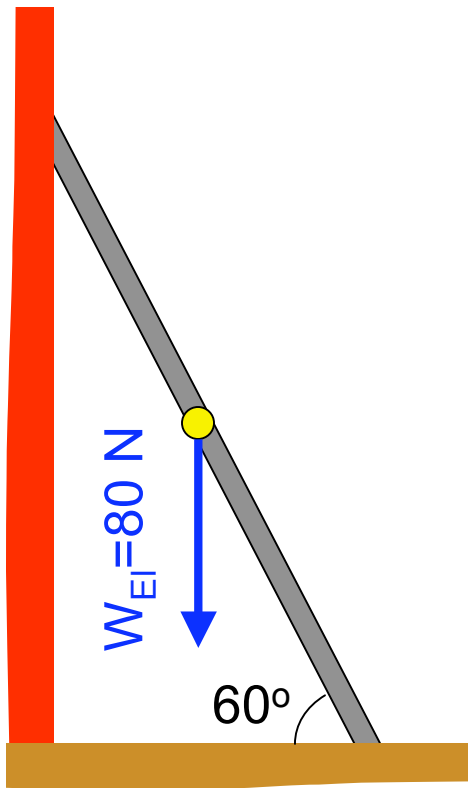


Equilibrium

Equilibrium



A ladder weighing 80 N leans against a frictionless wall, 60° from the horizontal. It is at rest. What are the magnitudes and directions of all forces on the ladder?

case of equilibrium

static equilibrium: $\vec{v}_{\text{cm}} = 0$ **and** $\vec{\omega} = 0$

more importantly: $\vec{a}_{\text{cm}} = 0$ **and** $\vec{\alpha} = 0$

$$\sum \vec{F} = 0 \quad \text{and} \quad \sum \vec{\tau} = 0$$

Any rigid body (not in equilb.) which has...

... a **net force** must also have a **net torque***.

~~$\sum \vec{F} \neq 0 \rightarrow \sum \vec{\tau} \neq 0$~~

A True

B false

contapositive:
... no **net torque***
will have no
net force

Think of a scenario where a body a **net force** and NO **net torque***.

* about its COM

Any rigid body (not in equilb.) which has...

... a **net torque*** must also have a **net force**.

~~$\sum \vec{\tau} \neq 0 \Rightarrow \sum \vec{F} \neq 0$~~

A True

B false

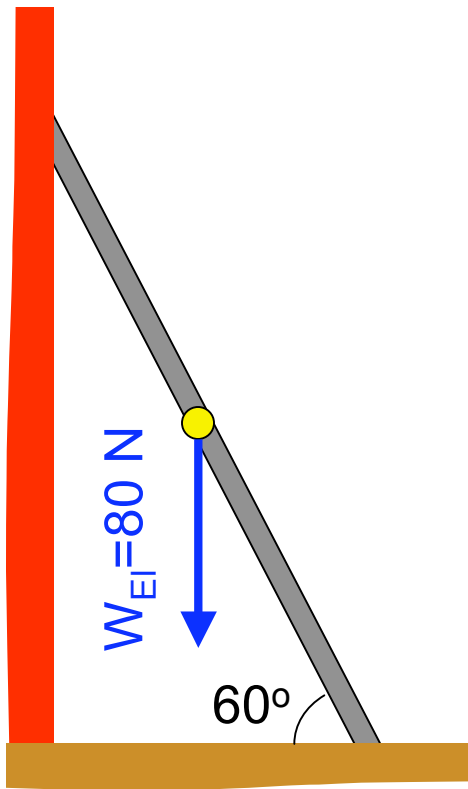
contapositive:

... no **net force**
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net torque*

Think of a scenario where a body a **net torque***
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* about its COM

Equilibrium

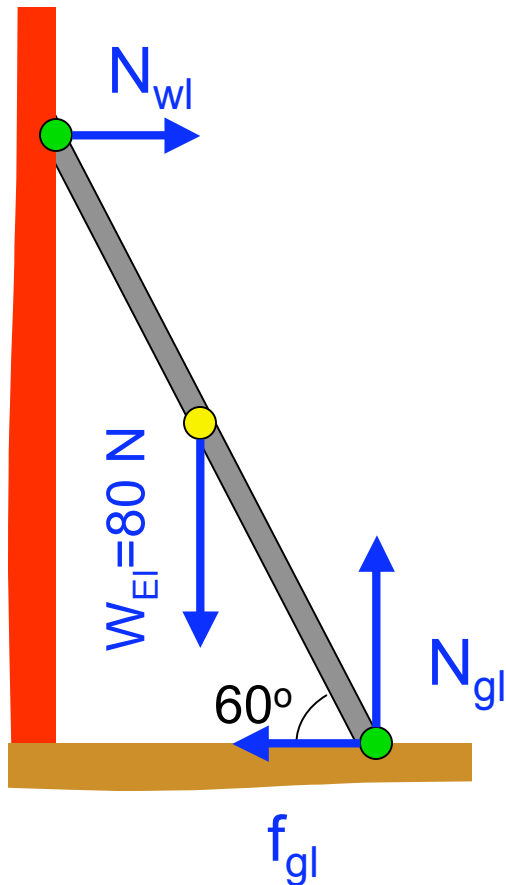


A ladder weighing 80 N leans against a frictionless wall, 60° from the horizontal. It is at rest. What are the magnitudes and directions of all forces on the ladder?

Not counting weight, how many forces act on the ladder?

- A 0
- B 1
- C 2
- D 3
- E 4

Equilibrium



A ladder weighing 80 N leans against a frictionless wall, 60° from the horizontal. It is at rest. What are the magnitudes and directions of all forces on the ladder?

Not counting weight, how many forces act on the ladder?

All unknown (as needed) forces

**3 unknowns require
3 equations**

A 0

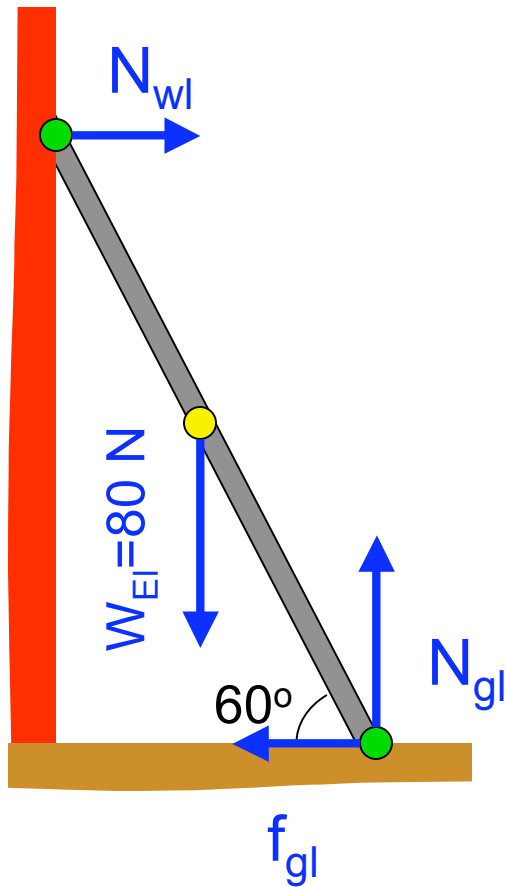
B 1

C 2

D 3

E 4

Equilibrium



A ladder weighing 80 N leans against a frictionless wall, 60° from the horizontal. It is at rest. What are the magnitudes and directions of all forces on the ladder?

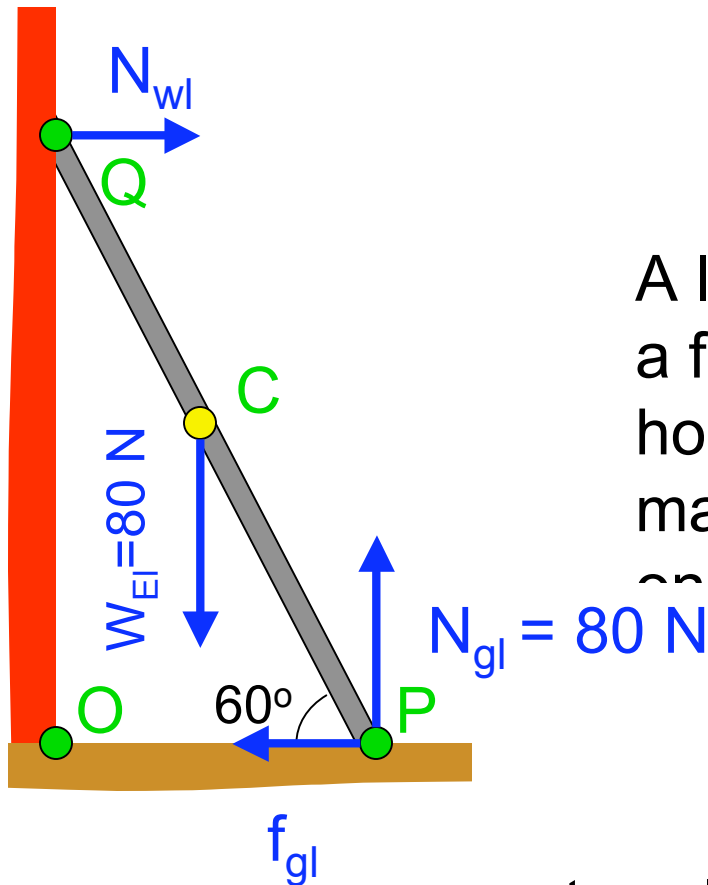
$$\sum \vec{F} = 0$$

$$\sum \vec{\tau} = 0$$

$$\sum F_y = N_{gl} - 80 \text{ N} = 0 \quad N_{gl} = 80 \text{ N}$$

$$\sum F_x = N_{wl} + f_{gl} = 0 \quad N_{wl} = -f_{gl} = ?$$

Equilibrium



A ladder weighing 80 N leans against a frictionless wall, 60° from the horizontal. It is at rest. What are the magnitudes and directions of all forces \vec{F} on the ladder?

