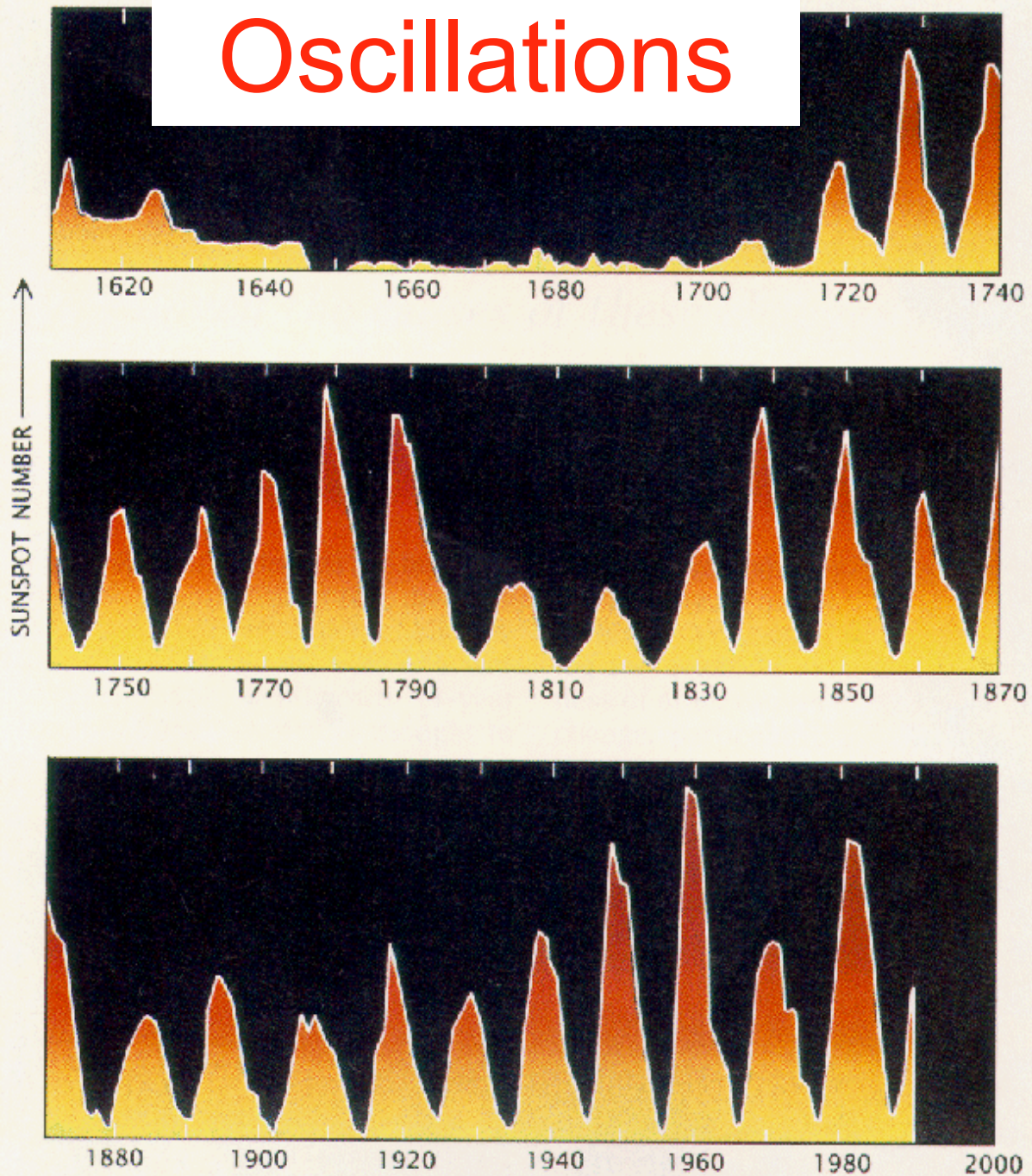
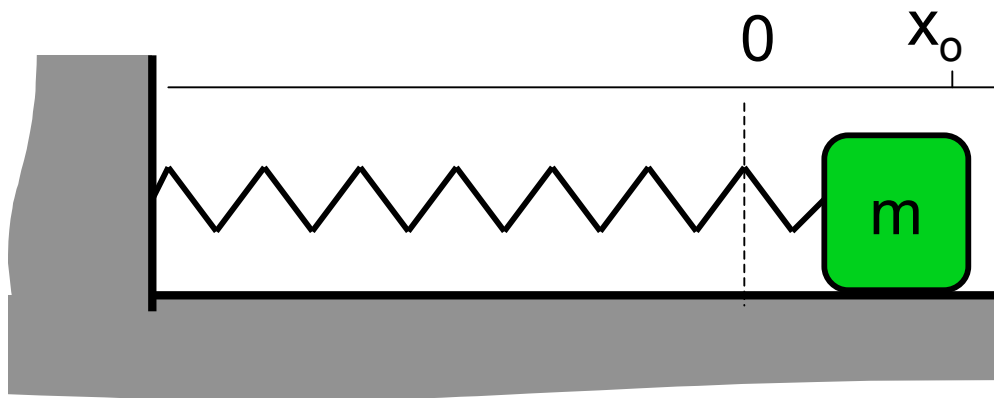


# Oscillations



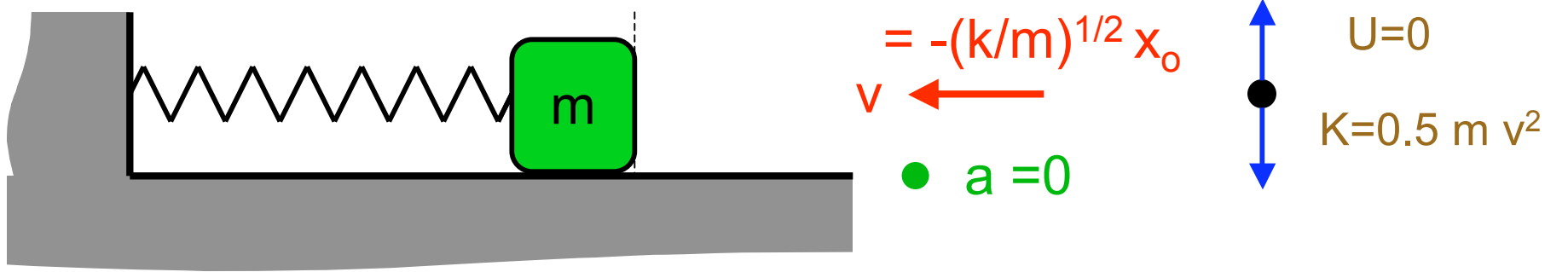
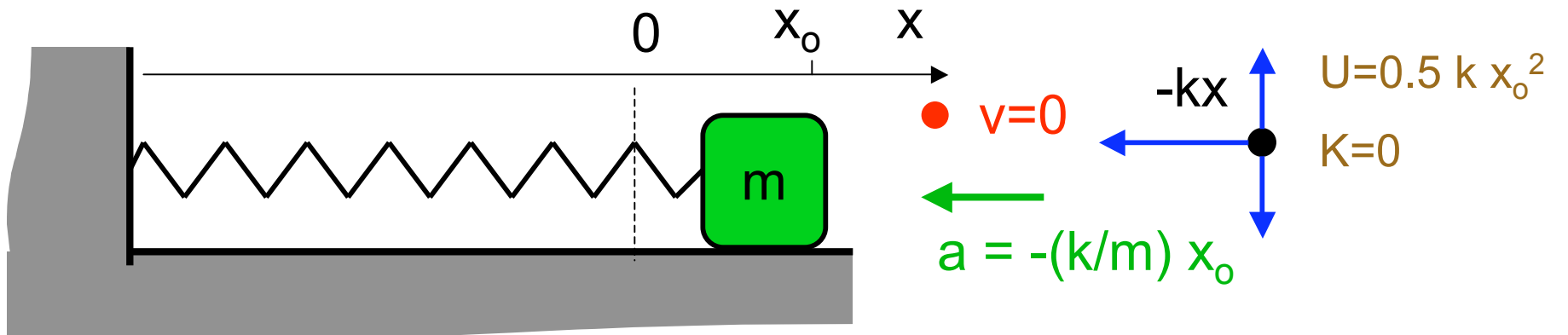


