

Standing waves



Chs. 16 & 17

- 16.1 - 16.6: traveling waves on strings
- 16.7 & 16.8:
- 16.9 & 16.10: superposition & interference
- 16.11:
- 16.12 & 16.13: standing waves

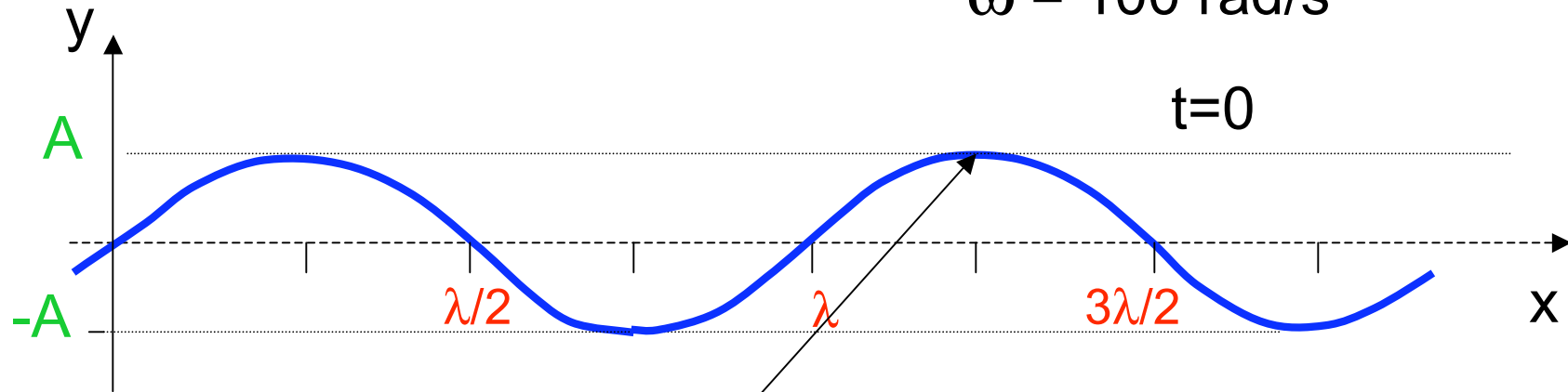
- 17.1 - 17.4: sound waves
- 17.5 - 17.6:
- 17.7: examples
- 17.8 - 17.10:

Sinusoidal Traveling Wave

$$y(x,t) = A \sin(kx + \omega t)$$

$$k = 10 \text{ rad/cm}$$

$$\omega = 100 \text{ rad/s}$$



Q: What is the velocity (w/ sign) of this crest?

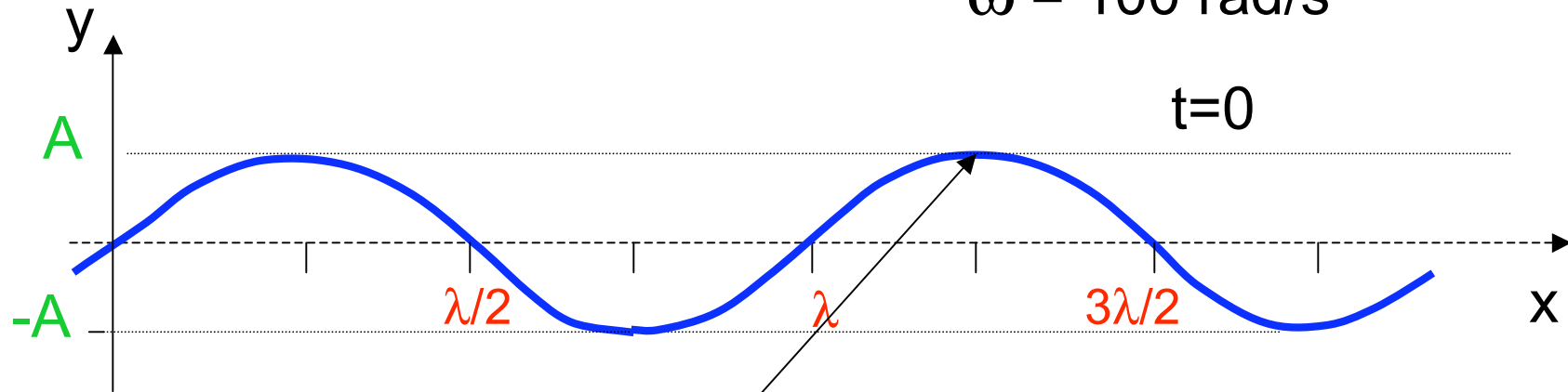
- A** -10 cm/s
- B** -0.1 cm/s
- C** 0.1 cm/s
- D** 10 cm/s

Sinusoidal Traveling Wave

$$y(x,t) = A \sin(kx - \omega t)$$

$$k = 10 \text{ rad/cm}$$

$$\omega = 100 \text{ rad/s}$$



Q: What is the velocity (w/ sign) of this crest?

