

Pendula



Simple Harmonic Motion

diff. eq.

$$\frac{d^2 y}{dt^2} = -Ky$$

1. Know frequency (& period)
immediately from diff. eq.

$$\omega = \sqrt{K}$$

2. Initial conditions: they will be of 2 kinds

A. at rest initially

$$y(0) = y_o$$

$$v_y(0) = 0$$

COSINE

$$y(t) = y_o \cos(\omega t)$$

B. at equilib. position

$$y(0) = 0$$

$$v_y(0) = v_o$$

SINE

$$y(t) = \frac{v_o}{\omega} \sin(\omega t)$$

Simple Harmonic Motion

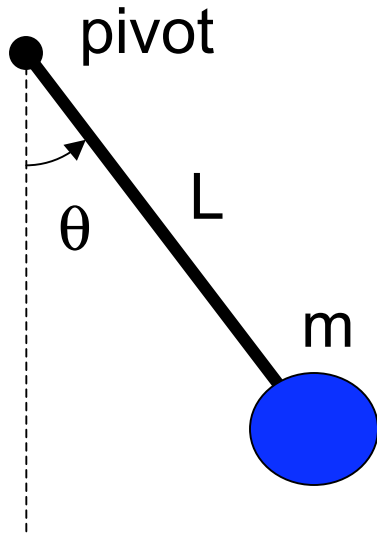
Q: How does the period of oscillation depend on amplitude in simple harmonic oscillation?

Doubling the amplitude will...

$$\frac{d^2 y}{dt^2} = -Ky$$

- A ... quadruple the period
- B ... double the period
- C ... not change the period**
- D ... half the period

critical aspect of SHM: period independent of amplitude



Simple Pendulum

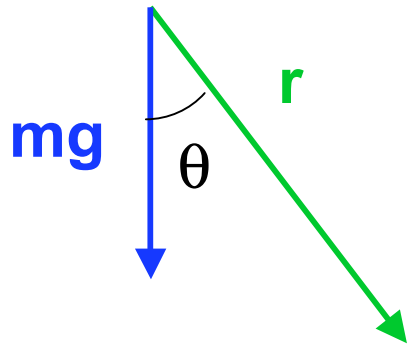
At the end of a light rod of length L is a mass: the “bob”. The other end of the rod is fixed to a frictionless pivot. Consider the system in the configuration shown.

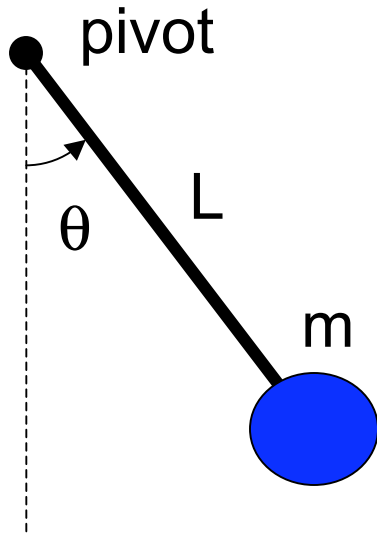
calculate the following:

$$I \text{ about pivot} = mL^2$$

$$\tau \text{ about pivot} = -mgL \sin\theta$$

$$\alpha = \tau/I = -(g/L) \sin\theta$$





Simple Pendulum

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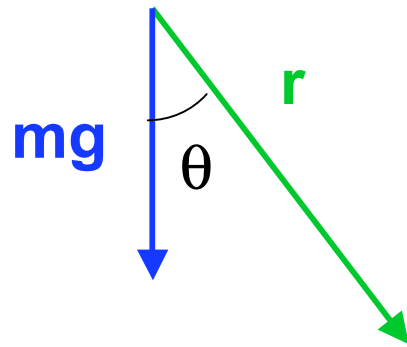
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$$\omega = d\theta/dt$$

$$\alpha = d\omega/dt = d^2\theta/dt^2$$



the other one

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L} \sin(\theta)$$

Harmonic Motion?

diff. eq.