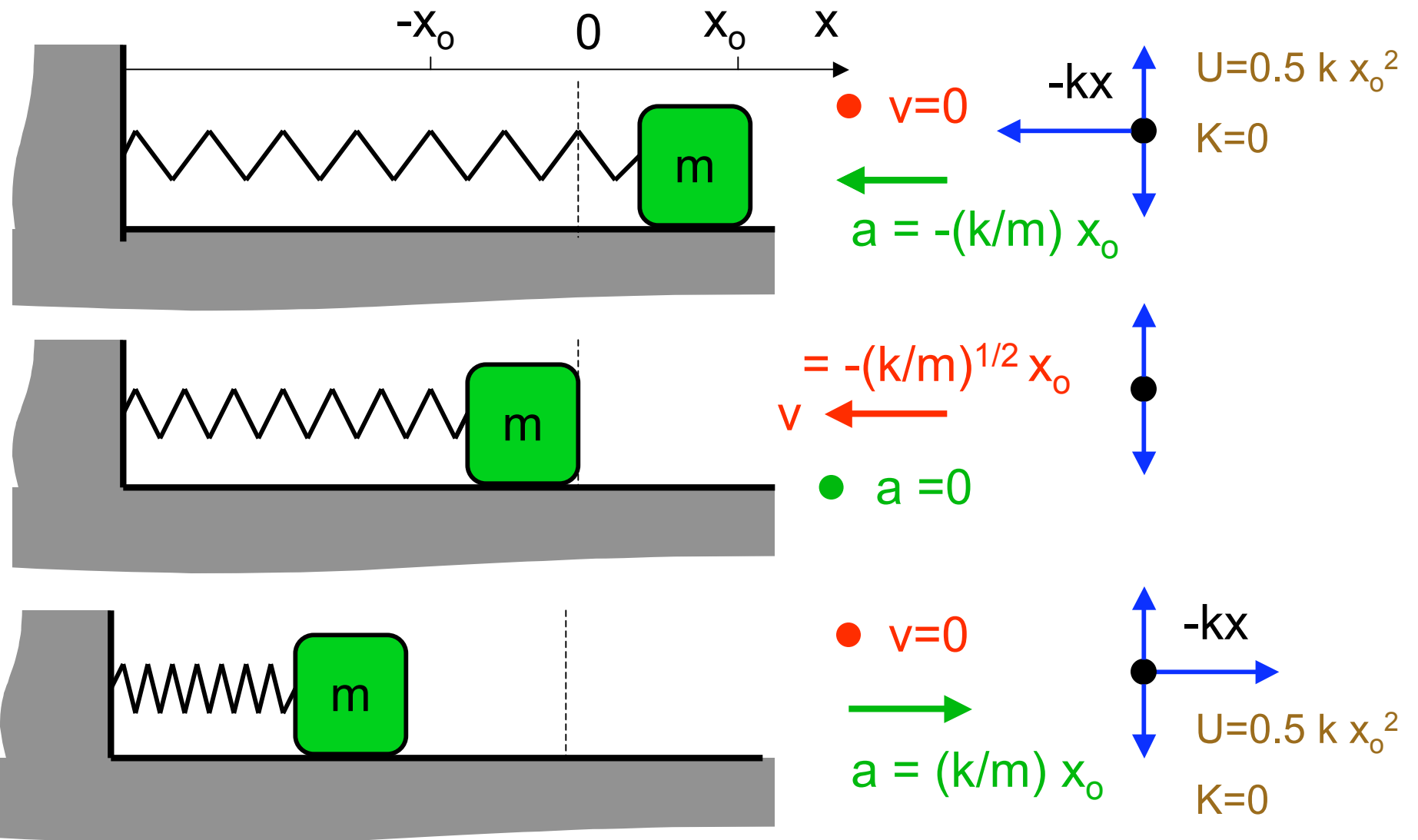
●  $v=0$

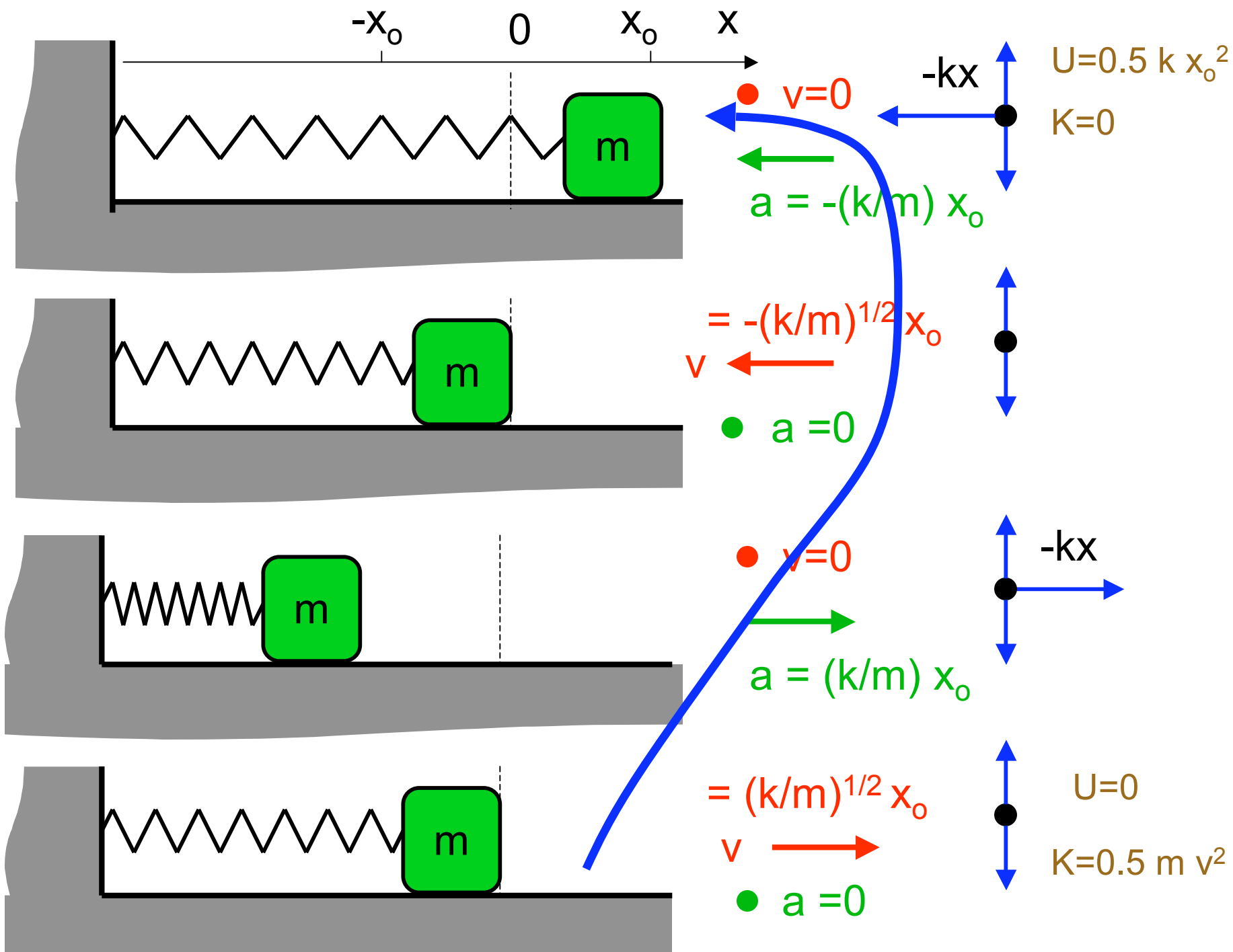
$a = -(k/m) x_0$

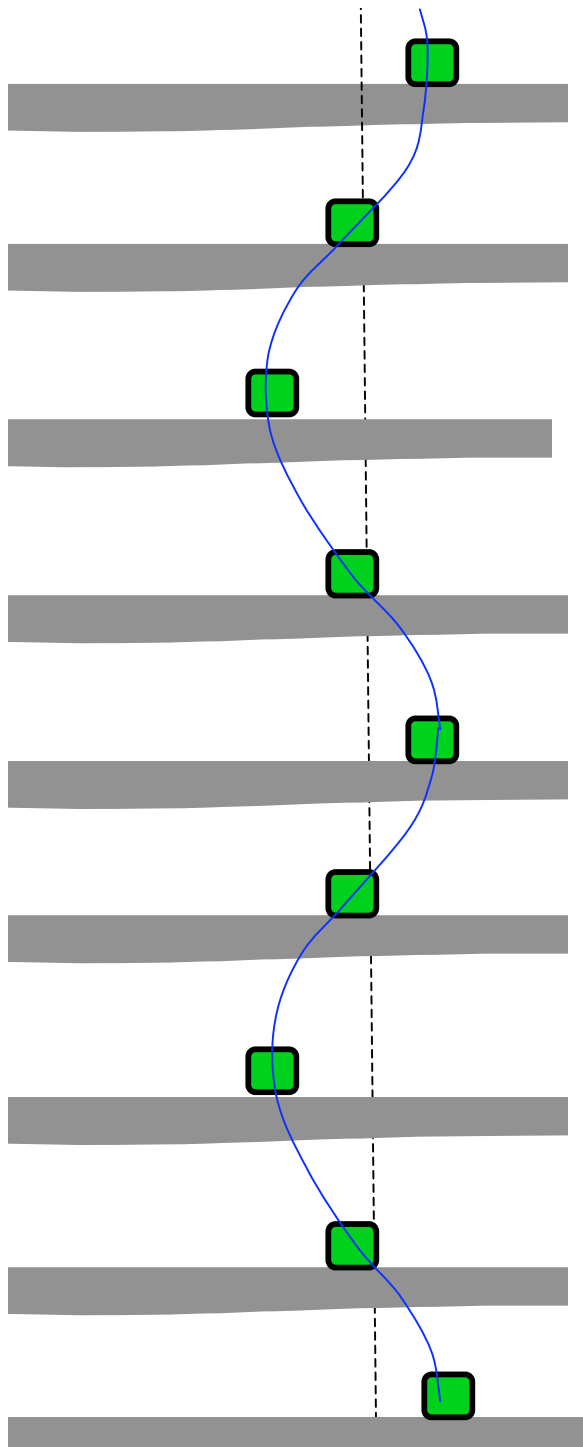