$$\sum \vec{F} = 0$$

$$\sum \vec{\tau} = 0$$

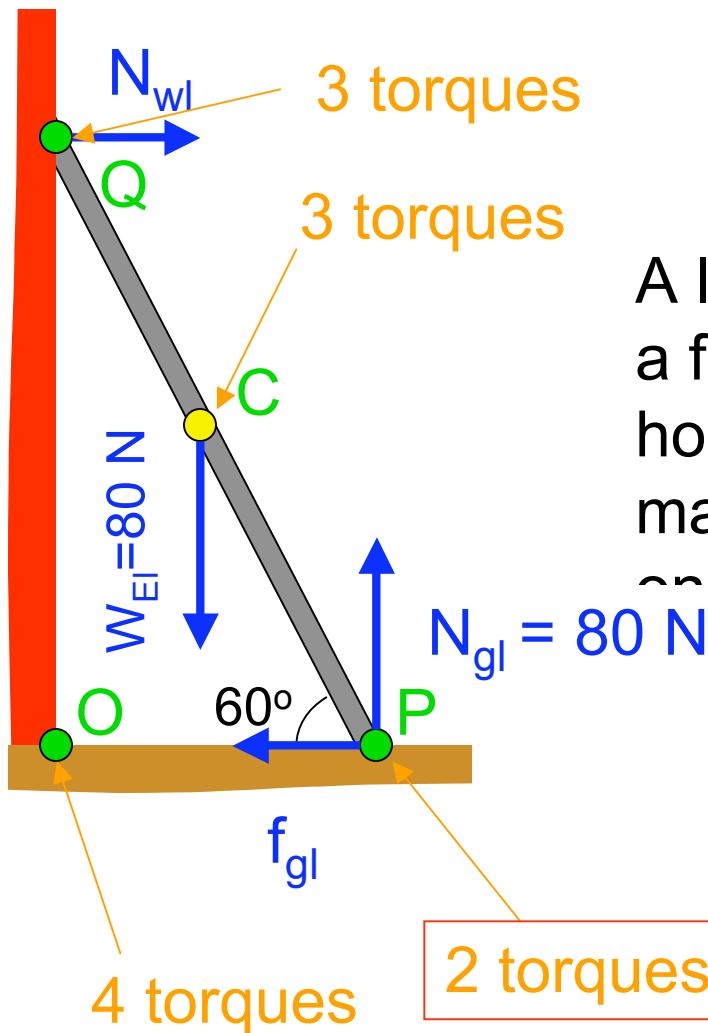
must evaluate torques... about which point?

Q: which point is ladder (not) rotating about?

A: about all points ... i.e. any point you choose

How do you choose? Choose to make life easiest

Equilibrium



A ladder weighing 80 N leans against a frictionless wall, 60° from the horizontal. It is at rest. What are the magnitudes and directions of all forces \vec{F} on the ladder?

$$\sum \vec{F} = 0$$

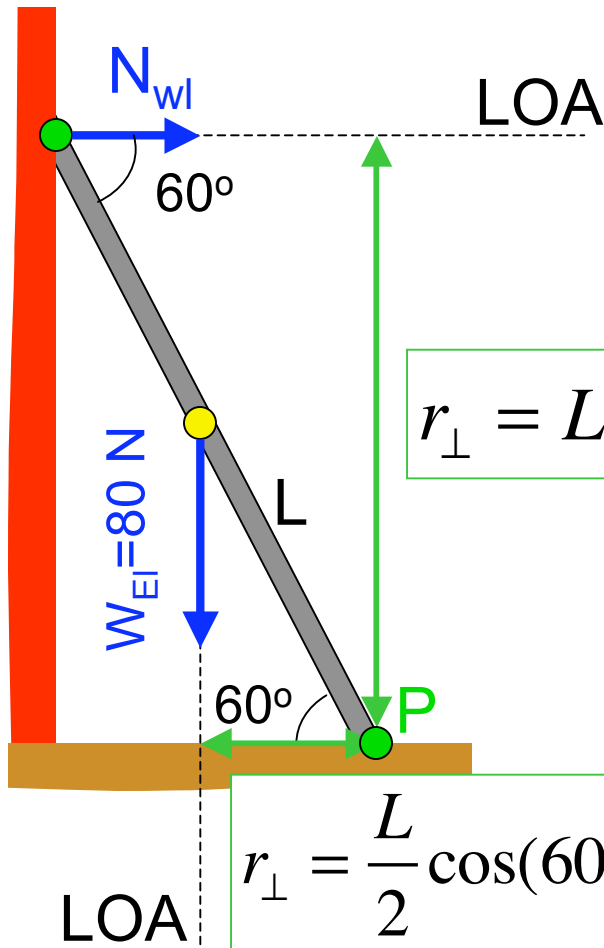
$$\sum \vec{\tau} = 0$$

How do you choose* point for τ ?
Choose to make life easiest.

A force creates no torque about the point it acts at... that force is removed from the calculation

* There are no wrong choices... only **harder** choices

Equilibrium



A ladder weighing 80 N leans against a frictionless wall, 60° from the ground. The ladder is at rest. What are the magnitudes and directions of all forces on the ladder?

$$r_{\perp} = L \sin(60^{\circ}) = \frac{\sqrt{3}}{2} L$$

$$\sum \vec{F} = 0$$

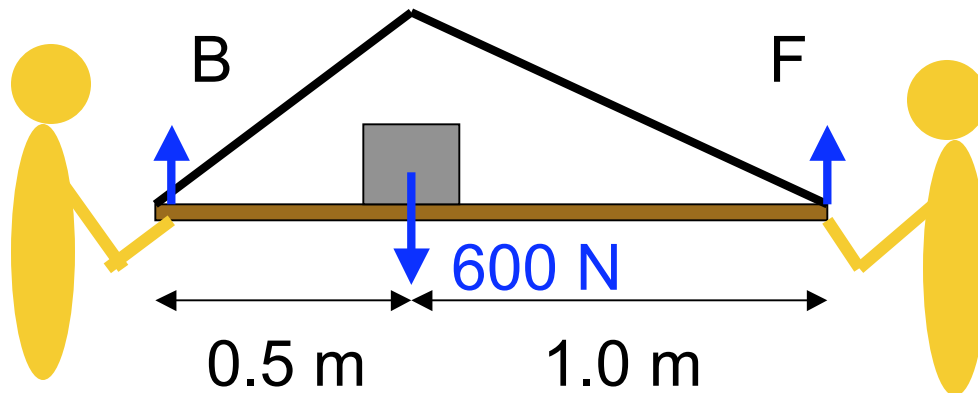
$$\sum \vec{\tau} = 0$$

$$r_{\perp} = \frac{L}{2} \cos(60^{\circ}) = \frac{1}{4} L$$

$$\tau_P = \left(\frac{1}{4} L\right)(80 \text{ N}) - \left(\frac{\sqrt{3}}{2} L\right)(N_{wl}) = 0$$

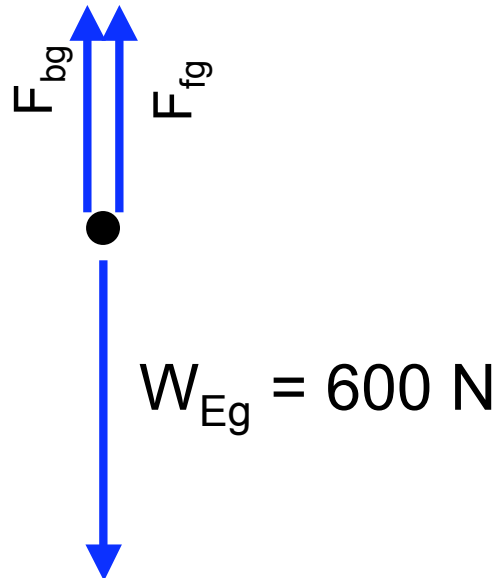
$$N_{wl} = \frac{1}{2\sqrt{3}} (80 \text{ N}) = 23 \text{ N}$$

$$f_{gl} = -N_{wl} = -23 \text{ N}$$



Graham asks you to help him lift a light go-cart frame into his truck. The only significant weight is 600 N from the engine located 0.5 m from the back (= 1.0 m from the front).

