

Projectile motion



Kinematics in 2d

one dimension: $\frac{dv}{dt} = a$

two dimensions: $\frac{d\vec{v}}{dt} = \vec{a}$ $\left\{ \begin{array}{l} \frac{dv_x}{dt} = a_x \\ \frac{dv_y}{dt} = a_y \end{array} \right.$

Kinematics in 2d


one
dimension:

$$x - x_i = v_i t + \frac{1}{2} a t^2$$

two
dimensions:

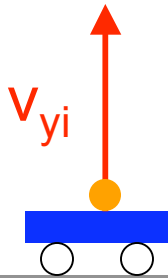
$$\vec{\mathbf{r}} - \vec{\mathbf{r}}_i = \vec{\mathbf{v}}_i t + \frac{1}{2} \vec{\mathbf{a}} t^2 \quad \left\{ \begin{array}{l} x - x_i = v_{xi} t + \frac{1}{2} a_x t^2 \\ y - y_i = v_{yi} t + \frac{1}{2} a_y t^2 \end{array} \right.$$

Decomposing motion


$$a = -gj$$

Ball launched* vertically upward

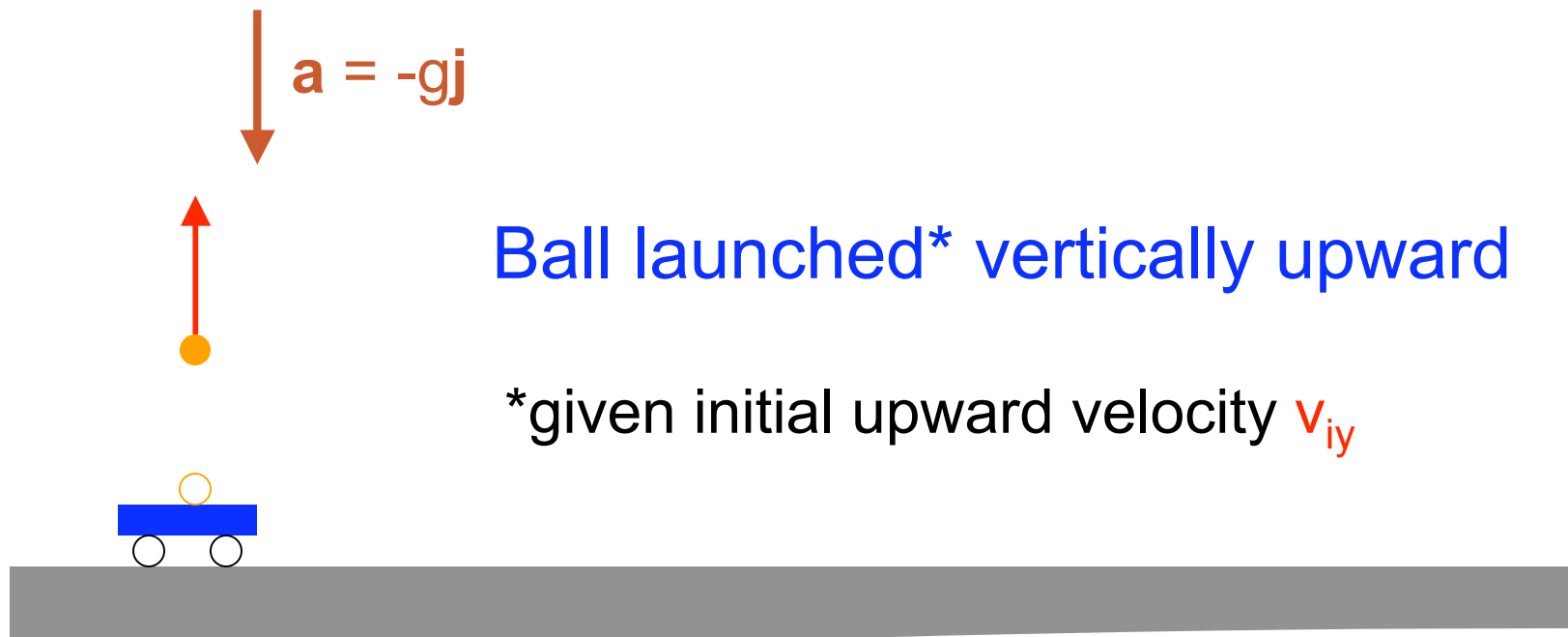
*given initial upward velocity v_{iy}



1d motion of ball:

$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

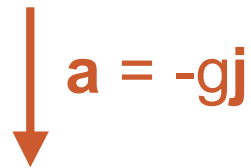
Decomposing motion



1d motion of ball:

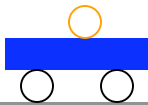
$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

Decomposing motion



Ball launched* vertically upward

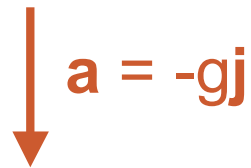
*given initial upward velocity v_{iy}



1d motion of ball:

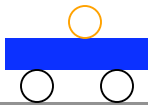
$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

Decomposing motion



Ball launched* vertically upward

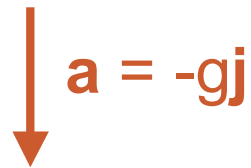
*given initial upward velocity v_{iy}



1d motion of ball:

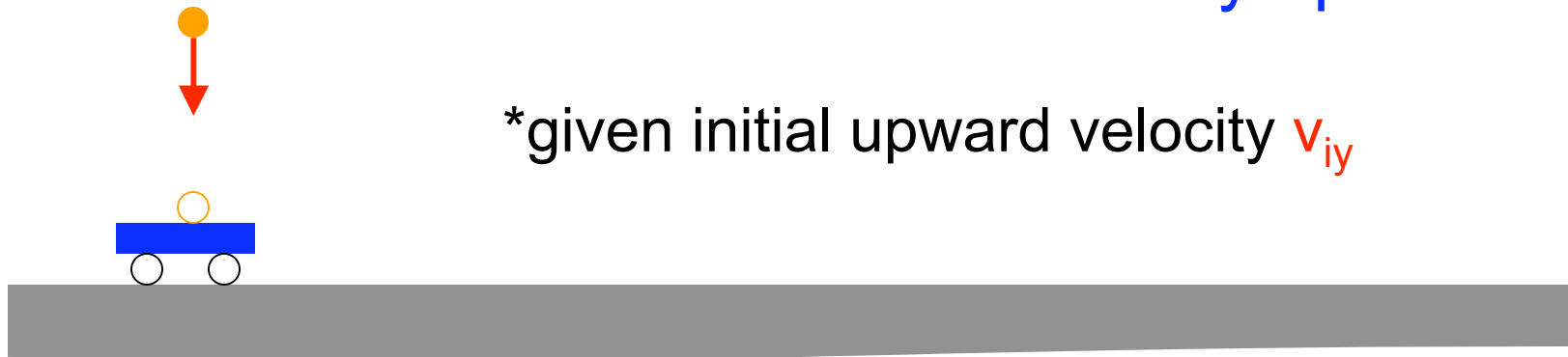
$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

Decomposing motion



Ball launched* vertically upward


*given initial upward velocity v_{iy}



1d motion of ball:

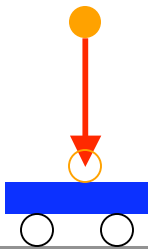
$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

Decomposing motion


$$a = -gj$$

Ball launched* vertically upward


*given initial upward velocity v_{iy}



1d motion of ball:

$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

Decomposing motion


$$a = -gj$$

Ball launched* vertically upward

*given initial upward velocity v_{iy}



1d motion of ball:

$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

Decomposing motion

Car pushed* rightward

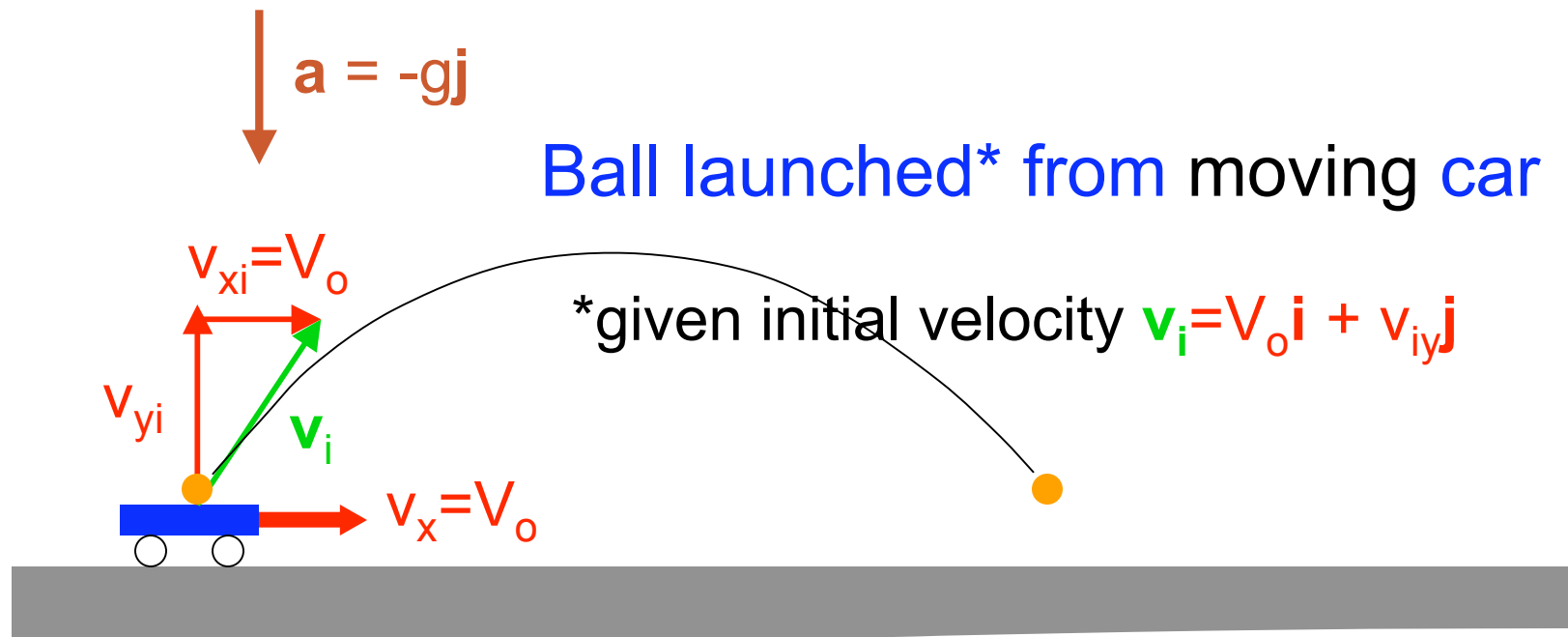
*given initial rightward velocity V_o



1d motion of car:

$$x(t) = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = x_i + V_o t$$

Decomposing motion

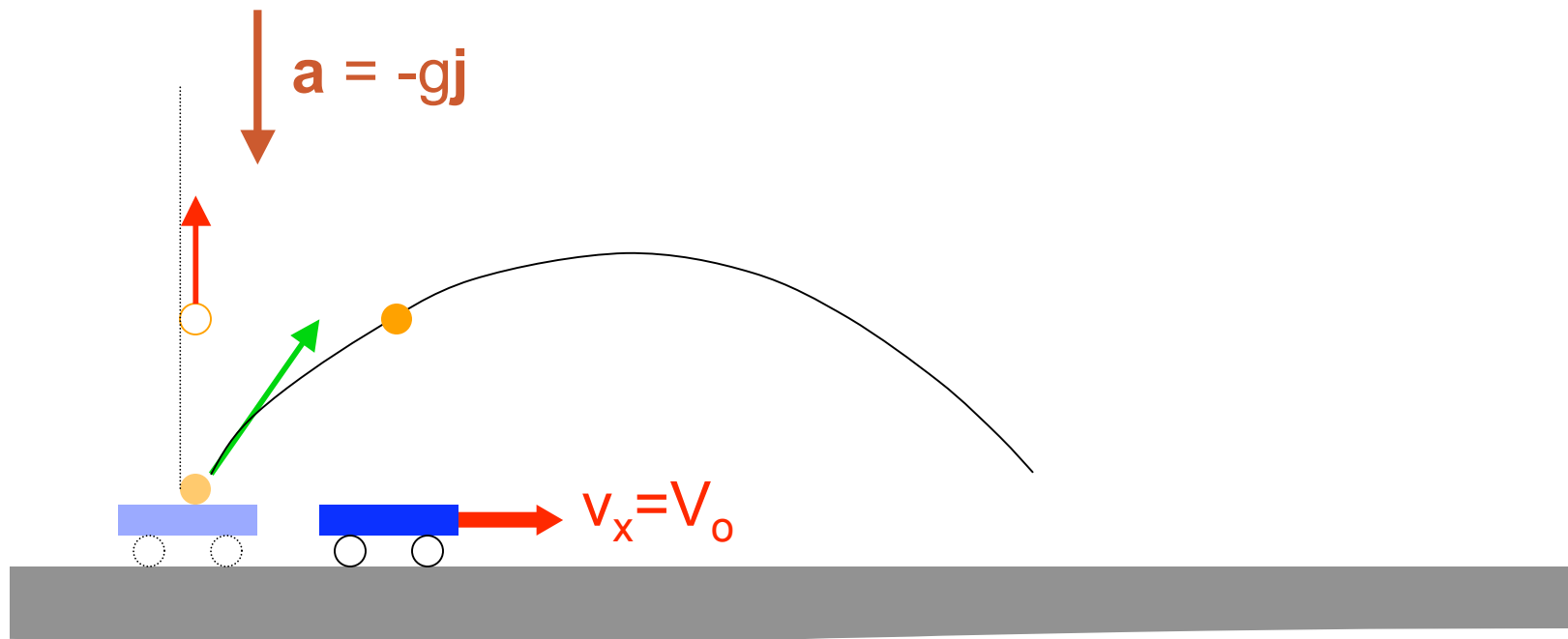


2d motion of ball:

$$x(t) = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = x_i + V_o t$$

$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

Decomposing motion



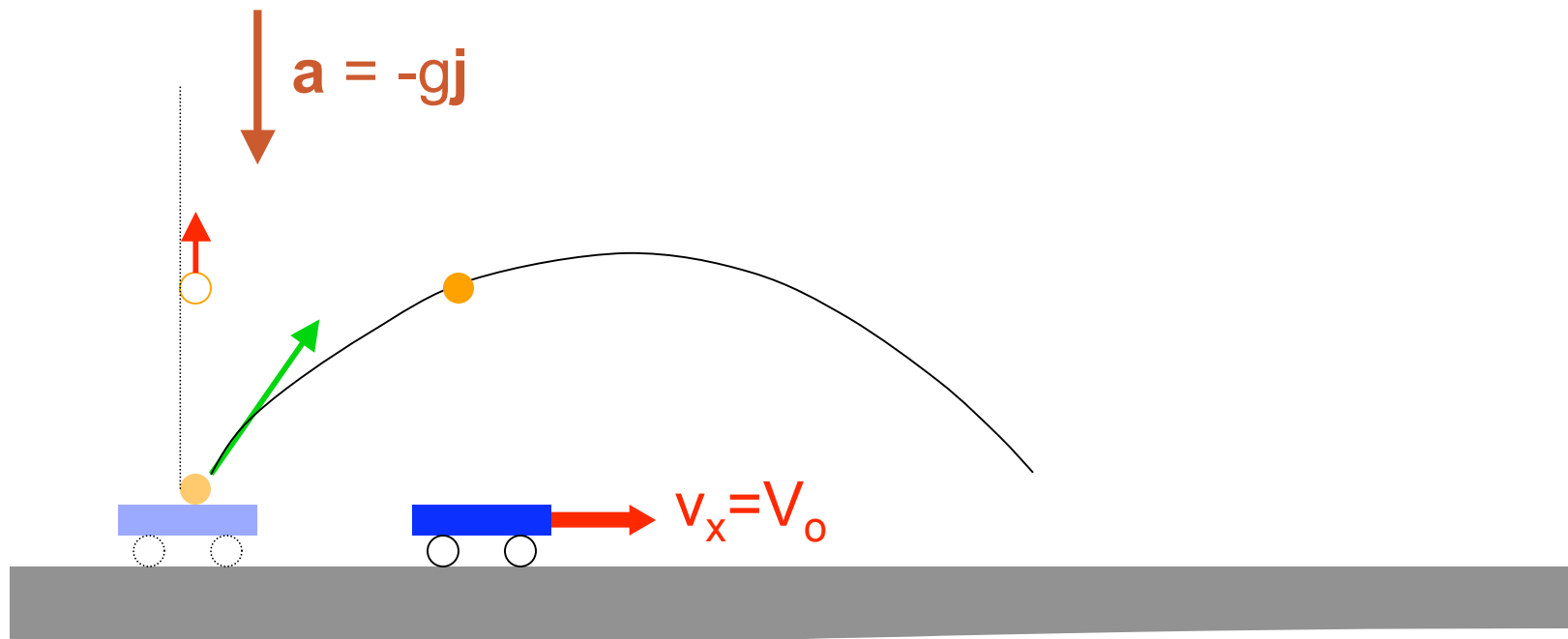
2d motion of ball:

$$x(t) = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = x_i + V_o t$$

$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

same as car

Decomposing motion



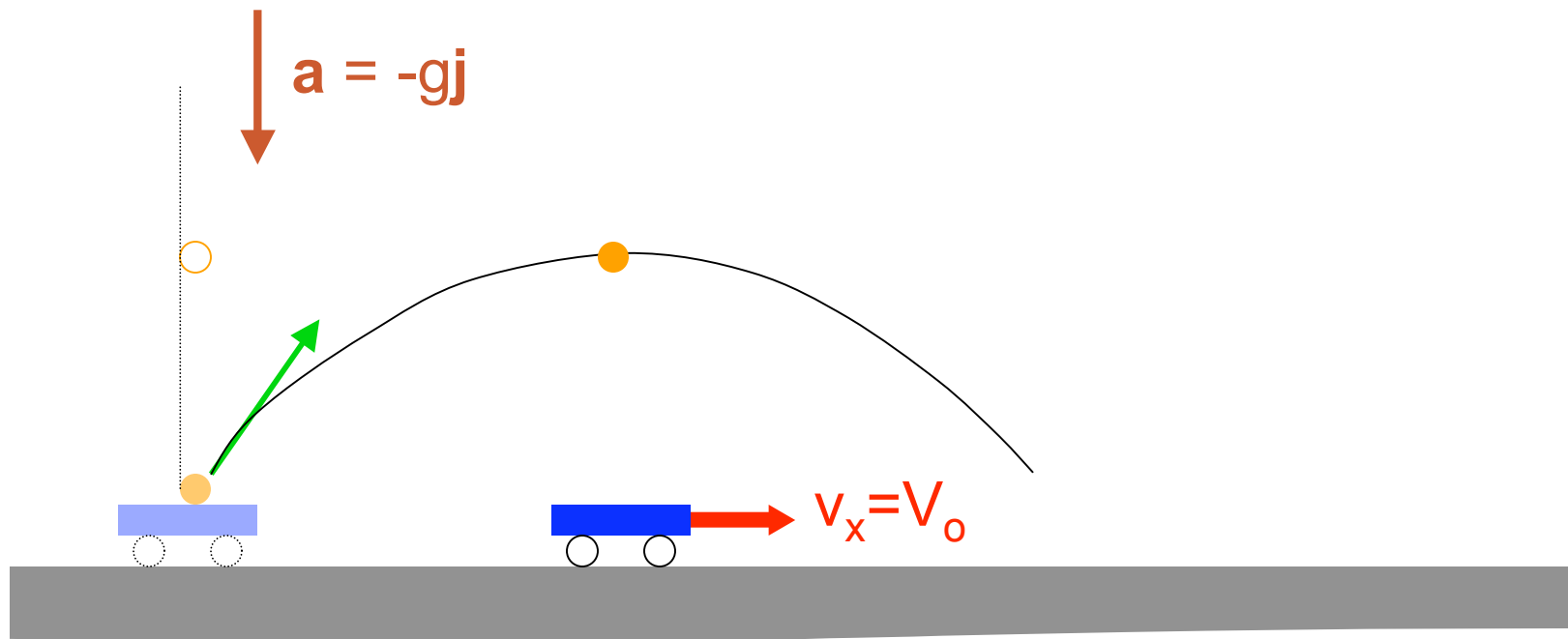
2d motion of ball:

$$x(t) = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = x_i + V_o t$$

$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

same as car

Decomposing motion



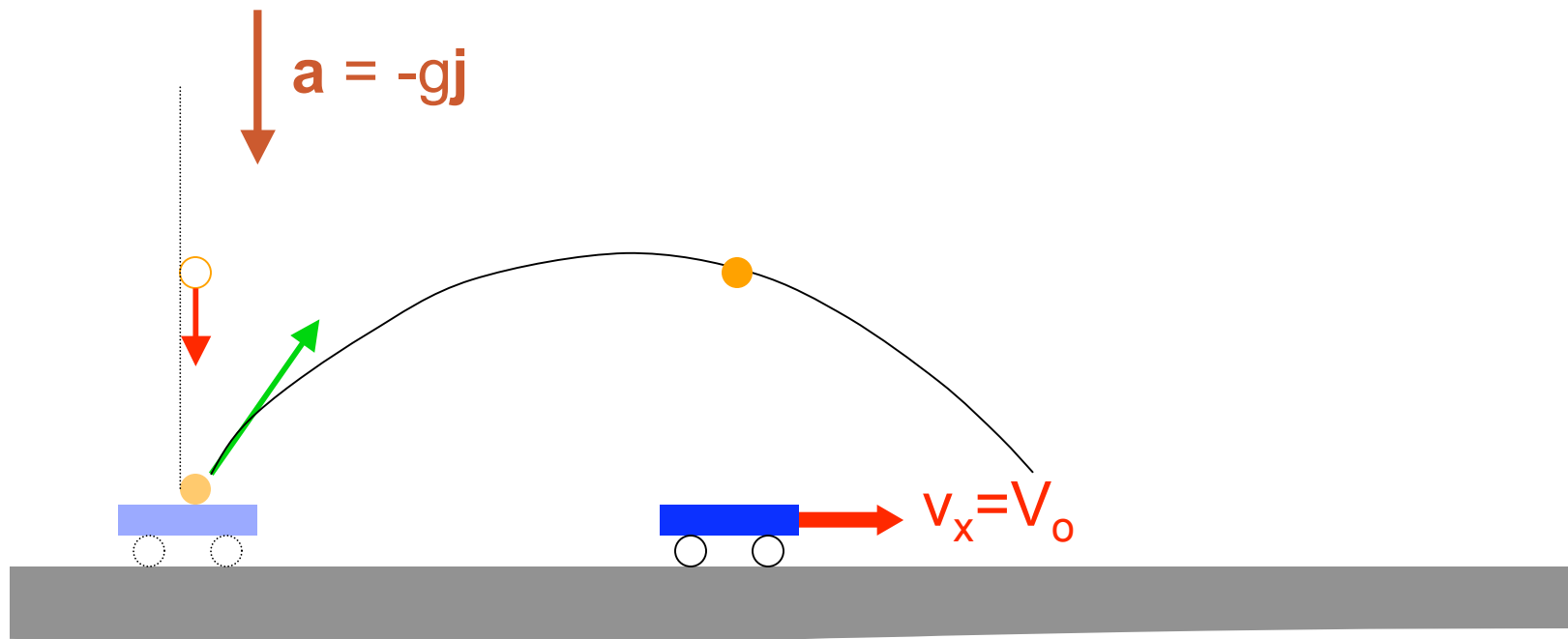
2d motion of ball:

$$x(t) = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = x_i + V_o t$$

$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

same as car

Decomposing motion



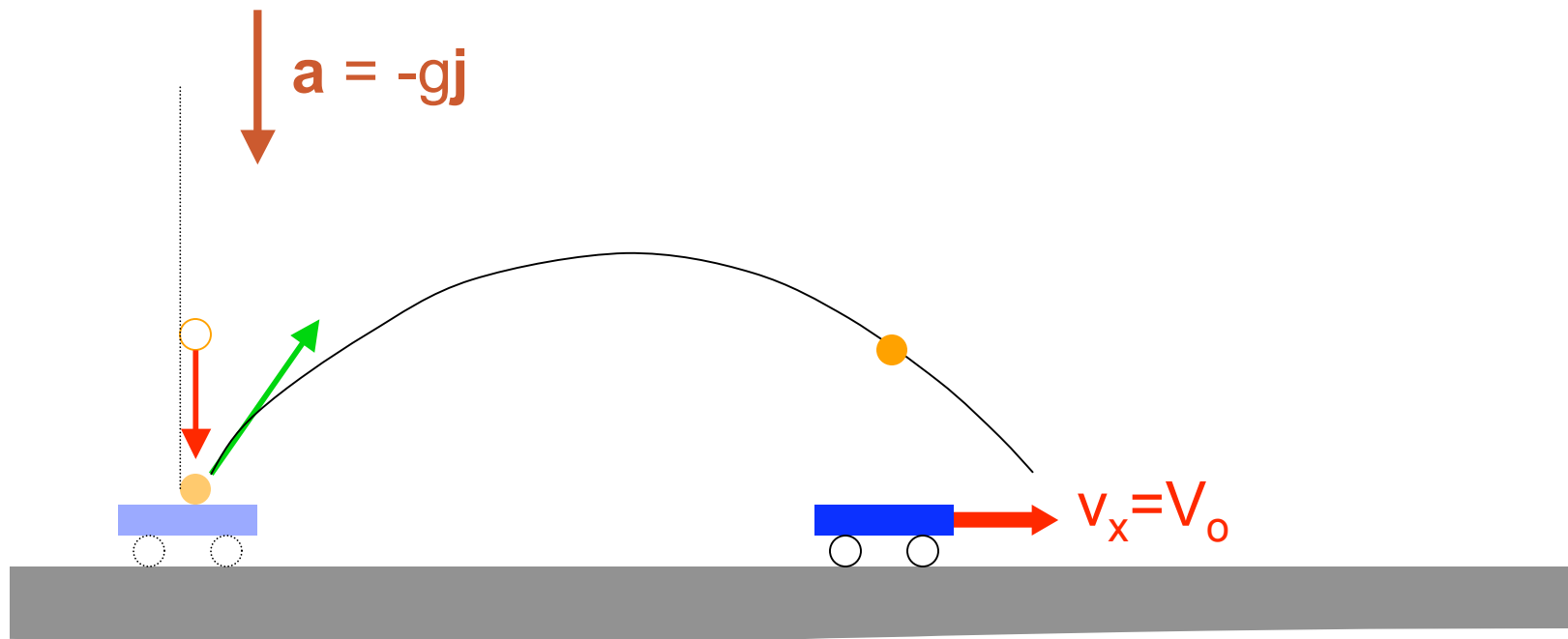
2d motion of ball:

$$x(t) = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = x_i + V_o t$$

$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

same as car

Decomposing motion



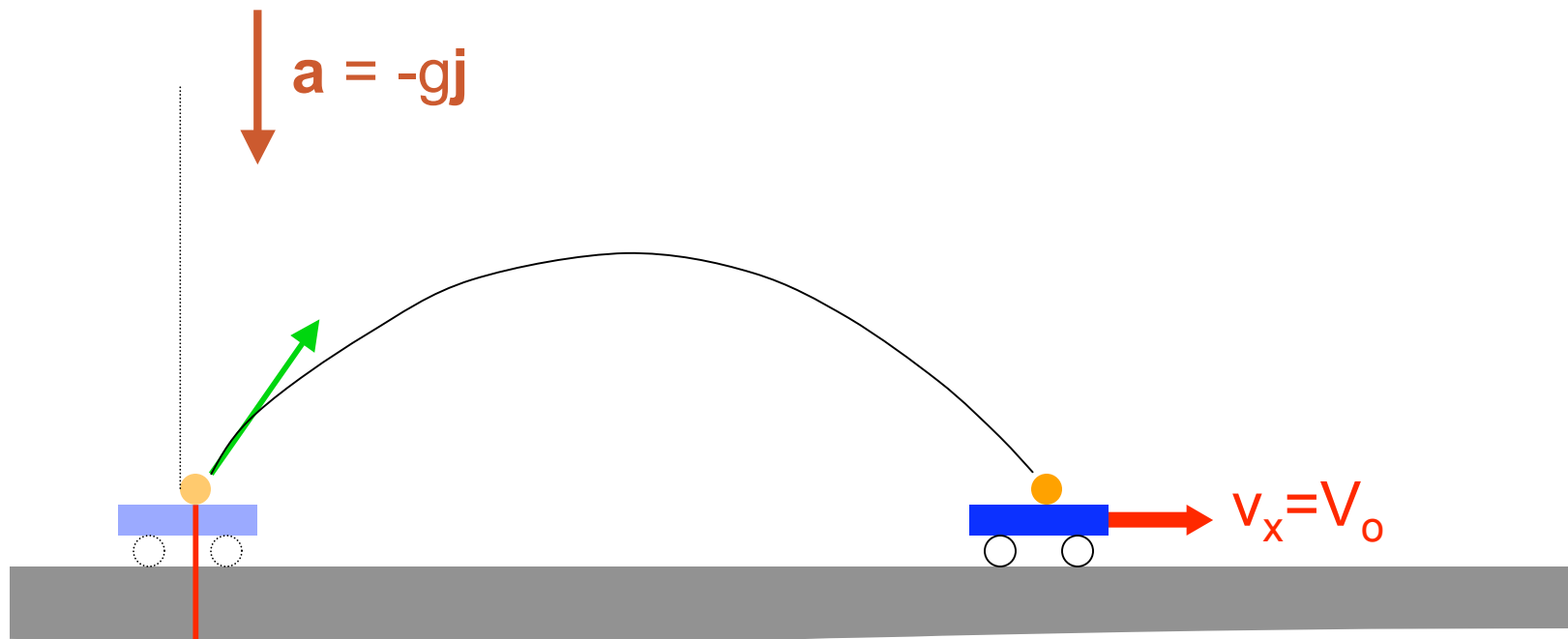
2d motion of ball:

$$x(t) = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = x_i + V_o t$$

$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

same as car

Decomposing motion



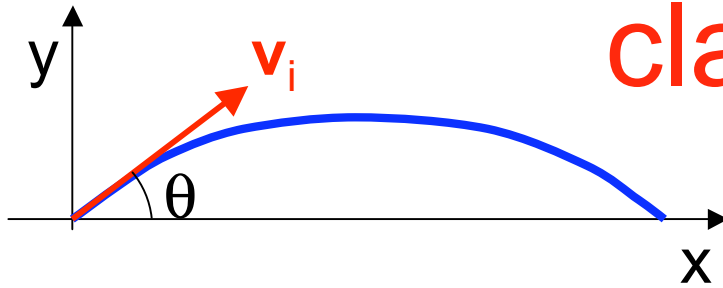
2d motion of ball:

$$x(t) = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = x_i + V_o t$$

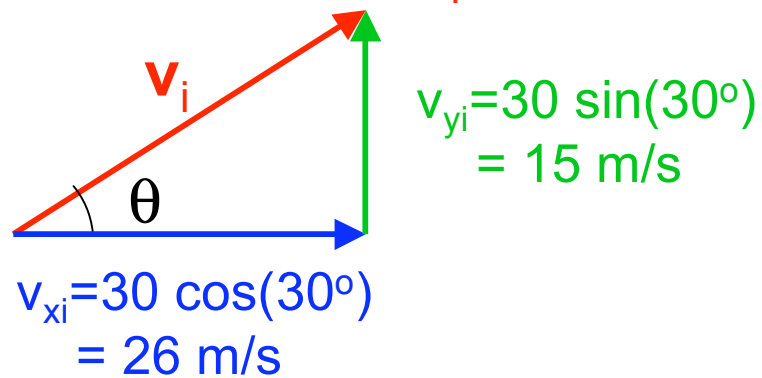
$$y(t) = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = v_{yi}t - \frac{g}{2}t^2$$

same as car

classic range problem



1. Decompose \mathbf{v}_i



2. Analyze one component

- A. x-motion for x_f
- B. x-motion for v_{xf}
- C. y-motion for t
- D. y-motion for v_{yf}

An outfielder throws a ball at speed $|\mathbf{v}_i| = 30 \text{ m/s}$ at an angle $\theta = 30^\circ$. How far does it travel?

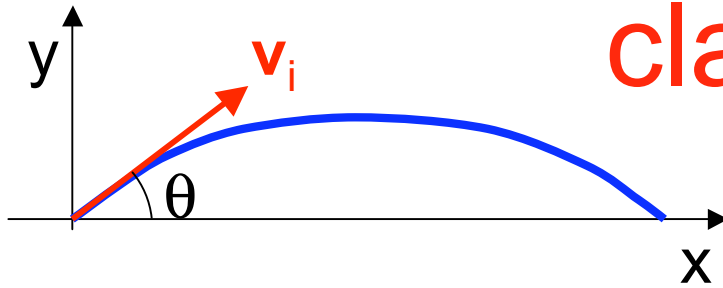
$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

$$0 = 0 + 15t - 4.9t^2$$

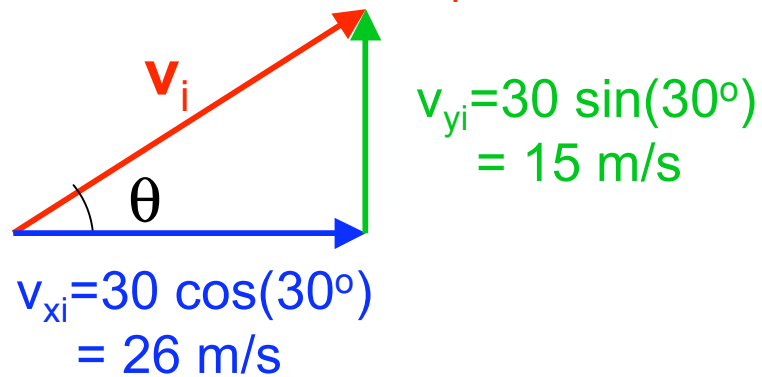
$$0 = t(15 - 4.9t)$$

$$t_f = 15/4.9 = 3.1 \text{ s}$$

classic range problem



1. Decompose \mathbf{v}_i



2. Analyze one component

- A. x-motion for x_f
- B. x-motion for v_{xf}
- C. y-motion for t
- D. y-motion for v_{yf}

An outfielder throws a ball at speed $|\mathbf{v}_i| = 30 \text{ m/s}$ at an angle $\theta = 30^\circ$. How far does it travel?