$ma = -kx_0$

$-kx$









t=0

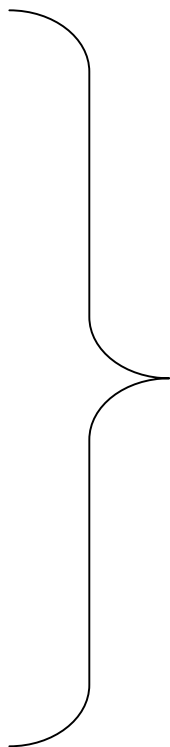
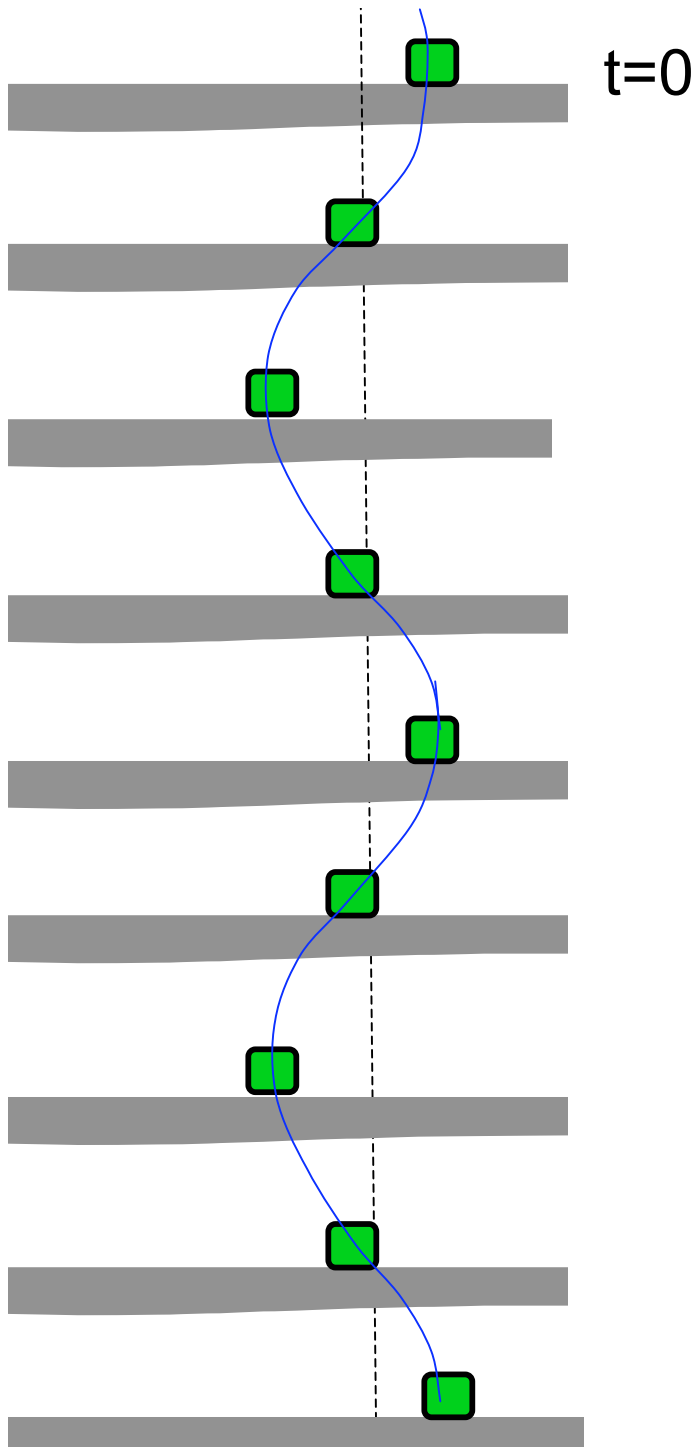
one period

t=?