$$\frac{d^2\theta}{dt^2} = -K \sin(\theta)$$

trial solution

$$\theta(t) = \sin(\sqrt{K} t)$$

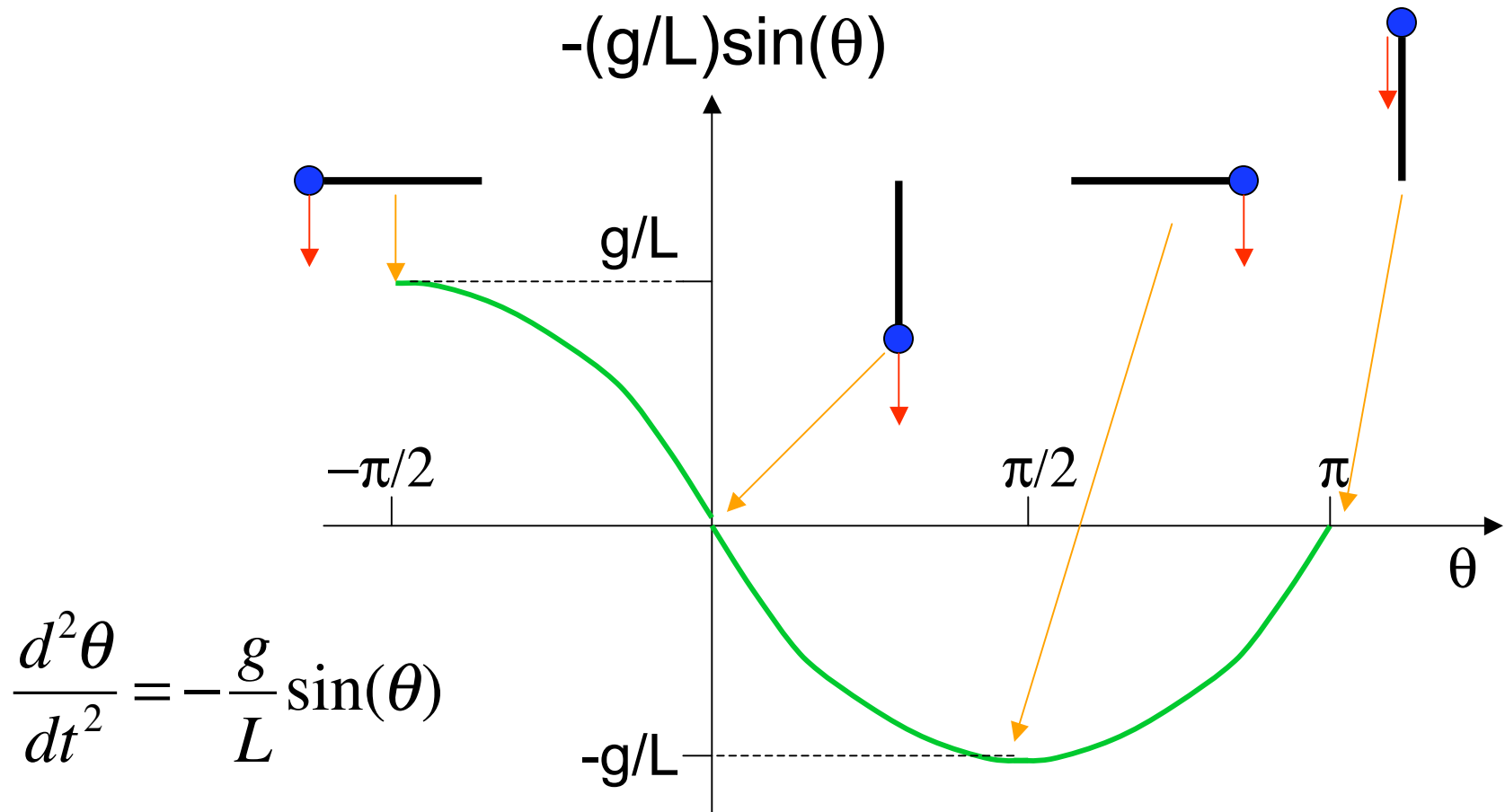
$$\frac{d^2\theta}{dt^2} = -K \sin(\sqrt{K} t)$$

↑ ?!