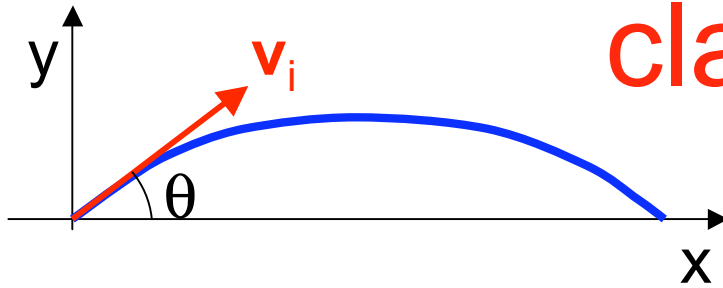
$$t_f = 15/4.9 = 3.1 \text{ s}$$

3. Analyze other component

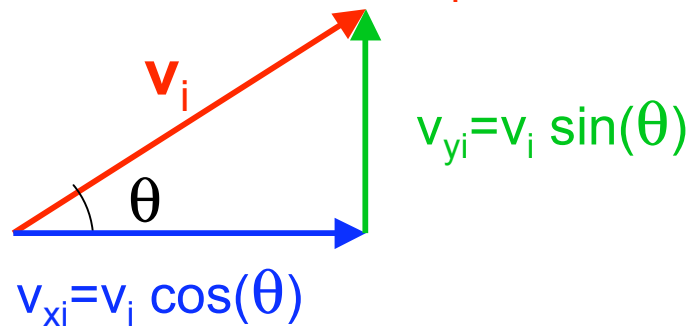
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$x_f = 0 + 26 t_f + 0 = 81 \text{ m}$$

classic range problem



1. Decompose v_i



2. Analyze one component

- A. x-motion for x_f
- B. x-motion for v_{xf}
- C. y-motion for t
- D. y-motion for v_{yf}

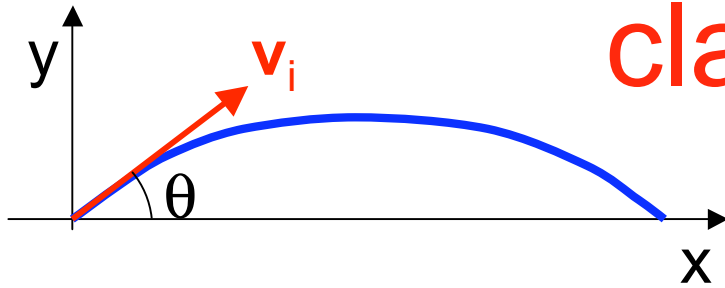
An outfielder throws a ball at speed v_i at an angle θ . How far does it travel?

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

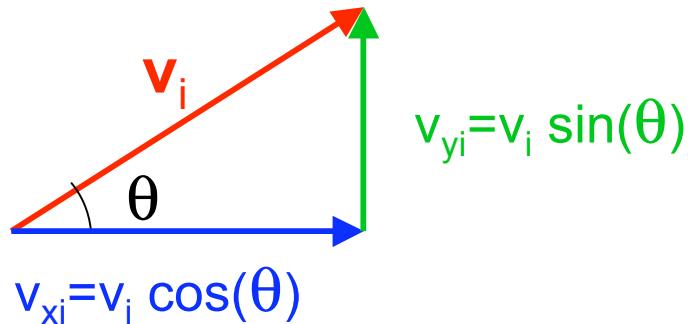
$$0 = t[v_i \sin(\theta) - \frac{1}{2}gt]$$

$$t_f = \frac{2v_i \sin(\theta)}{g}$$

classic range problem



1. Decompose v_i



2. Analyze one component

- A. x-motion for x_f
- B. x-motion for v_{xf}
- C. y-motion for t
- D. y-motion for v_{yf}

An outfielder throws a ball at speed v_i at an angle θ . How far does it travel?

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

$$0 = t[v_i \sin(\theta) - \frac{1}{2}gt]$$

$$t_f = \frac{2v_i \sin(\theta)}{g}$$

3. Analyze other component

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$x_f = v_i \cos(\theta)t_f$$

$$x_f = \frac{v_i^2}{g} 2 \sin(\theta) \cos(\theta) = \frac{v_i^2}{g} \sin(2\theta)$$

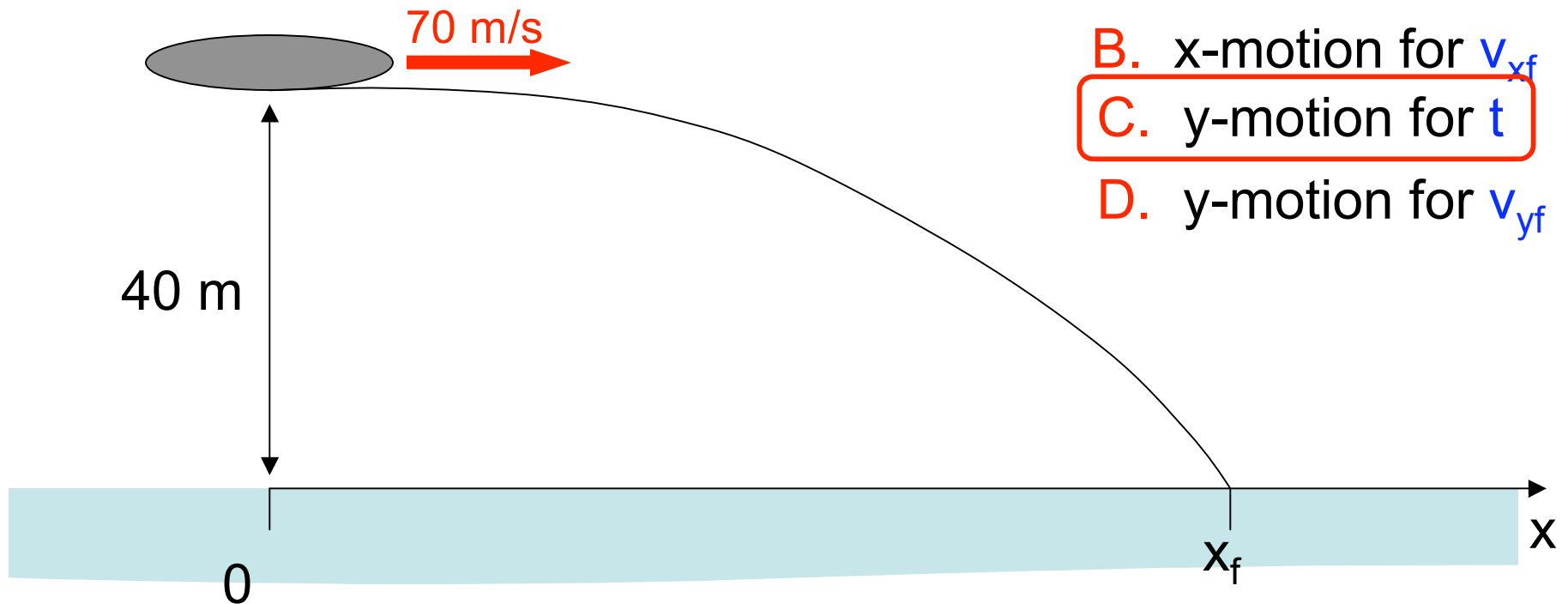
Old Exam Problem

A rescue plane is flying level at 40 m above the water, at a speed of 70 m/s. The pilot wishes to drop a deflated life raft to a person in the water.

- At what distance from his target should he release the raft?
- At what speed will the package hit the water?
- At what angle will the package hit the water?

First step is

- x-motion for t
- x-motion for v_{xf}
- y-motion for t
- y-motion for v_{yf}



Which Equation would you use?

A $v_{yf} = v_{yi} + a_y t$

B $y_f = y_i + \frac{1}{2}(v_{yf} + v_{yi})t$

C $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

D $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$

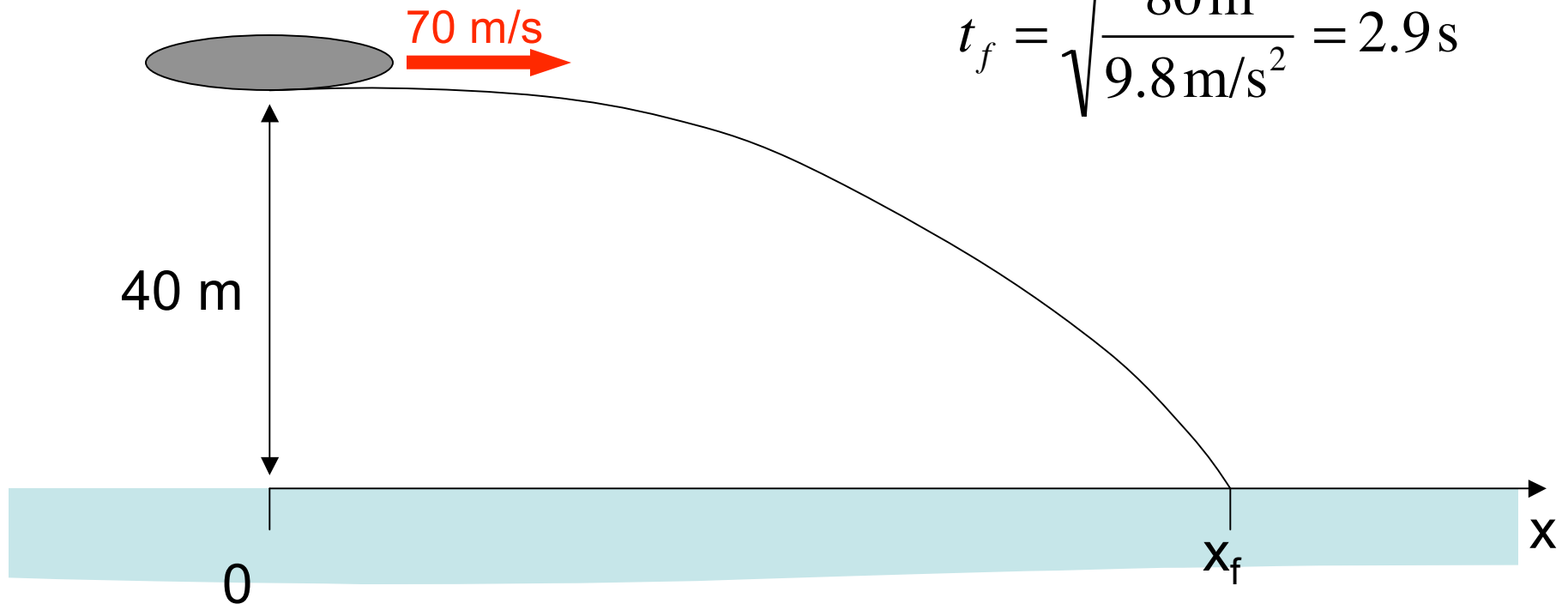
Old Exam Problem

A rescue plane is flying level at 40 m above the water, at a speed of 70 m/s. The pilot wishes to drop a deflated life raft to a person in the water.

- At what distance from his target should he release the raft?
- At what speed will the package hit the water?
- At what angle will the package hit the water?

$$0 = 40 \text{ m} - \frac{1}{2} g t_f^2$$

$$t_f = \sqrt{\frac{80 \text{ m}}{9.8 \text{ m/s}^2}} = 2.9 \text{ s}$$



Which Equation would you use?