A -10 cm/s

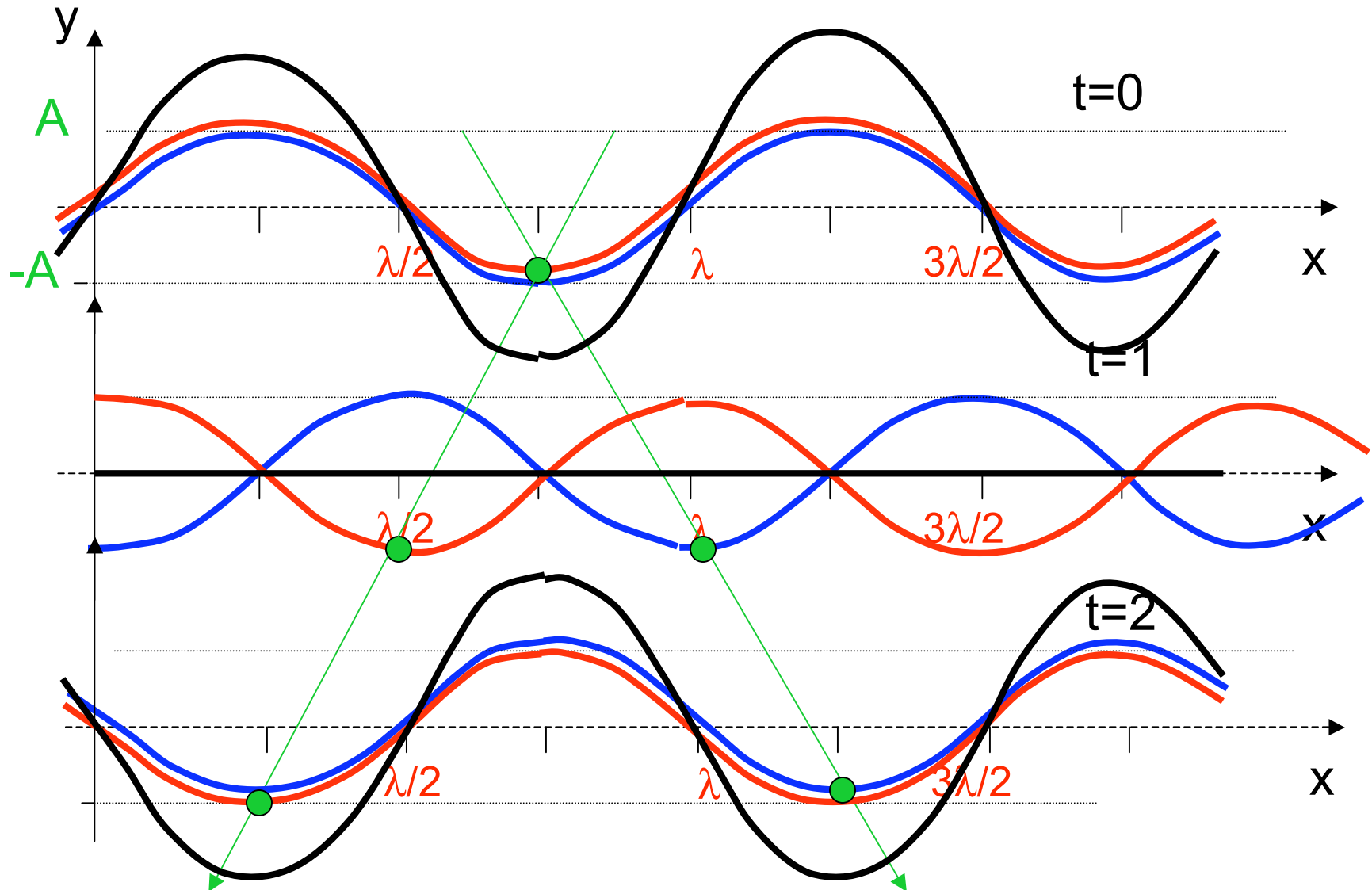
B -0.1 cm/s

C 0.1 cm/s

D 10 cm/s

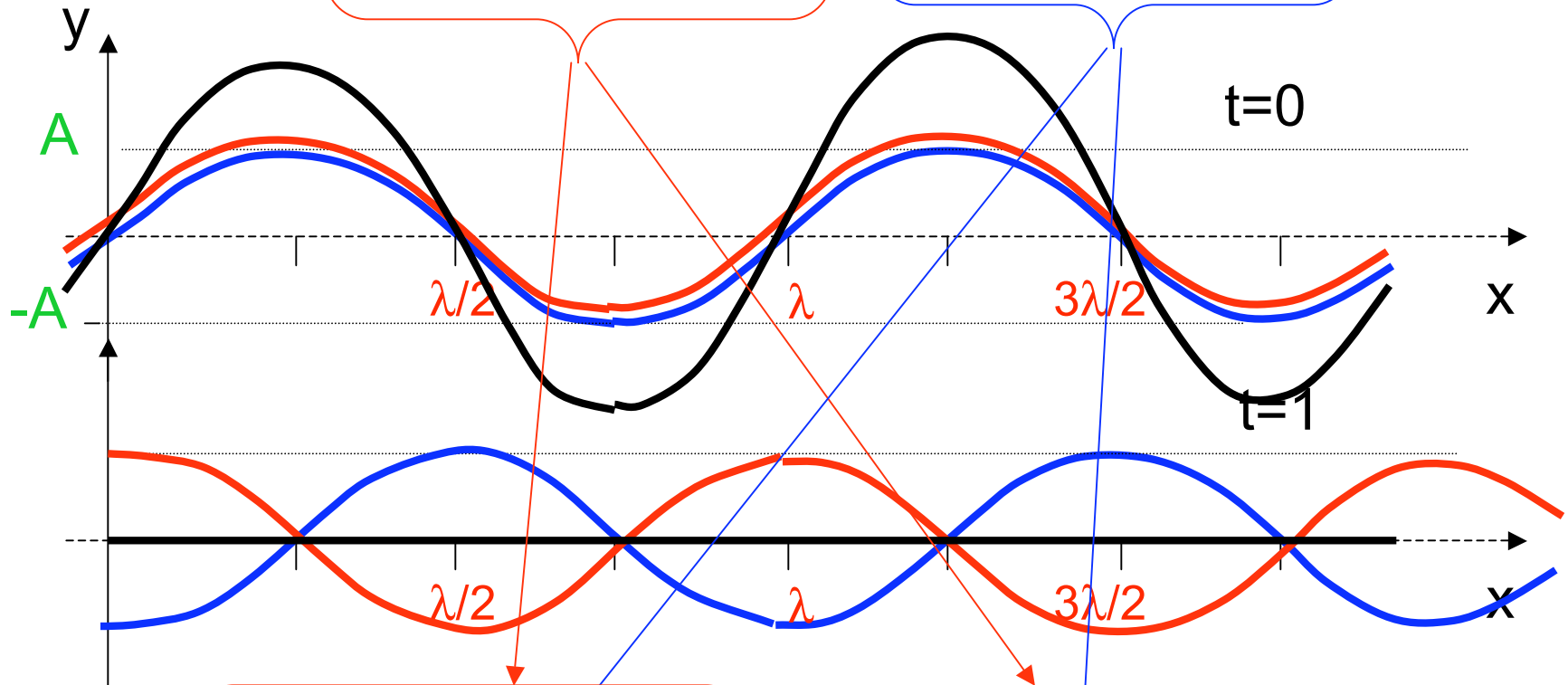
Superposition of Traveling Waves

$$y(x,t) = A \sin(kx + \omega t) + A \sin(kx - \omega t)$$



Superposition of Traveling Waves

$$y(x,t) = A \sin(kx + \omega t) + A \sin(kx - \omega t)$$

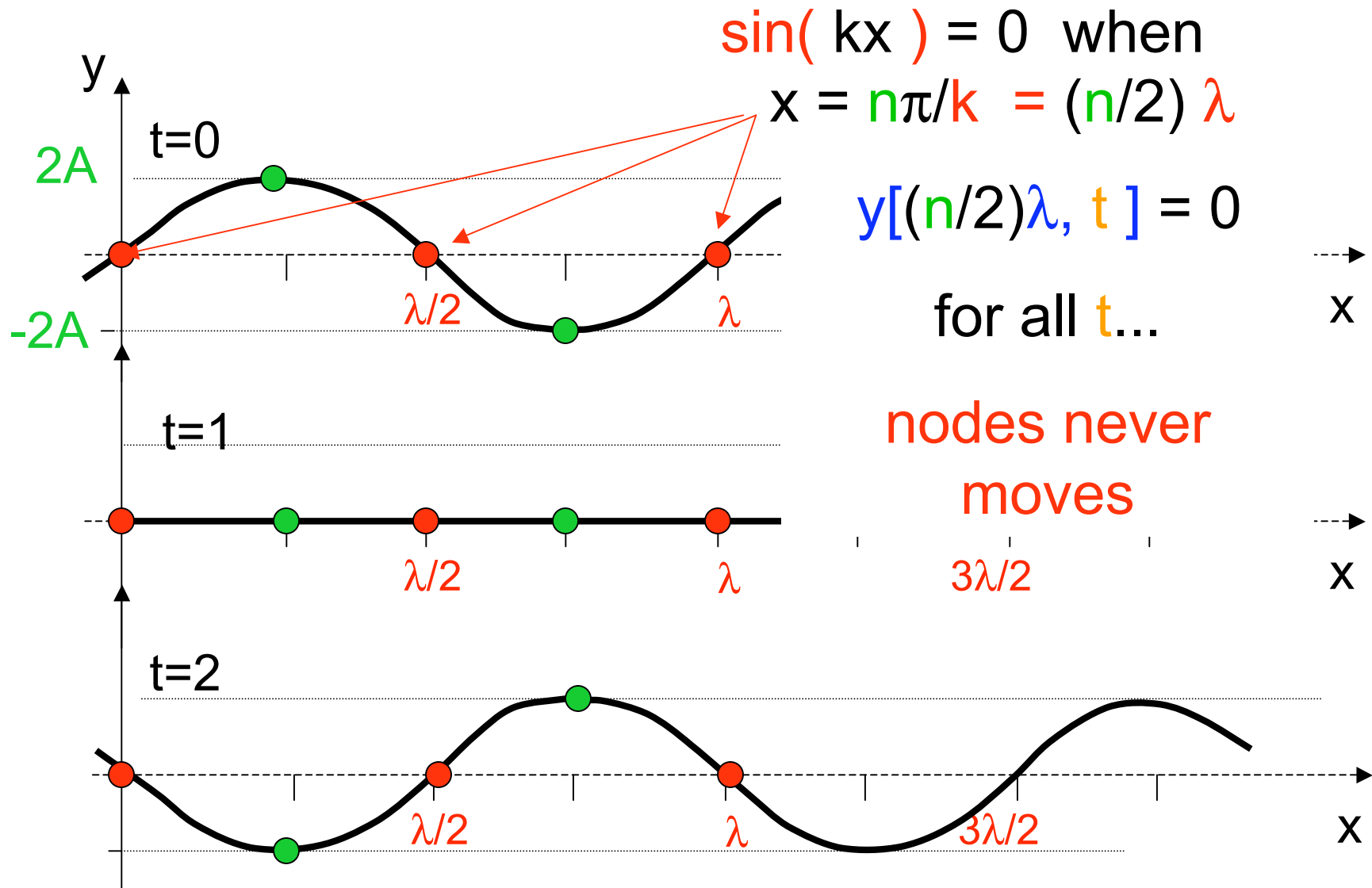


$$= A \sin(kx) \cos(\omega t) + A \cos(kx) \sin(\omega t)$$

$$+ A \sin(kx) \cos(\omega t) - A \cos(kx) \sin(\omega t)$$

$$y(x,t) = 2A \sin(kx) \cos(\omega t)$$

Standing wave $y(x,t) = 2A \sin(kx) \cos(\omega t)$



Standing wave $y(x,t) = 2A \sin(kx) \cos(\omega t)$

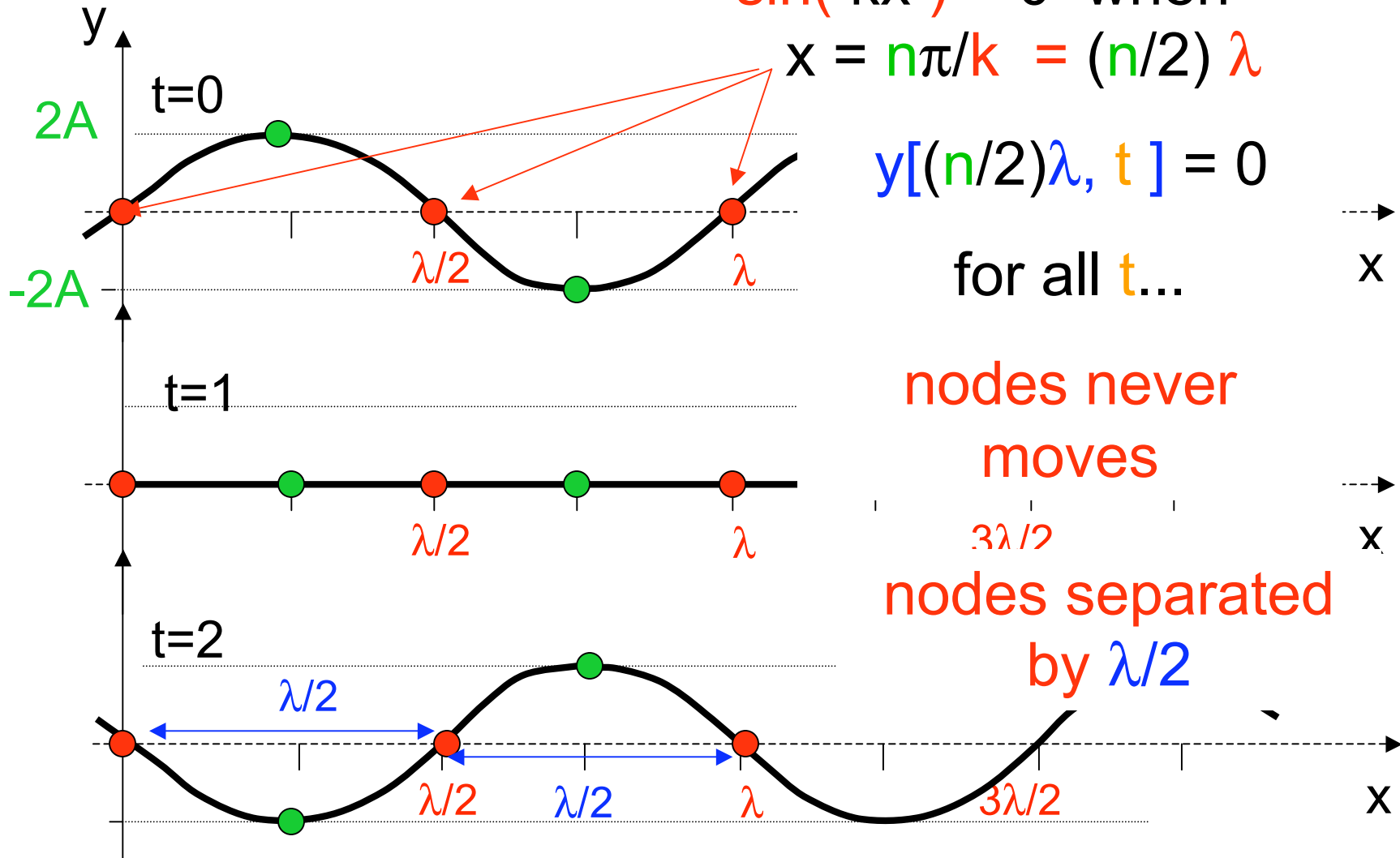
$\sin(kx) = 0$ when
 $x = n\pi/k = (n/2)\lambda$