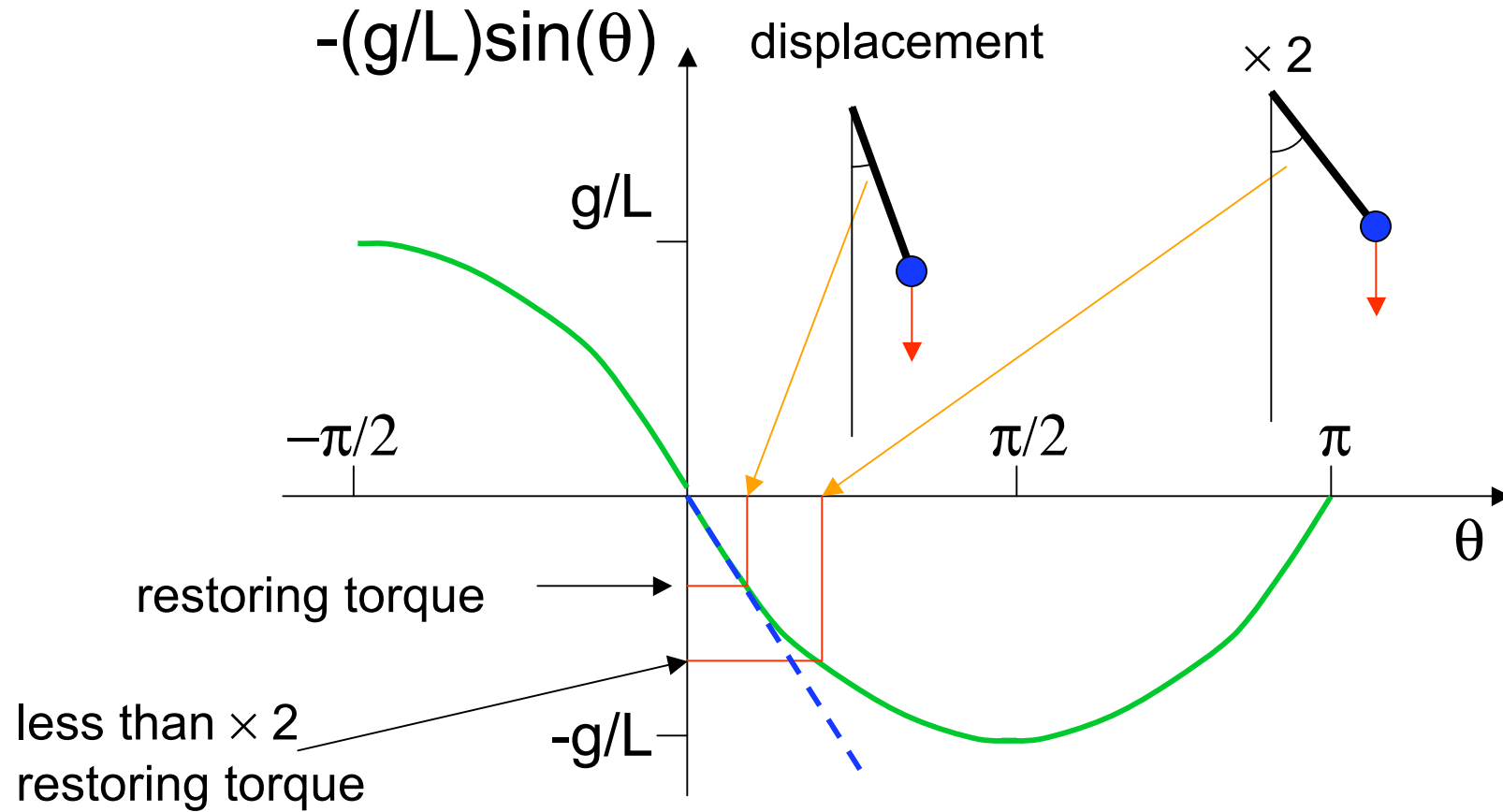
NOT SIMPLE

$$-K \sin(\theta) = -K \sin[\sin(\sqrt{K} t)]$$

What's happening?



What's happening?