A $v_{xf} = v_{xi} + a_x t$

B $x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})t$

C $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

D $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

Old Exam Problem

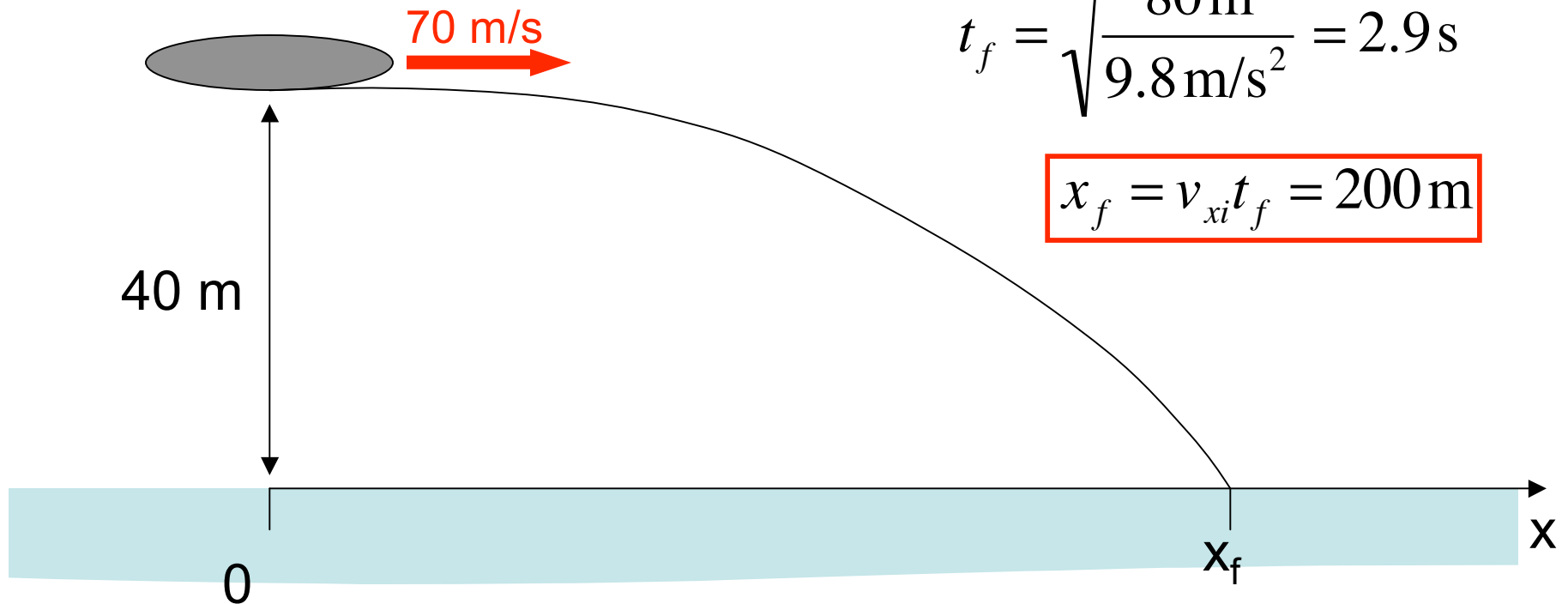
A rescue plane is flying level at 40 m above the water, at a speed of 70 m/s. The pilot wishes to drop a deflated life raft to a person in the water.

- At what distance from his target should he release the raft?
- At what speed will the package hit the water?
- At what angle will the package hit the water?

$$0 = 40 \text{ m} - \frac{1}{2} g t_f^2$$

$$t_f = \sqrt{\frac{80 \text{ m}}{9.8 \text{ m/s}^2}} = 2.9 \text{ s}$$

$$x_f = v_{xi} t_f = 200 \text{ m}$$



Old Exam Problem

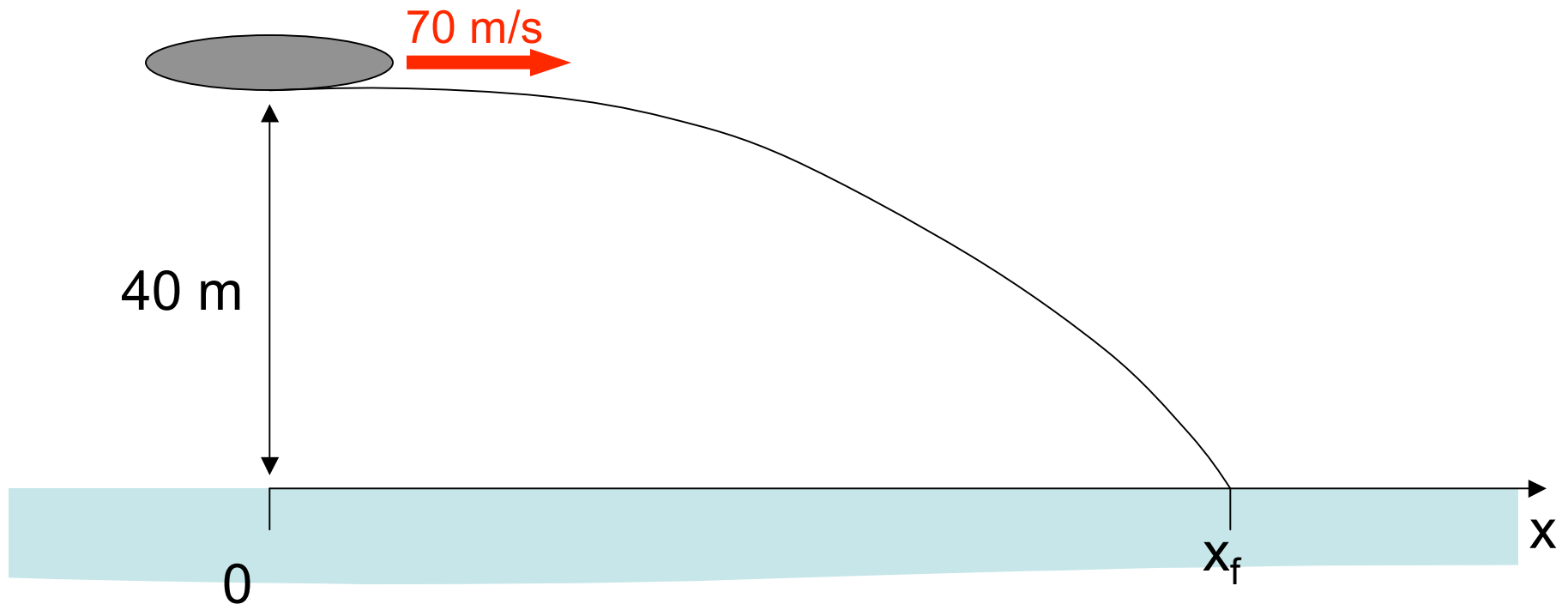
A rescue plane is flying level at 40 m above the water, at a speed of 70 m/s. The pilot wishes to drop a deflated life raft to a person in the water.

a. At what distance from his target should he release the raft?

b. At what speed will the package hit the water?

c. At what angle will the package hit the water?

$$t_f = \sqrt{\frac{80 \text{ m}}{9.8 \text{ m/s}^2}} = 2.9 \text{ s}$$



Which Equation would you use?

A $v_{yf} = v_{yi} + a_y t$

B $y_f = y_i + \frac{1}{2}(v_{yf} + v_{yi})t$

C $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

D $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$

Old Exam Problem

A rescue plane is flying level at 40 m above the water, at a speed of 70 m/s. The pilot wishes to drop a deflated life raft to a person in the water.

a. At what distance from his target should he release the raft?

b. At what speed will the package hit the water?

c. At what angle will the package hit the water?

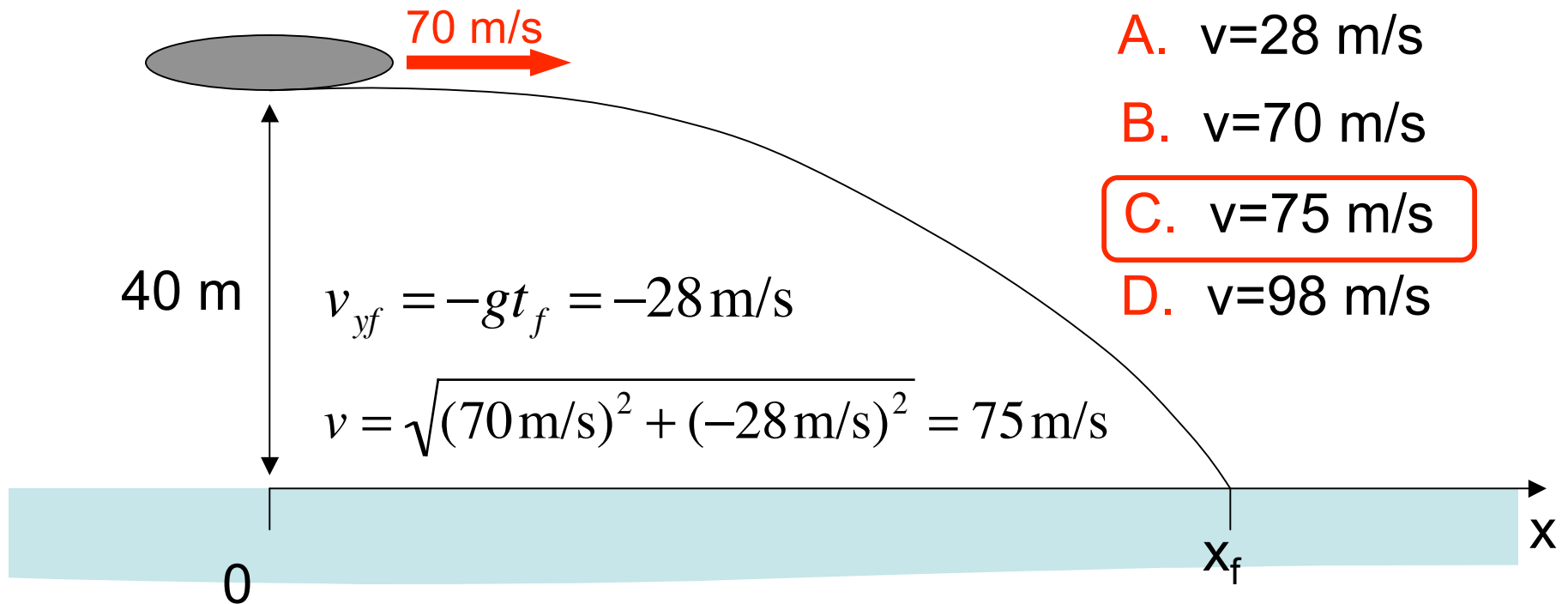
$$t_f = \sqrt{\frac{80\text{ m}}{9.8\text{ m/s}^2}} = 2.9\text{ s}$$

A. $v=28\text{ m/s}$

B. $v=70\text{ m/s}$

C. $v=75\text{ m/s}$

D. $v=98\text{ m/s}$



Old Exam Problem

A rescue plane is flying level at 40 m above the water, at a speed of 70 m/s. The pilot wishes to drop a deflated life raft to a person in the water.

a. At what distance from his target should he release the raft?

b. At what speed will the package hit the water?

c. At what angle will the package hit the water?