$y[(n/2)\lambda, t] = 0$

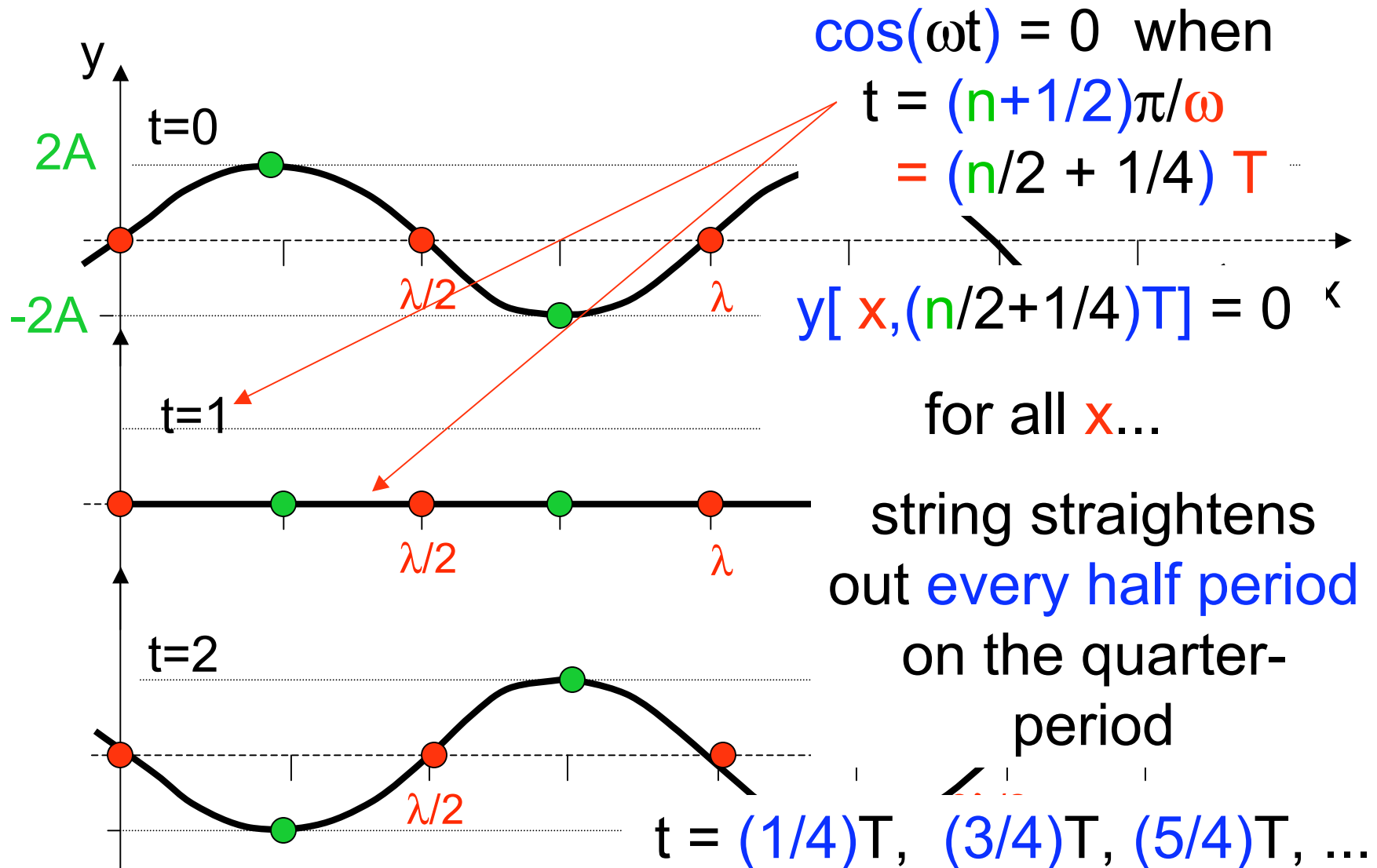
for all $t \dots$

nodes never
 moves

nodes separated
 by $\lambda/2$

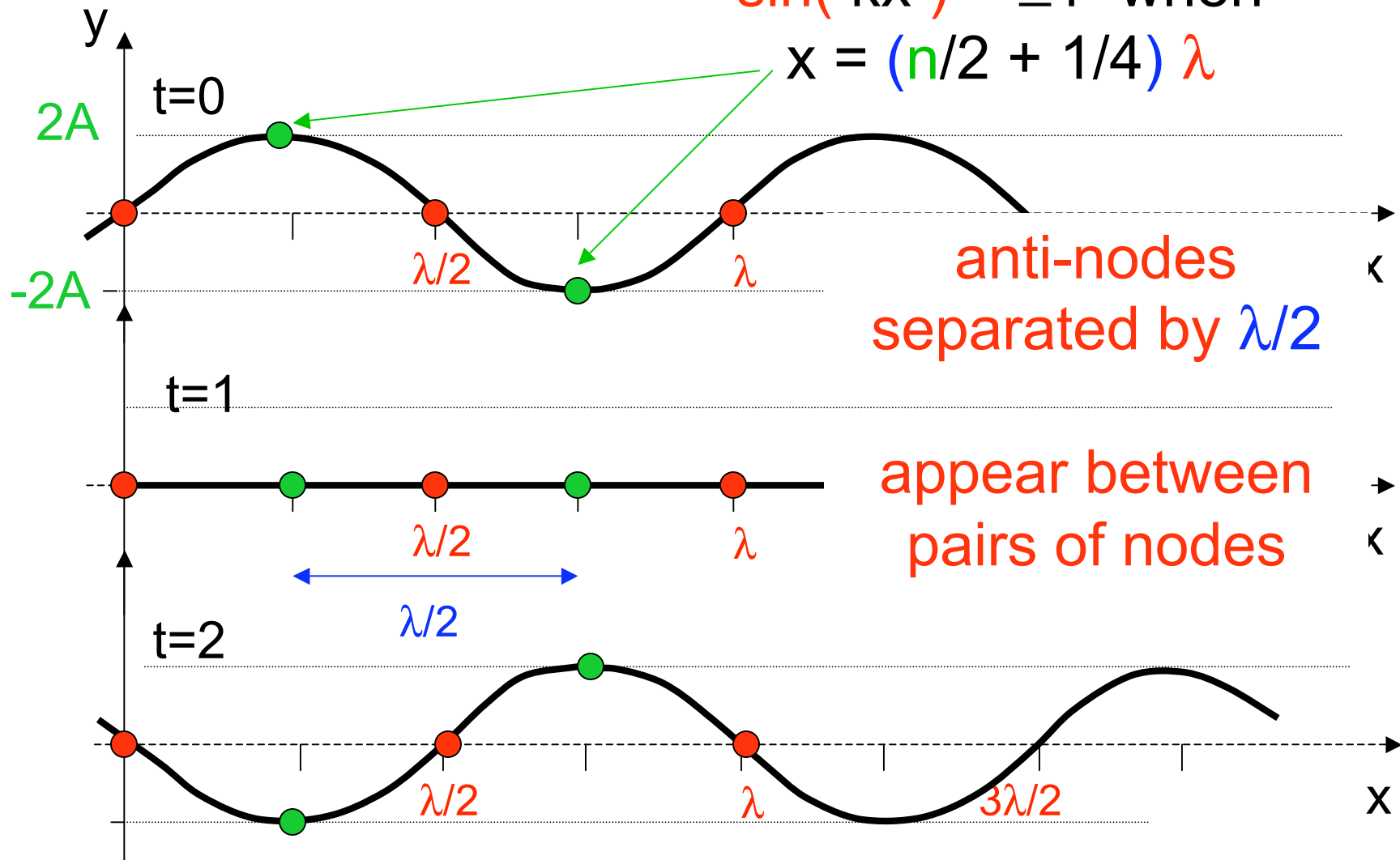


Standing wave $y(x,t) = 2A \sin(kx) \cos(\omega t)$



Standing wave $y(x,t) = 2A \sin(kx) \cos(\omega t)$

$\sin(kx) = \pm 1$ when
 $x = (n/2 + 1/4) \lambda$



Traveling vs. Standing

$y(x,t) = A \sin(kx \pm \omega t)$

$2A \sin(kx) \cos(\omega t)$

nodes: move @ $\pm v$

never move

string is
straight:

never

every $T/2$

crests/
anti-nodes

move @ $\pm v$

between nodes

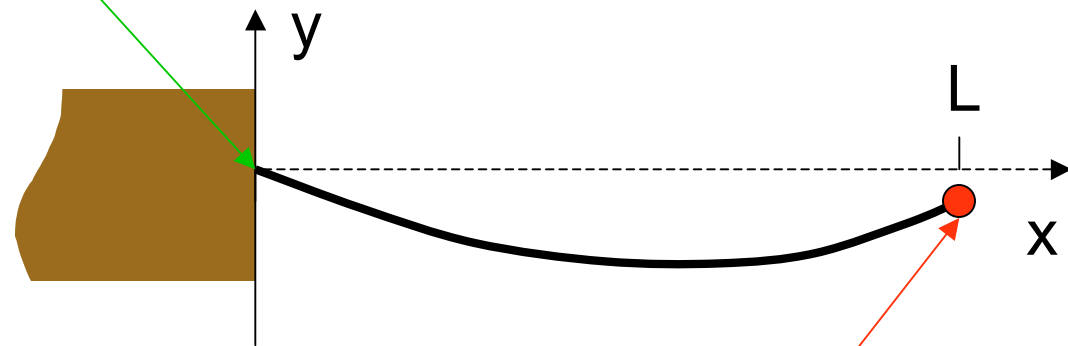
$\lambda/T =$

$$v = \sqrt{\frac{\tau}{\mu}}$$

$$v = \sqrt{\frac{\tau}{\mu}}$$

One fixed end

$$y(0,t) = 0 \text{ for all } t$$



what happens?