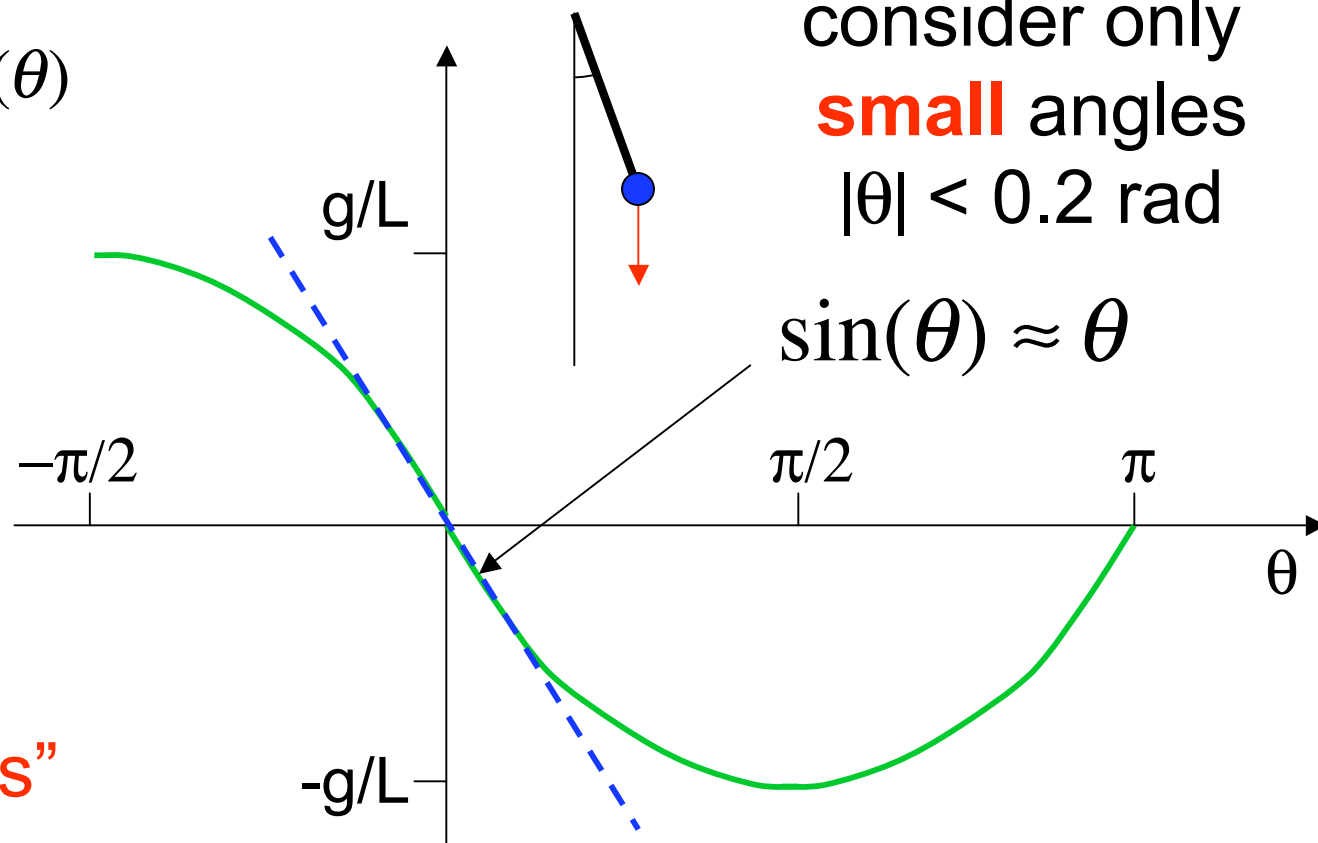


Way Forward

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin(\theta)$$

$$\frac{d^2\theta}{dt^2} \approx -\frac{g}{L}\theta$$

consider only
small angles
 $|\theta| < 0.2$ rad



$$\sin(\theta) \approx \theta$$

“Small
Oscillations”

$$-(g/L)\sin(\theta) \sim -(g/L)\theta$$

Simple Harmonic Motion

diff. eq.

