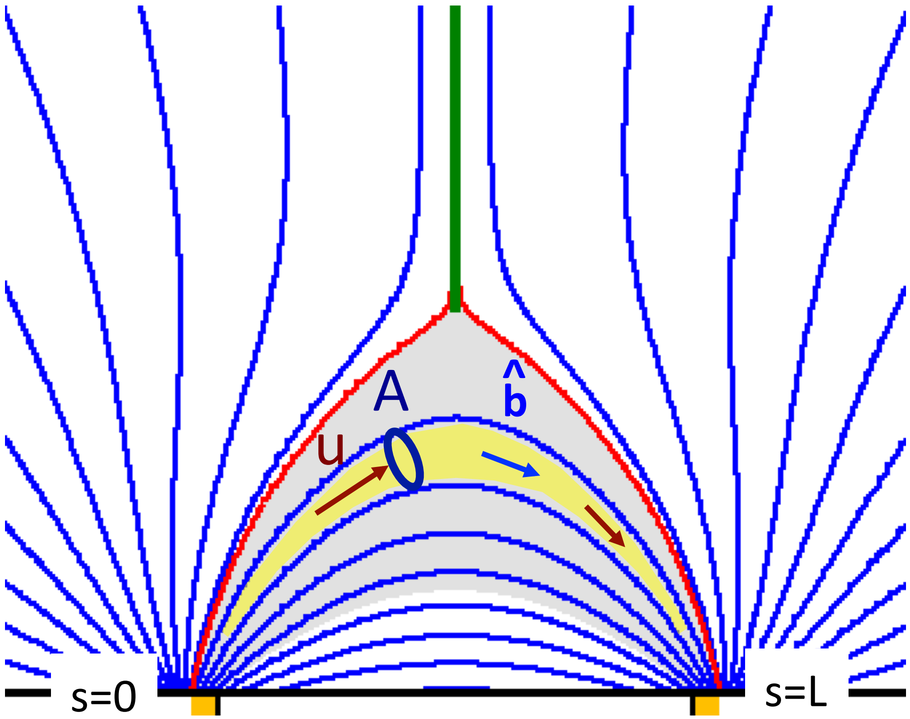
Flare Loops

1d Gas dynamics & 0-d models

Lecture 9

Feb. 15, 2017



$$\mathbf{E} = -\mathbf{u} \times \mathbf{B} = 0$$

$$\mathbf{B} \parallel \mathbf{u} = u\,\hat{\mathbf{b}}$$

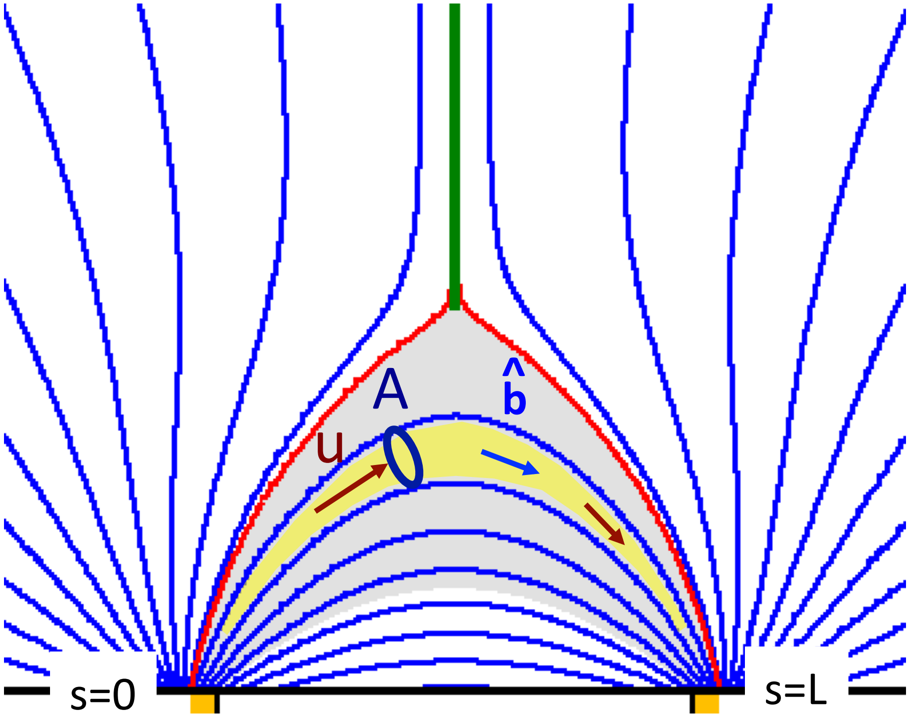
Flow inside a static tube: length coord s

X-section A(s)

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \quad \rightarrow \quad \boxed{\frac{\partial \rho}{\partial t} = -\frac{1}{A}\frac{\partial}{\partial s}(A\rho u)}$$

$$\rho\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{u} = -\nabla p + \frac{1}{c}\mathbf{J}\times\mathbf{B} + \rho\mathbf{g} + \nabla\cdot(\mu\nabla\mathbf{u})$$

$\hat{\mathbf{b}}$ component $\rightarrow$ $$\boxed{\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s}\right) = -\frac{\partial p}{\partial s} + \rho g_{\parallel} + \frac{\partial}{\partial s}\left(\tfrac{4}{3}\mu\frac{\partial u}{\partial s}\right)}$$



Energy equation

Thermal energy density

$$\tfrac{3}{2}p = \rho c_v T$$

conductive flux

radiative loss

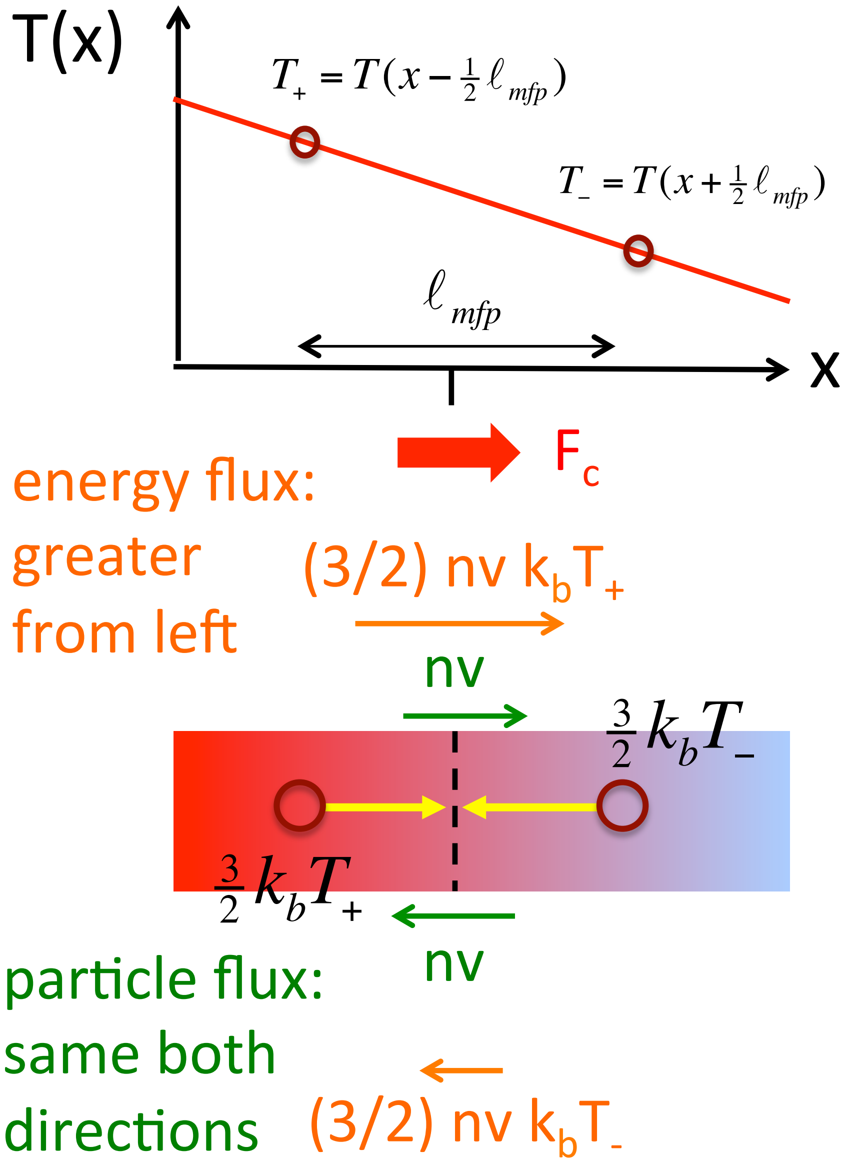
$$\rho c_v\left(\frac{\partial T}{\partial t} + \mathbf{u}\cdot\nabla T\right) = -p\left(\nabla\cdot\mathbf{u}\right) + \mu\left\|\nabla\mathbf{u}\right\|^2 - \nabla\cdot\mathbf{F}_c - L_{rad}$$

p dV work

viscous heat

$$\rightarrow \boxed{\rho c_v\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial s}\right) = -\frac{p}{A}\frac{\partial}{\partial s}\left(Au\right) + \tfrac{4}{3}\mu\left|\frac{\partial u}{\partial s}\right|^2 - \frac{1}{A}\frac{\partial}{\partial s}\left(AF_c\right) - L_{rad}}$$

Conductive flux



$$F_c = \tfrac{3}{2} n \mathrm{v} k_b (T_+ - T_-)$$

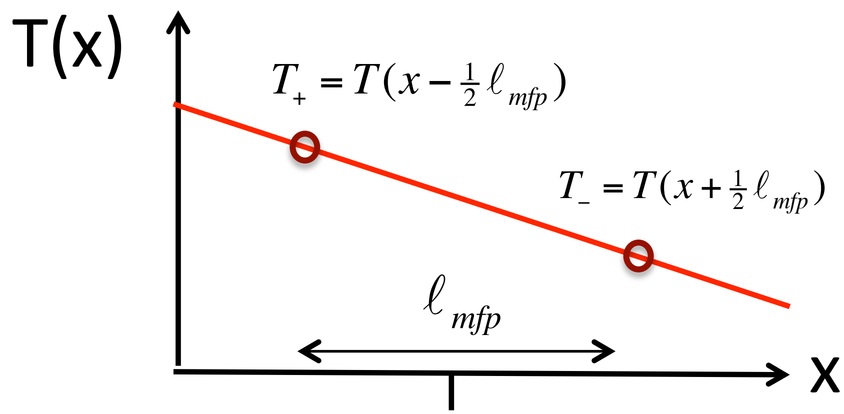
IF gradient is shallow or m.f.p. is small

$$F_c \approx -\underbrace{\boxed{\tfrac{3}{2} n \mathrm{v} k_b \ell_{mfp}}}_{=\,\kappa} \frac{\partial T}{\partial x}$$

$$\boxed{\vec{F}_c = -\kappa \nabla T}$$

Fourier's law – classical heat flux

Conductive flux



$$F_c = \tfrac{3}{2} n \mathrm{v}\, k_b \left(T_+ - T_-\right)$$

IF gradient is shallow or m.f.p. is small

$$F_c \approx -\boxed{\tfrac{3}{2} n \mathrm{v}\, k_b \ell_{mfp}} \frac{\partial T}{\partial x}$$

$= \kappa$

$$\kappa = \tfrac{3}{2} k_b \mathrm{v}_{th} n \ell_{mfp} = \frac{3}{2}\frac{k_b \mathrm{v}_{th}}{\sigma_{sc}}$$

$$\sigma_{sc} \sim \frac{q^4}{m^2 \mathrm{v}_{th}^4}$$ Rutherford scattering

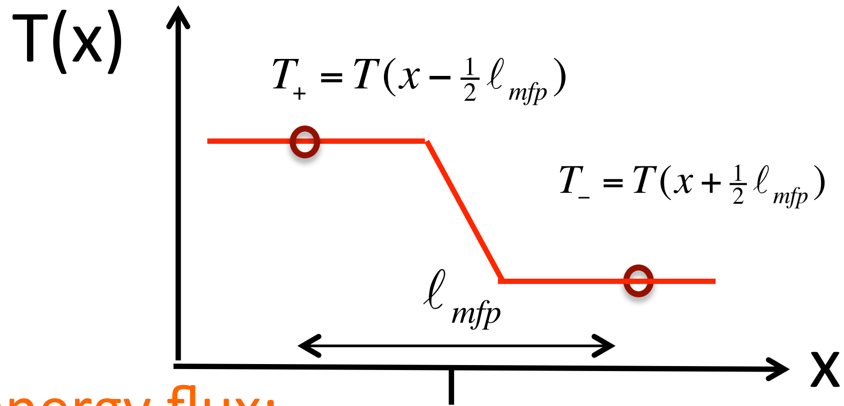
$$\kappa \sim \frac{k_b m^2 \mathrm{v}_{th}^5}{q^4} \sim \boxed{\frac{k_b^{7/2}}{m^{1/2} q^4}} T^{5/2}$$

e^{-}s : smallest m ➔ most heat flux

$\kappa = \kappa_0 T^{5/2}$ [erg cm^{-2} s^{-1} K^{-1}]