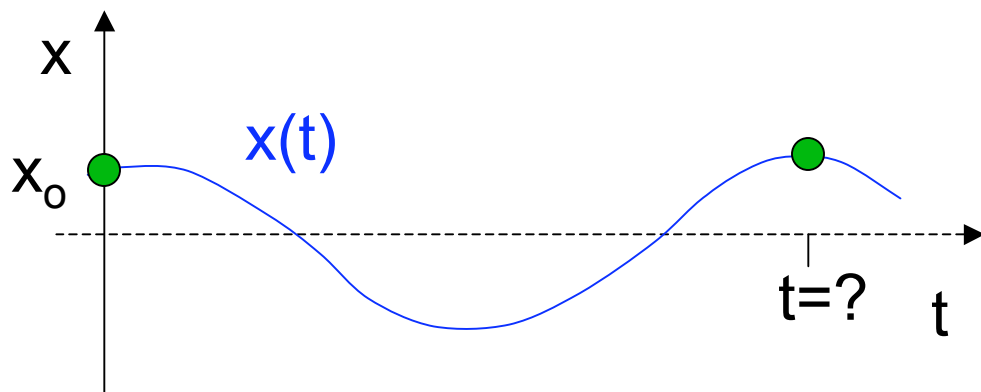
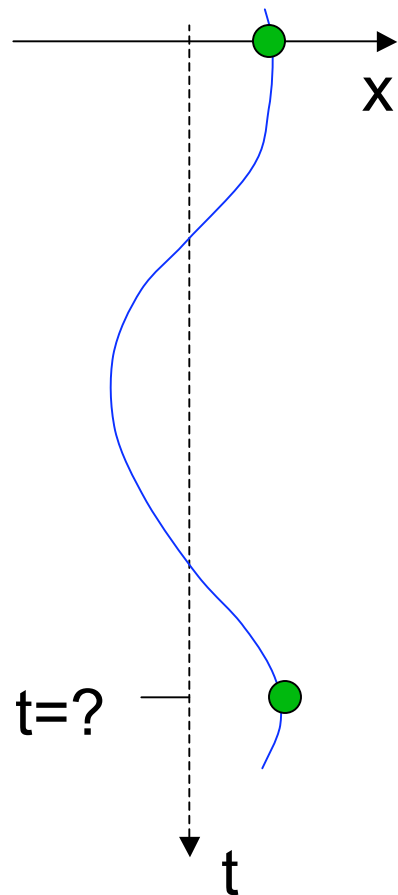
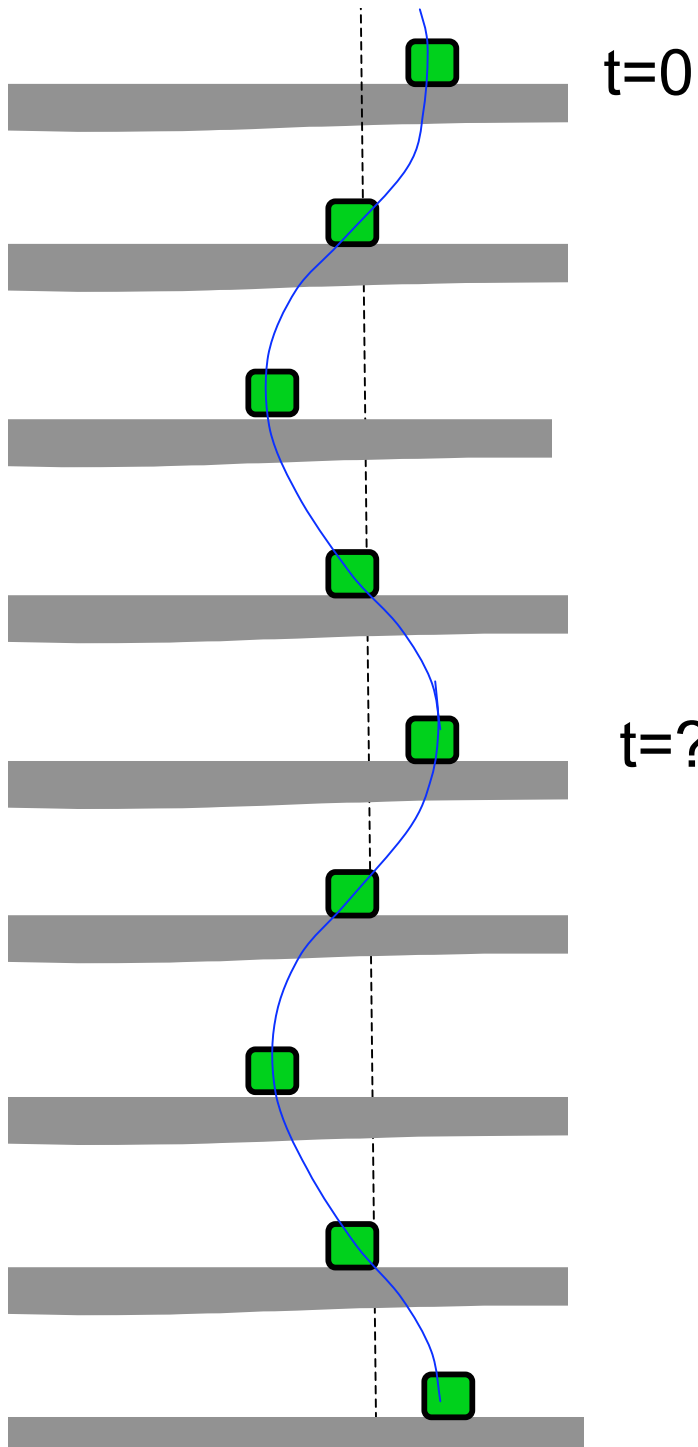
return to initial state...

... begin over again

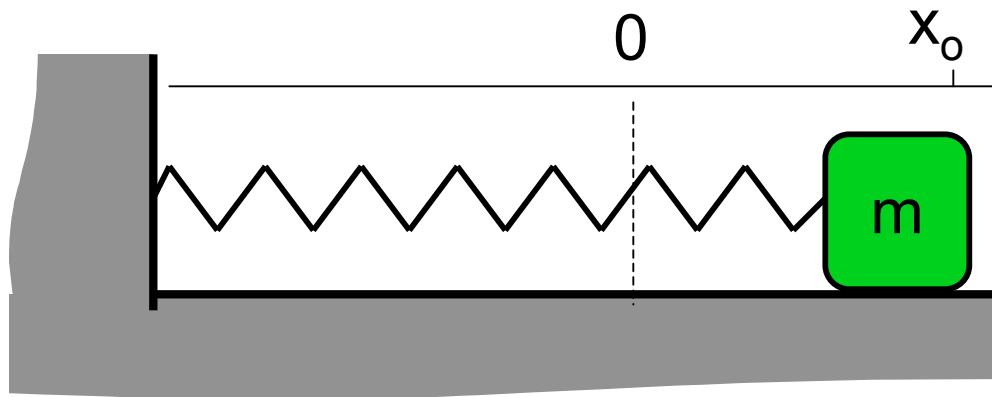
return to initial state...



also one period

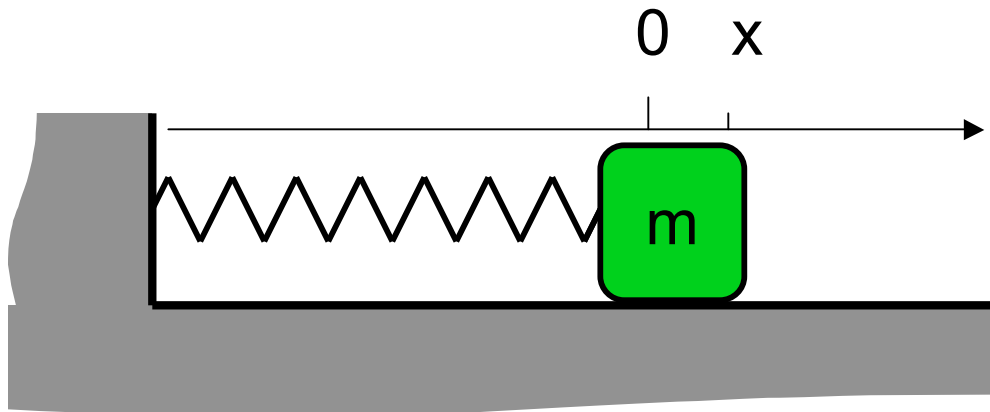


a more conventional view



$t=0$   
 •  $v(0)=0$   
 $x(0) = x_0$

} initial conditions



$a = F/m = -(k/m) x$

$$a = \frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

### differential equation

- depends on properties of system (not on IC).
- solution: function  $x(t)$

# Solving a Diff. Eq.

the question:

$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

does this really  
answer the question?