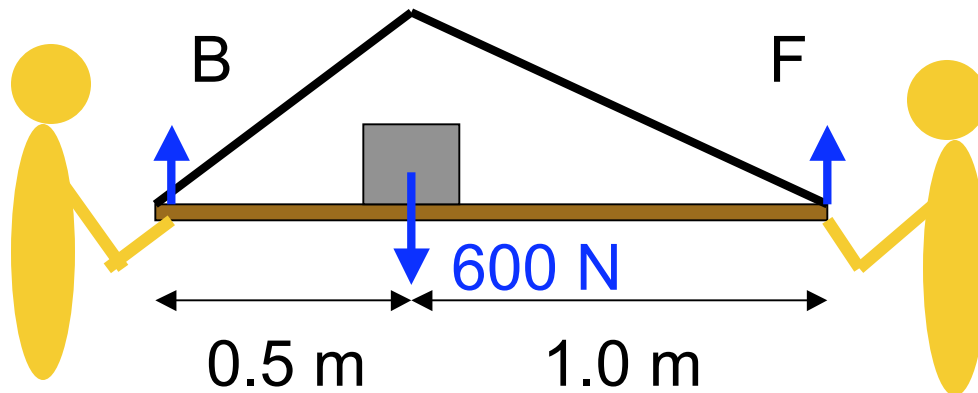
Forces
FBD:



$$F_{bg} + F_{fg} = 600 \text{ N}$$

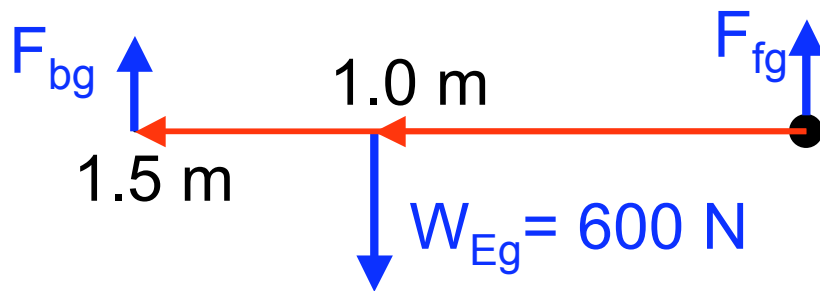
Which end is easiest to lift?

- A The front
- B The back
- C It doesn't matter
- D More information is needed



Graham asks you to help him lift a light go-cart frame into his truck. The only significant weight is 600 N from the engine located 0.5 m from the back (= 1.0 m from the front).

Torques about the front:



$$\tau_b = -F_{bg} \cdot 1.5 \text{ m}$$

$$\tau_W = 600 \text{ N} \cdot 1.0 \text{ m} = +600 \text{ N m}$$

$$\tau_f = F_{fg} \cdot 0 \text{ m} = 0 \text{ N m}$$

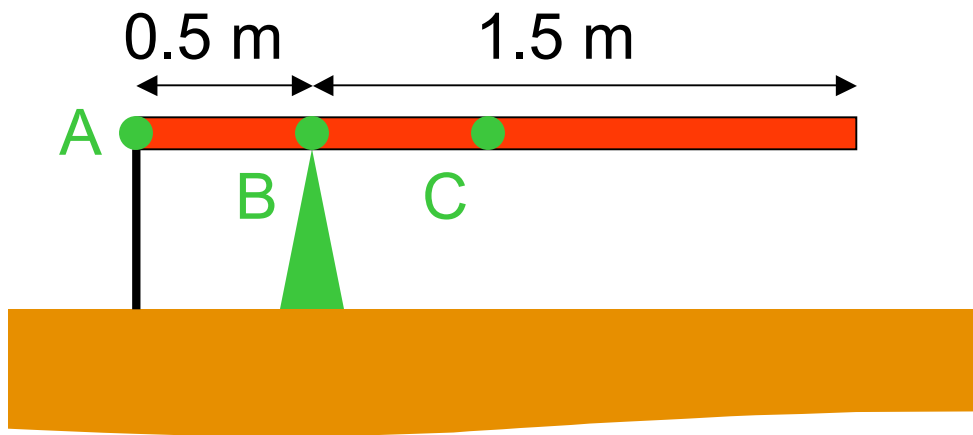
$$\sum \tau = 600 - 1.5 F_{bg} = 0$$

$$F_{bg} = 400 \text{ N} \quad F_{fg} = 200 \text{ N}$$

Which end is easiest to lift?

- A** The front
- B** The back
- C** It doesn't matter
- D** More information is needed

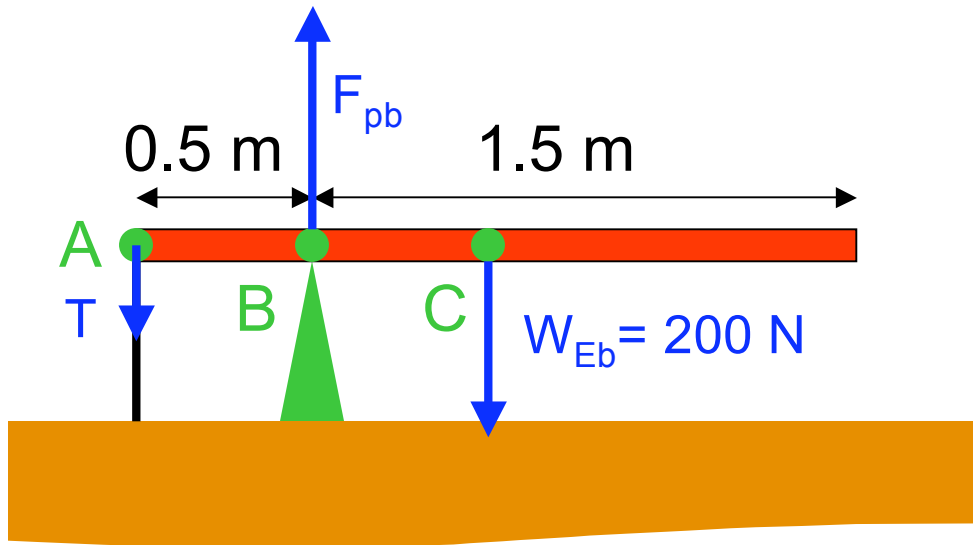
$$F_{bg} + F_{fg} = 600 \text{ N}$$



A 2 meter-long beam weighing 200 N is balanced on a pivot as shown. Its left end is tied to the ground by a cable. What is the force **on the pivot**?

What can be used to **most quickly** find the force?

- A torque about A
- B torque about B
- C torque about C
- D force balance in x
- E force balance in y



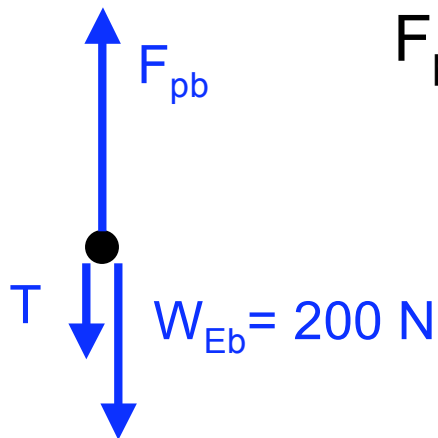
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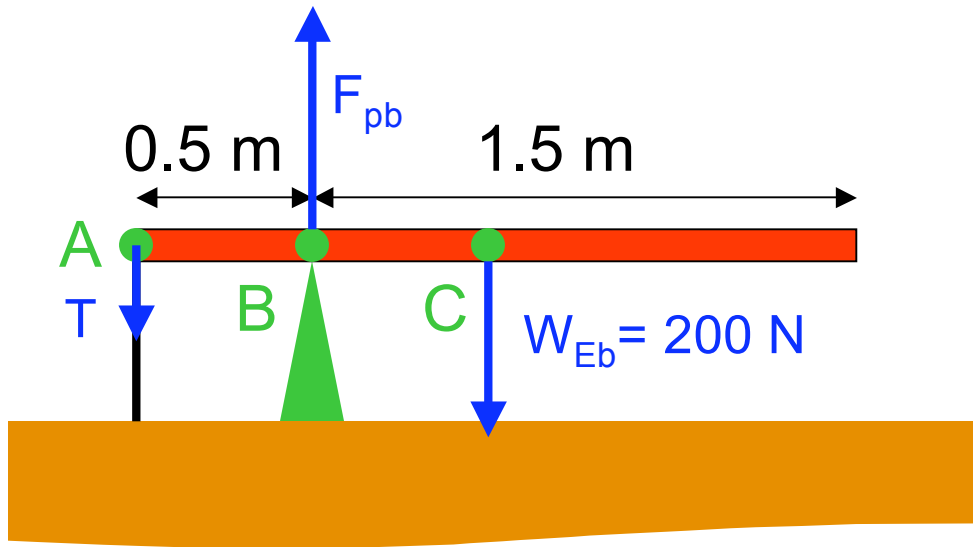
Forces
FBD:

$$F_{pb} - T - W_{Eb} = 0$$

$$F_{pb} = T + W_{Eb}$$



- A torque about A
- B torque about B
- C torque about C
- D force balance in x
- E force balance in y



