**Force-free magnetic fields — Problem set 5**

1. a. Find a constant-$\alpha$ field satisfying the following boundary conditions

\[
B_z(y, z = 0) = B_0 \cos\left(\frac{\pi}{L} y\right) \\
B_z(y, z = 2L) = 0 \\
B_y(y = -L, z) = 0 \\
B_y(y = +L, z) = 0
\]

(1) 
(2) 
(3) 
(4)

and $x$ is an ignorable coordinate in whose direction the domain extends infinitely. Do this for all allowable values of $\alpha$ and use a flux function $f(y, z)$ to express your answer.

b. Compute the magnetic energy, per length in $x$, as a function of $\alpha$.

c. For $\alpha > 0$ find the field line anchored to $x = \left(\frac{1}{2}L + \varepsilon\right)\hat{y}$, where $\varepsilon \ll L$. How does the location of the other footpoint (i.e. other point on the $z = 0$ plane) vary with $\alpha$? How does the apex height of this field line vary with $\alpha$?

d. Show that for certain values of $\alpha$ there are constant-$\alpha$ solutions with homogeneous versions of boundary conditions (1)–(4). Use a flux function to write these solutions.

e. For smallest\(^1\) value of $\alpha$ from part d. show that the field contains, somewhere in the interior ($0 < z < 2L$, $|y| < L$), a straight magnetic field line parallel to the $x$ axis. Find the field line which crosses the $x = 0$ plane a distance $\varepsilon$ below this straight field line ($\varepsilon \ll L$).

f. For the same $\alpha$ from part e. construct a constant-$\alpha$ field subject to the inhomogeneous boundary conditions (1)–(4), but with an arbitrary amount of the solution from d. added on. Find the energy as a function of the homogeneous contribution.

\(^1\)smallest in magnitude.