shake harmonically:
 $y(L,t) = A \cos(\omega t)$

I control these

One fixed end

$$k = \omega / v$$

$$\lambda = v / T$$

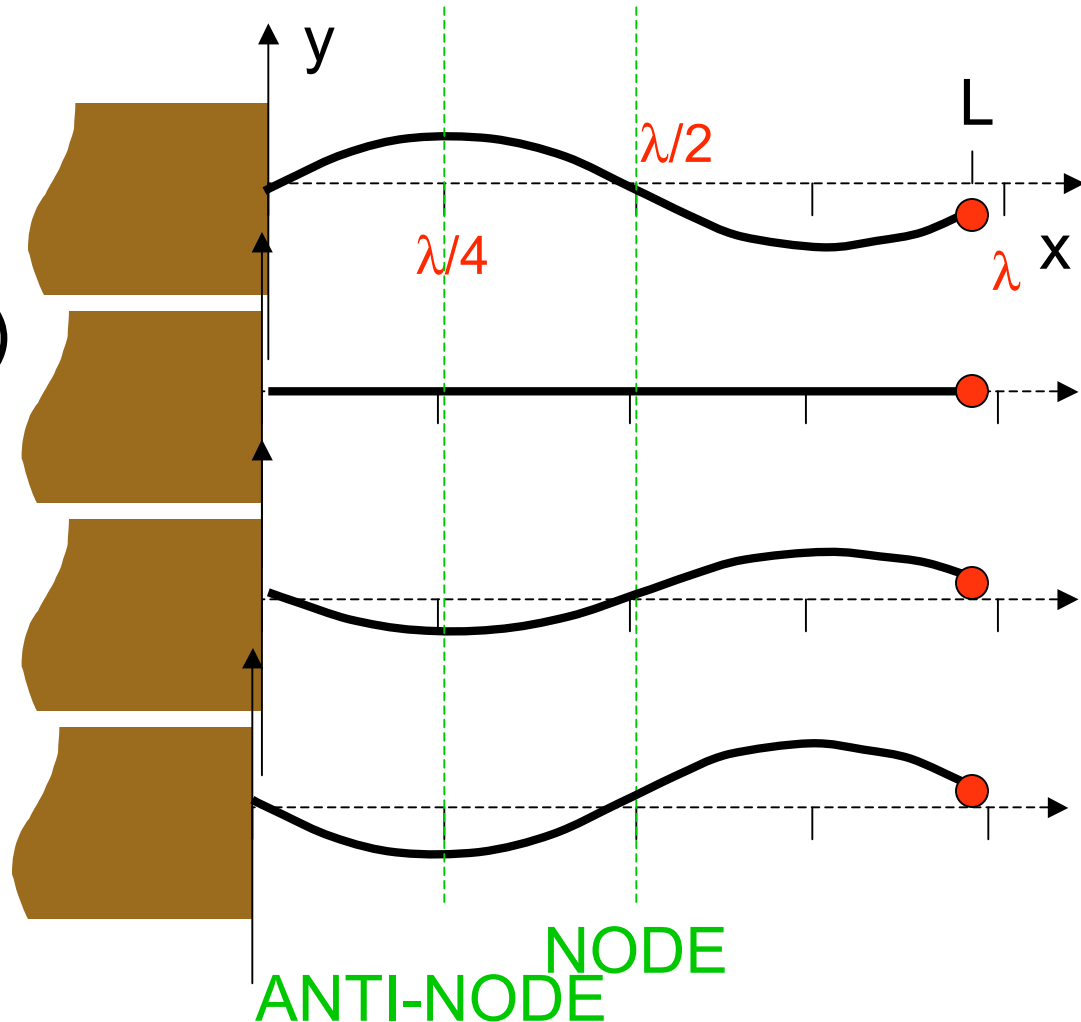
$$y(x,t) = y_m \sin(kx) \cos(\omega t)$$

$$y(L,t) = A \cos(\omega t)$$

$$= y_m \sin(kL) \cos(\omega t)$$

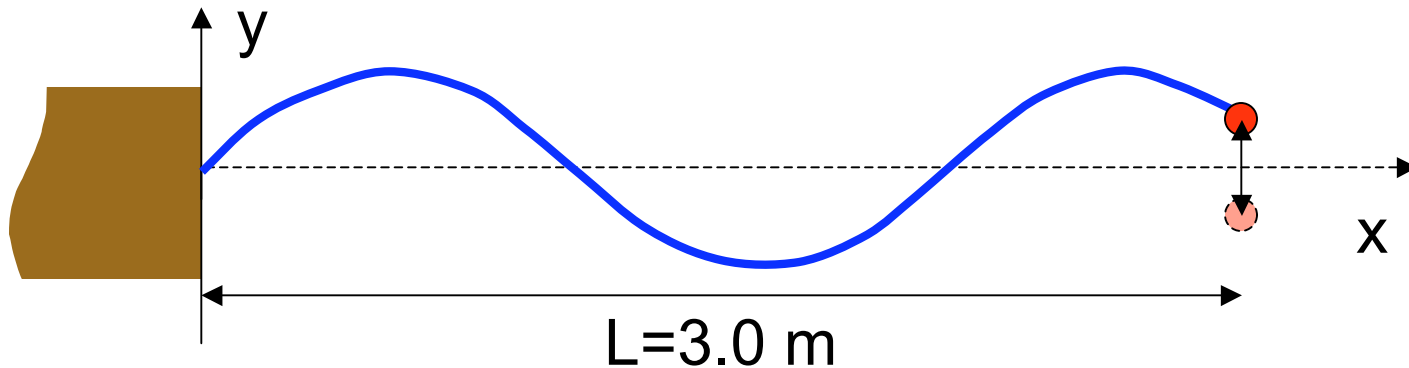
$$y_m = \frac{A}{\sin(kL)}$$

$$y_m = \frac{A}{\sin(2\pi L / \lambda)}$$



Standing Wave

$$y(x,t) = A \sin(kx) \cos(\omega t)$$



A 3-meter string (mass 0.30 kg) is stretched by 10 N of tension. The free end is shaken with a period of $T=0.25$ sec. What is the wavelength of oscillation?

A $\lambda = 1.0$ m

B $\lambda = 1.5$ m

C $\lambda = 2.5$ m

D $\lambda = 3.0$ m

E $\lambda = 6.0$ m

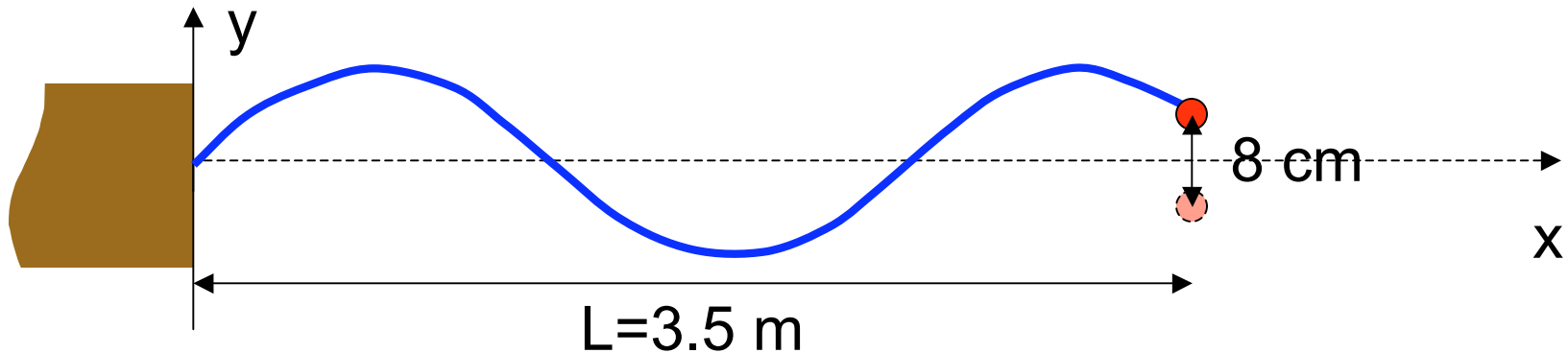
$$\mu = 0.3 \text{ kg}/3.0\text{m} = 0.1 \text{ kg/m}$$

$$v = (T/\mu)^{1/2} = 10 \text{ m/s}$$

$$\lambda = vT = (10 \text{ m/s})(0.25 \text{ s}) = 2.5 \text{ m}$$

Standing Wave

$$y(x,t) = A \sin(kx) \cos(\omega t)$$



A 3.5-meter string (mass 0.30 kg) is stretched by 10 N of tension. The free end is shaken with a period of $T = 0.25 \text{ sec}$ over 8.0 cm . What is the amplitude of oscillation at the antinodes?