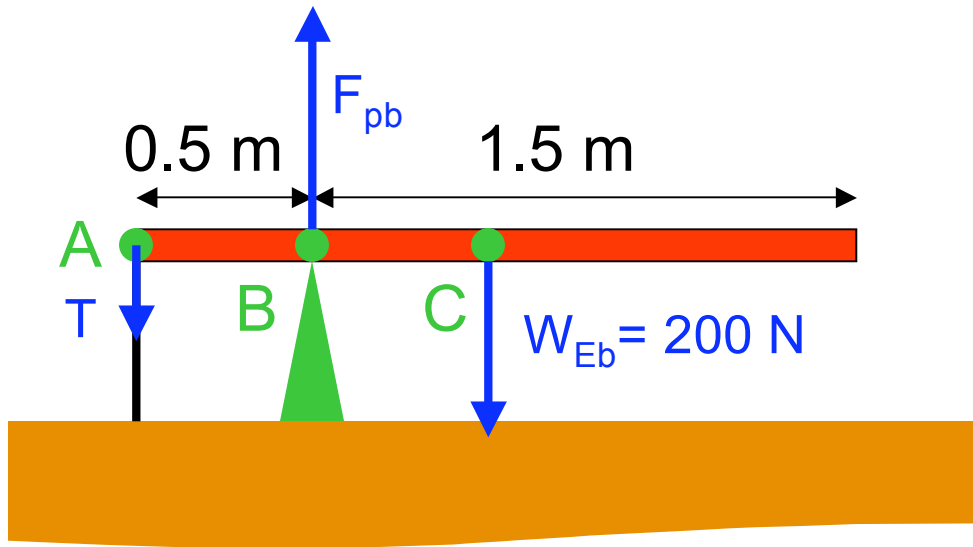
A 2 meter-long beam weighing 200 N is balanced on a pivot as shown. Its left end is tied to the ground by a cable. What is the force on the pivot?

What can be used to **most quickly** find the force?

- A** torque about A
- B** torque about B
- C** torque about C
- D** force balance in x
- E** force balance in y

unknown

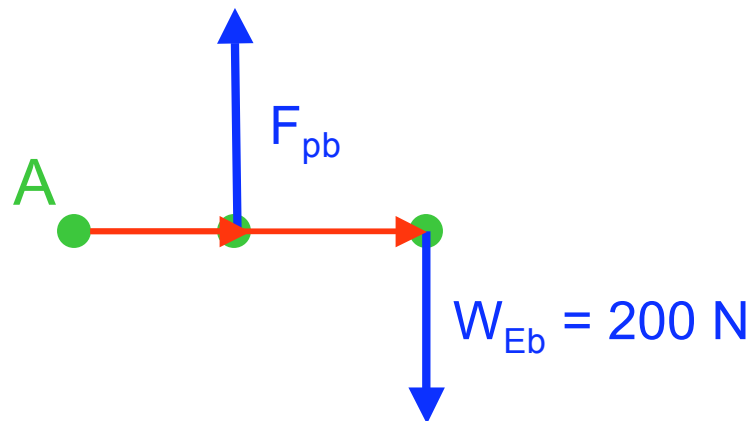
	T	F_{pb}	W_{Eb}
torque about	A	✗	✓
	B	✓	✓
	C	✓	✗



A 2 meter-long beam weighing 200 N is balanced on a pivot as shown. Its left end is tied to the ground by a cable. What is the force on the pivot?

What can be used to **most quickly** find the force?

- A torque about A
- B torque about B
- C torque about C
- D force balance in x
- E force balance in y

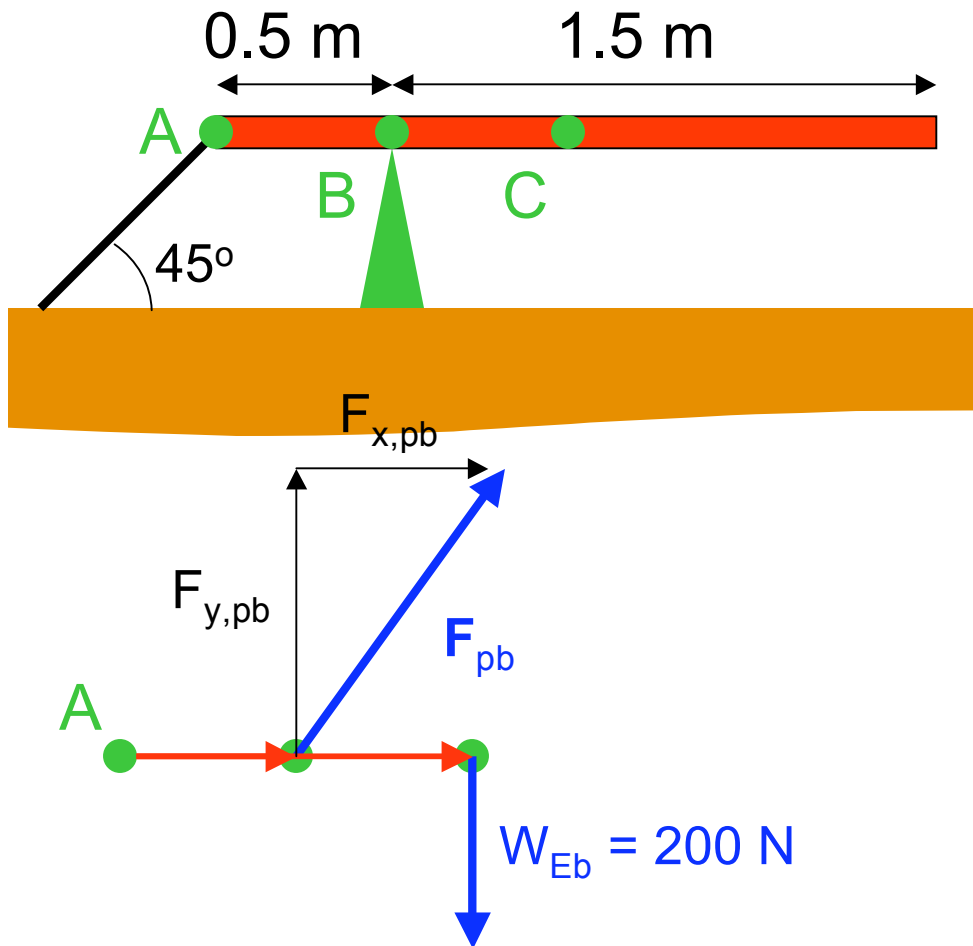


$$\tau_W = -200 \text{ N} \cdot 1.0 \text{ m} = -200 \text{ N} \cdot \text{m}$$

$$\tau_p = 0.5 \text{ m} F_{pb}$$

$$\sum \tau = -200 + 0.5 \text{ m} F_{pb} = 0$$

$$F_{pb} = 400 \text{ N}$$



The supporting cable is angled at 45° . What NOW is the force on the pivot?

Torque balance about A can be used to find

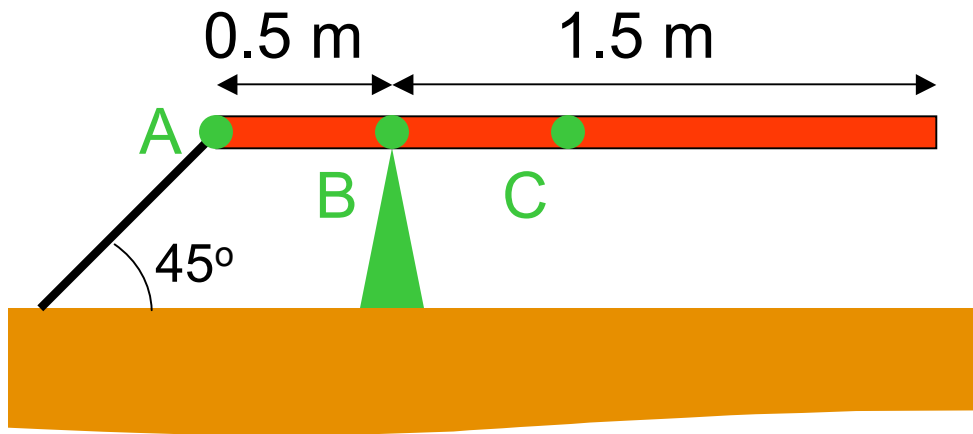
- A \mathbf{F} of the pivot
- B $|\mathbf{F}|$ of the pivot
- C F_x of the pivot
- D F_y of the pivot**