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta$$

1. Know frequency (& period) immediately from diff. eq.

$$\omega = \sqrt{\frac{g}{L}}$$

2. Initial conditions: they will be of 2 kinds

A. at rest initially

$$\theta(0) = \theta_o$$

$$\dot{\theta}(0) = 0$$

COSINE

$$\theta(t) = \theta_o \cos(\omega t)$$

B. at equilib. position

$$\theta(0) = 0$$

$$\dot{\theta}(0) = \frac{v_o}{L}$$

SINE

$$\theta(t) = \frac{v_o}{L\omega} \sin(\omega t)$$

2. Different angular frequencies for pendulums

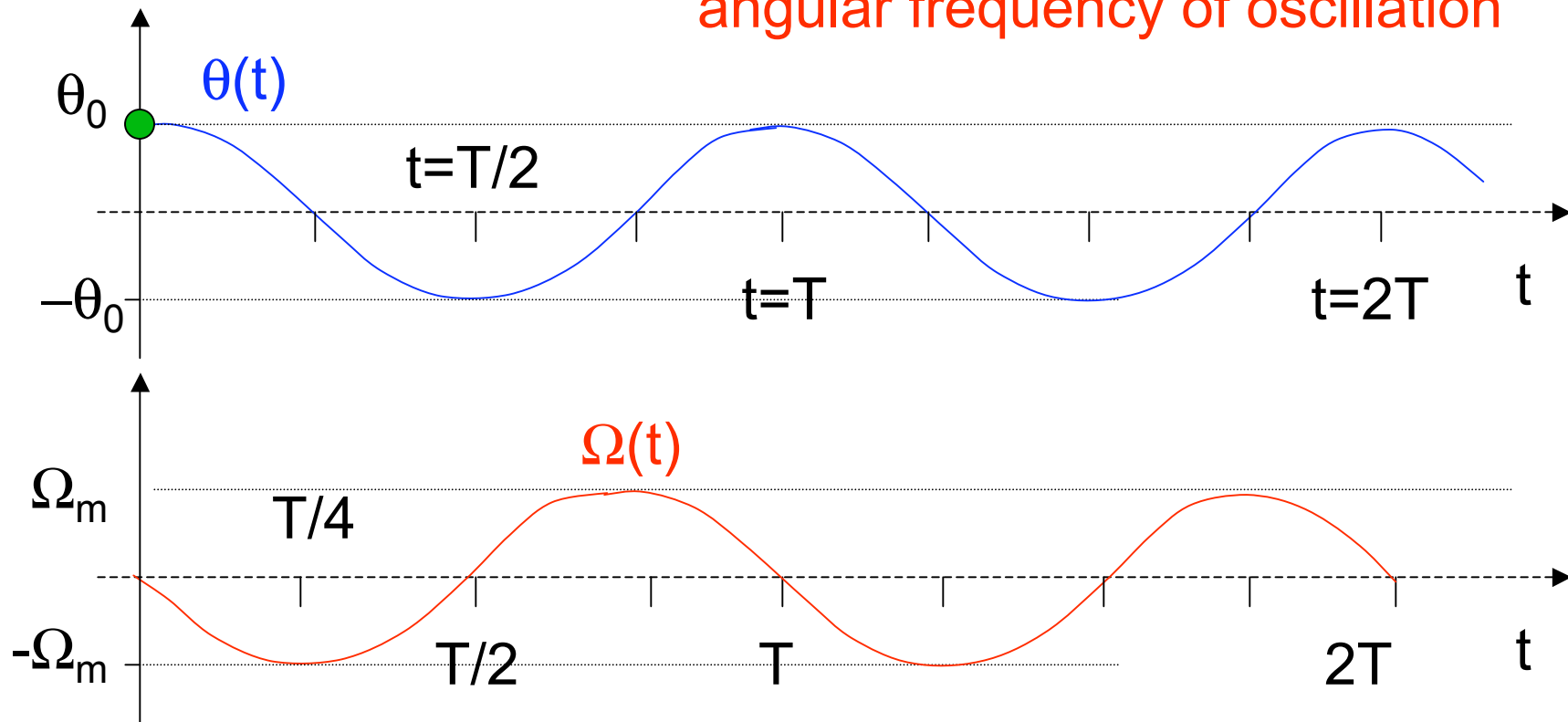
$$\omega = \sqrt{\frac{g}{L}}$$

angular frequency of oscillation

$$\theta(t) = \theta_o \cos(\omega t)$$

$$\frac{d\theta}{dt} = \Omega(t) = -\theta_o \omega \sin(\omega t)$$

angular frequency of oscillation



2. Different angular frequencies for pendulums

$$\omega = \sqrt{\frac{g}{L}}$$

angular frequency of oscillation

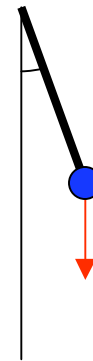
$$\theta(t) = \theta_o \cos(\omega t)$$

$$\frac{d\theta}{dt} = \Omega(t) = -\theta_o \omega \sin(\omega t)$$

angular frequency of oscillation

small angle assumption $|\theta(t)| \ll 1$

$$\Rightarrow |\theta_o| \ll 1$$



$$|\Omega(t)| = |\theta_o \omega \sin(\omega t)| \leq |\theta_o \omega| \ll \omega$$

Second Pendulum

You wish to construct a “seconds pendulum” (one whose period is exactly 1 second) using a light wire and a 5.0 kg bob. How long should the wire be?