$$\lambda = 2.5 \text{ m}$$

A $y_m = 2.0 \text{ cm}$

B $y_m = 3.4 \text{ cm}$

C $y_m = 6.8 \text{ cm}$

D $y_m = 13.6 \text{ cm}$

$$y(L,t) = 4.0 \text{ cm} \cos(2\pi t/T)$$

$$= y_m \sin(2\pi L/\lambda) \cos(2\pi t/T)$$

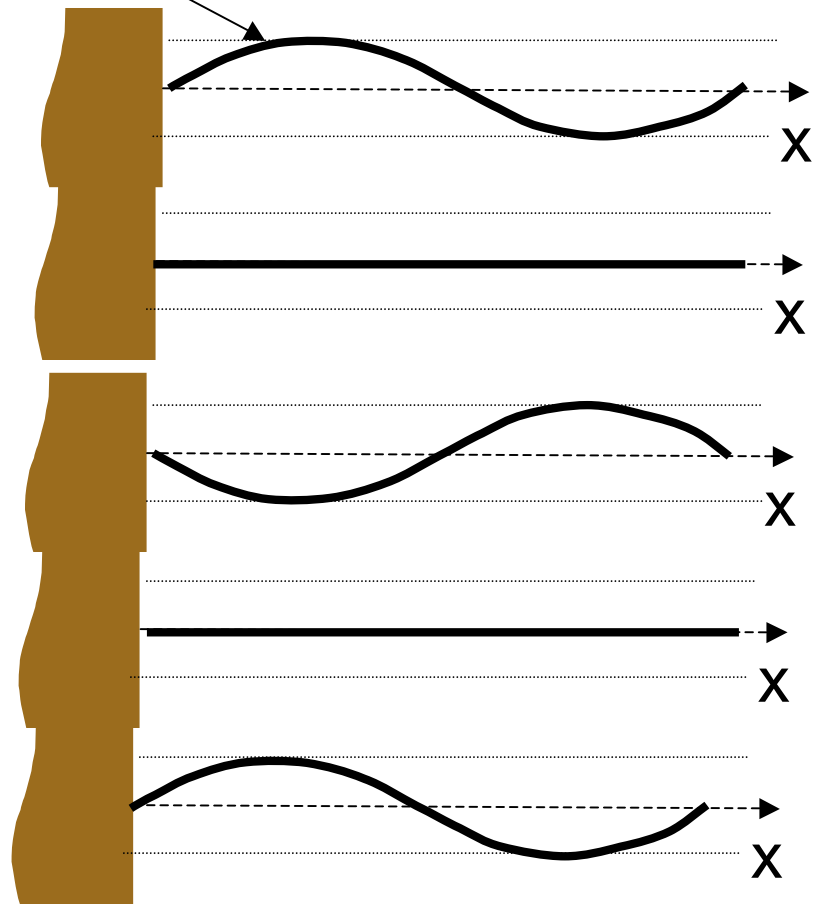
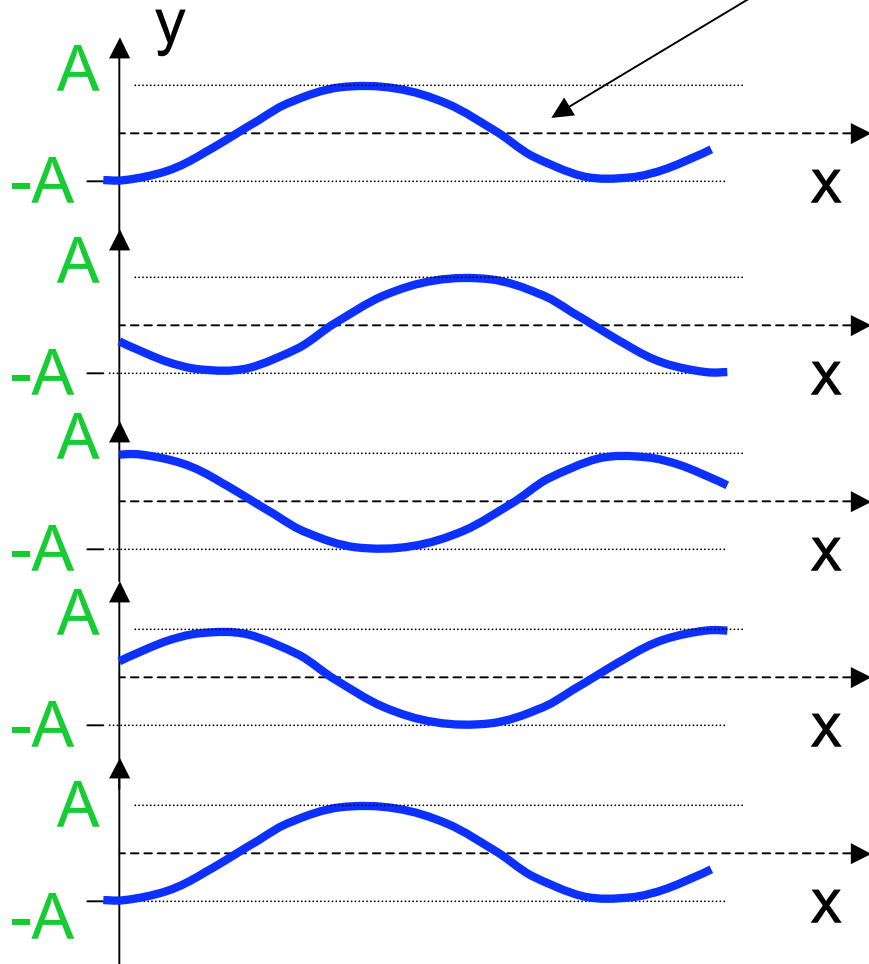
$$y_m = 4/\sin(2\pi L/\lambda) = 4/\sin(0.59) = 6.8 \text{ cm}$$