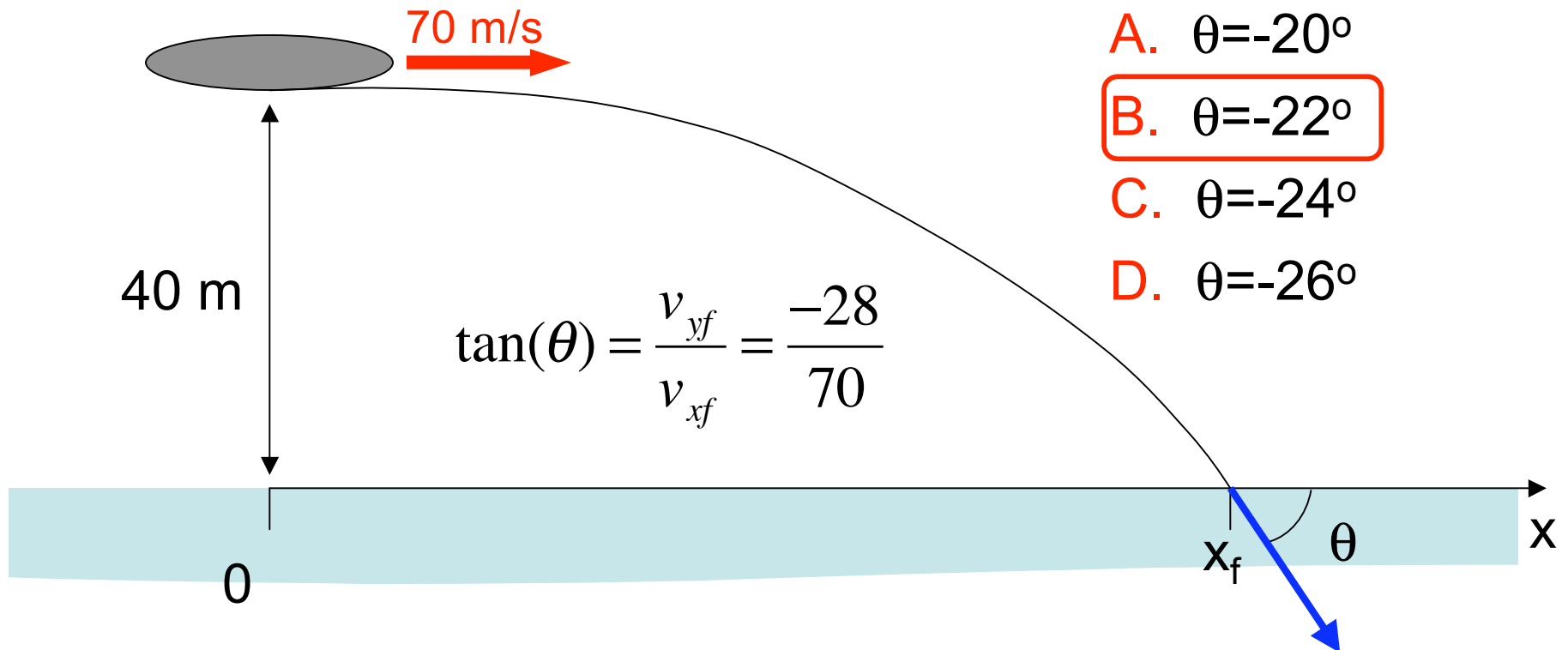
$$v_{yf} = -gt_f = -28 \text{ m/s}$$

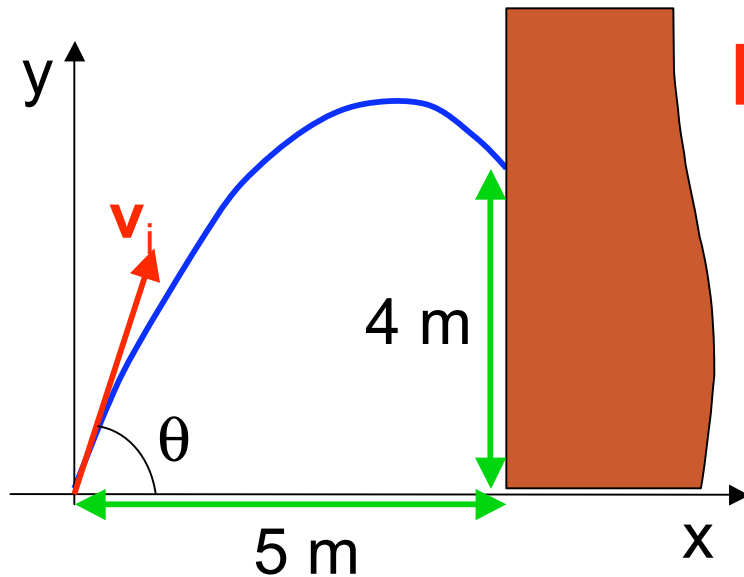
A. $\theta = -20^\circ$

B. $\theta = -22^\circ$

C. $\theta = -24^\circ$

D. $\theta = -26^\circ$

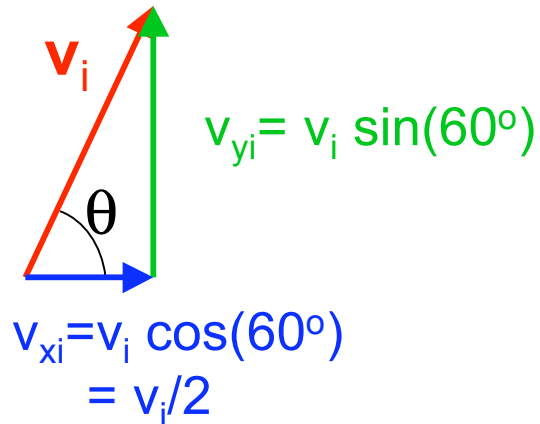




modified problem

Sandra manages to throw a spare set of keys into her open window 4.0 m off the ground. She throws from 5.0 m away at an angle of 60° . At what speed did the keys leave her hand?

1. Decompose \mathbf{v}_i



2. Analyze one component

- A. x-motion for x_f
- B. x-motion for t
- C. x-motion for v_{xf}
- D. y-motion for t

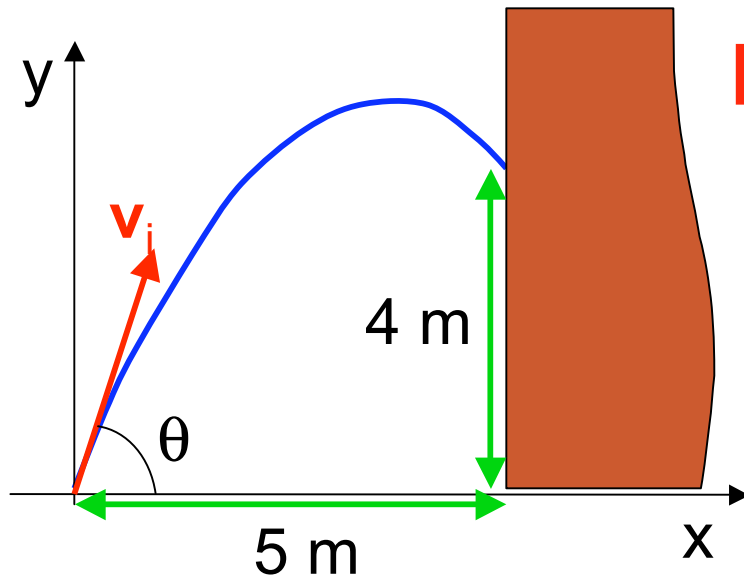
Which Equation would you use?

A $v_{xf} = v_{xi} + a_x t$

B $x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})t$

C $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

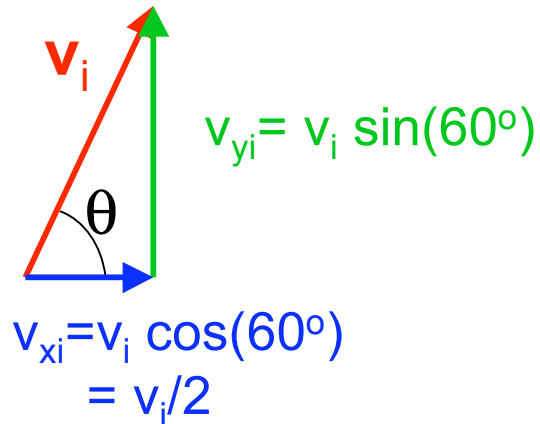
D $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$



modified problem

Sandra manages to throw a spare set of keys into her open window 4.0 m off the ground. She throws from 5.0 m away at an angle of 60° . At what speed did the keys leave her hand?

1. Decompose \mathbf{v}_i



2. Analyze one component

A. x-motion for x_f

B. x-motion for t

C. x-motion for v_{xf}

D. y-motion for t

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$5 = 0 + 0.5 v_i t_f + 0$$

$$t_f = 10 \text{ m}/v_i$$

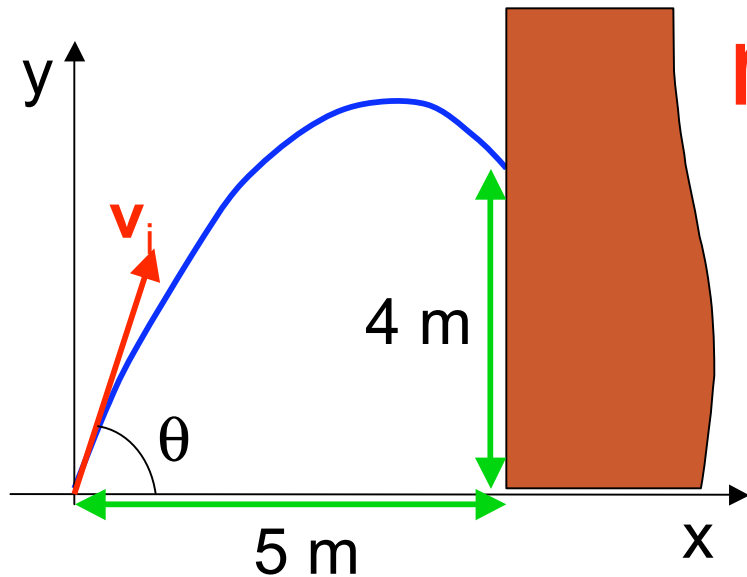
Which Equation would you substitute
the time expression into?

A $v_{yf} = v_{yi} + a_y t$

B $y_f = y_i + \frac{1}{2}(v_{yf} + v_{yi})t$

C $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

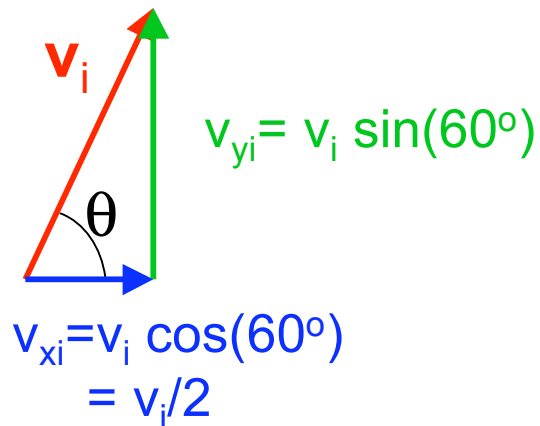
D $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$



modified problem

Sandra manages to throw a spare set of keys into her open window 4.0 m off the ground. She throws from 5.0 m away at an angle of 60° . At what speed did the keys leave her hand?