$\kappa_0 \simeq 10^{-6}$ erg cm^{-2} s^{-1} K$^{-7/2}$

Conductive flux



$$F_c = \frac{3}{2} n\mathrm{v}\, k_b \left(T_+ - T_-\right)$$

energy flux: greater from left

F_c

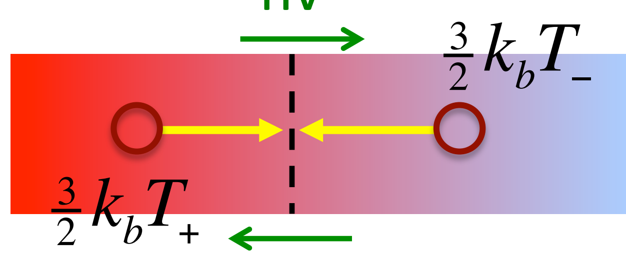
(3/2) nv $k_b T_+$

IF NOT

$$|F_c| < \frac{3}{2} n\mathrm{v}_{th} k_b T_+ = \boxed{\frac{3}{2} n_e \frac{k_b^{3/2}}{m_e^{1/2}} T_e^{3/2}}$$

$=F_{fs}$

nv



nv

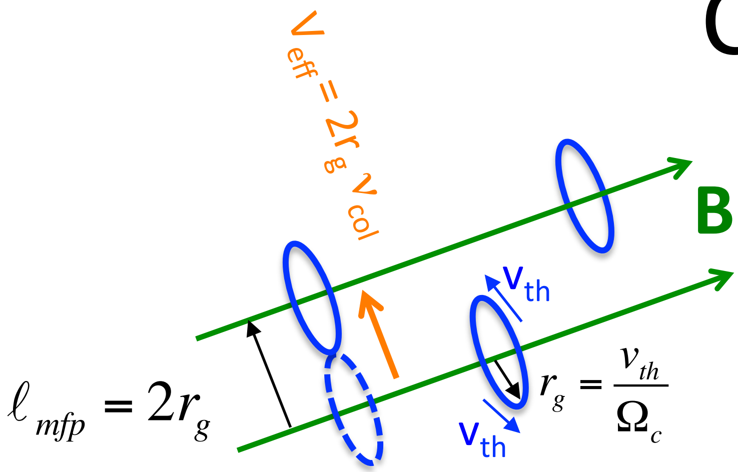
particle flux: same both directions

(3/2) nv $k_b T_-$

Free-streaming

heat flux: upper bound

Conductive flux $\perp$ to B



$$F_c \approx -\boxed{\tfrac{3}{2} n \mathrm{v}\, k_b \ell_{mfp}} \frac{\partial T}{\partial x}$$

$= \kappa$

$$\kappa_\perp = \tfrac{3}{2} k_b n \mathrm{v}_{eff} \ell_{mfp} = 6 k_b n r_g^2 \nu_{col} = 6 k_b n \frac{\mathrm{v}_{th}^2}{\Omega_c^2} \nu_{col}$$

$$= 6 k_b \mathrm{v}_{th} \boxed{\left(\frac{n \mathrm{v}_{th}}{\nu_{col}}\right)} \left(\frac{\nu_{col}}{\Omega_c}\right)^2$$

$1/\sigma_{sc}$

$$\kappa_\perp = 6 \frac{k_b \mathrm{v}_{th}}{\sigma_{sc}} \left(\frac{\nu_{col}}{\Omega_c}\right)^2 = 4 \kappa_\parallel \boxed{\left(\frac{\nu_{col}}{\Omega_c}\right)^2}$$

$\sim 10^{-14}$

Heat flows **ONLY** || to **B**

Constraints on heat flux:

Volumetric heating $\dot{Q} = -\nabla \cdot \vec{F}_c$

heat exchange across bndry

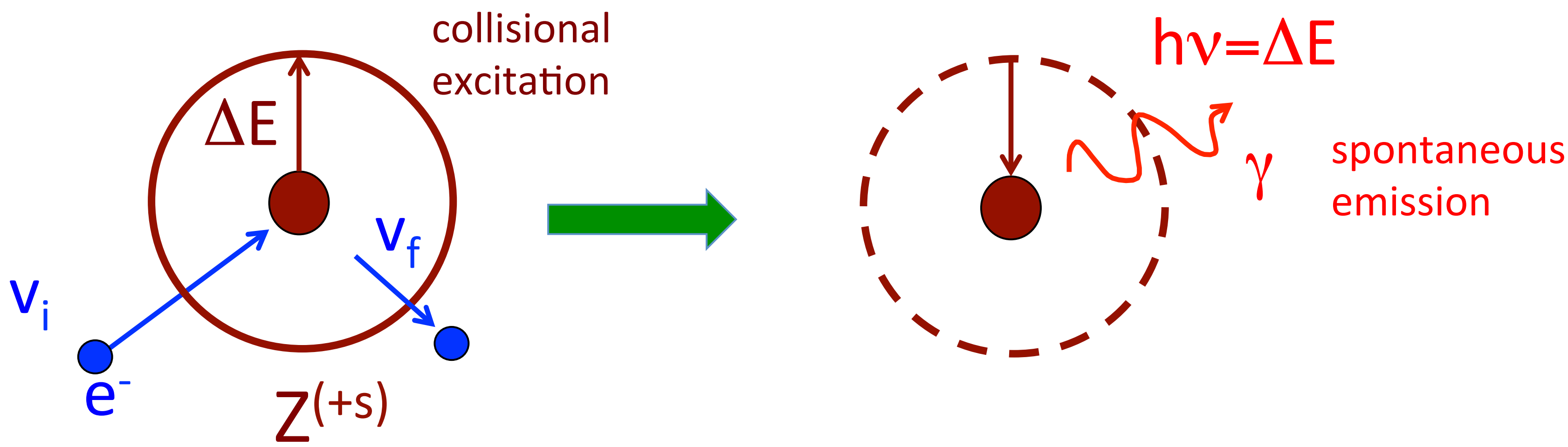
Entropy change $$\frac{dS}{dt} = \int_V \frac{\dot{Q}}{T} d^3x = -\int_V \frac{\nabla \cdot \vec{F}_c}{T} d^3x = \boxed{-\oint_{\partial V} \frac{\vec{F}_c}{T} \cdot d\vec{a}} - \int_V \frac{\vec{F}_c \cdot \nabla T}{T^2} d^3x$$

2nd law of thermo: $\boxed{\vec{F}_c \cdot \nabla T \leq 0}$

Classical flux: $$\nabla T \cdot \vec{F}_c = -(\nabla T) \cdot \overleftrightarrow{\kappa} \cdot (\nabla T)$$

κ must have **no negative e-values**

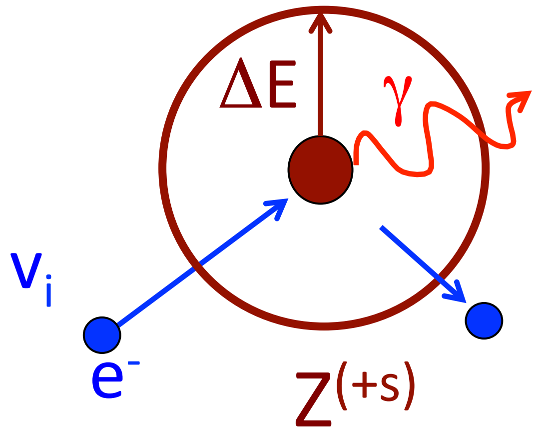
Energy loss by optically thin radiation



- e^- collides with element Z ionized +s
- e^- **loses** $\Delta E = m_e(v_i^2 - v_f^2)/2$
- excites ion to state @ ΔE
- ion de-excites, emitting γ w/ $h\nu=\Delta E$
- $\boxed{\gamma \text{ escapes to } \infty}$ carrying away ΔE

optically thin radiation

Energy loss by optically thin radiation



\# density: $n_{Z,s}$

- excitation cross section:

  $\sigma_{Z,s}(v_e)$

- average γ energy

  $\varepsilon_{Z,s}(v_e)$

sum over all transition!

- rate of energy loss **per e^-**

$$\dot{E}_e = \underbrace{n_{Z,s} \mathrm{v}_e \sigma_{Z,s}(\mathrm{v}_e)}_{\text{collision frequency}} \varepsilon_{Z,s}(\mathrm{v}_e)$$

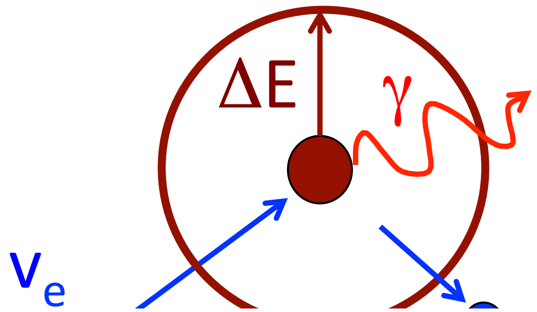
<u>Assumption I</u>: e^-s have **Maxwellian** dist'n w/ temp T_e

$$L_{Z,s} = n_{Z,s} n_e \boxed{\int \frac{e^{-m_e \mathrm{v}_e^2 / 2k_b T_e}}{(2\pi k_b T_e / m_e)^{3/2}} \mathrm{v}_e \sigma_{Z,s}(\mathrm{v}_e) \varepsilon_{Z,s}(\mathrm{v}_e)\, d^3\mathrm{v}_e}$$

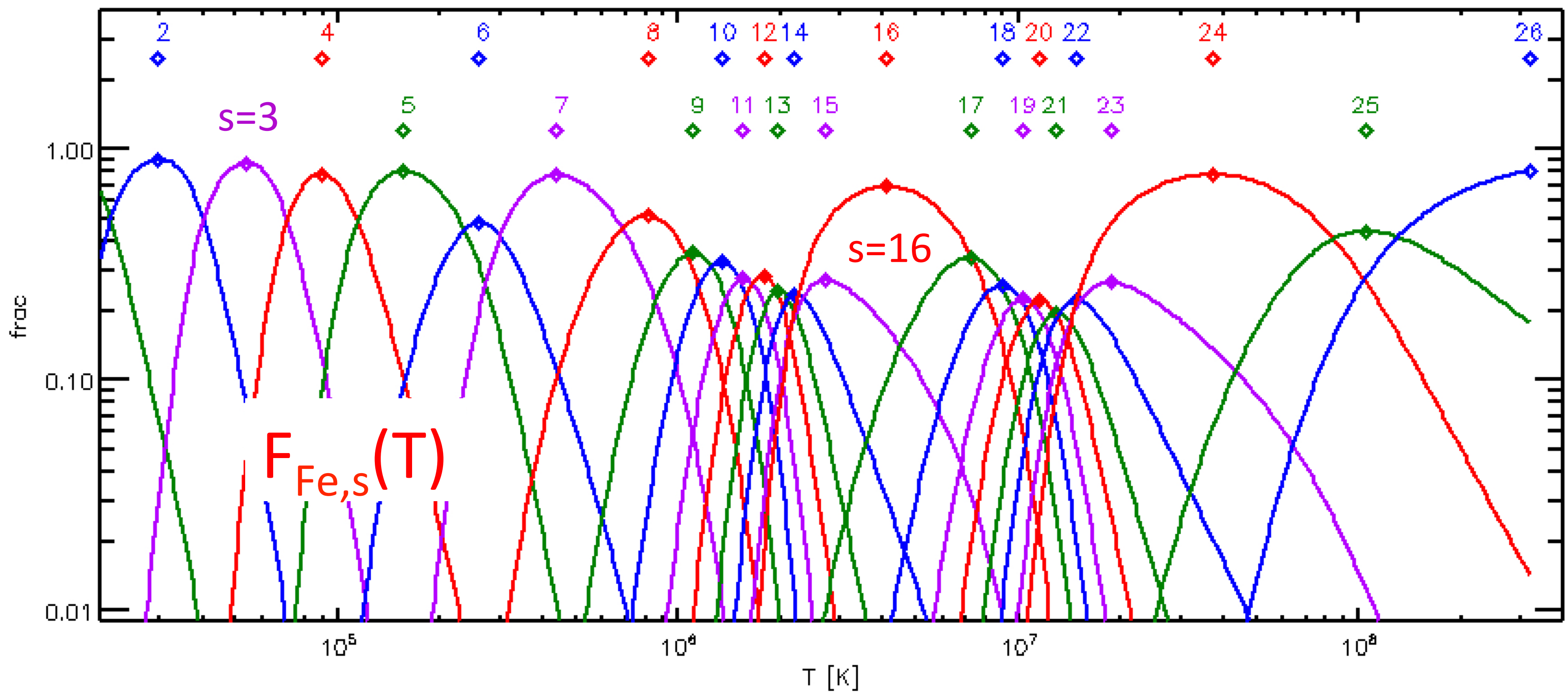
volumetric energy loss rate [erg cm^{-3} s^{-1}] to $Z^{(+s)}$