Standing vs. Traveling

$$y(x,t) = A \sin[k(x - vt)]$$

$$v = \sqrt{\frac{T}{\mu}}$$

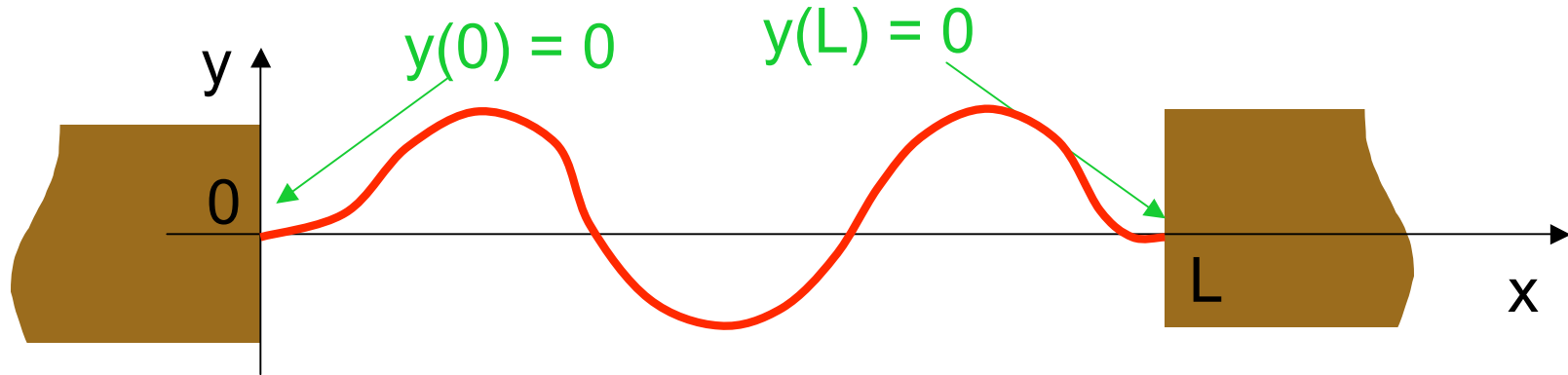
$$y(x,t) = A \sin(kx) \cos(kvt)$$



2 Fixed Ends

$$y(x,t) = A \sin(2\pi x/\lambda) \cos(2\pi ft)$$

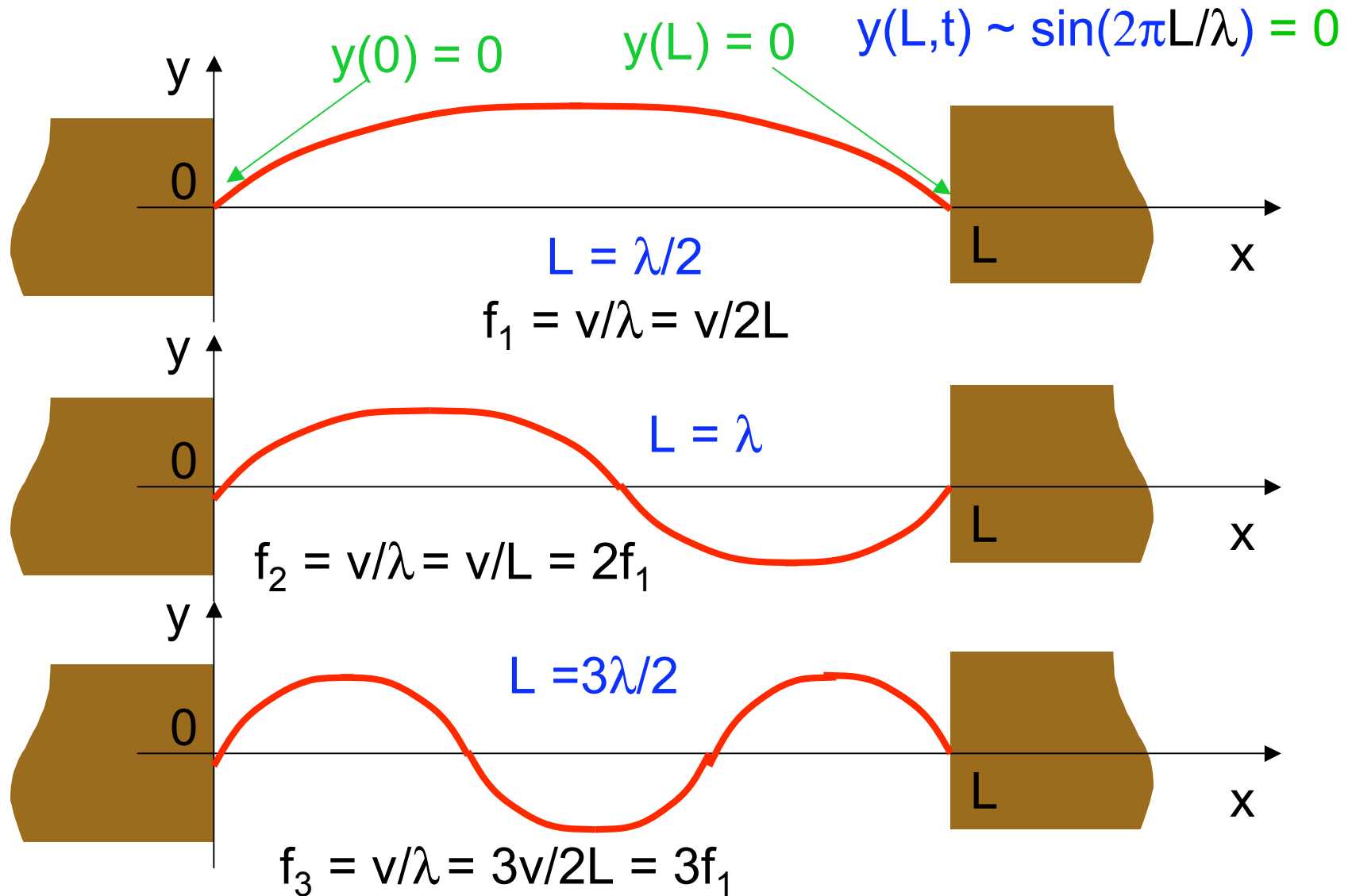
$x=0$ }
 $x=L$ } both nodes

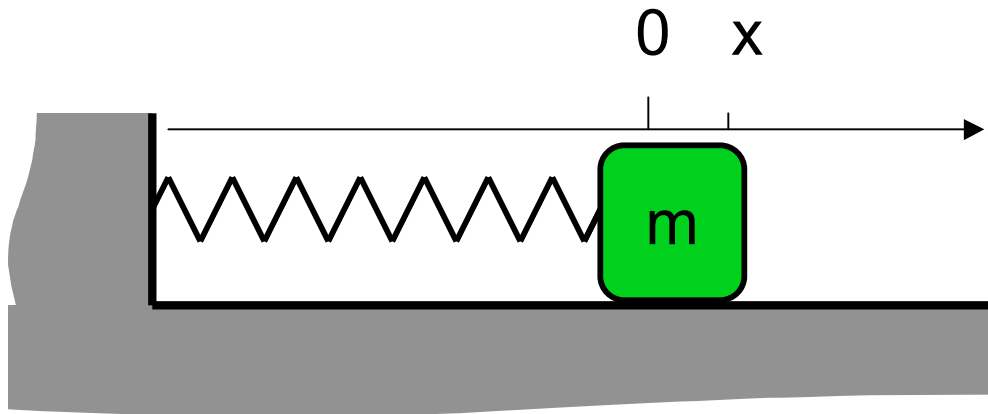


2 Fixed Ends

$$y(x,t) = A \sin(2\pi x/\lambda) \cos(2\pi ft)$$

$x=0$ } both nodes
 $x=L$ }



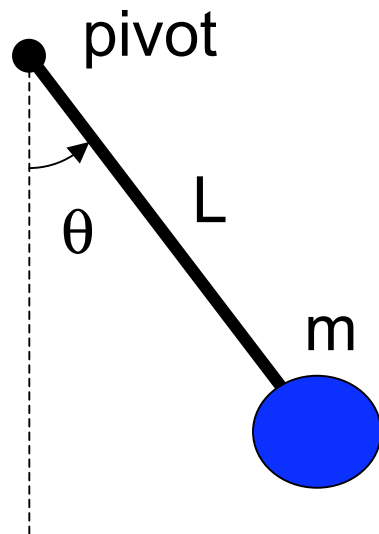


Oscillations

One element:
w/ restoring force
& inertia

→ oscillation

$$\theta(t) = \theta_o \cos(\omega t)$$

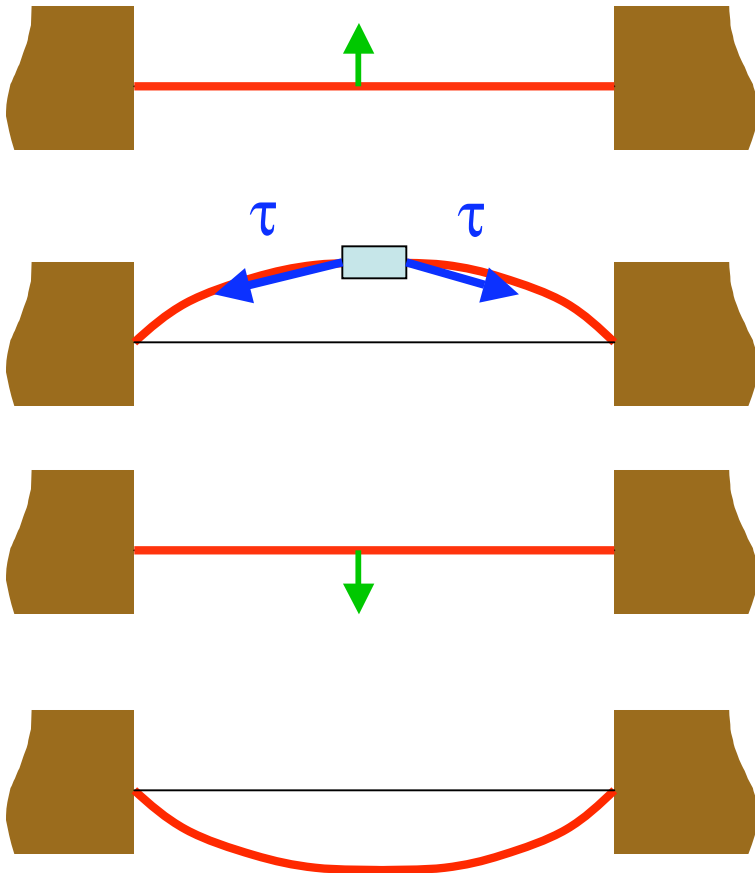


$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{g}{L}}$$

fixed by
system

Waves



restoring force on one piece: exerted by neighboring pieces.