$$\tau_W = -200\text{ N} \cdot 1.0\text{ m} = -200\text{ N m}$$

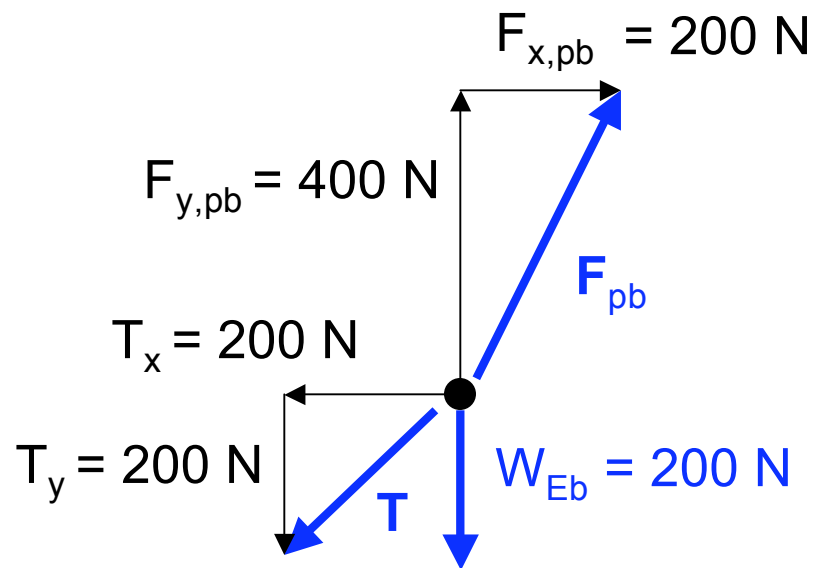
$$\tau_p = 0.5\text{ m} F_{y,pb}$$

$$\Sigma\tau = -200 + 0.5\text{ m} F_{y,pb} = 0$$

$$F_{y,pb} = 400\text{ N}$$

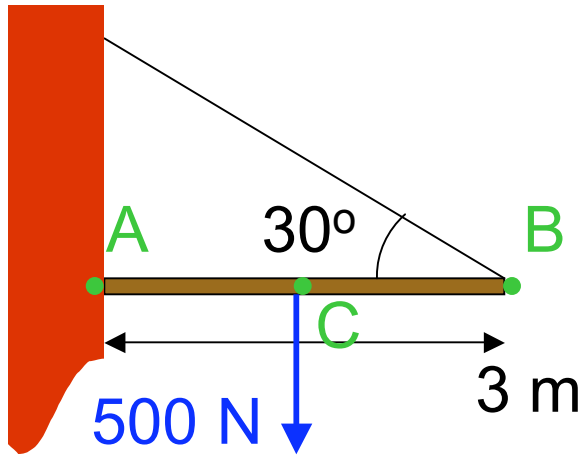


The supporting cable is angled at 45°. What NOW is the force on the pivot?



$$\mathbf{F}_{pb} = (200 \hat{\mathbf{i}} + 400 \hat{\mathbf{j}}) \text{ N}$$

A 3m beam (mass 50kg) is held horizontal against a wall by a cable.

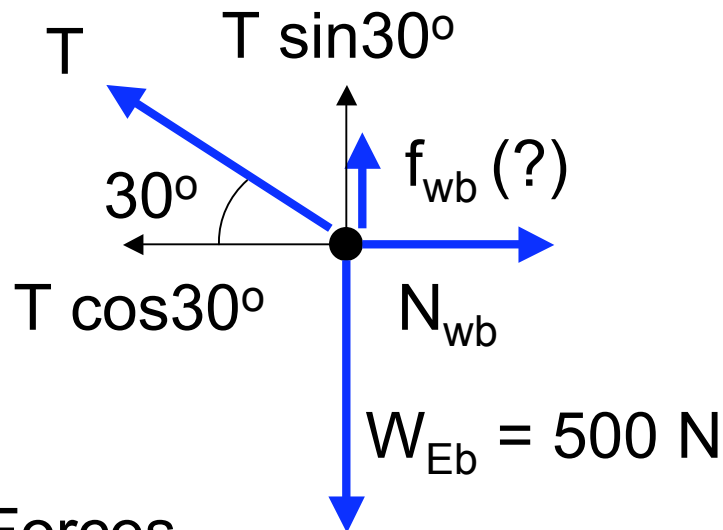


What forces will probably be present at A?

A Normal force from wall

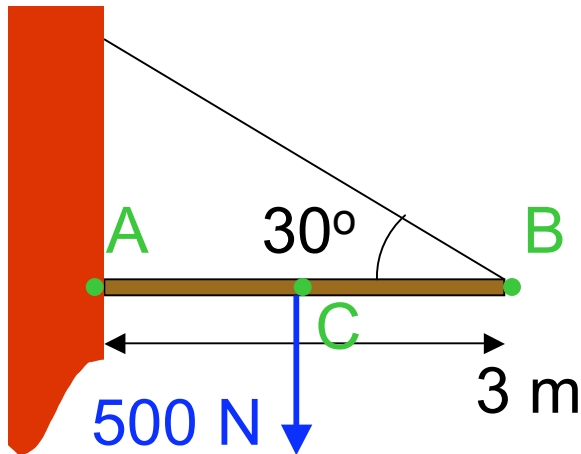
B Friction from wall

C Normal force and friction



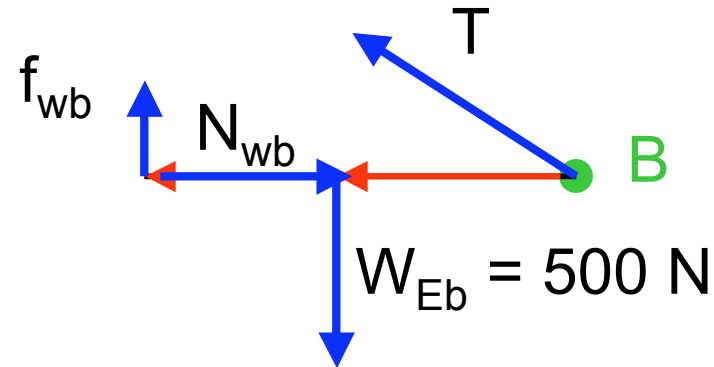
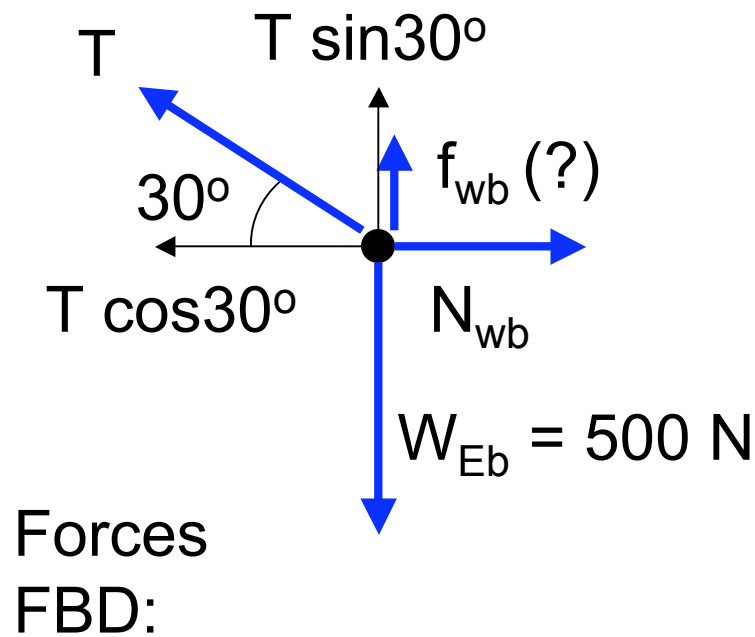
Forces
FBD:

A 3m beam (mass 50kg) is held horizontal against a wall by a cable.



What can be used to most quickly find f_{wb} ?

- A torque about A
- B torque about B**
- C torque about C
- D force balance



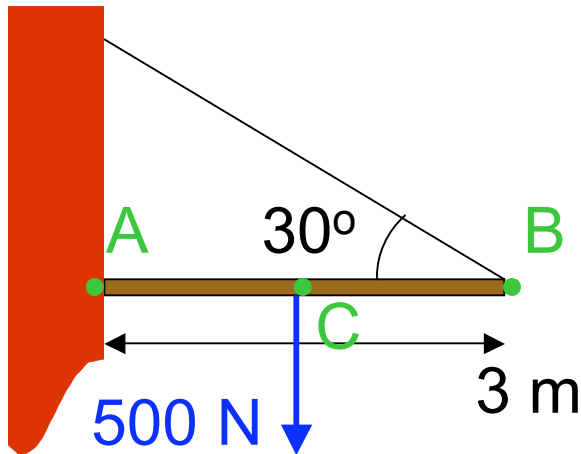
$$\tau_W = 500 \text{ N} \cdot 1.5 \text{ m} = 750 \text{ N m}$$