$R_{Z,s}(T_e)$ [erg cm^3 s^{-1}]

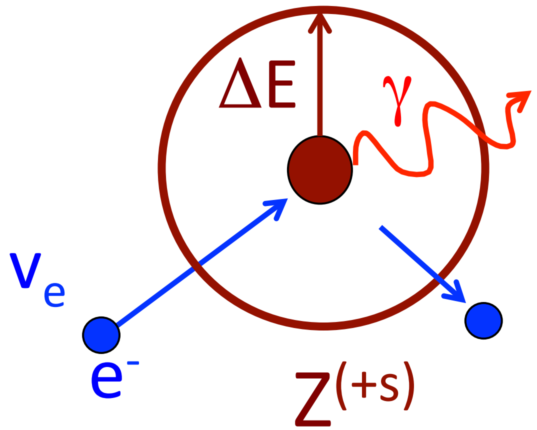
Energy loss by optically thin radiation



Assumption II: ionization equilibrium:
fraction $F_{Z,s}(T_e)$ of element Z ionized to +s ➔ $n_{Z,s} = F_{Z,s}(T_e)\, n_Z$



Energy loss by optically thin radiation



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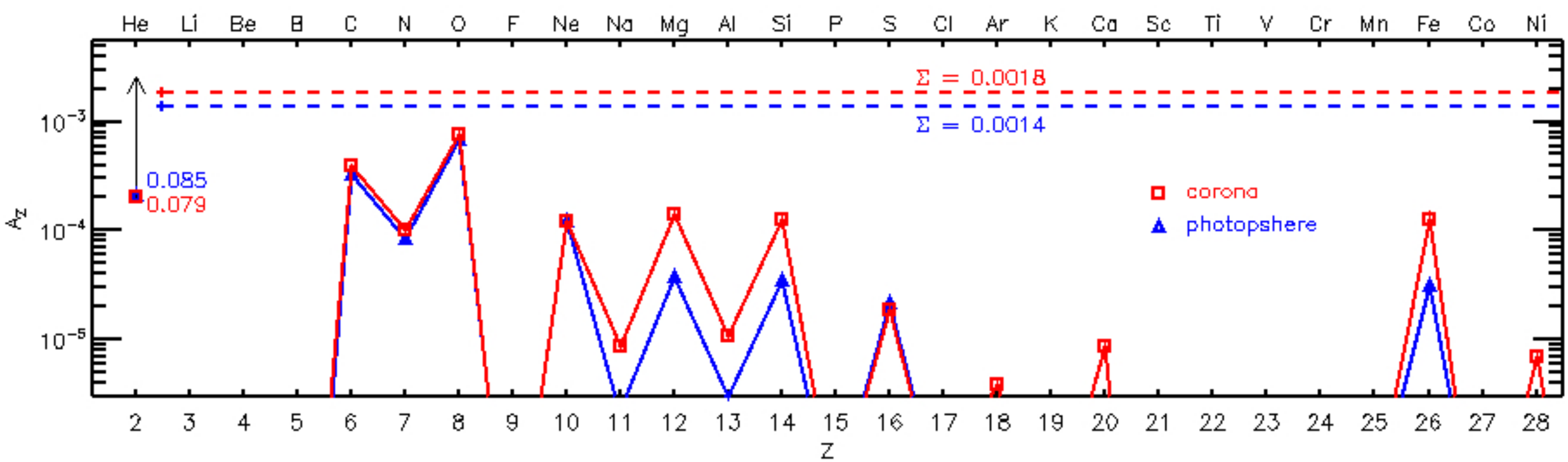
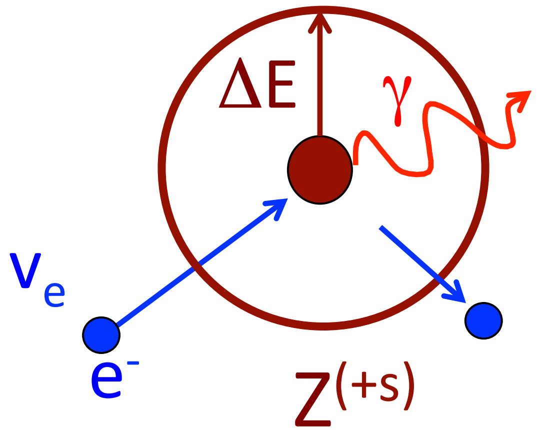
Assumption III: known abundances
$n_Z = A_Z\, n_H$

H & He fully ionized: $n_e = \sum_Z n_H A_Z \sum_s s F_{Z,s} \approx n_H \left(1 + 2A_{He}\right) \approx 1.17 n_H$



Energy loss by optically thin radiation



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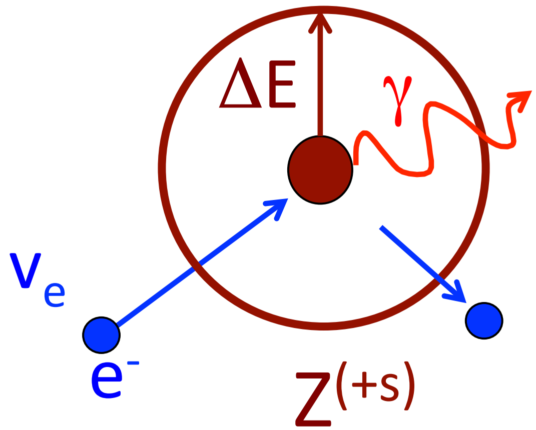
Assumption I: e^-s have **Maxwellian** dist'n w/ temp T_e

$$L_{Z,s} = n_{Z,s} n_e \boxed{\int \frac{e^{-m_e v_e^2/2k_b T_e}}{(2\pi k_b T_e / m_e)^{3/2}} v_e \sigma_{Z,s}(v_e) \varepsilon_{Z,s}(v_e)\, d^3 v_e}$$

volumetric energy loss rate [erg cm^{-3} s^{-1}] to $Z^{(+s)}$

$R_{Z,s}(T_e)$ [erg cm^3 s^{-1}]

Energy loss by optically thin radiation



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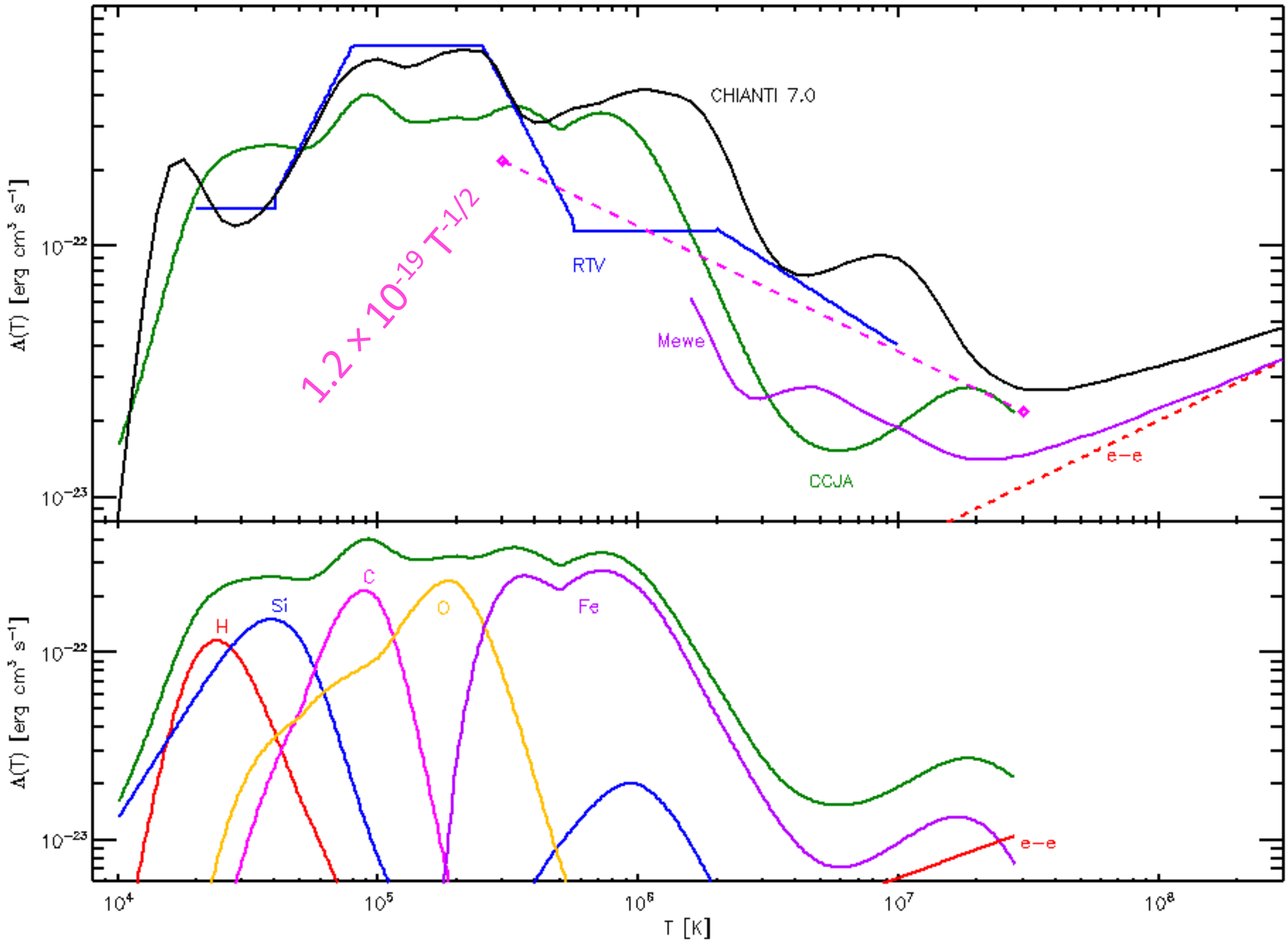
volumetric energy loss rate to $Z^{(+s)}$

$$L_{Z,s} = n_e n_{Z,s} R_{Z,s}(T_e) = \frac{n_e^2}{1.17} A_Z F_{Z,s}(T_e) R_{Z,s}(T_e)$$

volumetric energy loss rate [erg cm^{-3} s^{-1}]

$$L = n_e^2 \boxed{\sum_Z \frac{A_Z}{1.17} \sum_s F_{Z,s}(T_e) R_{Z,s}(T_e)} = n_e^2 \boxed{\Lambda(T_e)}$$

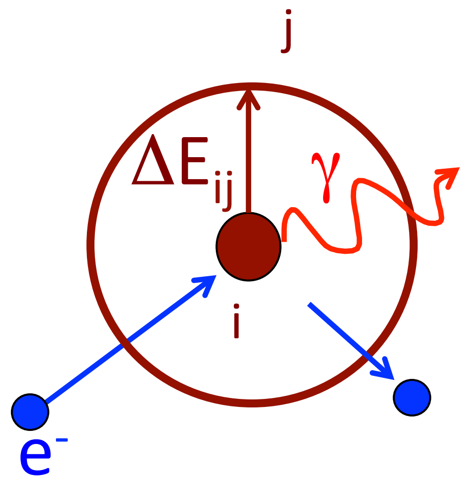
Radiative loss function $\Lambda(T_e)$



CHIANTI 7.0
RTV
Mewe
CCJA
e−e
$1.2\times10^{-19}\,T^{-1/2}$
$\Lambda(T)$ [erg cm^3 s^{-1}]
H
Si
C
O
Fe
e−e
10^{-22}
10^{-23}
10^4
10^5
10^6
10^7
10^8
T [K]

Emission measure

a sidebar



Corona & TR:
mostly optically thin

- Collisional excitation
- Spontaneous emission

$$I_\lambda = \int n_e^2(\mathbf{x})\, G_\lambda[T_e(\mathbf{x})]\, d^3x \approx G_\lambda(\bar{T}_e) \boxed{\int n_e^2(\mathbf{x})\, d^3x}$$