$$x(t) = A \sin(\omega t)$$

$$\frac{dx}{dt} = A \omega \cos(\omega t)$$

$$\frac{d^2 x}{dt^2} = -A \omega^2 \sin(\omega t) = -\omega^2 x(t)$$

some possible answers:

$$x(t) = A \sin(\omega t)$$

$$x(t) = B \cos(\omega t)$$

constants

constants

YES:

only  
if we  
make

$$\omega = \sqrt{\frac{k}{m}}$$

regardless of  
how we choose A

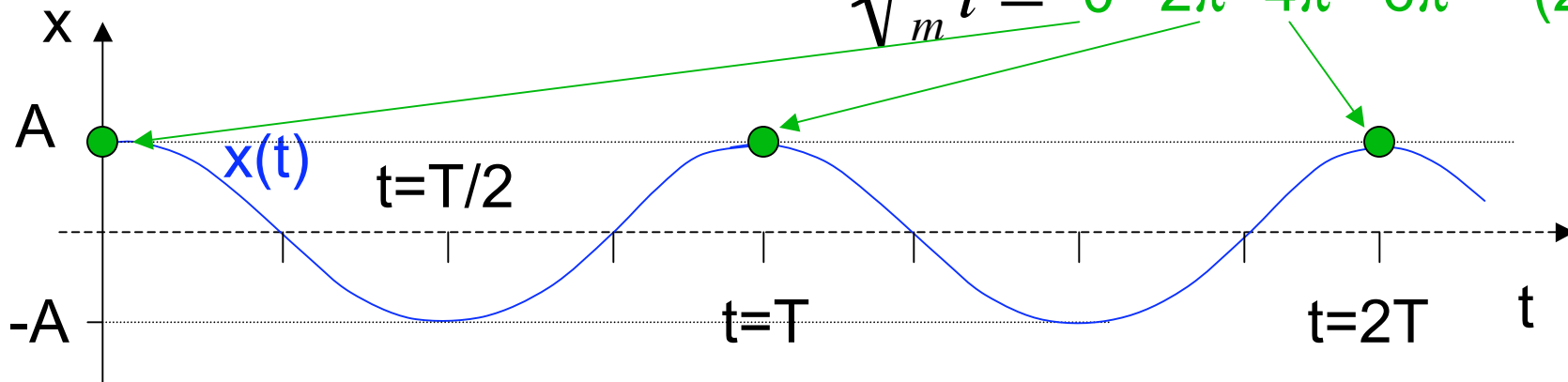
amplitude

frequency

$$x(t) = A \cos\left(\sqrt{\frac{k}{m}} t\right)$$

$$x=A \rightarrow \cos() = 1$$

$$\sqrt{\frac{k}{m}} t = 0 \quad 2\pi \quad 4\pi \quad 6\pi \quad (2\pi)n$$



$$T = \frac{2\pi}{\sqrt{k/m}}$$



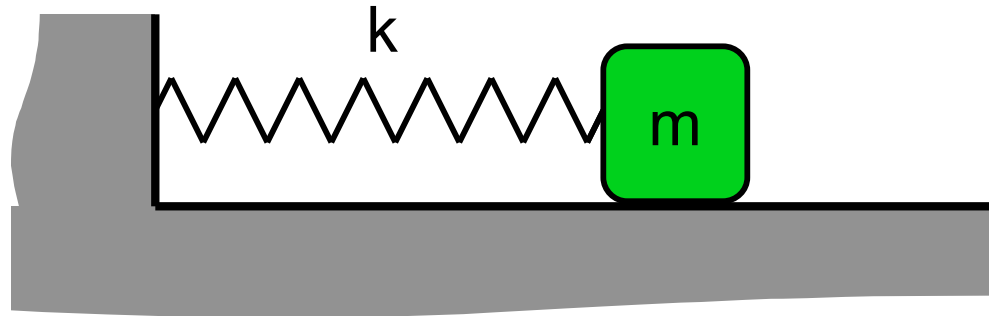
**Depends only on k & m**  
**... not on A**



$$\sqrt{\frac{k}{m}} = \frac{2\pi}{T} = \omega$$

angular frequency [ rad/s ]

# From the spring & mass\*



$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi f \quad \text{angular frequency [ rad/s ]}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{T} = \frac{\sqrt{k/m}}{2\pi} \quad \text{frequency [ s}^{-1} \text{ = Hz ]}$$

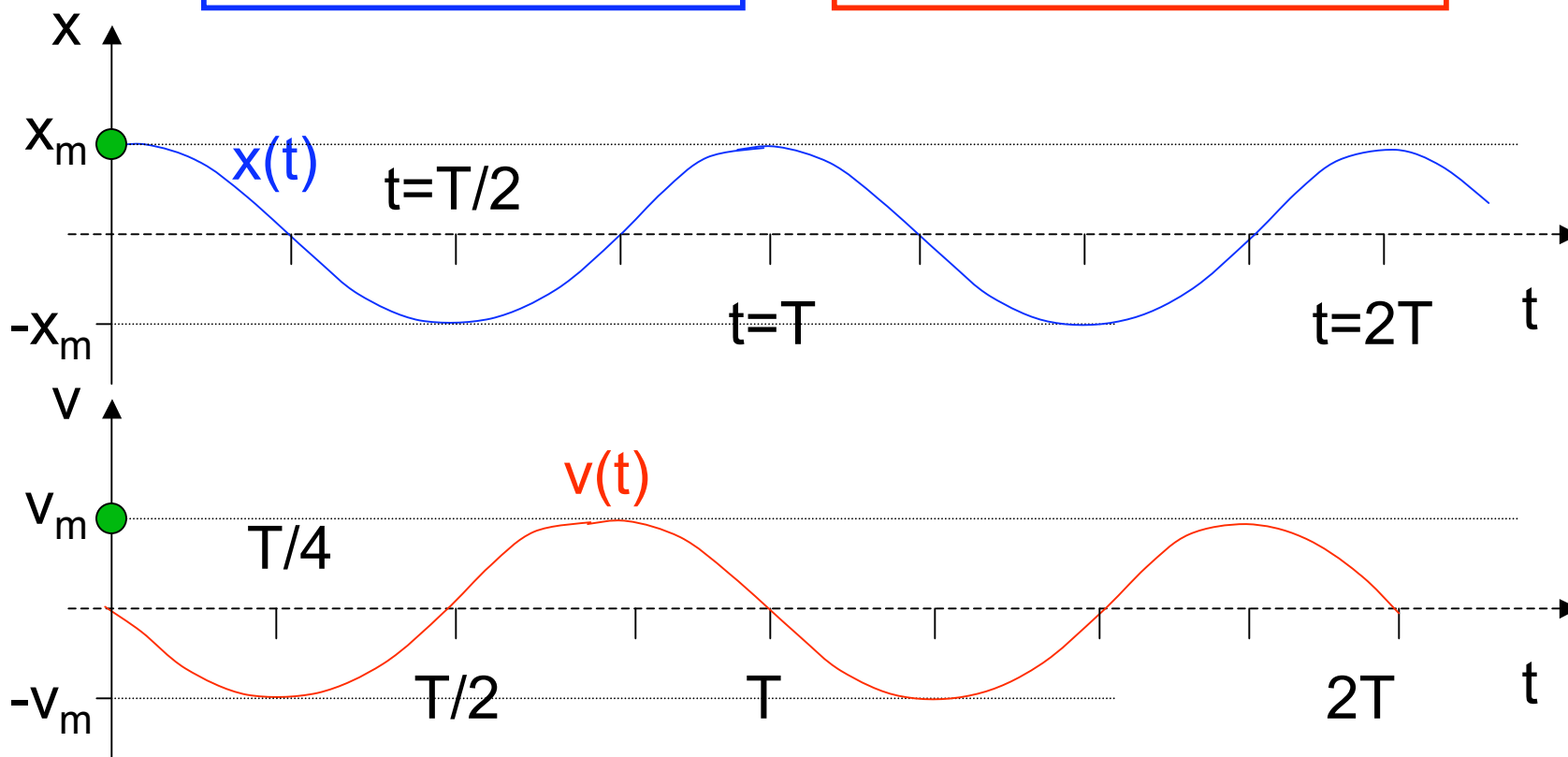
$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} \quad \text{oscillation period [ s ]}$$