$$\tau_f = -f_{wb} \cdot 3.0 \text{ m}$$

$$f_{wb} = 250 \text{ N}$$

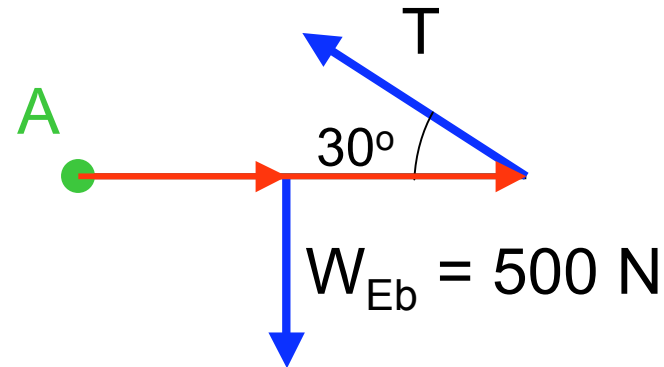
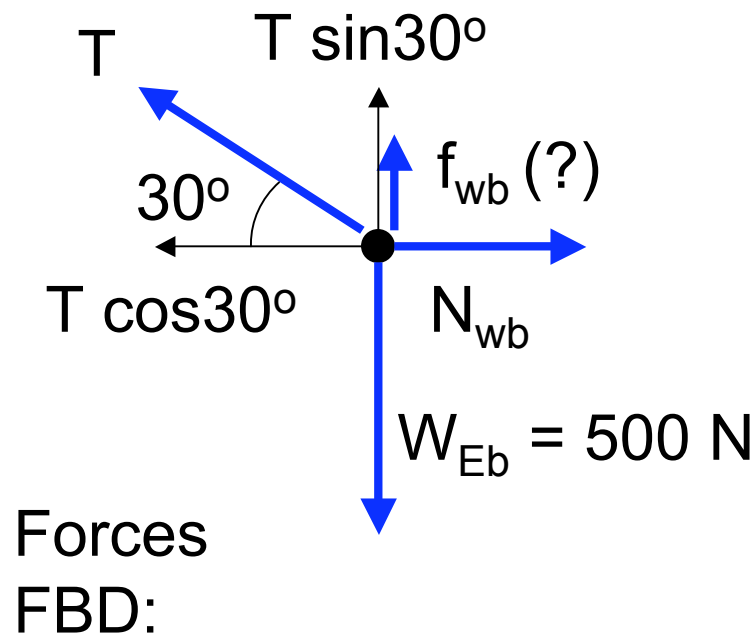
$$\sum \tau = 750 - 3 f_{wb} = 0$$

A 3m beam (mass 50kg) is held horizontal against a wall by a cable.



What can be used to most quickly find T ?

- A** torque about A
- B torque about B
- C torque about C
- D force balance



$$\tau_W = -500 \text{ N } 1.5 \text{ m} = -750 \text{ N m}$$

$$\tau_T = T 3.0 \text{ m } \sin(30^\circ) = +1.5 T$$

$$T = 500 \text{ N}$$

$$\sum \tau = -750 + 1.5 T = 0$$

A 3m beam (mass 50kg) is held horizontal against a wall by a cable.

The normal force is found from

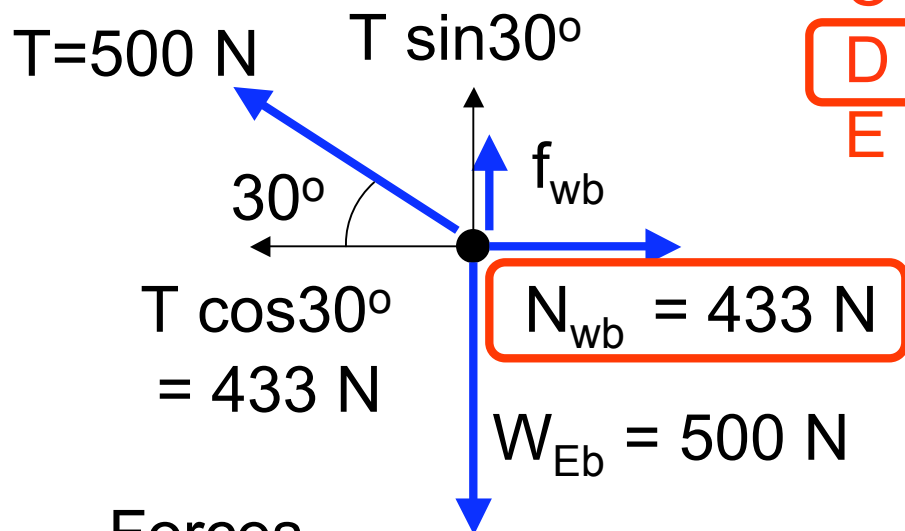
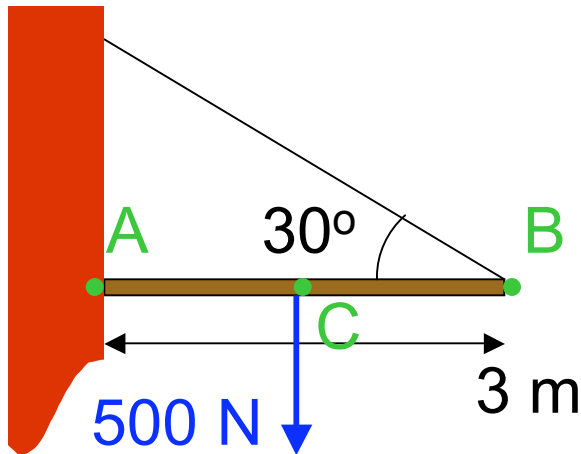
A torque about A

B torque about B

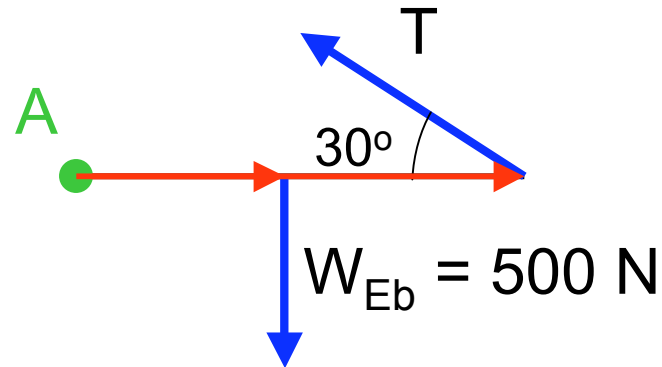
C torque about C

D force balance

E dividing f_{wb} by μ_s



Forces
FBD:

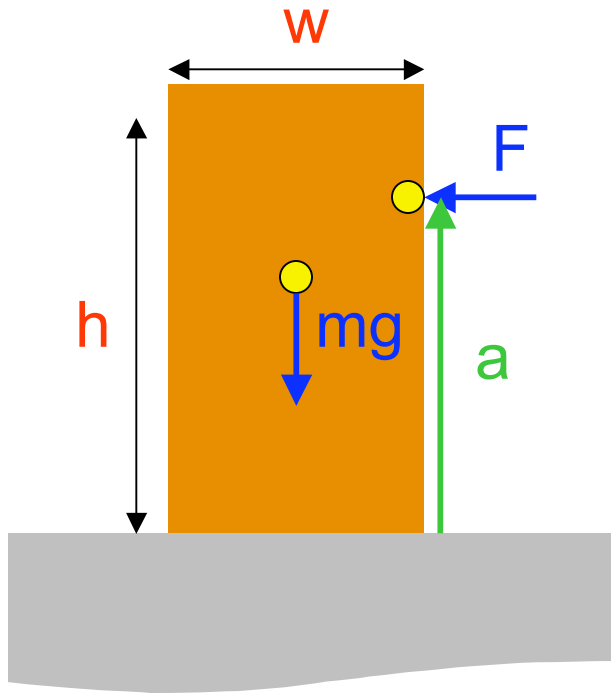


$$\tau_W = -500 \text{ N } 1.5 \text{ m} = -750 \text{ N m}$$

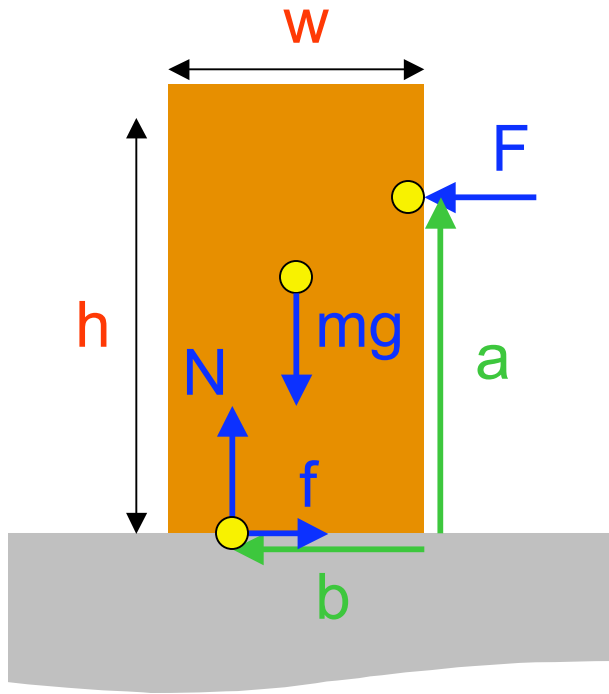
$$\tau_T = T 3.0 \text{ m } \sin(30^\circ) = +1.5 T$$

$$\sum \tau = -750 + 1.5 T = 0$$

$$T = 500 \text{ N}$$

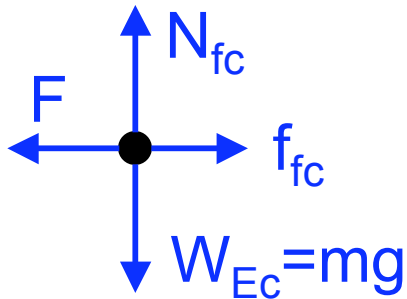


A $w \times h$ crate of mass m rests on the floor. A horizontal force F is applied at a height a .