1. Decompose \mathbf{v}_i



3. Eliminate t from y -motion

$$t_f = 10 \text{ m}/v_i$$

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

A. $v_i = 5.2 \text{ m/s}$

B. $v_i = 8.6 \text{ m/s}$

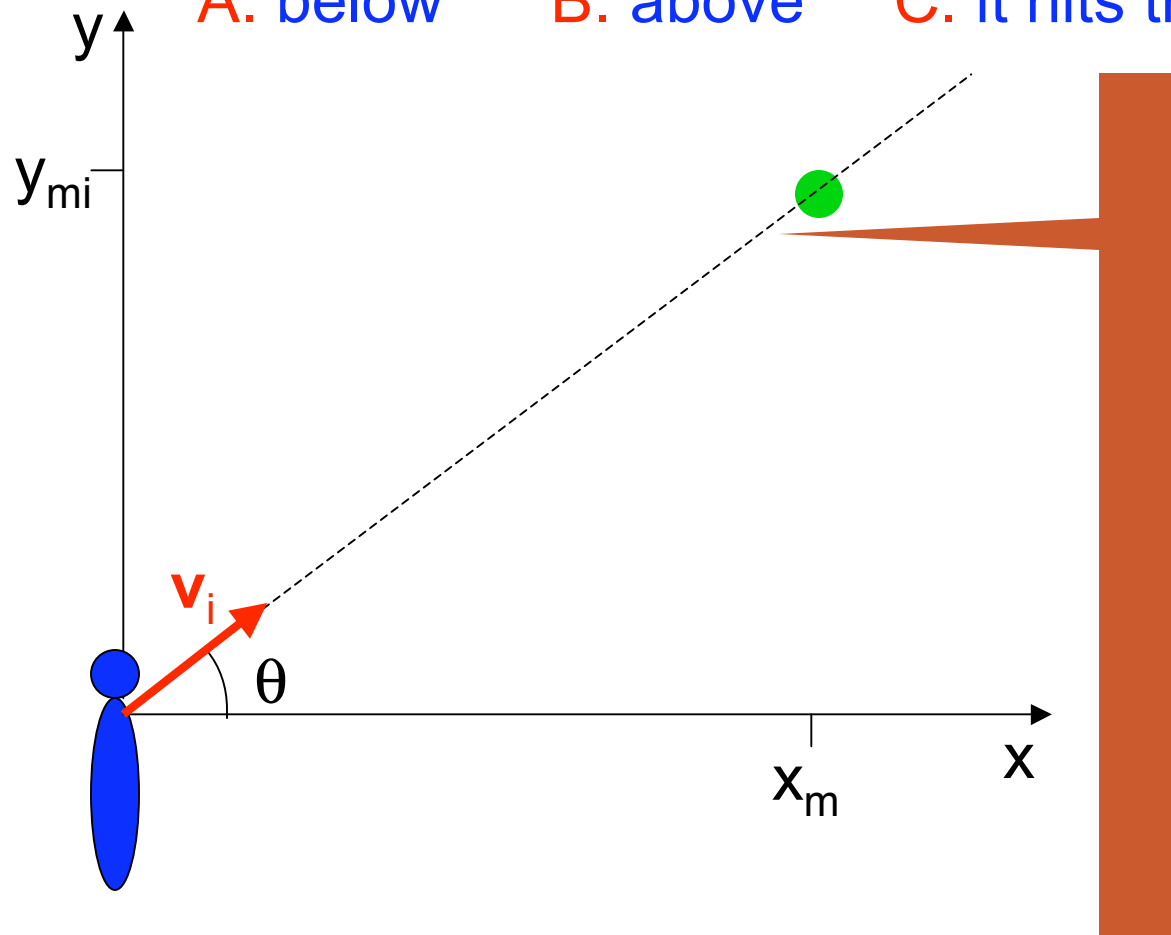
C. $v_i = 10.2 \text{ m/s}$

$$4 = 0 + v_i \sin(60^\circ) \frac{10}{v_i} - \frac{1}{2}g \frac{100}{v_i^2}$$

Monkey & Hunter

A monkey sits in a tree. A hunter having aimed directly at him fires a bullet at speed v_i . At that precise instant the monkey drops from the tree. Where does the bullet pass the wily monkey?

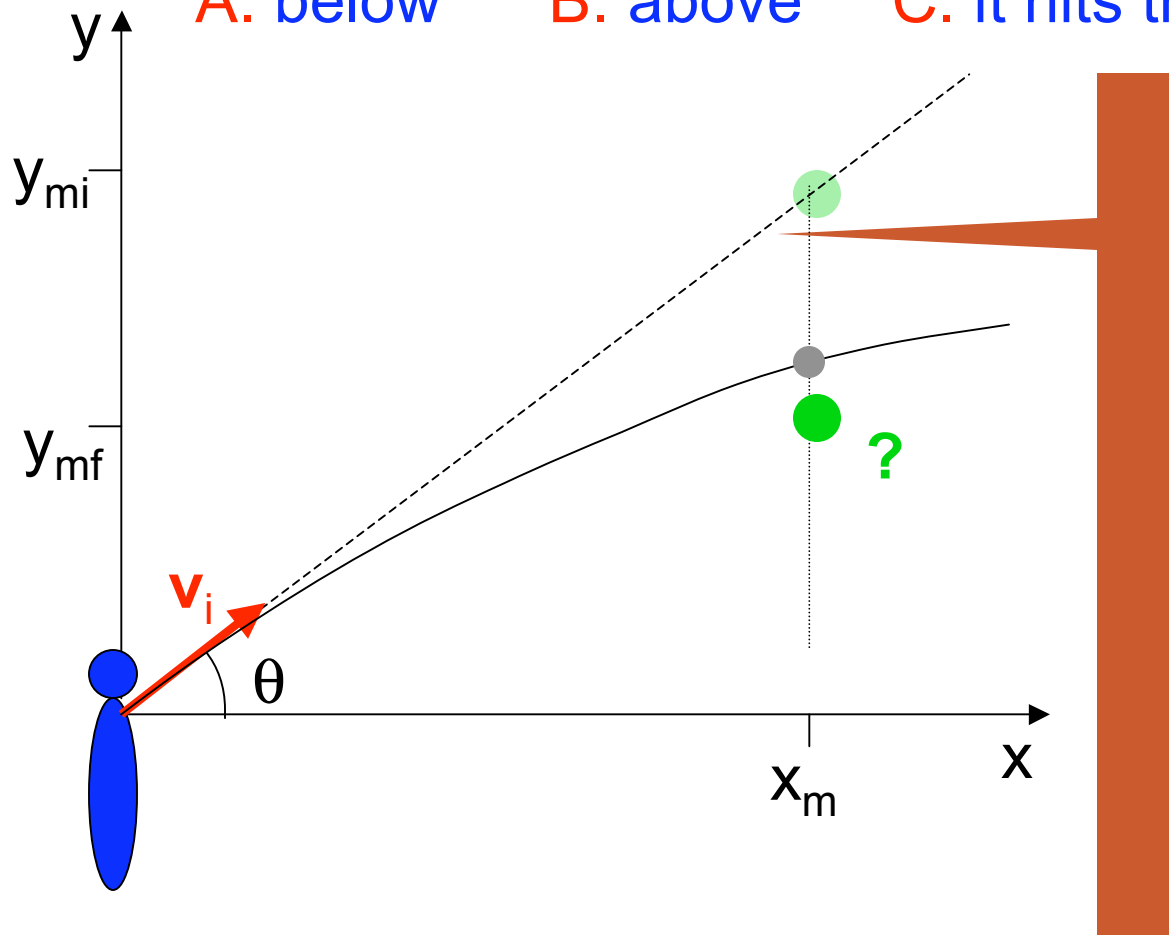
- A. below B. above C. it hits the monkey



Monkey & Hunter

A monkey sits in a tree. A hunter having aimed directly at him fires a bullet at speed v_i . At that precise instant the monkey drops from the tree. Where does the bullet pass the wily monkey?

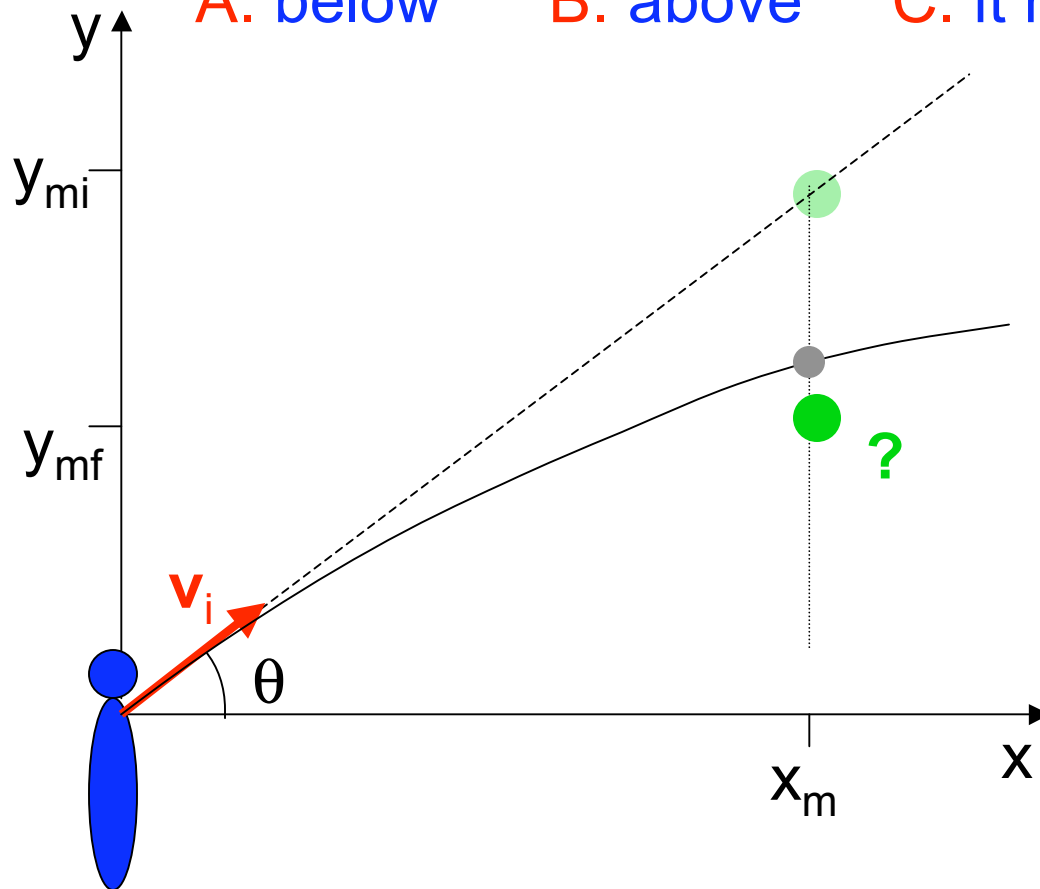
A. below B. above C. it hits the monkey



Monkey & Hunter

A monkey sits in a tree. A hunter having aimed directly at him fires a bullet at speed v_i . At that precise instant the monkey drops from the tree. Where does the bullet pass the wily monkey?

- A. below B. above C. it hits the monkey



First step is

- A. x-motion for t
- B. x-motion for v_{xf}
- C. y-motion for t
- D. y-motion for v_{yf}

Which Equation would you use?

A $v_{xf} = v_{xi} + a_x t$

B $x_f = x_i + \frac{1}{2}(v_{xf} + v_{xi})t$

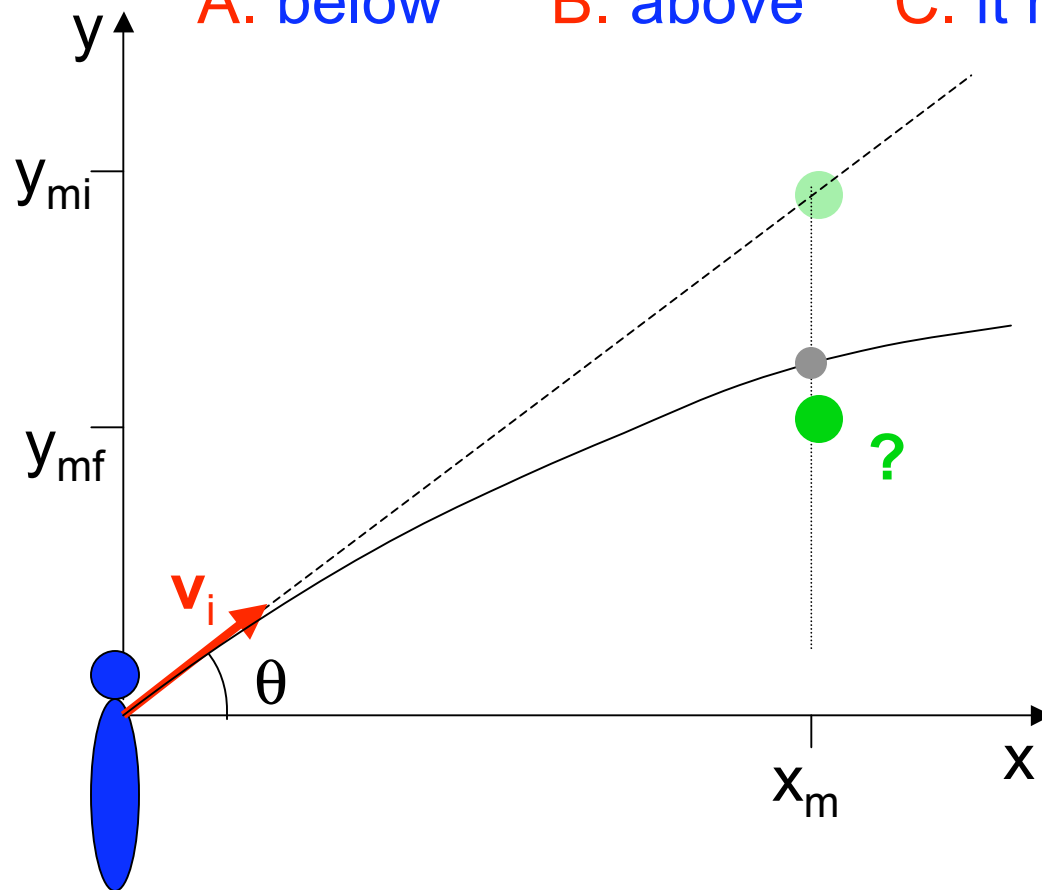
C $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

D $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$

Monkey & Hunter

A monkey sits in a tree. A hunter having aimed directly at him fires a bullet at speed v_i . At that precise instant the monkey drops from the tree. Where does the bullet pass the wily monkey?