disturbance propagates as wave

$$v = \sqrt{\frac{\tau}{\mu}}$$

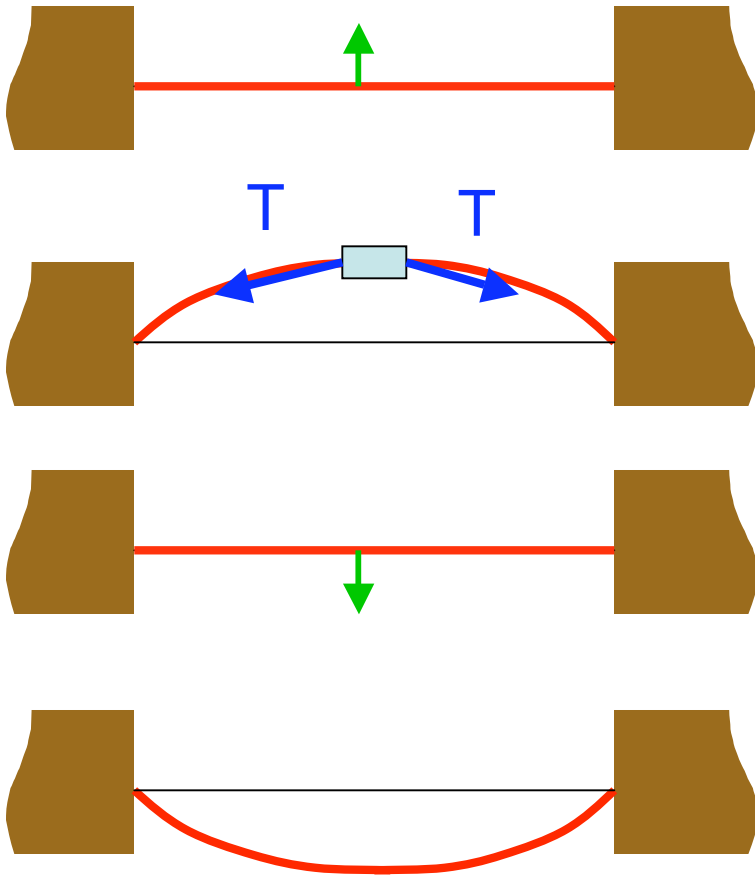
fixed by system

$$\omega = \frac{2\pi v}{\lambda}$$

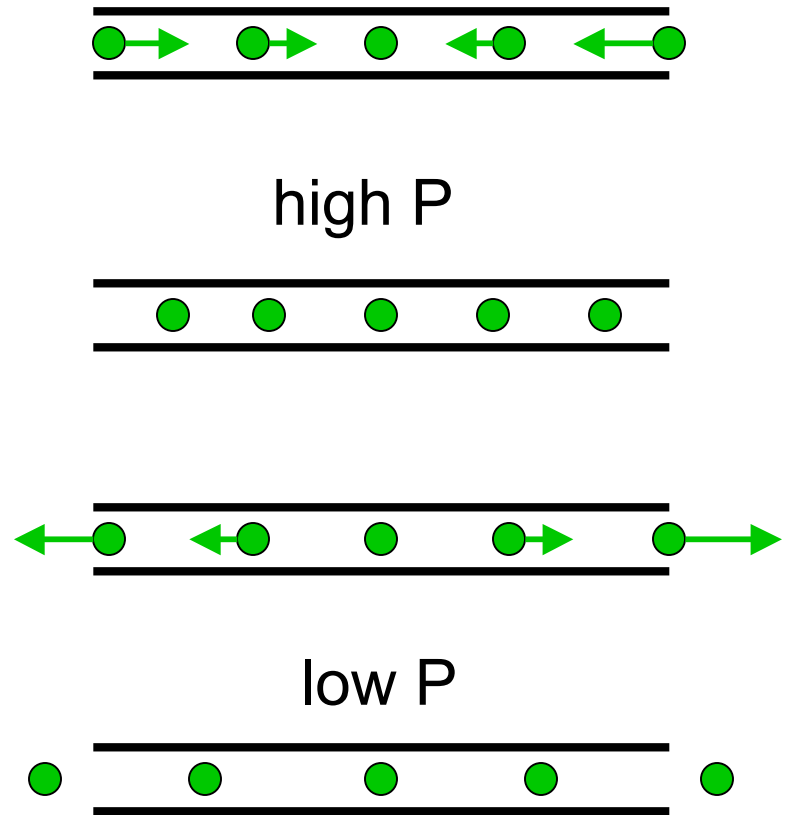
can vary

$$v = \sqrt{\frac{\tau}{\mu}}$$

Sound Waves



transverse

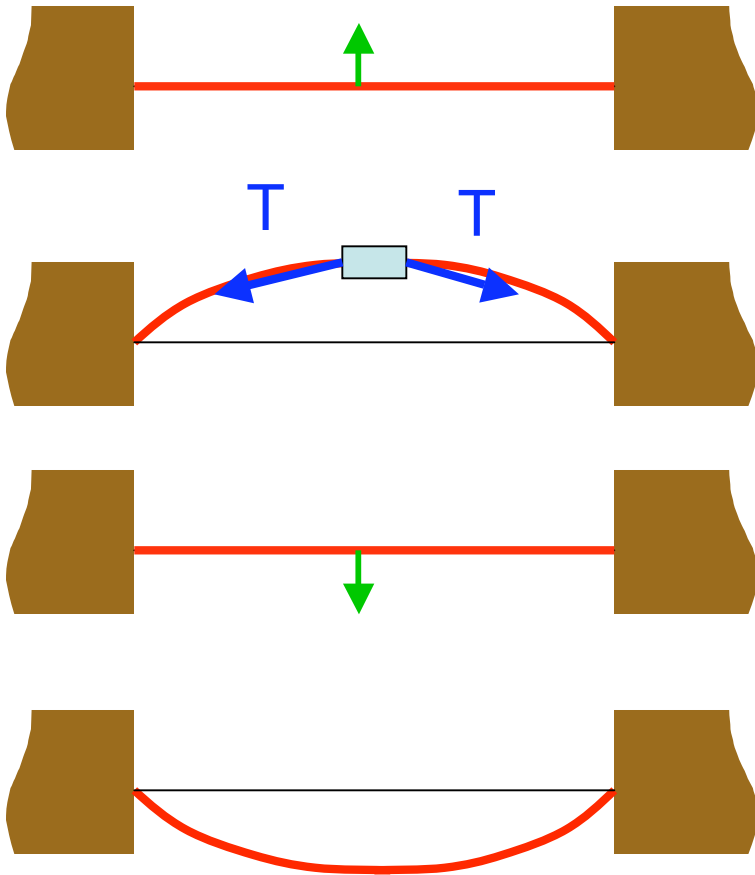


longitudinal

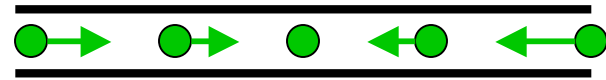
$$v = \sqrt{\frac{\tau}{\mu}}$$

Sound Waves

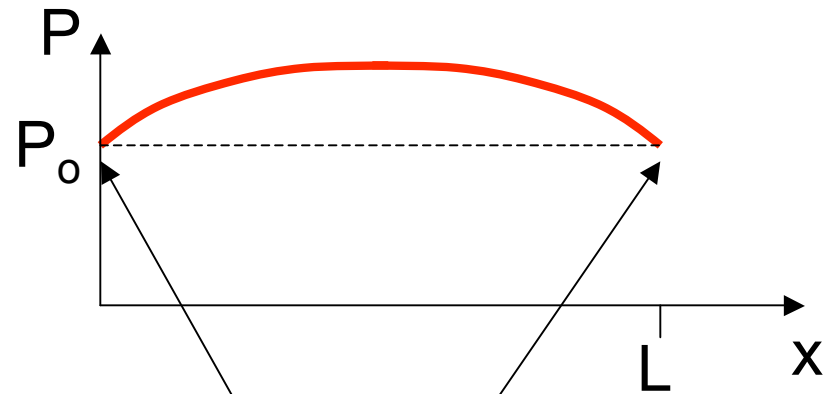
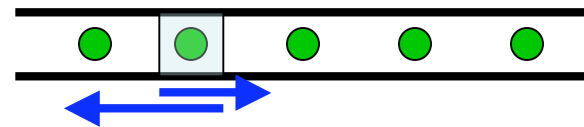
$$v \approx \sqrt{\frac{P_o}{\rho_o}}$$



transverse

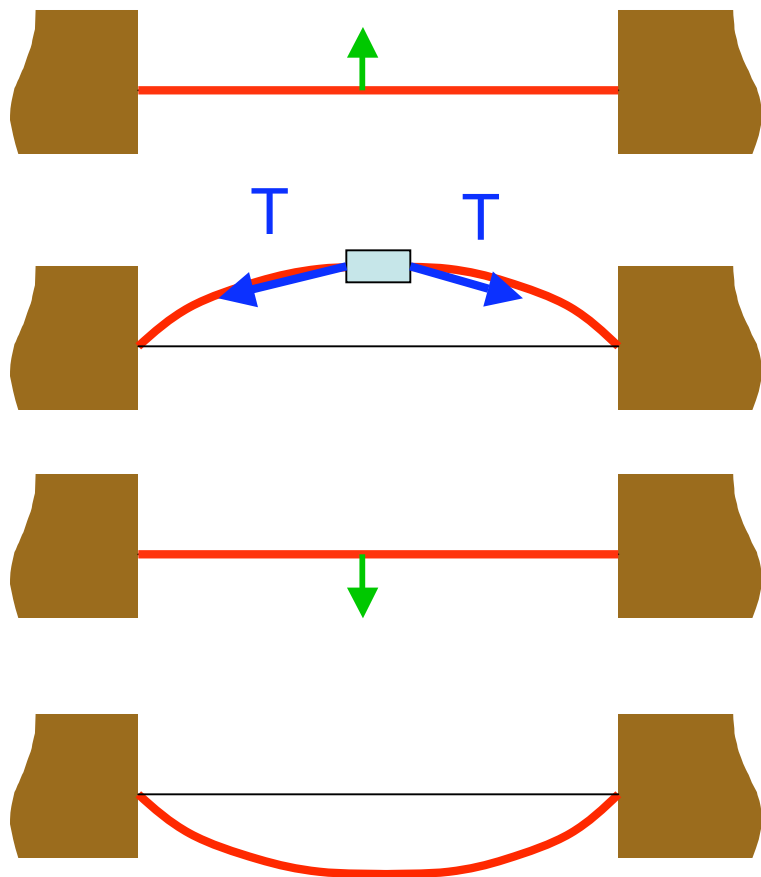


high P

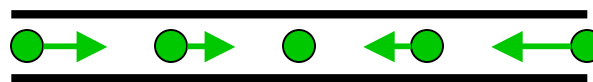
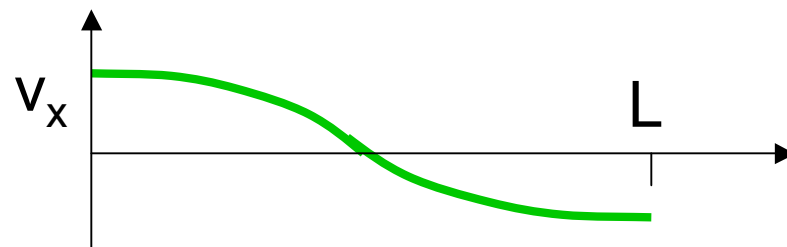


pressure nodes

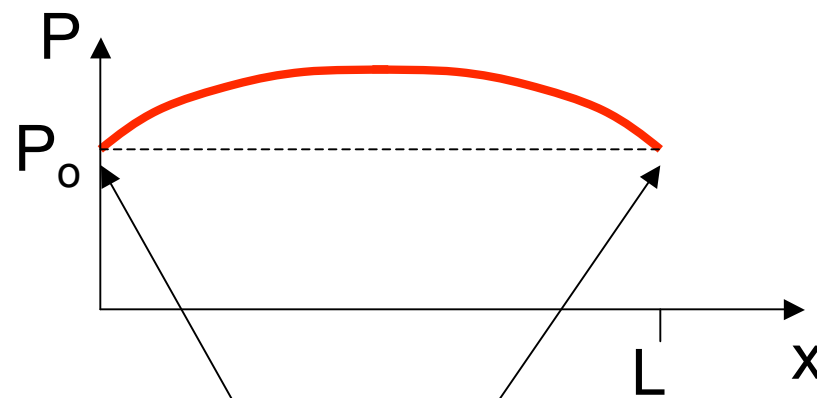
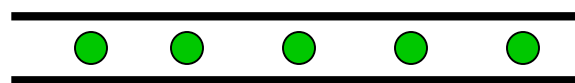
velocity
antinodes



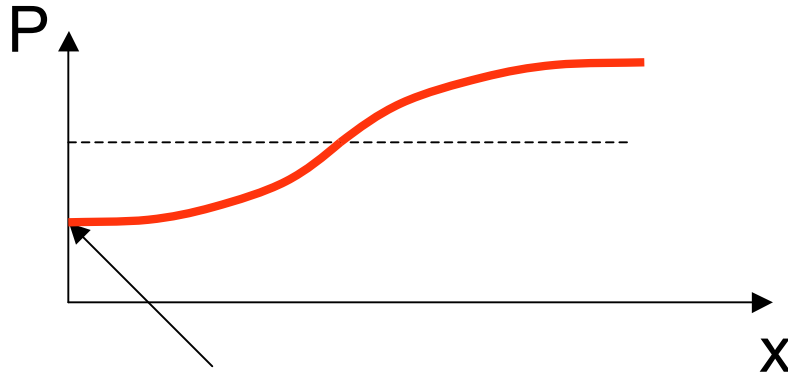
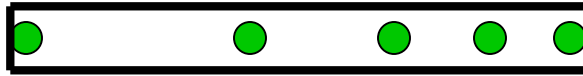
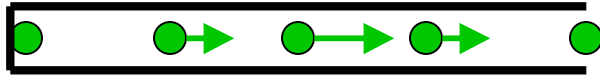
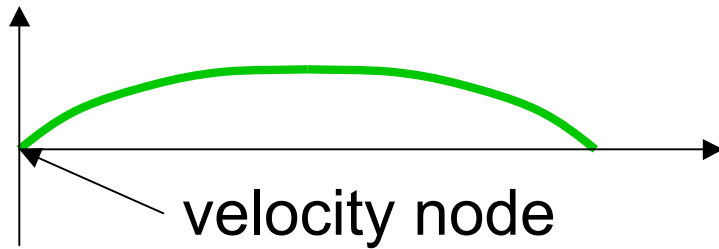
transverse



high P

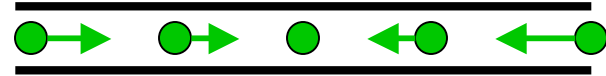
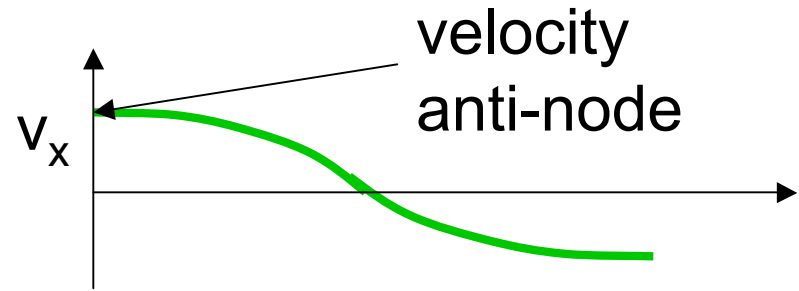