Emission measure: EM [cm^{-3}]

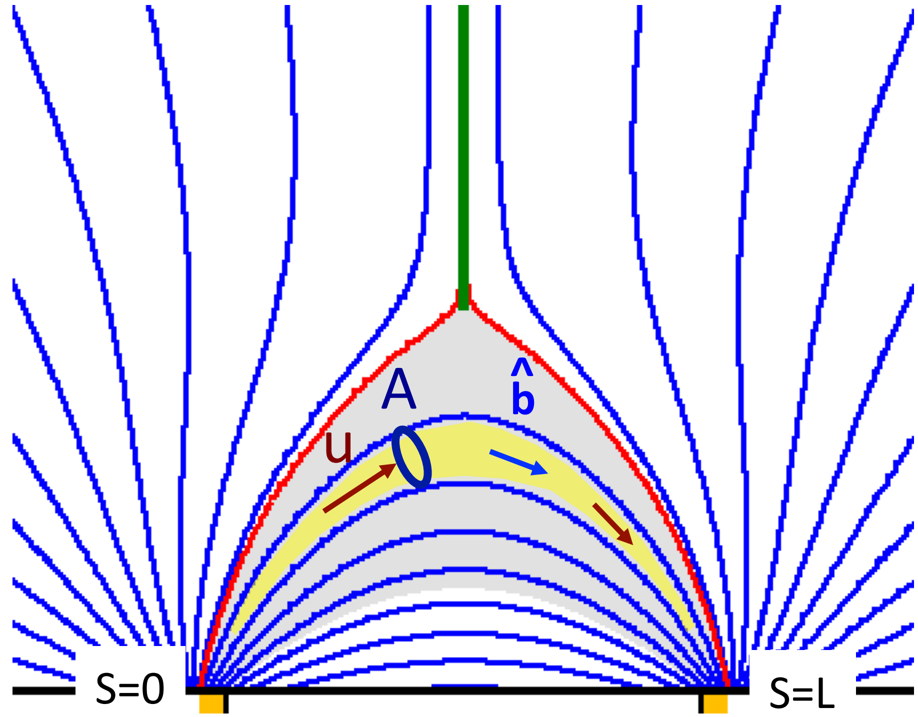
Regions of highest density emit most –

$n_e \uparrow \times 10 \quad \Rightarrow \quad I \sim$ EM $\uparrow \times 100$

The 1d flare loop

ρ(s,t), u(s,t) & T(s,t)

Fixed A(s) & $g_{||}$(s)



$$\frac{\partial \rho}{\partial t} = -\frac{1}{A}\frac{\partial}{\partial s}(A\rho u)$$

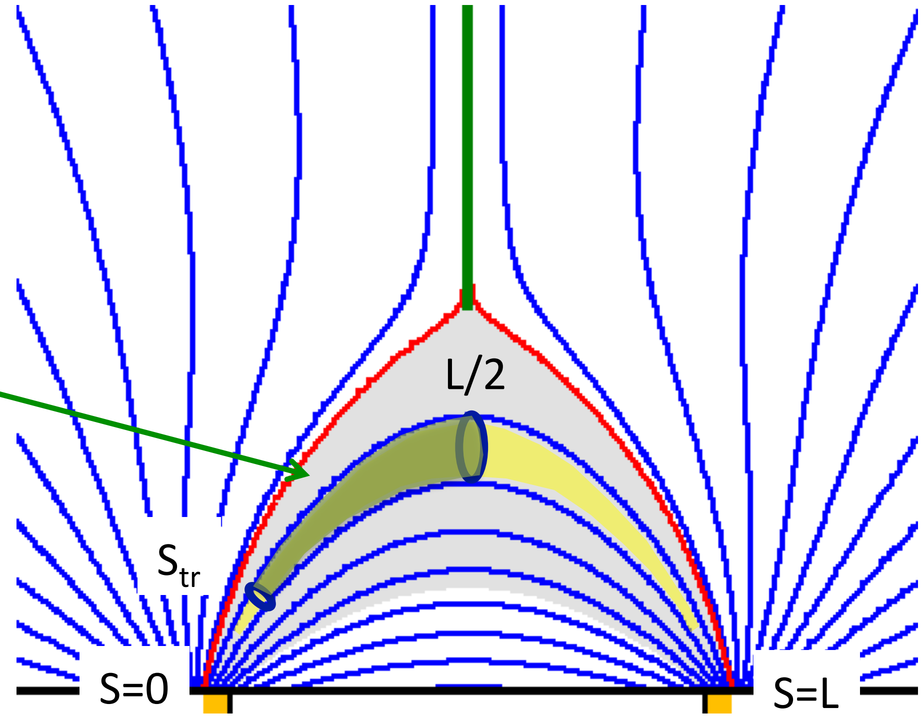
$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s}\right) = -\frac{\partial p}{\partial s} + \rho g_{\parallel} + \frac{\partial}{\partial s}\left(\tfrac{4}{3}\mu\frac{\partial u}{\partial s}\right)$$

Source of flare energy

$$\rho c_v\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial s}\right) = -\frac{p}{A}\frac{\partial}{\partial s}(Au) + \tfrac{4}{3}\mu\left|\frac{\partial u}{\partial s}\right|^2 + \frac{1}{A}\frac{\partial}{\partial s}\left[A\kappa\frac{\partial T}{\partial s}\right] - n_e^2\Lambda(T) + h$$

$$p = \frac{k_b}{\bar{m}}\rho T \qquad c_v = \frac{3}{2}\frac{k_b}{\bar{m}}$$

Integral quantities



$$M = \int_{s_{tr}}^{L/2} \rho(s,t)\, A(s)\, ds$$

$$\frac{dM}{dt} = \rho(s_{tr})\, u(s_{tr})\, A(s_{tr})$$

$$E_{tot} = \int_{s_{tr}}^{L/2} \left[\tfrac{1}{2}\rho u^2 + \tfrac{3}{2} p + \rho \Psi \right] A\, ds$$

$$\frac{dE_{tot}}{dt} \approx \underbrace{-\int_{s_{tr}}^{L/2} n_e^2 \Lambda(T) A\, ds}_{\text{radiative loss}} + \tfrac{1}{2} u^3 A \Big|_{tr} + \underbrace{\tfrac{5}{2} p u A \Big|_{tr}}_{\text{enthalpy flux}} \underbrace{- \kappa \frac{\partial T}{\partial s} A \Big|_{tr}}_{\text{conductive flux}} + \overbrace{\int_{s_{tr}}^{L/2} h\, A\, ds}^{\text{flare heat}}$$

Their evolution: 0d models

$$M = \int_{s_{tr}}^{L/2} \rho(s,t)\, A(s)\, ds \equiv \tfrac{L}{2} A\, m_p \bar{n}_e$$

$$\boxed{\frac{d\bar{n}_e}{dt} = \tfrac{2}{L} n_{e,tr} u_{tr}}$$

$$E_{tot} = \int_{s_{tr}}^{L/2} \left[\cancel{\tfrac{1}{2}\rho u^2} + \tfrac{3}{2} p\right] A\, ds \equiv \tfrac{3L}{4} A\, \bar{p}$$

$$\frac{d}{dt}\left(\tfrac{3}{2}\bar{p}\right) \approx -\tfrac{2}{L}\int_{s_{tr}}^{L/2} n_e^2 \Lambda(T)\, ds + \overbrace{\tfrac{5}{L} p_{tr}\, u_{tr}}^{\text{enthalpy flux}} - \tfrac{2}{L}\kappa \left.\frac{\partial T}{\partial s}\right|_{tr} + \tfrac{2}{L}\overbrace{\int_{s_{tr}}^{L/2} h\, ds}^{F_{fl}}$$

radiative loss

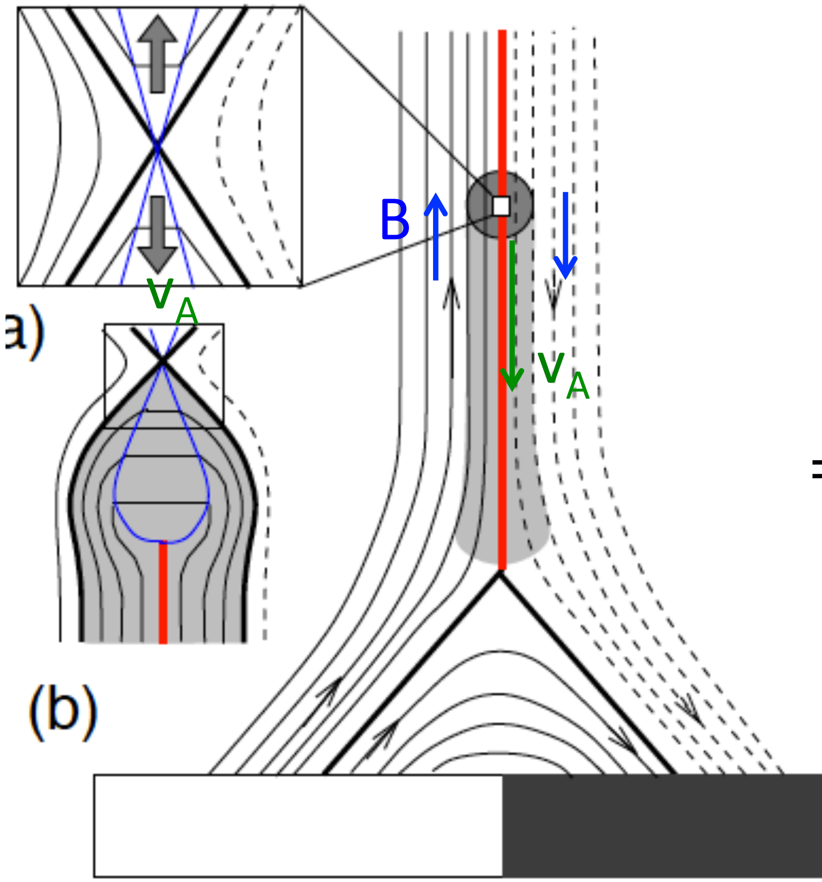
$$\approx -\bar{n}_e^2 \Lambda(\bar{T})$$

conductive flux

$$\approx \frac{8\kappa_0}{7L^2}\bar{T}^{7/2}$$

$$\boxed{\frac{d}{dt}\left(\tfrac{3}{2}\bar{p}\right) \approx -\bar{n}_e^2\, \Lambda(\bar{T}) + \tfrac{5}{L} p_{tr}\, u_{tr} - \frac{8\kappa_0}{7L^2}\bar{T}^{7/2} + \tfrac{2}{L} F_{fl}}$$

What will be the value of F_{fl}?



$$F_{fl} = \varsigma \frac{B^2}{8\pi} v_A = \frac{\varsigma}{2\sqrt{\rho}} \left(\frac{B^2}{4\pi} \right)^{3/2}$$

B = 100 G

$\rho = 10^{-15}$ g cm^{-3}

$$\Rightarrow F_{fl} = \varsigma\, 3 \times 10^{11} \,\mathrm{erg\, s^{-1}\, cm^{-2}}$$

Compare to upward fluxes:

- Steady AR:
  F ~ 10^7 erg cm^{-2} s^{-1}
- Luminosity (white light)
  $L_\odot / 4\pi R_\odot^2 = 6 \times 10^{10}$

$$\frac{d}{dt}\left(\tfrac{3}{2}\bar{p}\right) \approx \underbrace{-\bar{n}_e^{\,2}\,\Lambda(\bar{T})} + \tfrac{5}{L}\,p_{tr}\,u_{tr} - \underbrace{\frac{8\kappa_0}{7L^2}\bar{T}^{7/2}} + \tfrac{2}{L}F_{\mathrm{fl}}$$

$$\tau_{rad} \equiv \frac{\tfrac{3}{2}\bar{p}}{\bar{n}_e^{\,2}\Lambda(\bar{T})}$$

$$= \frac{3\bar{n}_e k_b \bar{T}}{\bar{n}_e^{\,2}\Lambda(\bar{T})} \sim \frac{3k_b}{1.2\times10^{-19}}\frac{\bar{T}^{3/2}}{\bar{n}_e}$$

$$\tau_{cond} \equiv \frac{\tfrac{3}{2}\bar{p}}{\frac{8\kappa_0}{7L^2}\bar{T}^{7/2}}$$