\* without knowledge of IC

# Simple Harmonic Motion: cos

$$x(t) = x_m \cos(\omega t)$$

$$v(t) = -x_m \omega \sin(\omega t)$$

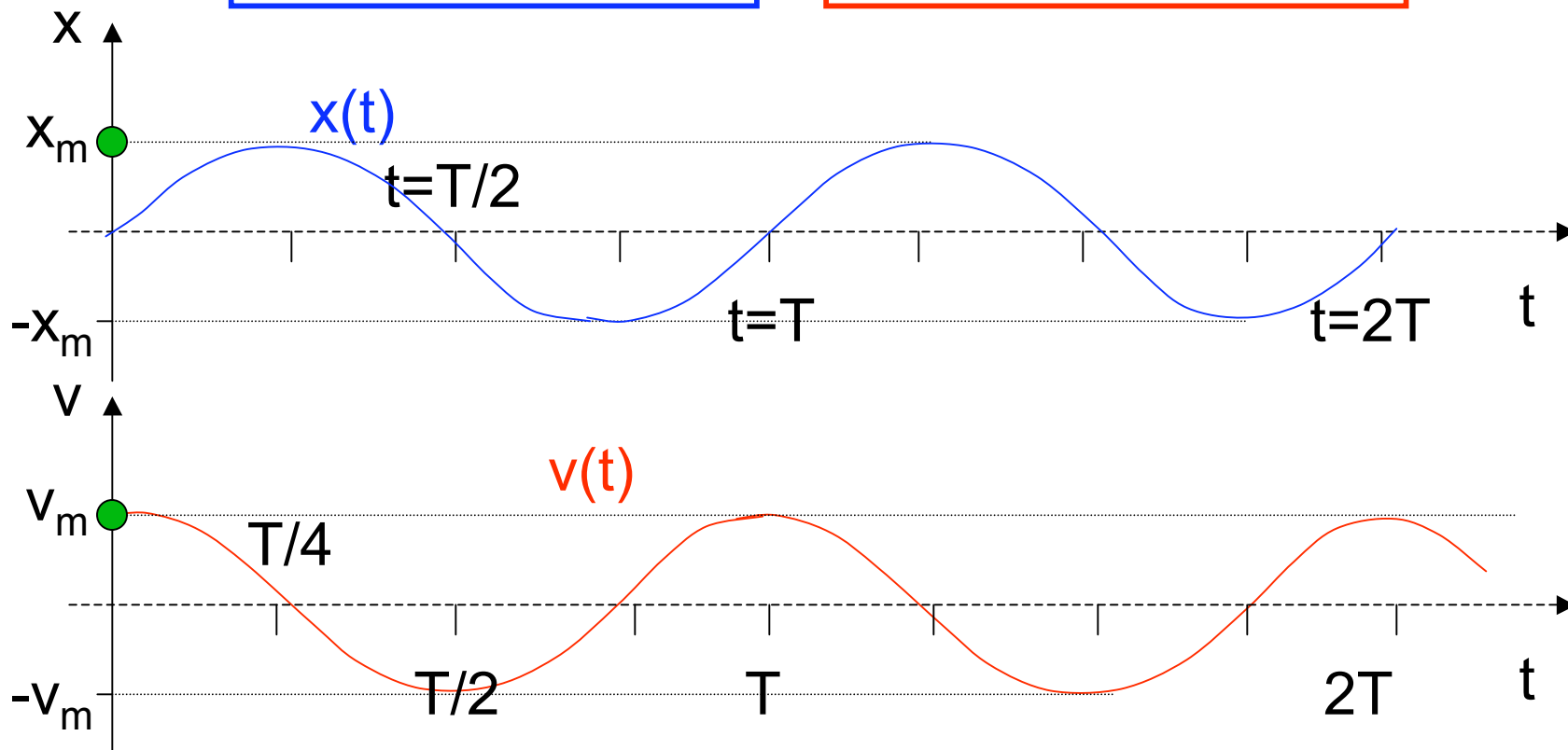


starts: from rest ( $v=0$ )

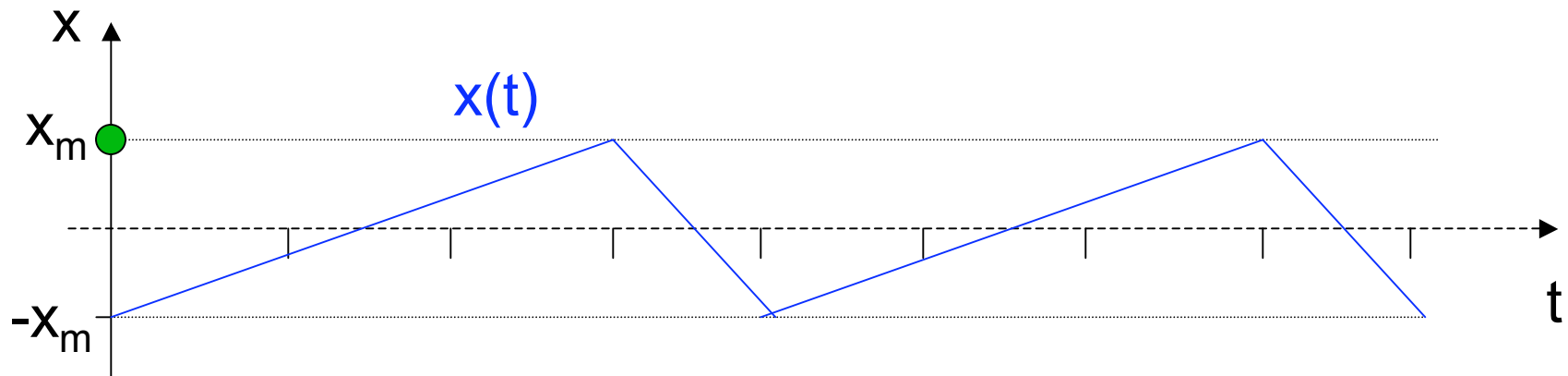
# Simple Harmonic Motion: sin

$$x(t) = x_m \sin(\omega t)$$

$$v(t) = x_m \omega \cos(\omega t)$$

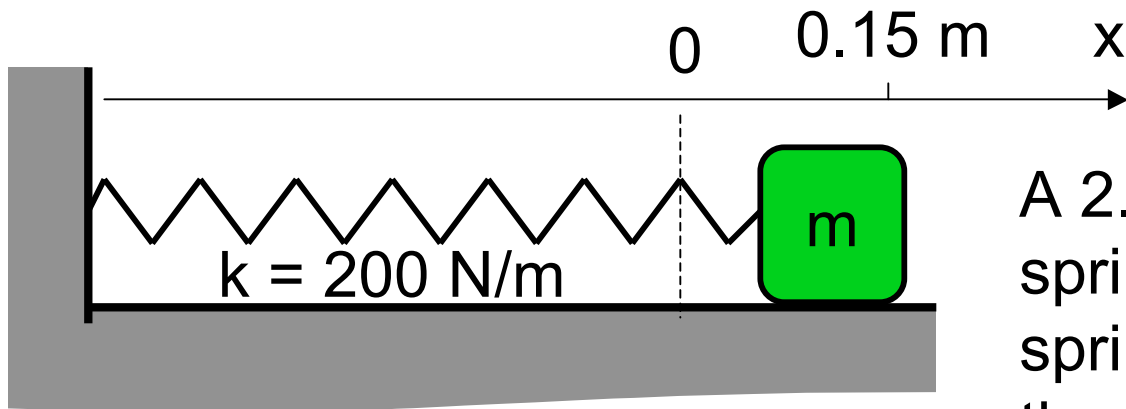


starts: from equilibrium position ( $x=0$ ) but moving



The position vs. time curve above depicts

- A periodic motion
- B simple harmonic motion
- C both A & B
- D neither A nor B
- E the motion is not possible



## Example

A 2.0 kg mass is attached to a spring with  $k=200$  N/m. The spring is stretched 15.0 cm and the mass is released. How long before the spring is fully compressed?