- A 0.25 m
- B 1.00 m
- C 1.56 m
- D 9.80 m
- E 61.6 m

Second Pendulum

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$$T = 1 \text{ sec}$$

$$\omega = 2\pi \text{ rad/sec} = \sqrt{\frac{g}{L}}$$

$$L = \frac{g}{(2\pi)^2} = 0.25 \text{ m}$$

You build the pendulum with $L=0.25$ m of wire, but use a **heavier bob**: $m=10.0$ kg. This is twice as heavy as you had planned for. The period will now be

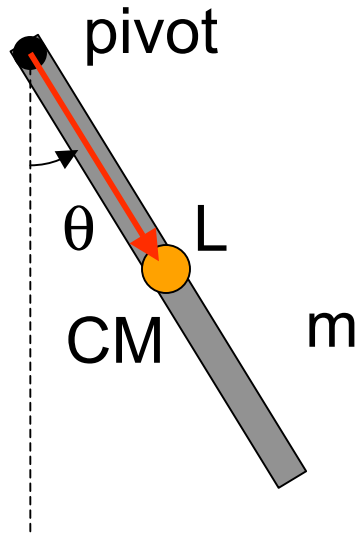
- A 0.5 sec
- B 0.71 sec
- C 1.00 sec
- D 1.41 sec
- E 2.00 sec

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- A 0.5 sec
- B 0.71 sec
- C 1.00 sec
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$$\omega = \sqrt{\frac{g}{L}}$$

does not depend on m



Physical Pendulum

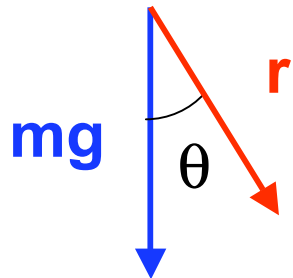
A uniform bar of length L and mass m is fixed to a frictionless pivot. Consider the system in the configuration shown.

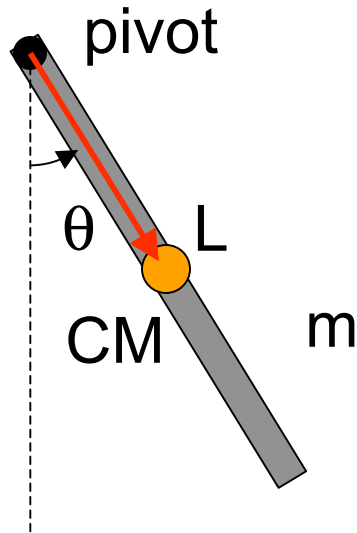
calculate the following:

$$I \text{ about pivot} = (1/3)mL^2$$

$$\tau \text{ about pivot} = -(1/2)mgL \sin\theta$$

$$\alpha = \tau/I = -(1.5g/L) \sin\theta$$





Physical Pendulum

A uniform bar of length L and mass m is fixed to a frictionless pivot. Consider the system in the configuration shown.

calculate the following:

$$\frac{d^2\theta}{dt^2} \approx -\frac{3g}{2L}\theta$$

$$I \text{ about pivot} = (1/3)mL^2$$

$$\tau \text{ about pivot} = -(1/2)mgL \sin\theta$$

$$\alpha = \tau/I = -(1.5g/L) \sin\theta$$

$$\omega = d\theta/dt$$

$$\alpha = d\omega/dt = d^2\theta/dt^2$$

$$\frac{d^2\theta}{dt^2} = -\frac{3g}{2L} \overbrace{\sin(\theta)}^{\sim\theta}$$

Simple Harmonic Motion

diff. eq.

$$\frac{d^2\theta}{dt^2} = -\frac{3g}{2L}\theta$$

1. Know frequency (& period) immediately from diff. eq.

$$\omega = \sqrt{\frac{3g}{2L}}$$

2. Initial conditions: they will be of 2 kinds

A. at rest initially

$$\theta(0) = \theta_o$$

$$\dot{\theta}(0) = 0$$

COSINE

$$\theta(t) = \theta_o \cos(\omega t)$$

B. at equilib. position

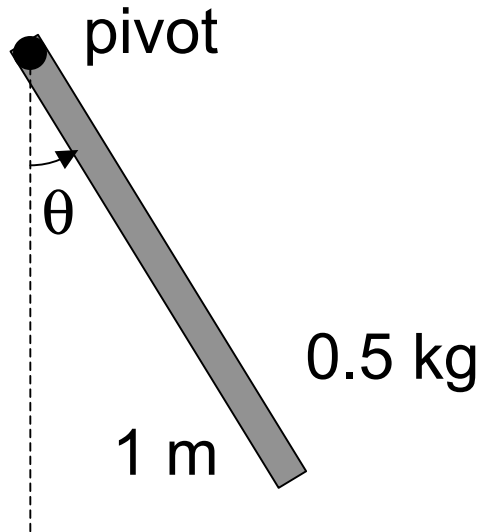
$$\theta(0) = 0$$

$$\dot{\theta}(0) = \frac{v_o}{L}$$

SINE

$$\theta(t) = \frac{v_o}{L\omega} \sin(\omega t)$$

Example



A uniform 1.0-meter rod whose mass is 500 g is released from an angle of $\theta_0 = 0.1$ rad. Write down the angle as a function of time $\theta(t)$.

