

Physics 411 — Problem set 5: Dirac Notation & Hermitian Operators

Due Through Feb. 17

Reading: Liboff Chapter 4

15. A particle in a box, $0 < x < a$, has energy eigenstates, $|\phi_n\rangle = \phi_n(x)$ given by Liboff eq. (4.15). Consider a particle whose wave function is

$$\psi(x) = \sqrt{\frac{2}{a}} \left| \sin\left(\frac{2\pi x}{a}\right) \right| .$$

Note that due to the absolute value signs this is *not* the same as $|\phi_2\rangle$.

- a. Find the expectation value $\langle \psi | \hat{H} | \psi \rangle$ where the Hamiltonian operator for a particle of mass m_p is

$$\hat{H} = -\frac{\hbar^2}{2m_p} \frac{\partial^2}{\partial x^2} ,$$

- b. Find the first 3 coefficients, c_1, c_2, c_3 , in the expansion

$$|\psi\rangle = \sum_{n=1}^{\infty} c_n |\phi_n\rangle .$$

[You may state in words why a particular coefficient vanishes, if it does, and you don't need to show the integral.]

16. The angular momentum eigenstates of a particle on a ring are

$$|\phi_n\rangle = \frac{1}{\sqrt{2\pi}} e^{in\theta} , \quad n = 0, \pm 1, \pm 2, \dots$$

with eigenvalue $p_\theta = n\hbar$. [FYI the angular momentum operator is $\hat{p}_\phi = -i\hbar\partial/\partial\theta$.]

- a. Show explicitly that $\langle \phi_m | \phi_n \rangle = \delta_{mn}$. Be careful about the sign of n . In particular $|\phi_{-1}\rangle \neq |\phi_1\rangle$ (and so on for other pairs of positive/negative integers). In fact your result must apply to the case $\langle \phi_{-1} | \phi_1 \rangle = 0$ etc.
- b. For the operator $\hat{C} = \cos(\theta)$ find the matrix elements

$$\langle \phi_m | \hat{C} | \phi_n \rangle$$

for an arbitrary pair of integers (m, n) . [HINT: Use the decomposition $\cos\theta = (e^{i\theta} + e^{-i\theta})/2$ inside the integral.]

- c. Use the $|\phi_n\rangle$ basis set to expand the wave function

$$\psi(\theta) = \begin{cases} \frac{1}{\sqrt{2\pi}} & , \quad 0 < \theta < \pi \\ -\frac{1}{\sqrt{2\pi}} & , \quad \pi < \theta < 2\pi \end{cases}$$

Be clear about which values the integer n may take on in your series.

17. Consider arbitrary operators \hat{A} and \hat{B} and arbitrary *Hermitian* operators \hat{F} and \hat{G} . Which of the following operators are Hermitian?

- (a) $\hat{F} + i\hat{G}$
- (b) $\hat{A} + \hat{A}^\dagger$
- (c) $\hat{A}^\dagger \hat{A}$
- (d) $\hat{A}\hat{B} + \hat{A}^\dagger \hat{B}^\dagger$
- (e) $\hat{F}\hat{G} - \hat{G}\hat{F}$
- (f) $i(\hat{F}\hat{G} - \hat{G}\hat{F})$
- (g) $\hat{A}^\dagger \hat{F} \hat{A}$