

Force-free magnetic fields — Problem set 5: due Friday Nov. 4

1. a. Find a constant- α field satisfying the following boundary conditions

$$B_z(y, z = 0) = B_0 \cos\left(\frac{\pi}{L} y\right) \quad (1)$$

$$B_z(y, z = 2L) = 0 \quad (2)$$

$$B_y(y = -L, z) = 0 \quad (3)$$

$$B_y(y = +L, z) = 0 \quad (4)$$

and x is an ignorable coordinate in whose direction the domain extends infinitely. Do this for all allowable values of α and use a flux function $f(y, z)$ to express your answer.

- b. Compute the magnetic energy, per length in x , as a function of α .
- c. For $\alpha > 0$ find the field line anchored to $\mathbf{x} = (\frac{1}{2}L + \varepsilon)\hat{\mathbf{y}}$, where $\varepsilon \ll L$. How does the location of the other footpoint (i.e. other point on the $z = 0$ plane) vary with α ? How does the apex height of this field line vary with α ?
- d. Show that for certain values of α there are constant- α solutions with *homogeneous* versions of boundary conditions (1)–(4). Use a flux function to write these solutions.
- e. For smallest¹ value of α from part d. show that the field contains, somewhere in the interior ($0 < z < 2L$, $|y| < L$), a straight magnetic field line parallel to the x axis. Find the field line which crosses the $x = 0$ plane a distance ε below this straight field line ($\varepsilon \ll L$).
- f. For the same α from part e. construct a constant- α field subject to the inhomogeneous boundary conditions (1)–(4), but with an arbitrary amount of the solution from d. added on. Find the energy as a function of the homogeneous contribution.

¹smallest in magnitude.