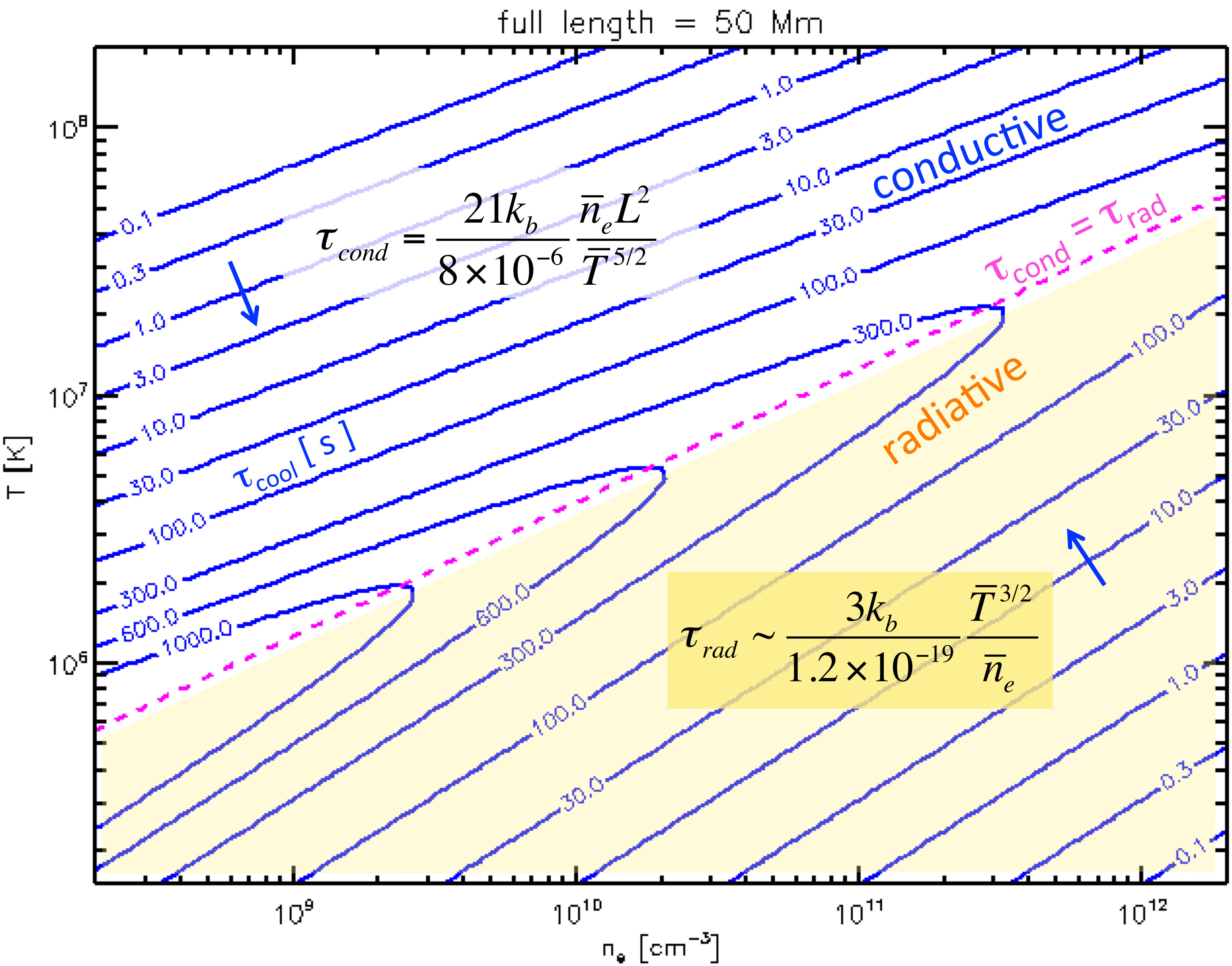
$$= \frac{21k_b}{8\times10^{-6}}\frac{\bar{n}_e L^2}{\bar{T}^{5/2}}$$

Fully-ionized H plasma

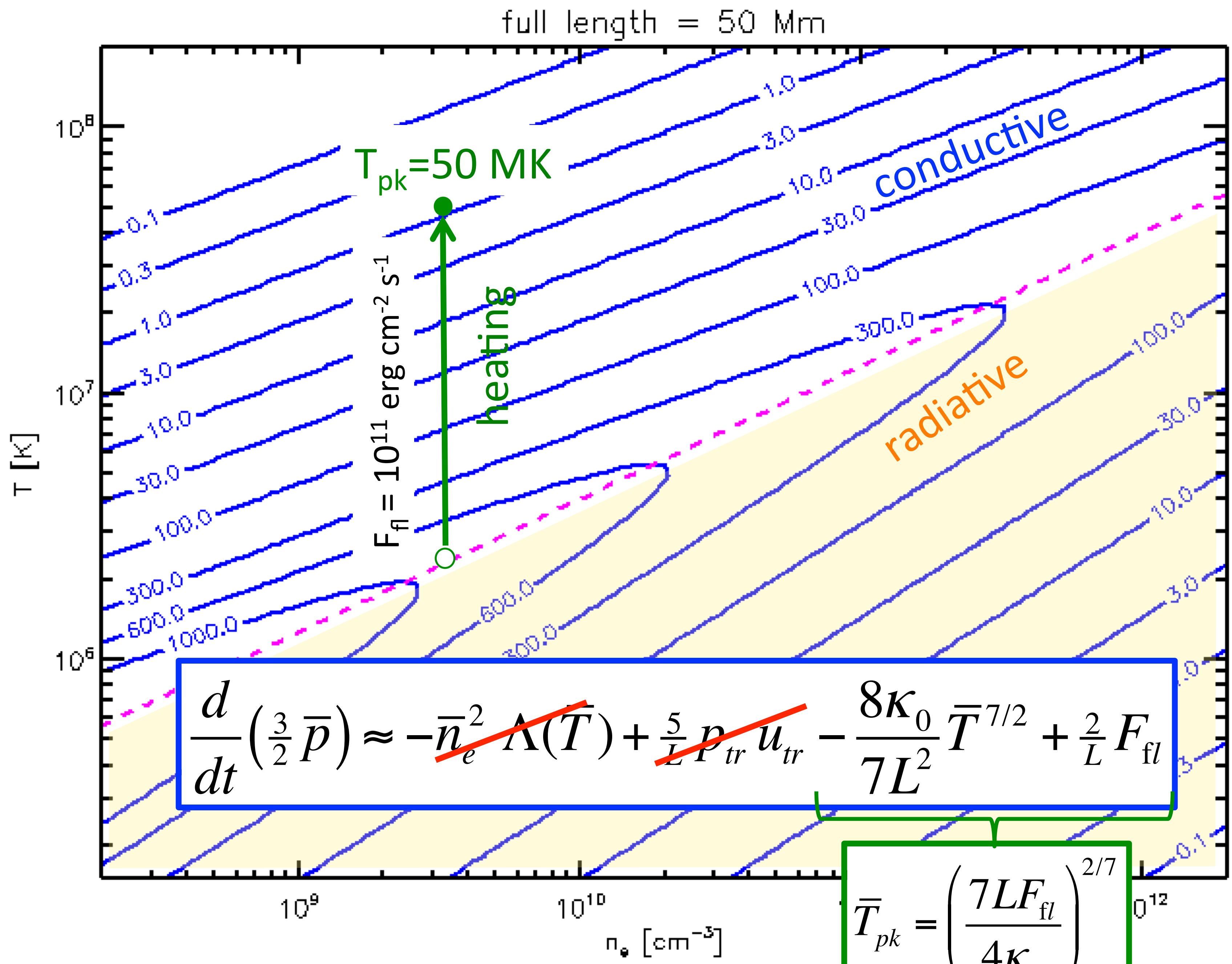
$$\bar{p} = 2\bar{n}_e k_b \bar{T}$$

Cooling rate:

$$\boxed{\frac{1}{\tau_{cool}} = \frac{1}{\tau_{rad}} + \frac{1}{\tau_{cond}}}$$



full length = 50 Mm
$\tau_{cond} = \frac{21k_b}{8\times10^{-6}} \frac{\bar{n}_e L^2}{\bar{T}^{5/2}}$
conductive
$\tau_{cond} = \tau_{rad}$
τ_{cool} [s]
radiative
$\tau_{rad} \sim \frac{3k_b}{1.2\times10^{-19}} \frac{\bar{T}^{3/2}}{\bar{n}_e}$
T [K]
n_e [cm^{-3}]



$$\frac{d}{dt}\left(\tfrac{3}{2}\,\bar{p}\right) \approx -\cancel{\bar{n}_e^2\,\Lambda(\bar{T})} + \cancel{\tfrac{5}{L}\,p_{tr}\,u_{tr}} \underbrace{-\frac{8\kappa_0}{7L^2}\,\bar{T}^{7/2} + \tfrac{2}{L}\,F_{\rm fl}}$$

$$\bar{T}_{pk} = \left(\frac{7LF_{\rm fl}}{4\kappa_0}\right)^{2/7}$$

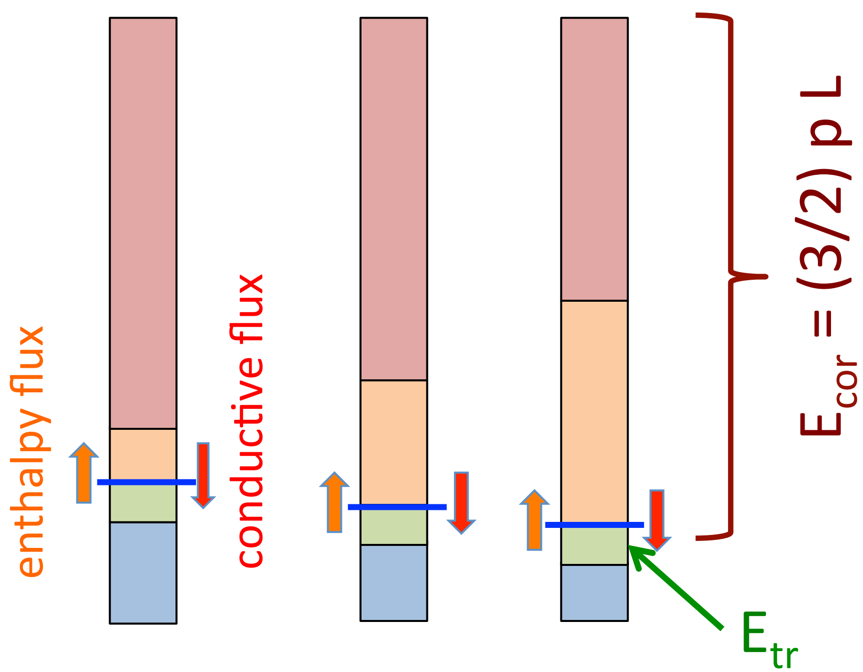
Evaporation*:

$$\frac{d\bar{n}_e}{dt} = \tfrac{2}{L} n_{e,tr} u_{tr}$$

mass flux increases n_e

á la Antiochos & Sturrock 1978; Cargill *et al.* 1995

$$\frac{d}{dt}\underbrace{\left(\tfrac{3}{2}\bar{p}\right)}_{E_{cor}} \approx \cancel{-\bar{n}_e^2 \Lambda(\bar{T})} + \overbrace{\tfrac{5}{L} p_{tr} u_{tr}}^{F_e} - \overbrace{\frac{8\kappa_0}{7L^2}\bar{T}^{7/2}}^{F_c} + \cancel{\tfrac{2}{L} F_{fl}}$$