pressure
nodes

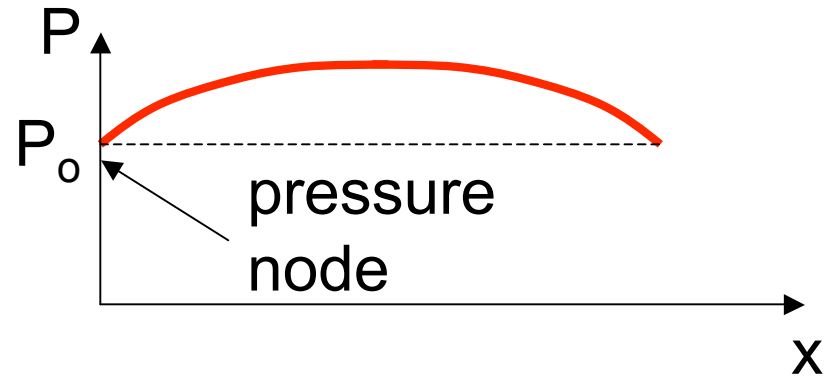
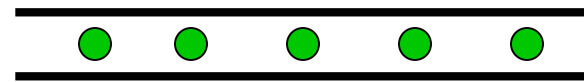


pressure anti-node

closed end



high P



open end

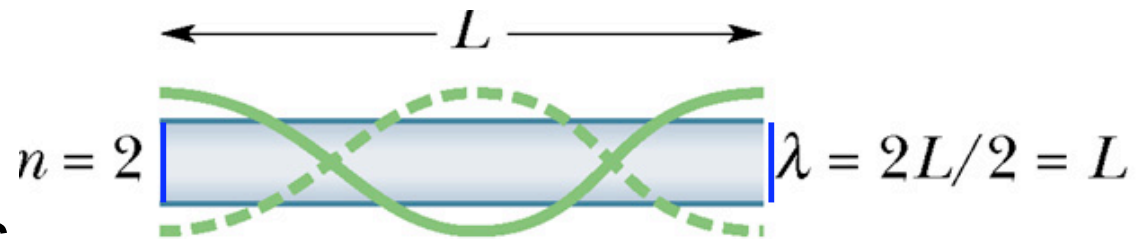
A penny whistle:

two closed ends

closed

closed

$n=1$ (not shown)



The green curves represent:



A pressure

B velocity

C music

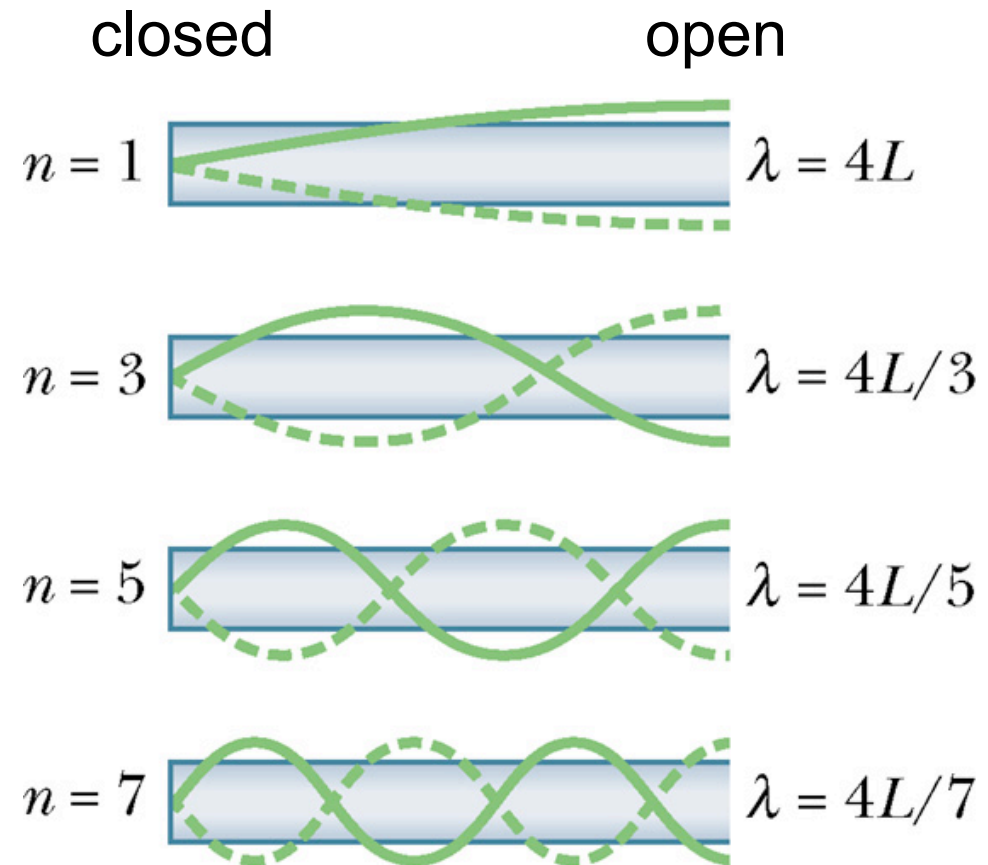


A flute:

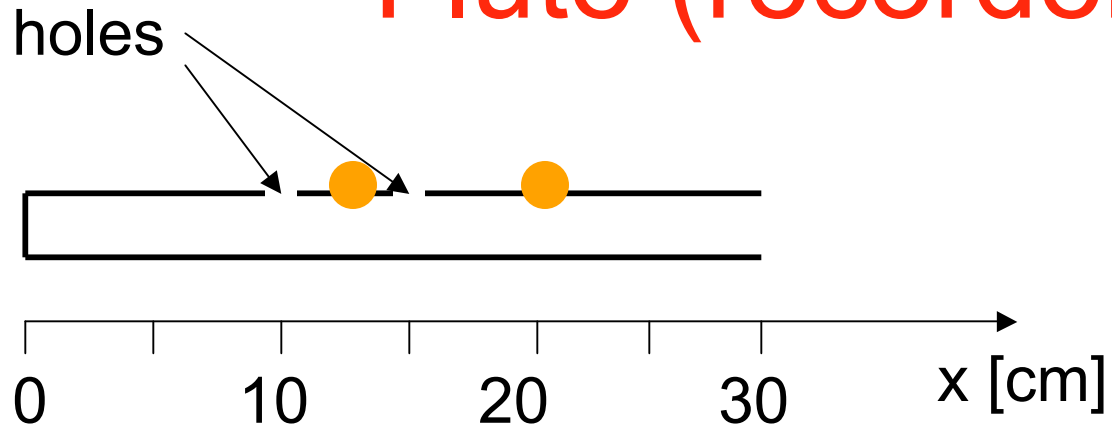
one open end,
one closed end

The green curves
represent:

- A pressure
- B velocity
- C music

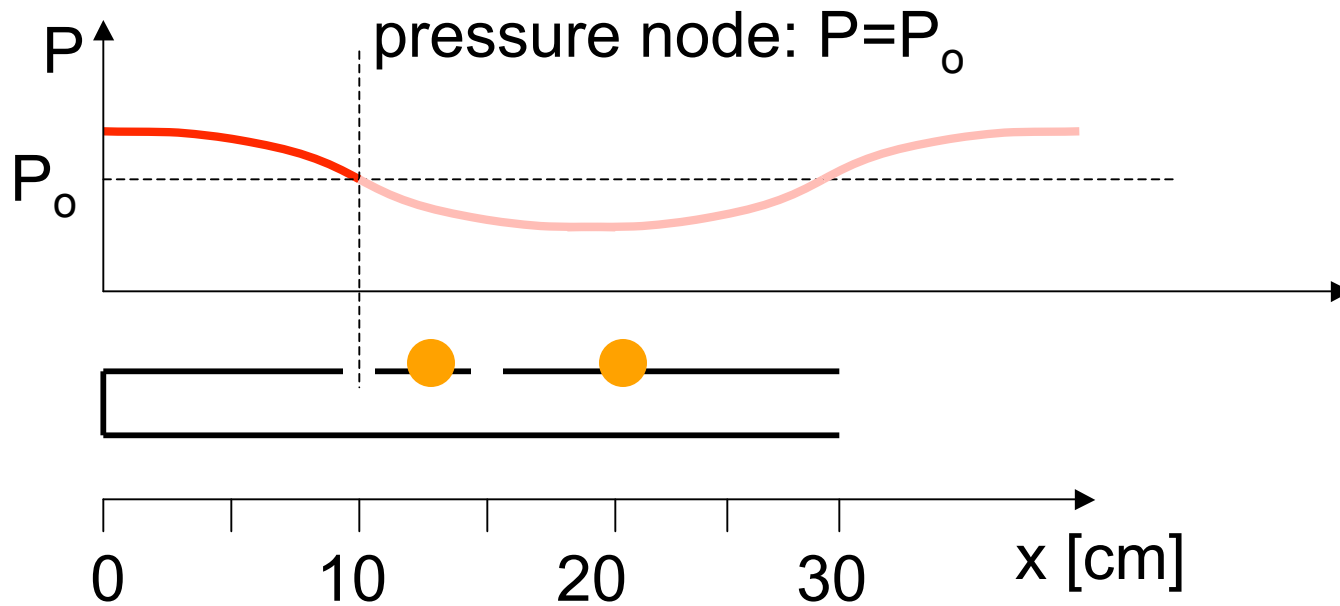


Flute (recorder)



The wavelength of the fundamental is

- A 20 cm
- B 30 cm
- C 40 cm
- D 60 cm
- E 120 cm



The wavelength of the fundamental is

- A 20 cm
- B 30 cm
- C 40 cm**
- D 60 cm
- E 120 cm

$$\lambda = 4L = 40 \text{ cm}$$