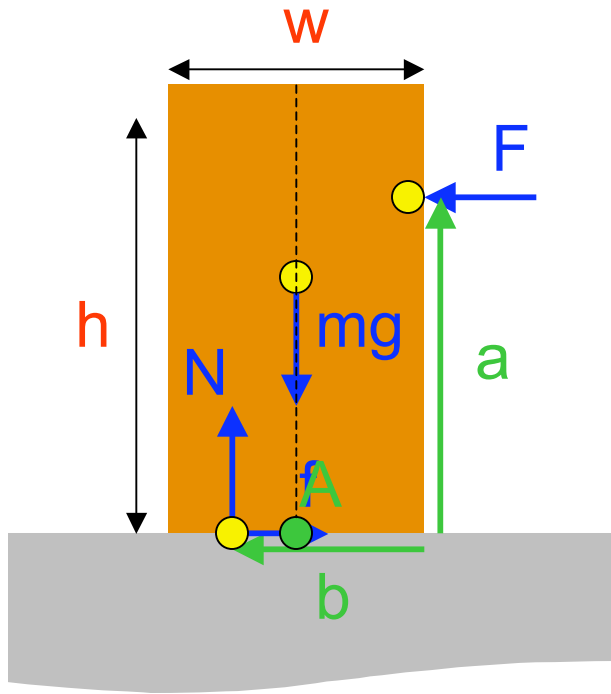


A $w \times h$ crate of mass m rests on the floor. A horizontal force F is applied at a height a . The normal force from the floor may be considered to be acting at a single point (the center of pressure).

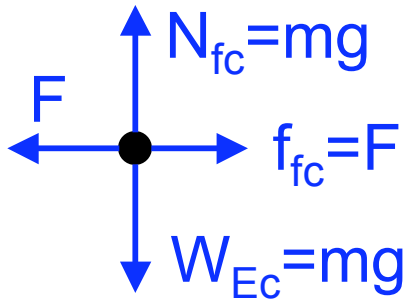
How many unknowns are in this problem?



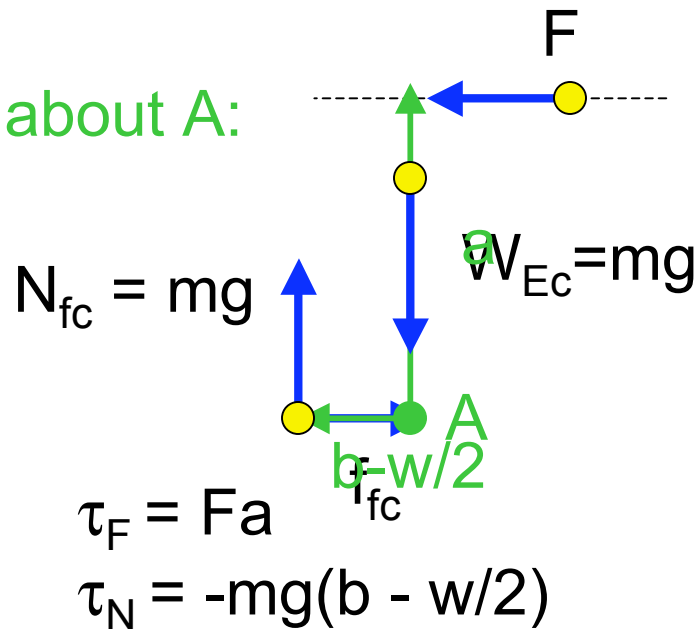
- unknown:
- | | | | |
|----------|---|---|---|
| N_{fc} | } | A | 0 |
| f_{fc} | | B | 1 |
| b | | C | 2 |
| | | D | 3 |
| | | E | 4 |



A $w \times h$ crate of mass m rests on the floor. A horizontal force F is applied at a height a . The normal force from the floor may be considered to be acting at a single point (the center of pressure).



torques about A:

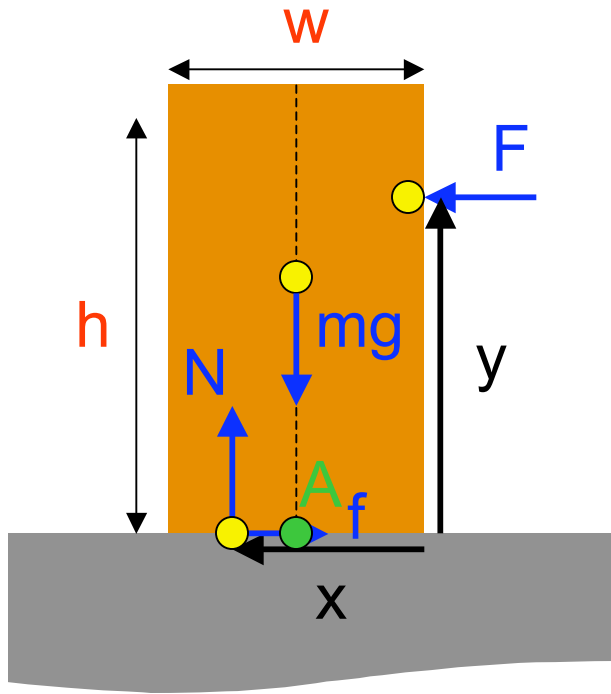


$$\tau_F = Fa$$

$$\tau_N = -mg(b - w/2)$$

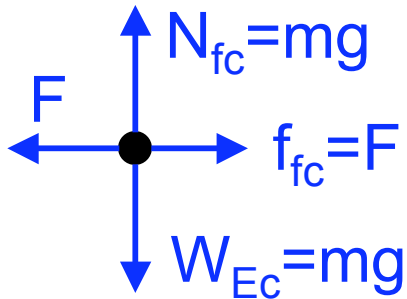
$$b = \frac{w}{2} + \frac{F}{mg}a$$

$$\sum \tau = Fa - mg(b - w/2) = 0$$



A $w \times h$ crate of mass m rests on the floor. A horizontal force F is applied at a height y . The normal force from the floor may be considered to be acting at a single point (the center of pressure).

How much force is required to tip the crate?



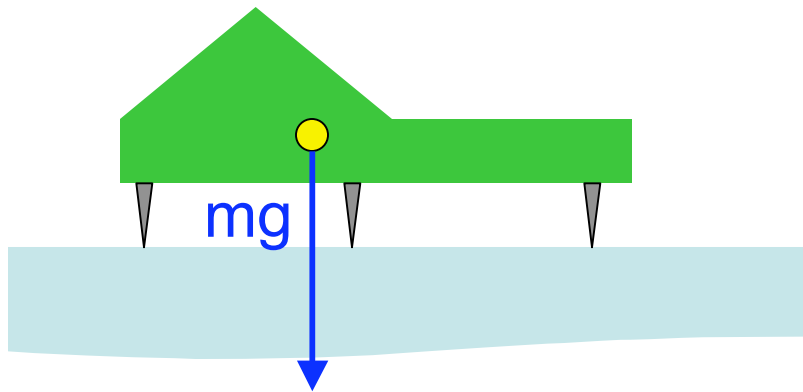
equilibrium only for $b \leq w$

Easier to tip if you push up high

$$b = \frac{w}{2} + \frac{F}{mg} a$$

no torque balance possible when

$$F > \frac{w}{2a} mg$$

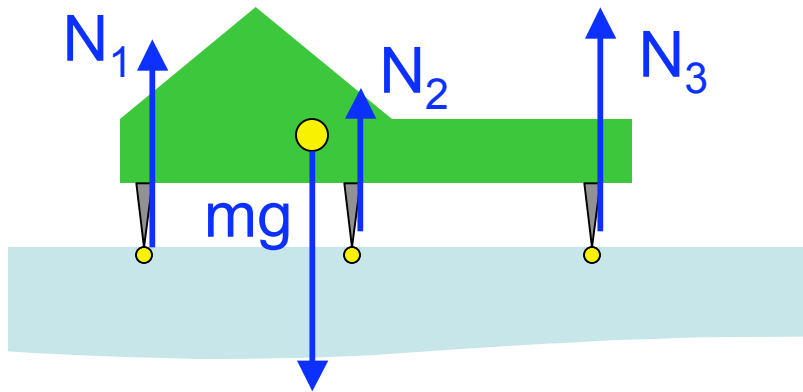


A delicate scale is placed on a frictionless table. It is supported on 3 knife-edges. What are the forces on these?

Not counting: $\sum F_y = 0$ (automatic since $f_s = 0$) :

How many Eqs/unknowns are there?

- A 2 Eqs / 2 unknowns
- B 2 Eqs / 3 unknowns
- C 3 Eqs / 2 unknowns
- D 3 Eqs / 3 unknowns



A delicate scale is placed on a frictionless table. It is supported on 3 knife-edges. What are the forces on these?

Not counting: $\sum F_y = 0$ (automatic since $f_s = 0$) :

How many Eqs/unknowns are there?