$$\omega = \sqrt{\frac{k}{m}} = 10 \text{ rad/s}$$

$$T = 2\pi/\omega = 0.63 \text{ sec}$$

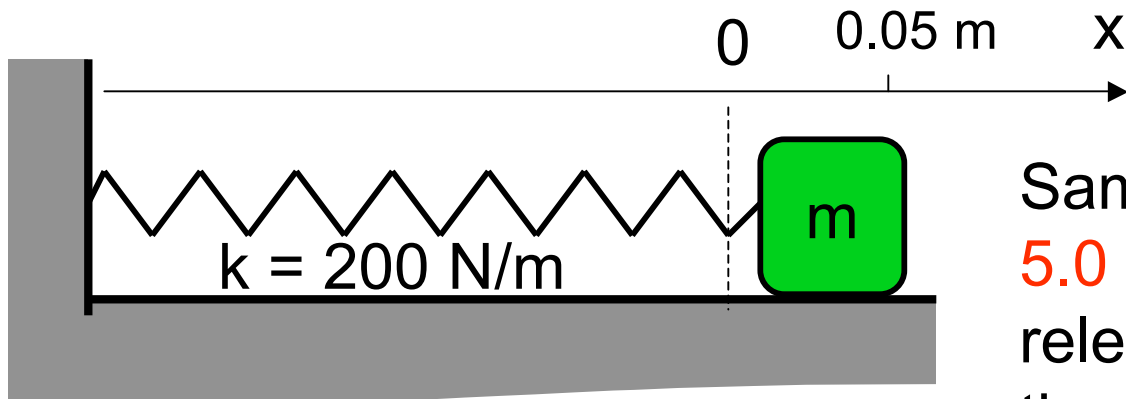
A 0.05 sec

B 0.1 sec

C 0.15 sec

D 0.31 sec

E 0.63 sec



# Example

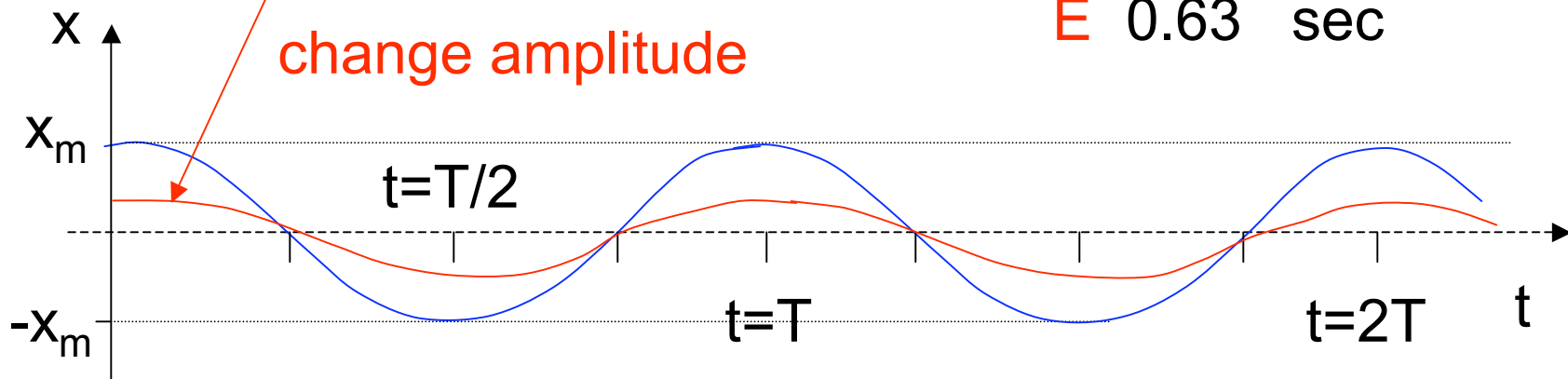
Same mass/spring is stretched **5.0 cm** and the mass is released. How long **NOW** until the spring is full compressed?

$$\omega = \sqrt{\frac{k}{m}}$$

same frequency

$$x(t) = x_m \cos(\omega t)$$

change amplitude



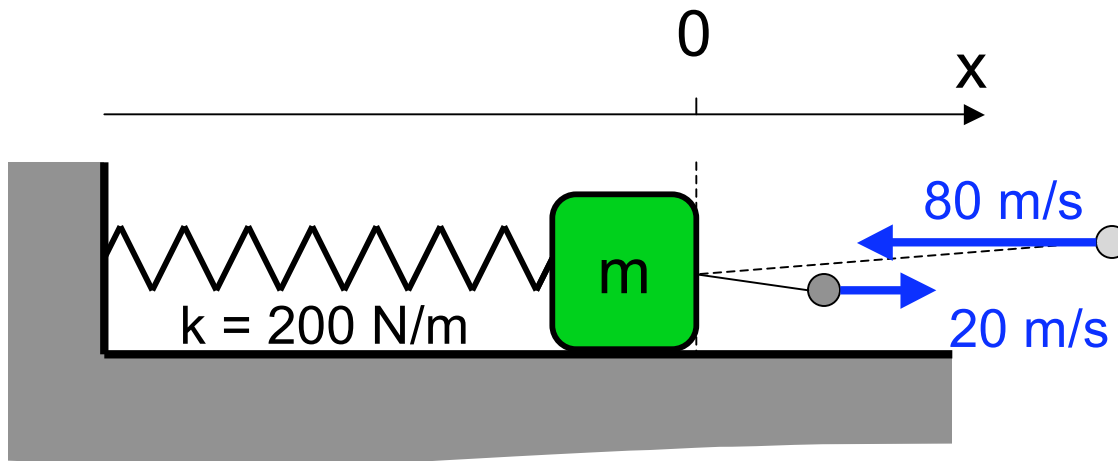
A 0.05 sec

B 0.1 sec

C 0.15 sec

**D 0.31 sec**

E 0.63 sec



## Variant

Same mass/spring is initially in equilibrium. A 50 g metal ball hits the mass travelling 80 m/s, and rebounds at 20 m/s. The collision lasts for 15 msec. How fast is the mass traveling after it is hit?

$$p_i = (0.05 \text{ kg})(-80 \text{ m/s}) = -4 \text{ kg m/s}$$

$$p_f = (0.05 \text{ kg})(20 \text{ m/s}) + (2 \text{ kg}) v_f$$

$$p_f = 1 \text{ kg m/s} + (2 \text{ kg}) v_f = -4 \text{ kg m/s}$$

$$(2 \text{ kg}) v_f = -5 \text{ kg m/s}$$

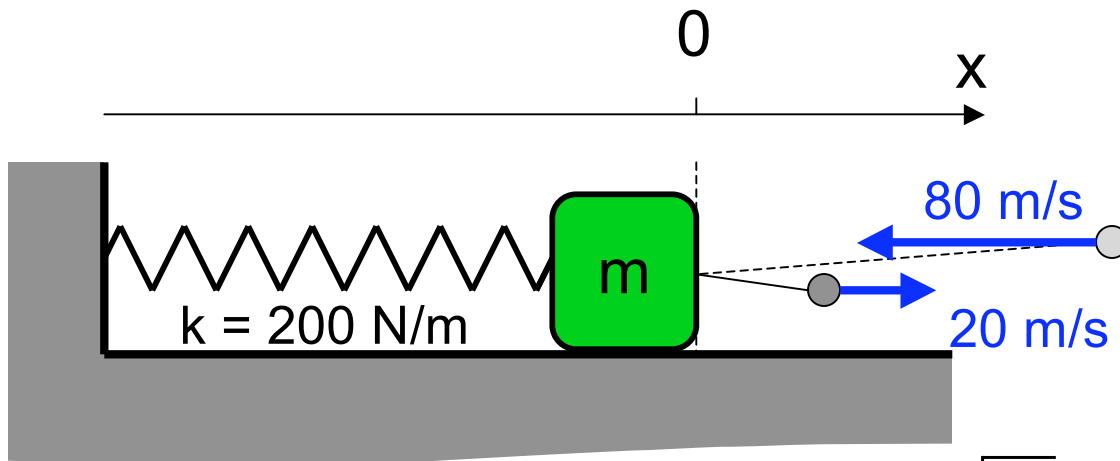
$$v_f = -2.5 \text{ m/s}$$

A 0.31 m/s

B 0.63 m/s

C 1.5 m/s

**D 2.5 sec**



# Variant

How long after the mass is hit is the spring fully compressed?