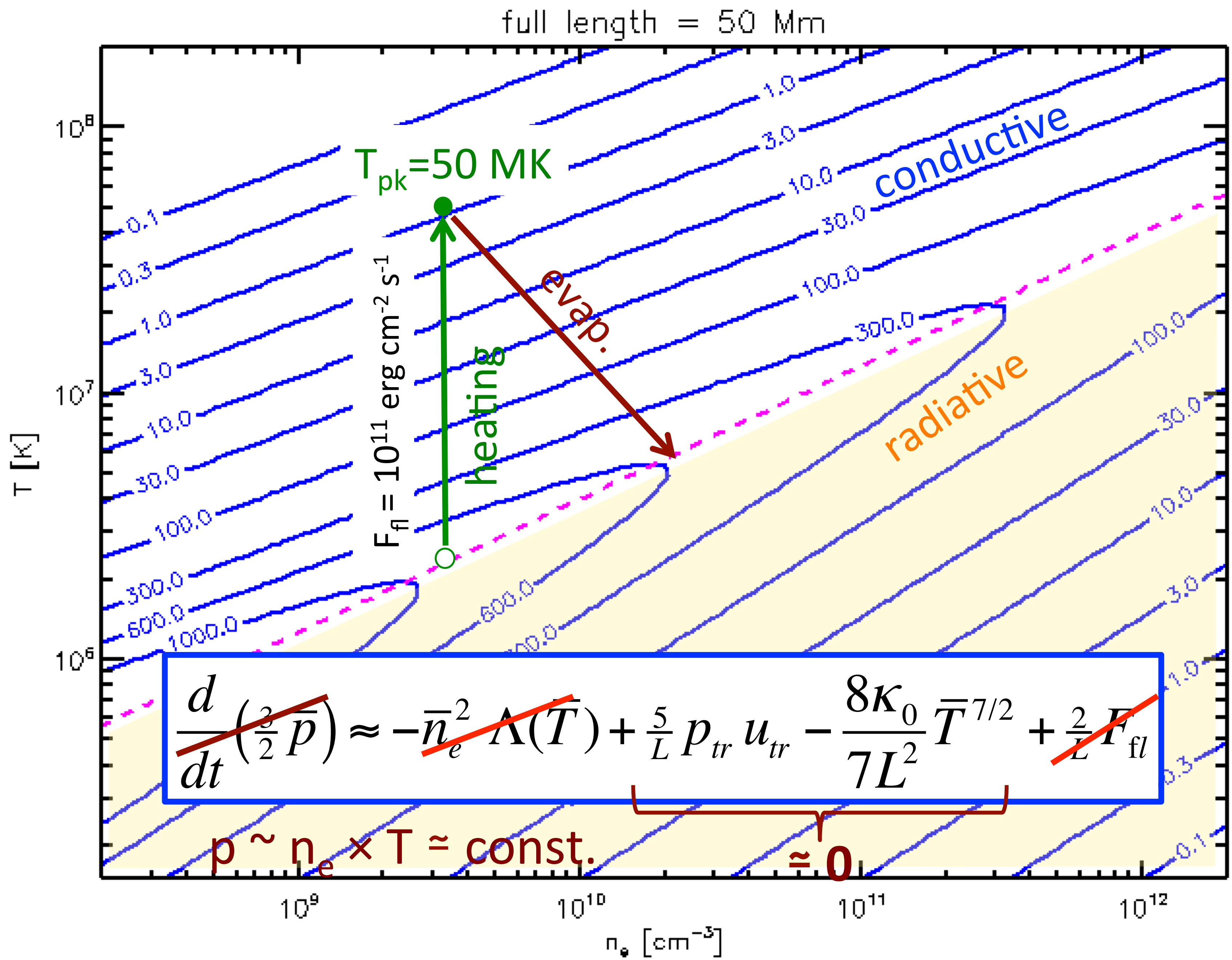


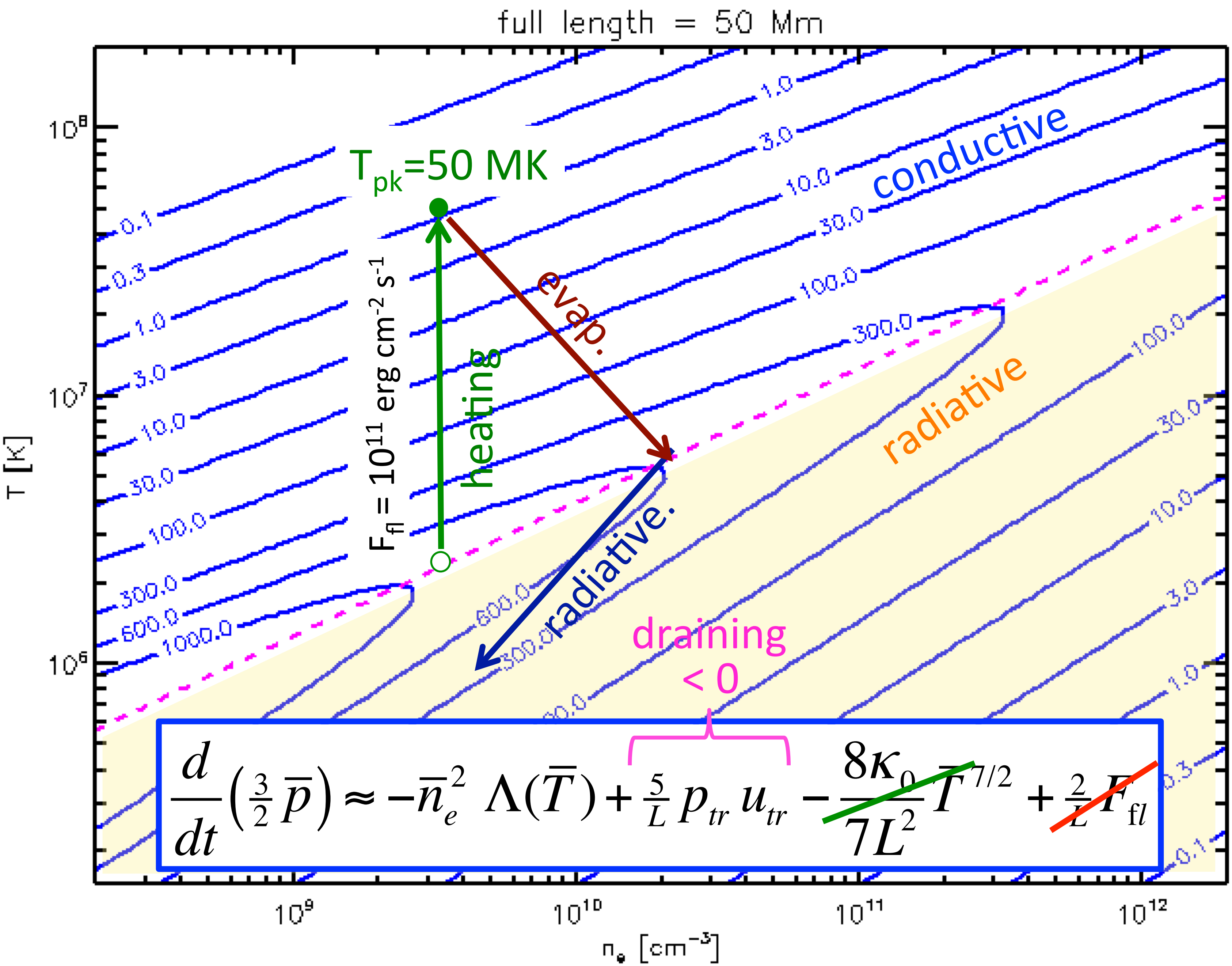
- Heat flows into TR – conductive flux: **F_c**
- TR expands into corona – enthalpy flux: **F_e**
- No change in E_{tr} & **no losses** (i.e. rad)

➔ No change in E_{cor}

➔ $p \sim n_e \times T$ = const.

* Historical term based on analogy. Not genuine evaporation


full length = 50 Mm
T [K]
n_e [cm^{-3}]
T_{pk}=50 MK
$F_{fl} = 10^{11}$ erg cm^{-2} s^{-1}
heating
evap.
conductive
radiative
$$\frac{d}{dt}\left(\frac{3}{2}\bar{p}\right) \approx -\bar{n}_e^2\Lambda(\bar{T}) + \underbrace{\frac{5}{L}p_{tr}u_{tr} - \frac{8\kappa_0}{7L^2}\bar{T}^{7/2}}_{\simeq 0} + \frac{2}{L}F_{\mathrm{fl}}$$
p ~ n_e × T ≃ const.


full length = 50 Mm
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n_e [cm^{-3}]
T_{pk}=50 MK
$F_{fl} = 10^{11}$ erg cm^{-2} s^{-1}
heating
evap.
radiative.
conductive
radiative
draining
< 0
$$\frac{d}{dt}\left(\frac{3}{2}\bar{p}\right) \approx -\bar{n}_e^2\,\Lambda(\bar{T}) + \overbrace{\frac{5}{L}p_{tr}u_{tr}}^{\text{draining} < 0} - \cancel{\frac{8\kappa_0}{7L^2}\bar{T}^{7/2}} + \cancel{\frac{2}{L}F_{\mathrm{fl}}}$$

Evaporation:

$$\frac{d\bar{n}_e}{dt} = \tfrac{2}{L} u_{tr} n_{e,tr} \approx \tfrac{2}{L} u_{tr} \bar{n}_e$$

Antiochos & Sturrock 1978;
Cargill et al. 1995

$$\cancel{\frac{d}{dt}\left(\tfrac{3}{2}\bar{p}\right)} \approx \cancel{-\bar{n}_e^2 \Lambda(\bar{T})} + \underbrace{\tfrac{5}{L} p_{tr} u_{tr} - \frac{8\kappa_0}{7L^2}\bar{T}^{7/2}}_{\simeq 0} + \cancel{\tfrac{2}{L} F_{fl}}$$

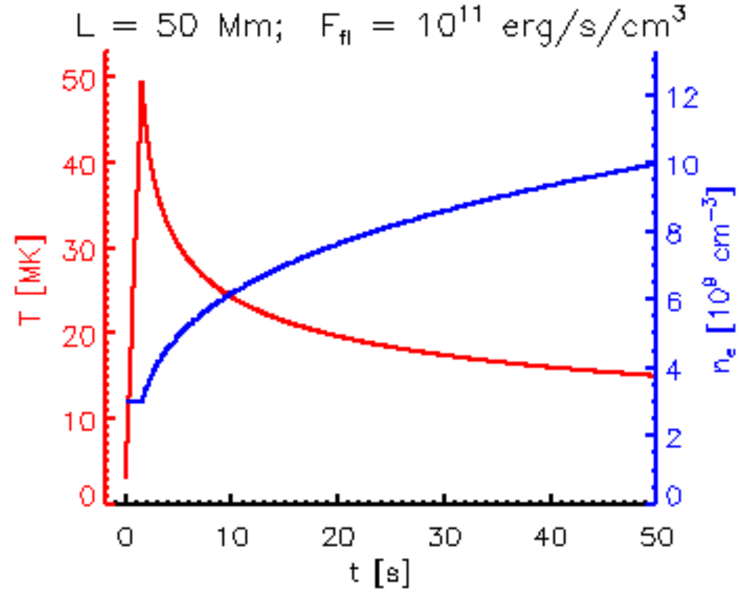
p ~ n_e × T ≃ const.

$$\tfrac{5}{L} p_{tr} u_{tr} = \tfrac{5}{L} p u_{tr} = \frac{8\kappa_0}{7L^2}\bar{T}^{7/2} = \frac{\tfrac{3}{2}p}{\tau_{c,0}}\left(\frac{\bar{T}}{\bar{T}_0}\right)^{7/2} = \frac{\tfrac{3}{2}p}{\tau_{c,0}}\left(\frac{\bar{n}_e}{\bar{n}_{e,0}}\right)^{-7/2} \rightarrow \boxed{\tfrac{2}{L} u_{tr} = \tfrac{3}{5}\frac{1}{\tau_{c,0}}\left(\frac{\bar{n}_e}{\bar{n}_{e,0}}\right)^{-7/2}}$$

$$\frac{d}{dt}\left(\frac{\bar{n}_e}{\bar{n}_{e,0}}\right) = \frac{\tfrac{3}{5}}{\tau_{c,0}}\left(\frac{\bar{n}_e}{\bar{n}_{e,0}}\right)^{-5/2} \rightarrow \boxed{\bar{n}_e(t) = \bar{n}_{e,0}\left(\tfrac{21}{10}\frac{t-t_0}{\tau_{c,0}} + 1\right)^{2/7}}$$

$$\downarrow$$

$$\boxed{\bar{T}(t) = \bar{T}_0\left(\tfrac{21}{10}\frac{t-t_0}{\tau_{c,0}} + 1\right)^{-2/7}}$$