A. below B. above C. it hits the monkey



First step is

A. x-motion for t

B. x-motion for v_{xf}

C. y-motion for t

D. y-motion for v_{yf}

$$x_m = v_i \cos(\theta) t_f$$

$$t_f = \frac{x_m}{v_i \cos(\theta)}$$

Which Equation would you substitute
the time expression into?

A $v_{yf} = v_{yi} + a_y t$

B $y_f = y_i + \frac{1}{2}(v_{yf} + v_{yi})t$

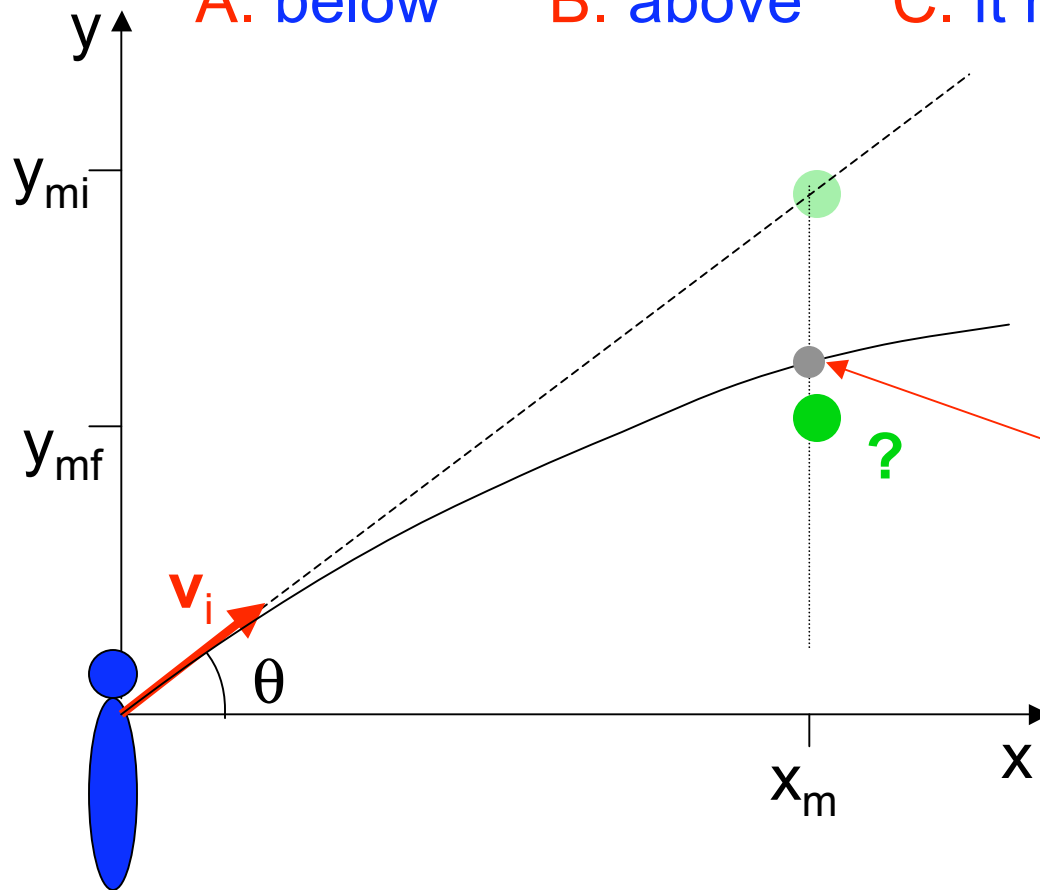
C $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

D $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$

Monkey & Hunter

A monkey sits in a tree. A hunter having aimed directly at him fires a bullet at speed v_i . At that precise instant the monkey drops from the tree. Where does the bullet pass the wily monkey?

A. below B. above C. it hits the monkey



$$t_f = \frac{x_m}{v_i \cos(\theta)}$$

$$y_f = v_i \sin(\theta) t_f - \frac{1}{2} g t_f^2$$

$$y_f = x_m \tan(\theta) - \frac{1}{2} g t_f^2$$

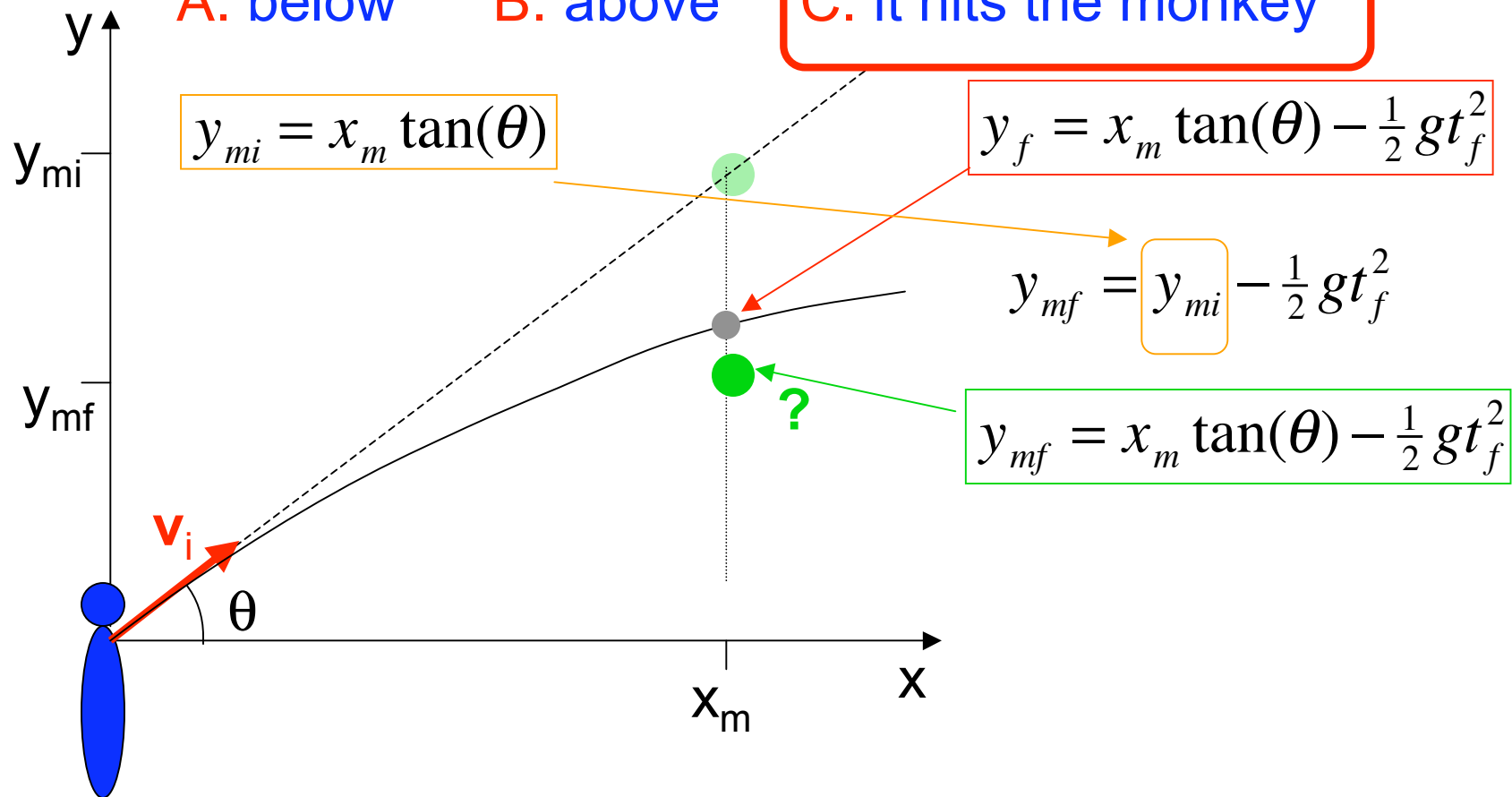
Monkey & Hunter

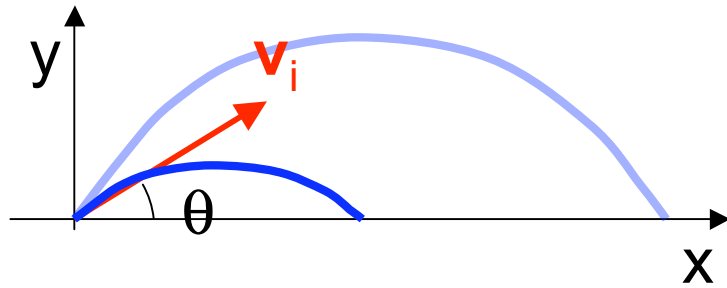
A monkey sits in a tree. A hunter having aimed directly at him fires a bullet at speed v_i . At that precise instant the monkey drops from the tree. Where does the bullet pass the wily monkey?

A. below

B. above

C. it hits the monkey



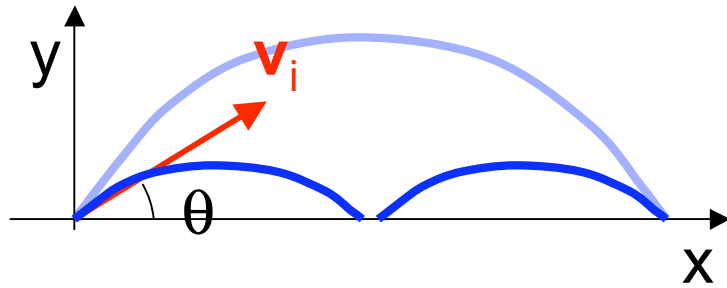


What angle should the outfielder throw the ball, with initial speed $|\mathbf{v}_i|=30 \text{ m/s}$, to go half as far? (40 m)

$$x_f = \frac{v_i^2}{g} \sin(2\theta)$$

$$\frac{x_f g}{v_i^2} = \sin(2\theta) = \frac{40 \text{ m} \times 9.8 \text{ m/s}^2}{(30 \text{ m/s}^{-1})^2} = 0.39$$

$$\sin(2\theta) = 0.39, \theta = 11^\circ$$



What angle should the outfielder throw the ball, with initial speed $|\mathbf{v}_i|=30 \text{ m/s}$, to go half as far? (40 m)

$$x_f = \frac{v_i^2}{g} \sin(2\theta)$$

$$\frac{x_f g}{v_i^2} = \sin(2\theta) = \frac{40 \text{ m} \times 9.8 \text{ m/s}^2}{(30 \text{ m/s}^{-1})^2} = 0.39$$

$$\sin(2\theta) = 0.39, \theta = 11^\circ$$

The ball bounces perfectly & repeats the trajectory. How long does it take to go 80.1 m this way?

- A. 1.2 s
- B. 2.4 s**
- C. 3.1 s
- D. 4.2 s

$$t_f = \frac{2v_i \sin(\theta)}{g} = 1.2 \text{ s}$$