- A 0.05 sec
- B 0.16 sec
- C 0.31 sec
- D 0.4 sec
- E 0.63 sec

$$v_o = -2.5 \text{ m/s} \quad \leftarrow \quad \omega = \sqrt{\frac{k}{m}}$$

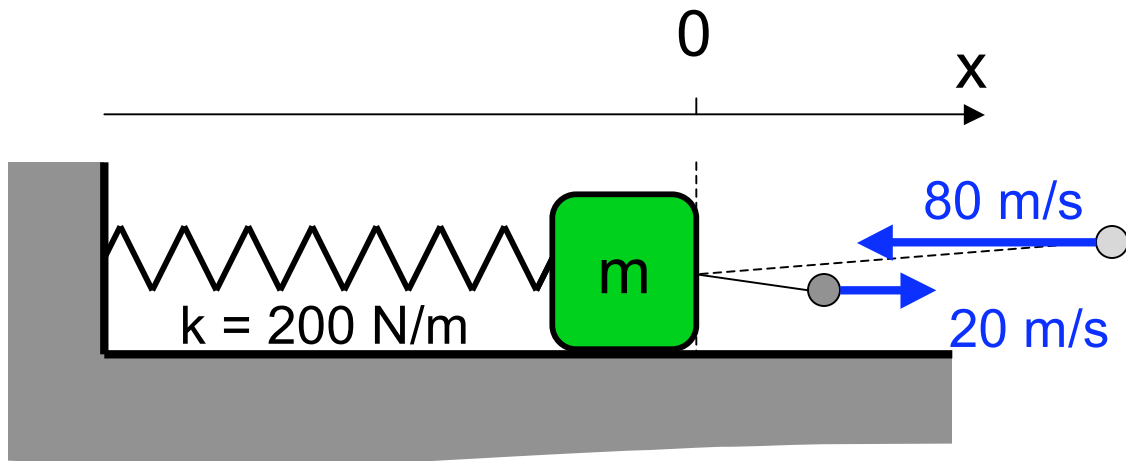
$$x(t) = x_m \sin(\omega t) \quad \leftarrow \text{choose sin}$$

$$\frac{dx}{dt} = x_m \omega \cos(\omega t)$$

$$v(0) = x_m \omega \cos(0) = -x_m \omega = v_o$$

$$x(t) = -\frac{v_o}{\omega} \sin(\omega t)$$

$$v(t) = v_o \cos(\omega t)$$



# Variant

How long after the mass is hit is the spring fully compressed?

$$v_o = -2.5 \text{ m/s}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$x(t) = \frac{v_o}{\omega} \sin(\omega t)$$

$$v(t) = v_o \cos(\omega t)$$

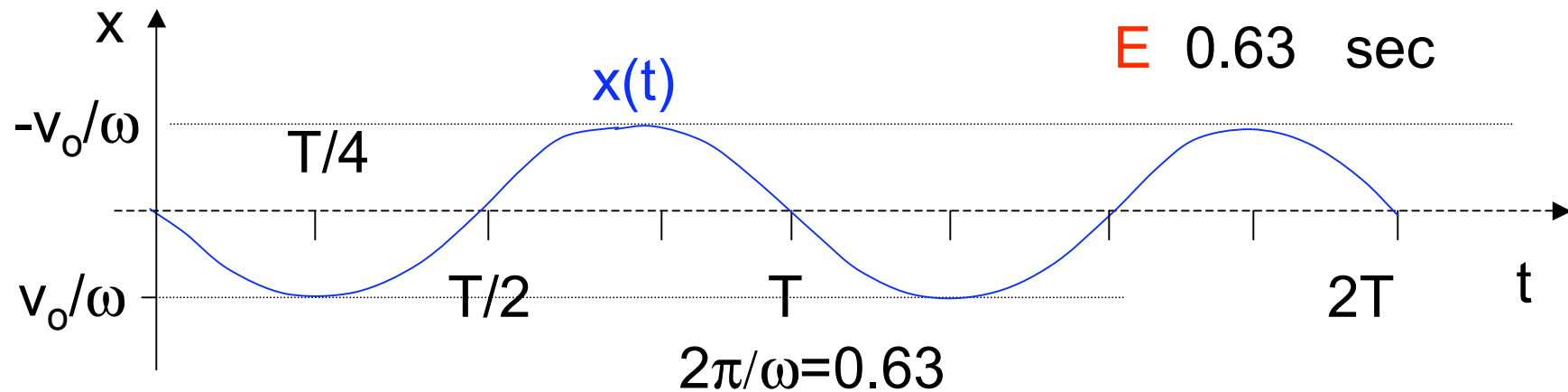
A 0.05 sec

**B 0.16 sec**

C 0.31 sec

D 0.4 sec

E 0.63 sec



# Simple Harmonic Motion

diff. eq.

$$\frac{d^2 y}{dt^2} = -Ky$$

General solution

$$y(t) = A \cos(\sqrt{K}t + \phi)$$

amplitude frequency phase

$$y(t) = \underbrace{A \cos(\phi)}_B \cos(\sqrt{K}t) - \underbrace{A \sin(\phi)}_C \sin(\sqrt{K}t)$$

$$y(t) = B \cos(\sqrt{K}t) + C \sin(\sqrt{K}t)$$

alternative form

$$y(0) = B$$

$$v_y(0) = C\sqrt{K}$$

Initial conditions

# Simple Harmonic Motion

diff. eq.

$$\frac{d^2 y}{dt^2} = -Ky$$

1. Know frequency (& period)  
immediately from diff. eq.

$$\omega = \sqrt{K}$$

2. Initial conditions: they will be of 2 kinds

A. at rest initially

$$y(0) = y_o$$

$$v_y(0) = 0$$

**COSINE**

$$y(t) = y_o \cos(\omega t)$$

B. at equilib. position

$$y(0) = 0$$

$$v_y(0) = v_o$$

**SINE**

$$y(t) = \frac{v_o}{\omega} \sin(\omega t)$$

# Energy of SH oscillator

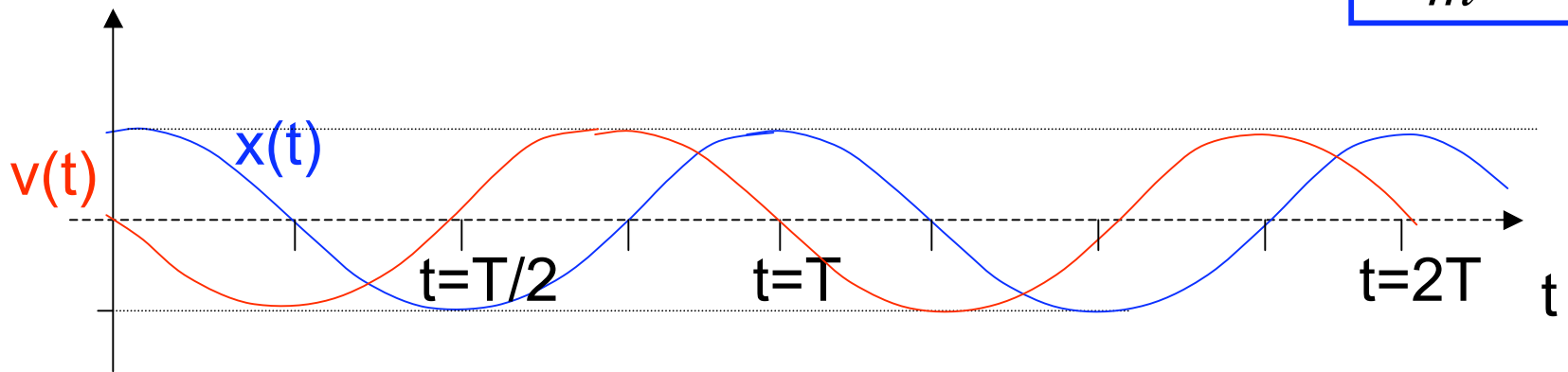
$$x(t) = x_m \cos(\omega t)$$

$$v(t) = -x_m \omega \sin(\omega t)$$

$$U = \frac{1}{2} kx^2 = \frac{1}{2} kx_m^2 \cos^2(\omega t)$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 x_m^2 \sin^2(\omega t)$$

$$m \frac{k}{m} = k$$



# Energy of SH oscillator

$$x(t) = x_m \cos(\omega t)$$

$$U = \frac{1}{2} kx^2 = \frac{1}{2} kx_m^2 \cos^2(\omega t)$$

$$v(t) = -x_m \omega \sin(\omega t)$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 x_m^2 \sin^2(\omega t)$$

$$m \frac{k}{m} = k$$

$$KE + U = \frac{1}{2} kx_m^2 [\cos^2(\omega t) + \sin^2(\omega t)]$$