Radiative cooling

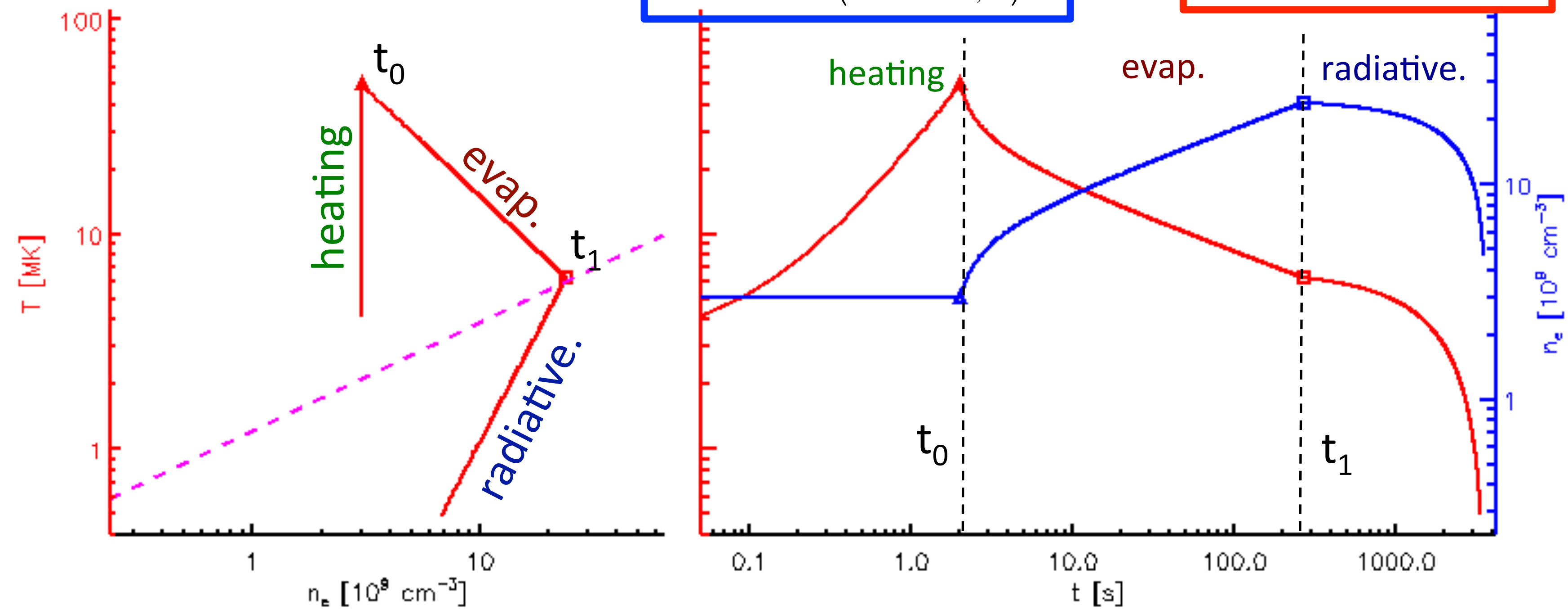
Cargill et al. 1995

$$\frac{d}{dt}\left(\tfrac{3}{2}\bar{p}\right) \approx -\bar{n}_e^2\,\Lambda(\bar{T}) + \cancel{\tfrac{5}{L}p_{tr}u_{tr}} - \cancel{\frac{8\kappa_0}{7L^2}}\bar{T}^{7/2} + \cancel{\tfrac{2}{L}F_{fl}}$$

I. draining (empirical)

$$\bar{T} \propto \bar{n}_e^2 \Rightarrow \bar{p} = 2\bar{n}_e k_b \bar{T} = \bar{p}_1\left(\frac{\bar{n}_e}{\bar{n}_{e,1}}\right)^3 \qquad \tau_{rad} = \tau_{r,1}\left(\frac{\bar{n}_e}{\bar{n}_{e,1}}\right)^{-1}\left(\frac{\bar{T}}{\bar{T}_1}\right)^{3/2} = \tau_{r,1}\left(\frac{\bar{n}_e}{\bar{n}_{e,1}}\right)^2$$

$$3\left(\frac{\bar{n}_e}{\bar{n}_{e,1}}\right)^2\frac{d}{dt}\left(\frac{\bar{n}_e}{\bar{n}_{e,1}}\right) = -\frac{1}{\tau_{r,1}}\left(\frac{\bar{n}_e}{\bar{n}_{e,1}}\right) \Rightarrow \boxed{\bar{n}_e(t) = \bar{n}_{e,1}\left(1-\tfrac{2}{3}\frac{t-t_1}{\tau_{r,1}}\right)^{1/2}} \Rightarrow \boxed{\bar{T}(t) = \bar{T}_1\left(1-\tfrac{2}{3}\frac{t-t_1}{\tau_{r,1}}\right)}$$



Radiative cooling

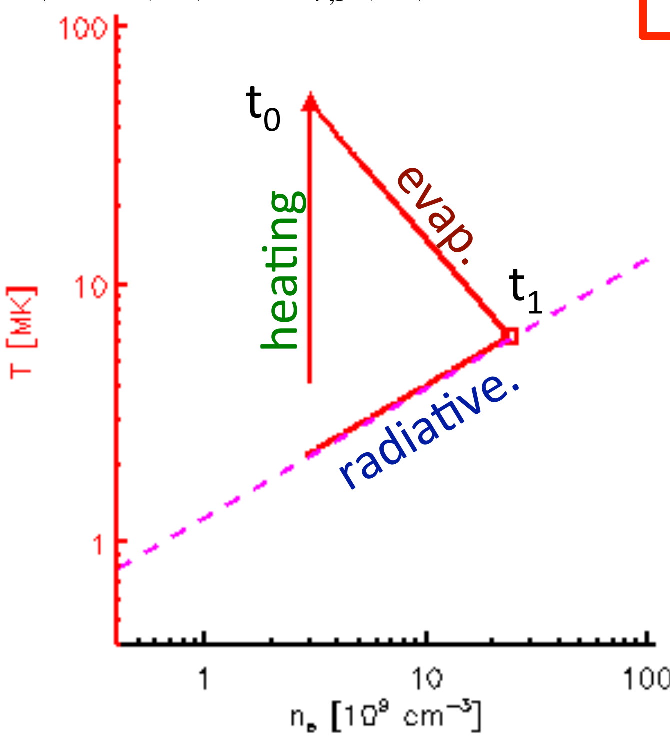
remain equal

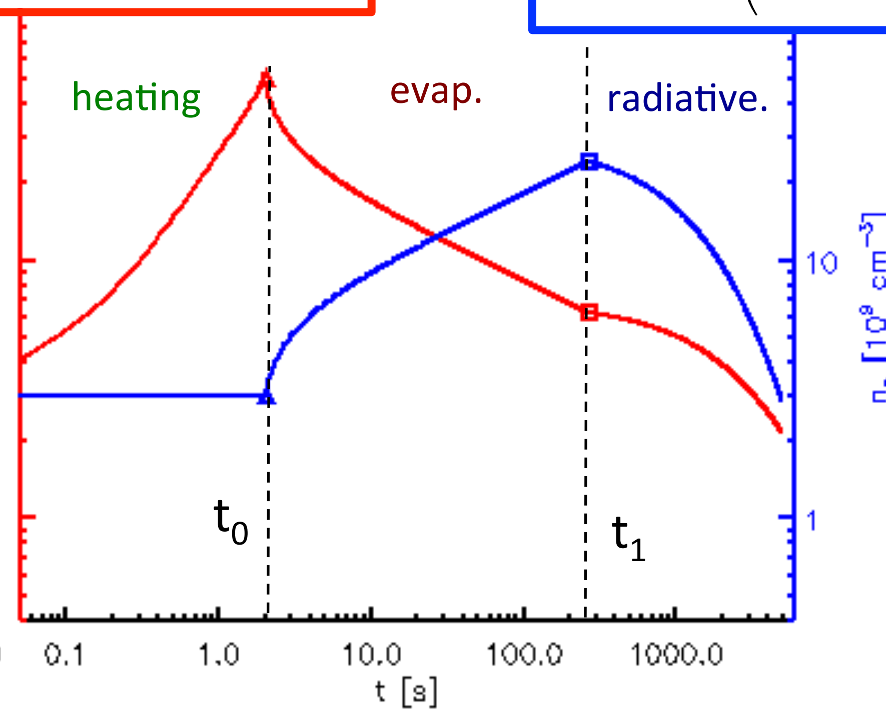
$$\frac{d}{dt}\left(\tfrac{3}{2}\bar{p}\right) \approx -\bar{n}_e^2\,\Lambda(\bar{T}) + \cancel{\tfrac{5}{L}p_{tr}u_{tr}} - \frac{8\kappa_0}{7L^2}\bar{T}^{7/2} + \cancel{\tfrac{2}{L}F_{fl}}$$

II. mechanical equilibrium

$$\bar{T} \propto \bar{n}_e^{1/2} \;\rightarrow\; \bar{p} = 2\bar{n}_e k_b \bar{T} = \bar{p}_1\left(\frac{\bar{T}}{\bar{T}_1}\right)^3 \qquad \tau_{rad} = \tau_{r,1}\left(\frac{\bar{n}_e}{\bar{n}_{e,1}}\right)^{-1}\left(\frac{\bar{T}}{\bar{T}_1}\right)^{3/2} = \tau_{r,1}\left(\frac{\bar{T}}{\bar{T}_1}\right)^{-1/2}$$

$$3\left(\frac{\bar{T}}{\bar{T}}\right)^2 \frac{d}{dt}\left(\frac{\bar{T}}{\bar{T}}\right) = -\frac{2}{\tau_{r,1}}\left(\frac{\bar{T}}{\bar{T}}\right)^{7/2} \;\rightarrow\; \boxed{\bar{T}(t) = \bar{T}_1\left(1 + \tfrac{1}{3}\frac{t-t_1}{\tau_{r,1}}\right)^{-2}} \;\rightarrow\; \boxed{\bar{n}_e(t) = \bar{n}_{e,1}\left(1 + \tfrac{1}{3}\frac{t-t_1}{\tau_{r,1}}\right)^{4}}$$

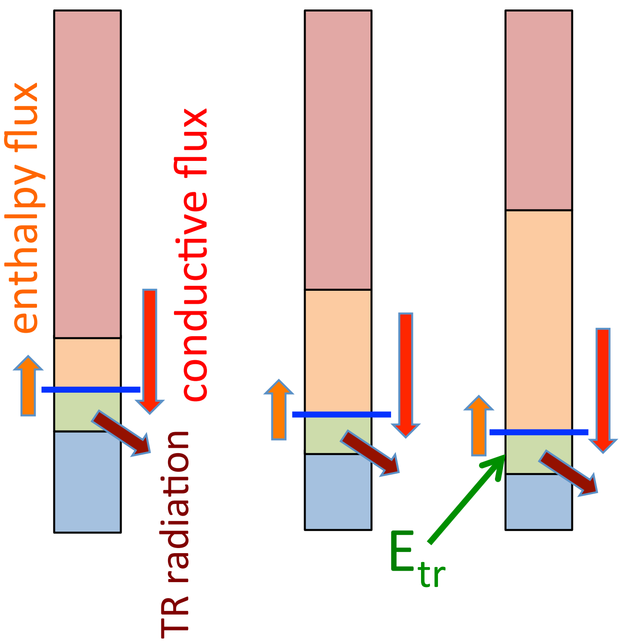




Evaporation again:

EBTEL – Klimchuk *et al.* 2008

$$\frac{d}{dt}\left(\tfrac{3}{2}\bar{p}\right) \approx \underbrace{-\bar{n}_e^2\,\Lambda(\bar{T})}_{R_{co}} + \underbrace{\tfrac{5}{L}p_{tr}\,u_{tr}}_{F_e} - \overbrace{\frac{8\kappa_0}{7L^2}\bar{T}^{7/2}}^{F_c} + \tfrac{2}{L}F_{fl}$$



- Heat flows into TR – conductive flux: F_c
- TR expands into corona – enthalpy flux: F_e
- TR radiates $R_{tr} = c_1 R_{co}$
- No change in E_{tr}