A 2 Eqs / 2 unknowns

B 2 Eqs / 3 unknowns

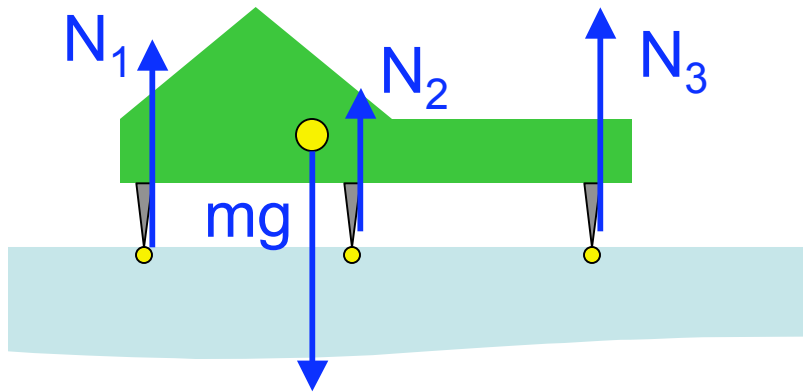
C 3 Eqs / 2 unknowns

D 3 Eqs / 3 unknowns

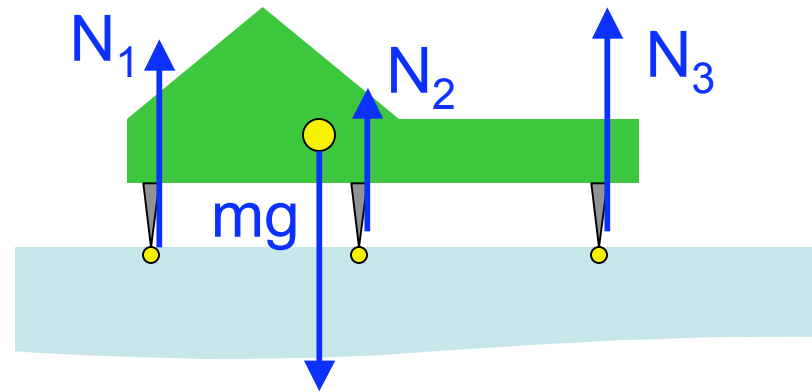
$$\sum F_x = 0$$

$$\sum \tau_z = 0$$

solution is non-unique (indeterminate)



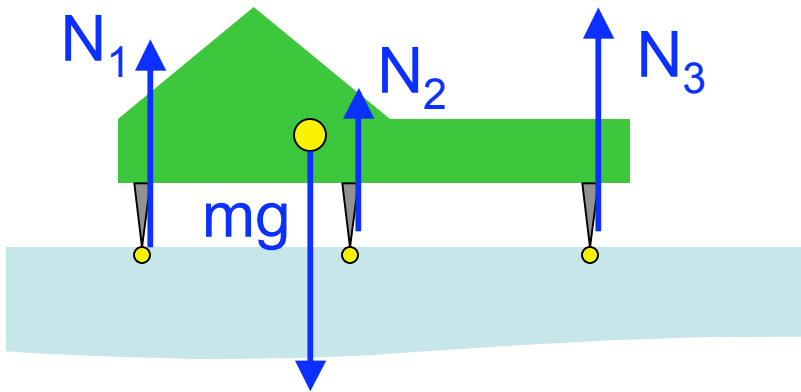
one possibility: $N_2=0$



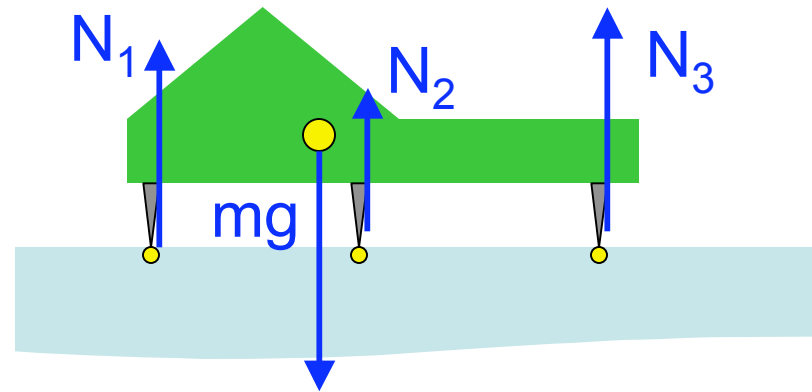
2nd possibility: $N_3=0$

and many, many more

solution is non-unique (indeterminate)



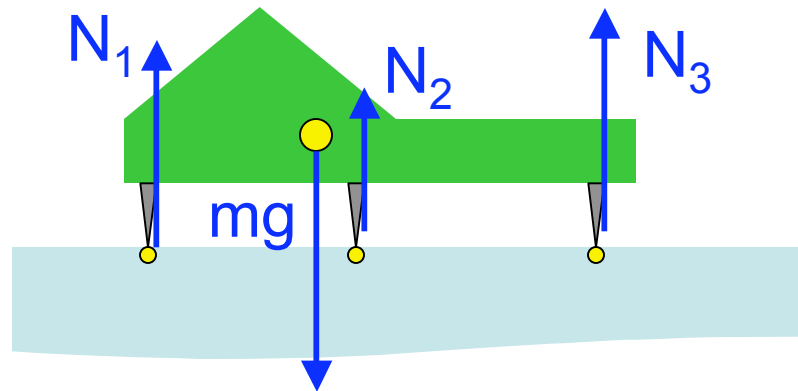
one possibility: $N_2=0$



2nd possibility: $N_3=0$

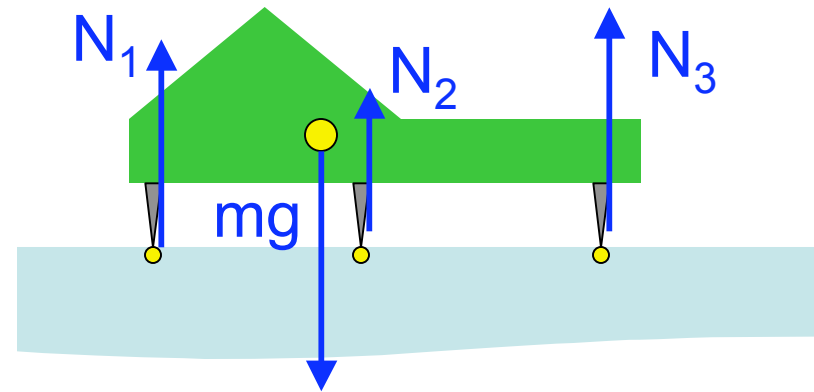
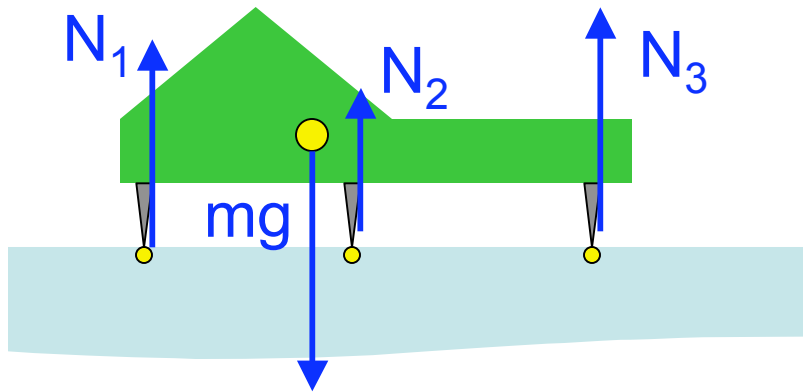
$$\sum F_x = 0$$

$$\sum \tau_z = 0$$



could you **solve equations** with $N_1=0$?

solution is non-unique (indeterminate)

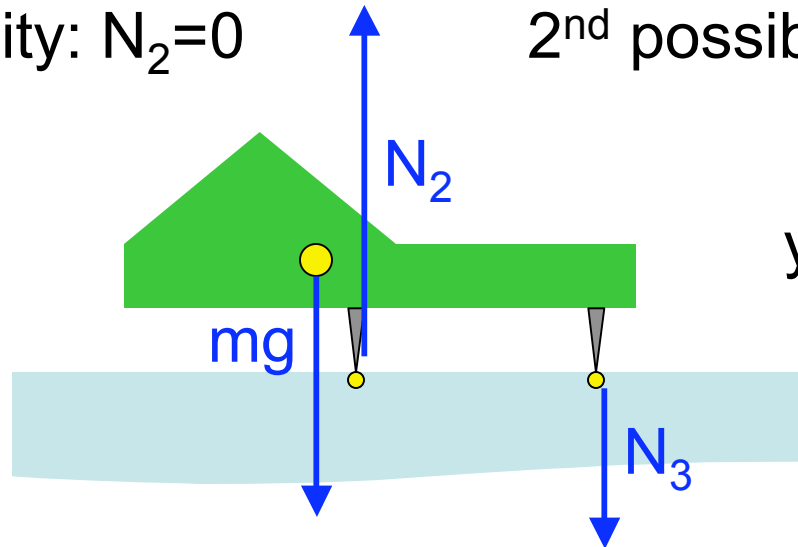


one possibility: $N_2=0$

2nd possibility: $N_3=0$

$$\sum F_x = 0$$

$$\sum \tau_z = 0$$



yes BUT $N_3 < 0$

could you **solve equations** with $N_1=0$?

solution is non-unique (indeterminate)