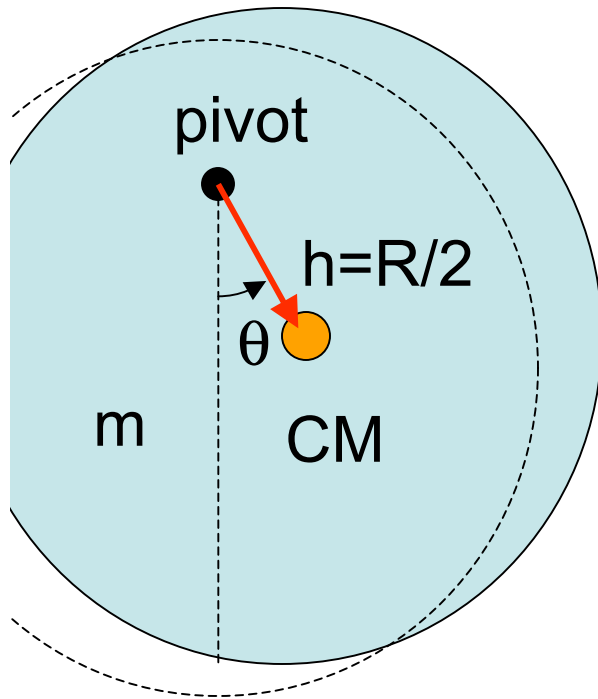
$$\omega = \sqrt{\frac{3g}{2L}} = 3.83 \text{ rad/s}$$

$$\theta(t) = \theta_0 \cos(\omega t)$$

$$\theta(t) = 0.1 \cos(3.83 t)$$

Physical Pendulum

A circular metal disk of radius R and mass m pivots about a screw $h=R/2$ from the center.

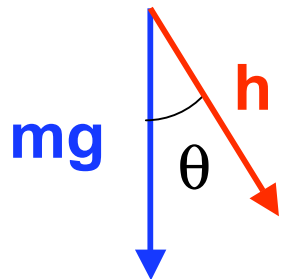


calculate the following:

I about pivot

τ about pivot = $-mgh \sin\theta$

$\alpha = \tau/I = -(mgh/I) \sin\theta$

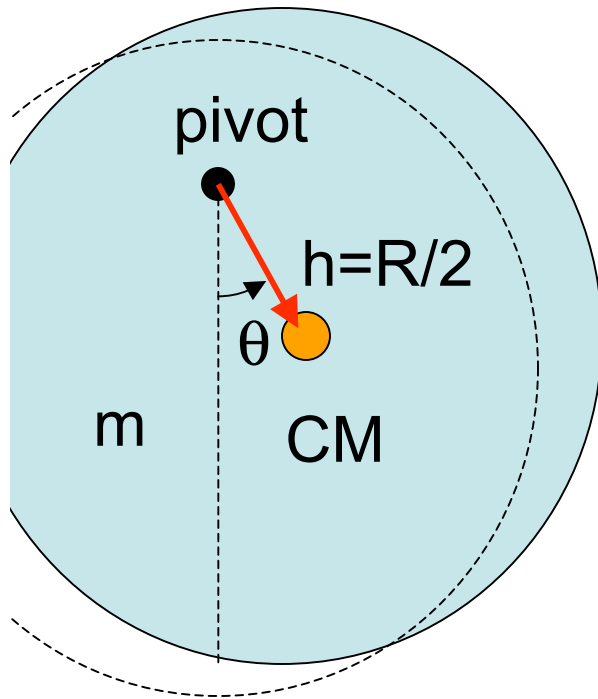


$$\frac{d^2\theta}{dt^2} = \alpha \approx -\frac{mgh}{I}\theta$$

$$\omega = \sqrt{\frac{mgh}{I}}$$

Physical Pendulum

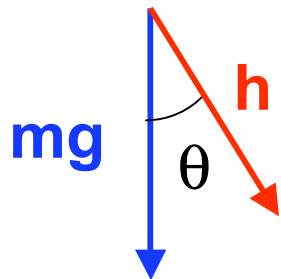
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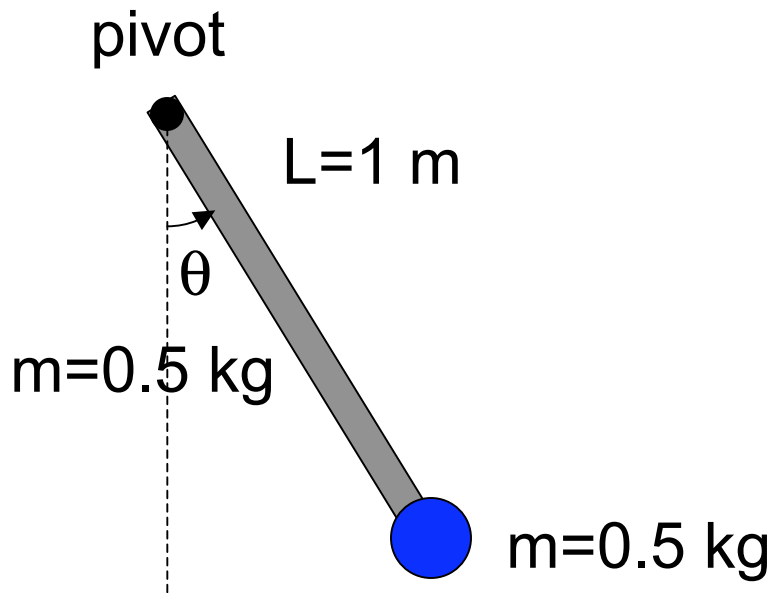
calculate the following:

I about pivot $I=0.5 mR^2 + m(R/2)^2$

$$I = 0.75 mR^2$$



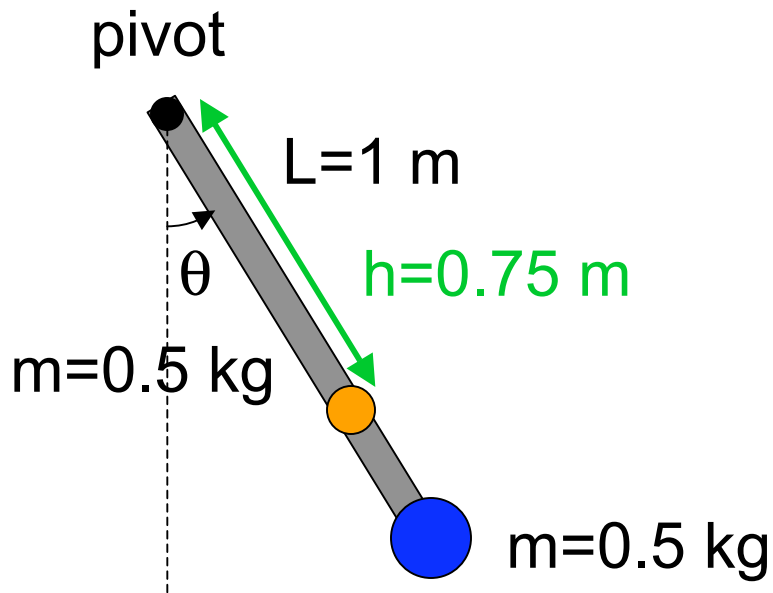
$$\omega = \sqrt{\frac{mgh}{I}} = \sqrt{\frac{2}{3} \frac{g}{R}}$$



Example

A uniform 1.0-meter rod has a mass $m = 0.50 \text{ kg}$. A small bob of the same mass is attached to its end. What is the angular frequency of its oscillation?

- A 1.76 rad/s
- B 2.35 rad/s
- C 2.95 rad/s
- D 3.13 rad/s
- E 4.43 rad/s



Example

A uniform 1.0-meter rod has a mass $m=0.50\text{ kg}$. A small bob of the same mass is attached to its end. What is the angular frequency of its oscillation?

$$I = (mL^2) + (mL^2)/3$$

$$I = (4/3) mL^2$$

$$h = (3/4) L$$

$$M = 2 m \quad \text{total mass}$$

$$\omega = \sqrt{\frac{Mgh}{I}} = \sqrt{\frac{18 g}{16 L}}$$

A 1.76 rad/s

B 2.35 rad/s

C 2.95 rad/s

D 3.13 rad/s

E 4.43 rad/s