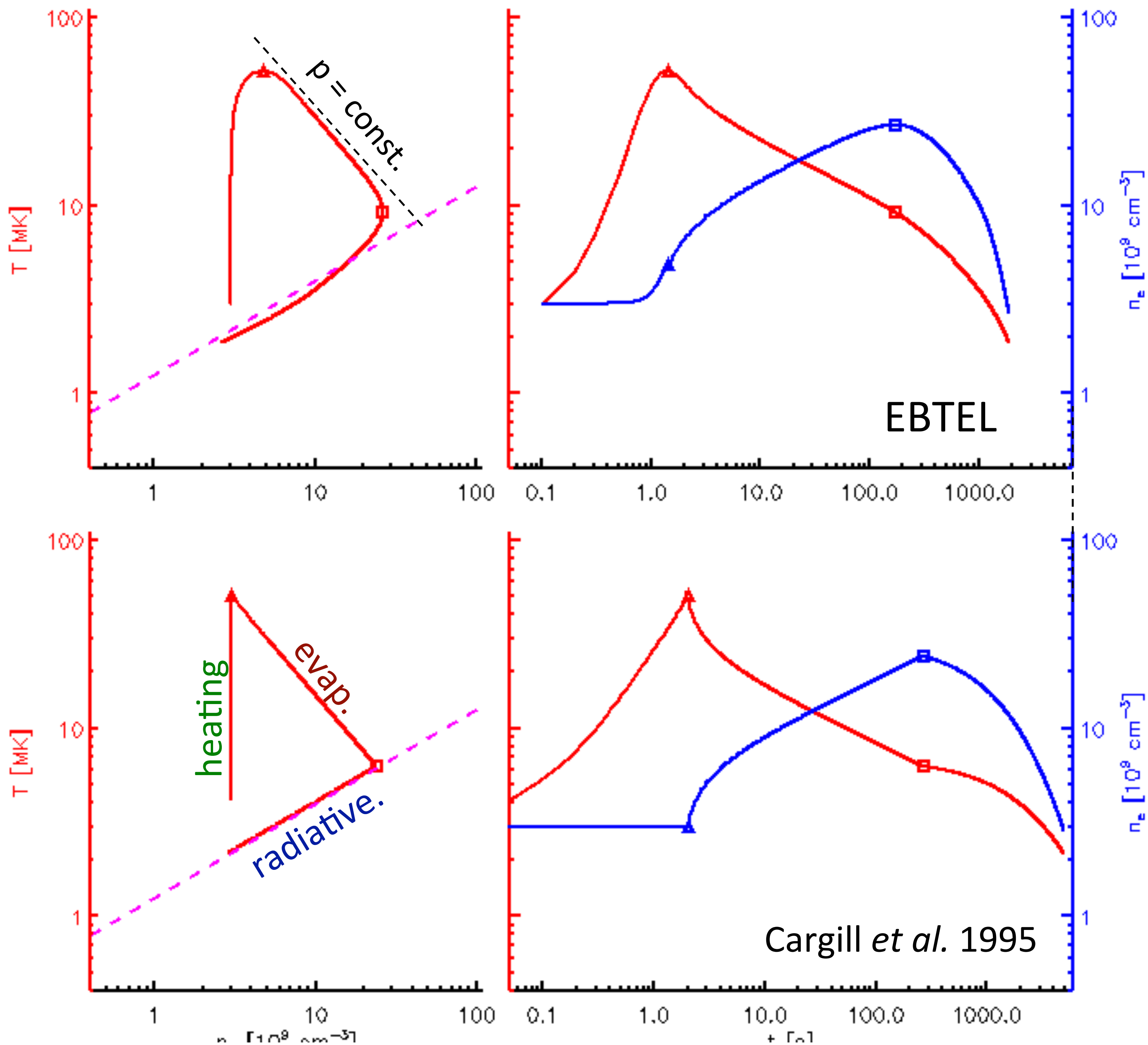
➔ $F_e = F_c - R_{tr} = F_c - c_1 R_{co}$

$$\tfrac{2}{L}u_{tr} = \frac{2}{5\bar{p}}F_e = \frac{2}{5\bar{p}}\left(\frac{8\kappa_0}{7L^2}\bar{T}^{7/2} - c_1\bar{n}_e^2\,\Lambda(\bar{T})\right)$$

$$\boxed{\frac{d}{dt}\left(\tfrac{3}{2}\bar{p}\right) \approx -(1+c_1)\bar{n}_e^2\,\Lambda(\bar{T}) + \tfrac{2}{L}F_{fl}}$$

$$\boxed{\frac{d\bar{n}_e}{dt} = \tfrac{2}{L}u_{tr}\bar{n}_e = \frac{1}{5k_b\bar{T}}\left(\frac{8\kappa_0}{7L^2}\bar{T}^{7/2} - c_1\bar{n}_e^2\,\Lambda(\bar{T})\right)}$$

2 eqns. for unknowns $p(t)$ & $n_e(t)$ — $T = p/(2k_b n_e)$



p = const.
heating
evap.
radiative.
EBTEL
Cargill et al. 1995
T [MK]
n_e [10^9 cm^{-3}]
t [s]
100
10
1
0.1
1.0
10.0
100.0
1000.0

$$\sqrt{\frac{\tau_{rad}}{\tau_{cond}}} = \sqrt{\frac{8\times10^{-6}}{7\times1.2\times10^{-19}}}\,\frac{\bar{T}^2}{\bar{n}_e L} = \frac{3\times10^{6}}{(2k_b)^2}\,\frac{\bar{p}^2}{\bar{n}_e^3 L} = 4\times10^{37}\,\frac{\bar{p}^2}{\bar{n}_e^3 L}$$

$\tau_{cond} = \tau_{rad}$

$$\boxed{\bar{n}_e = 3\times10^{12}\,\frac{\bar{p}^{2/3}}{L^{1/3}}}$$

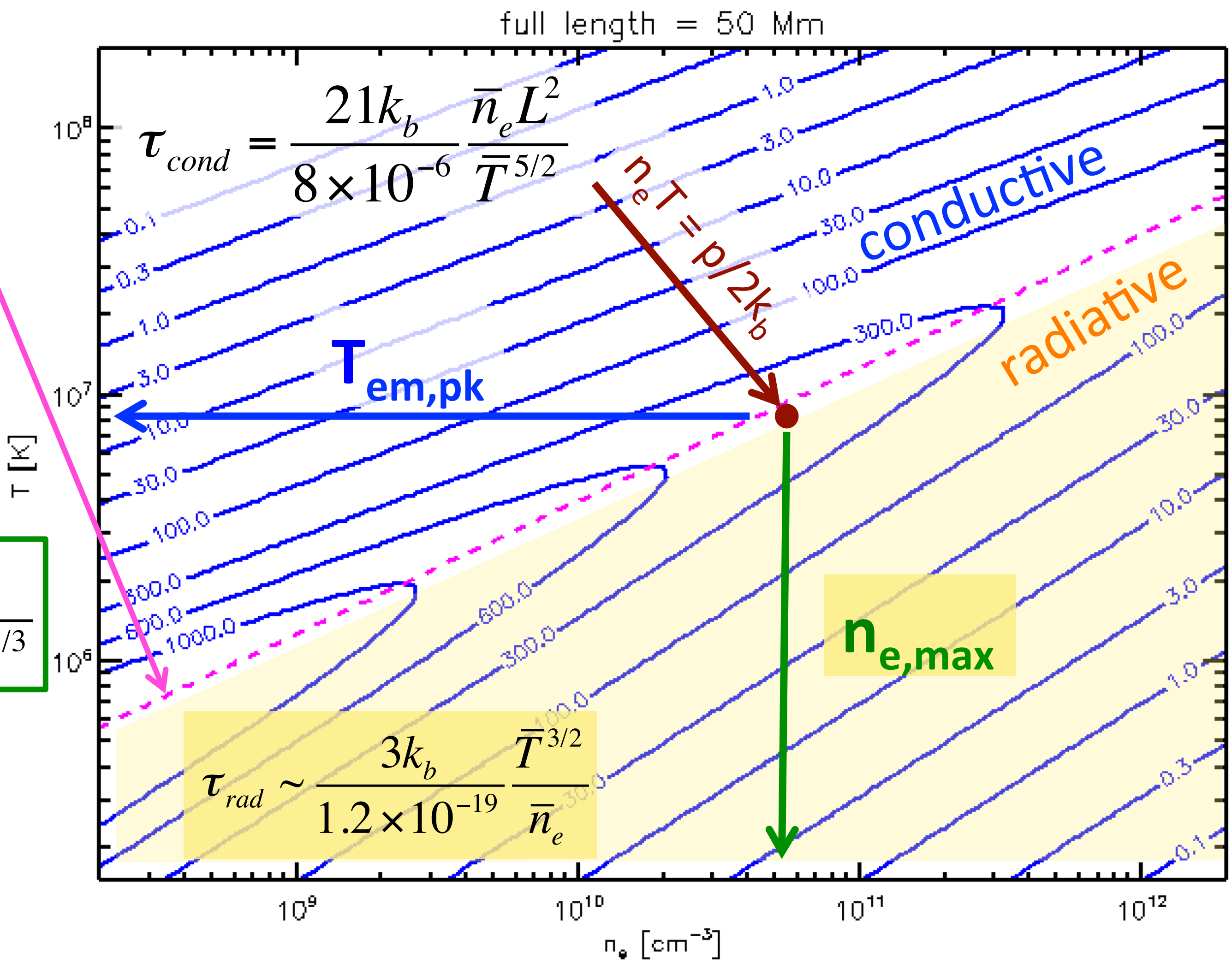
All flare energy, E, kept through evap phase

$$\bar{p} = \frac{2E}{3V}$$

$$\boxed{\bar{n}_{e,\max} = 2.6\times10^{12}\,\frac{E^{2/3}}{V^{2/3}L^{1/3}}}$$

Warren & Antiochos 2004

$$\boxed{\bar{T}_{em,pk} = 930\left(\frac{EL}{V}\right)^{1/3}}$$

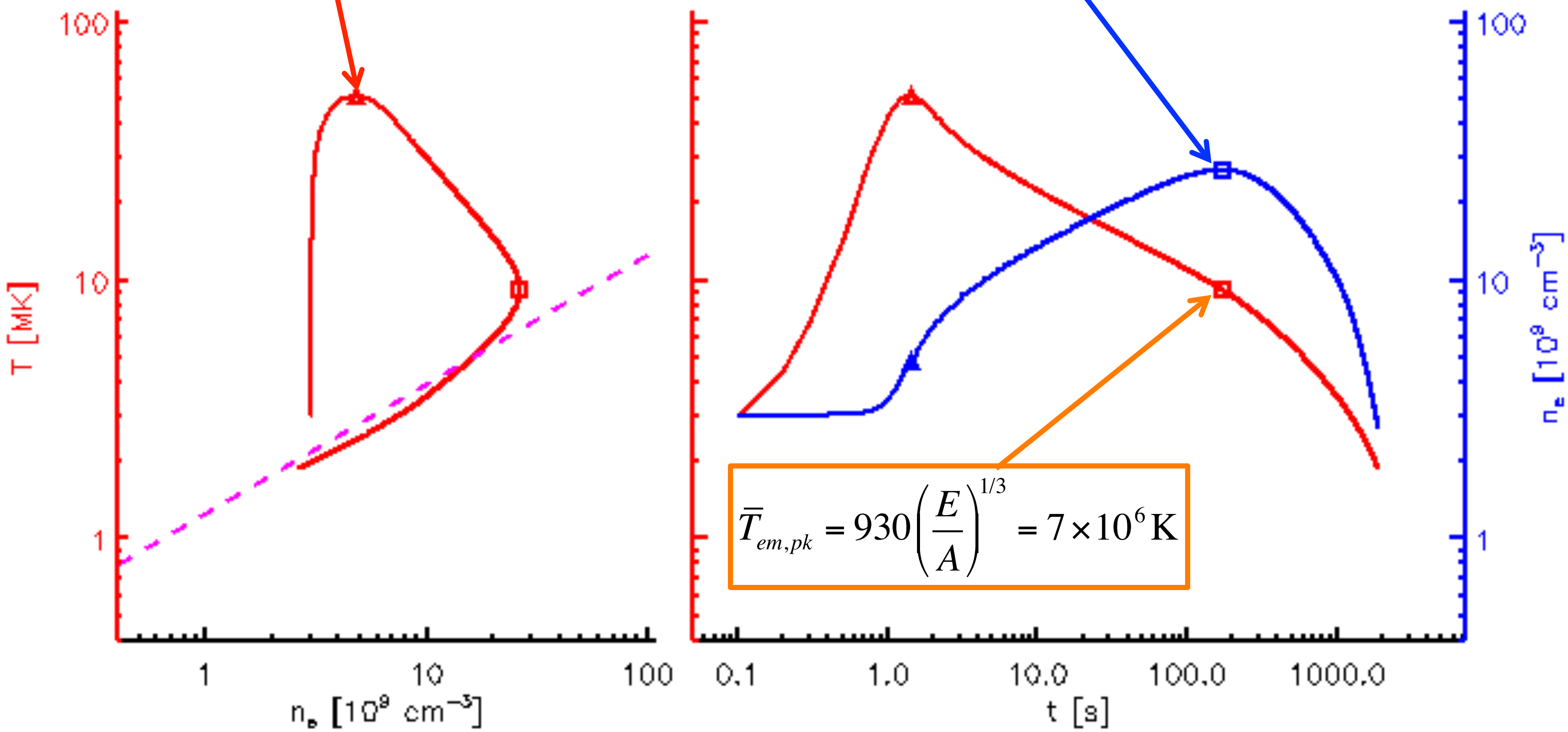


$F_{fl} = 10^{11}$ erg/s/cm^2 L = 5×10^9 cm = 50 Mm

$$\frac{E}{A} = 2\int F_{fl}\,dt \;=\; 4\times10^{11}\ \mathrm{erg\,cm^{-2}}$$

$$\bar{T}_{pk} = \left(\frac{7LF_{fl}}{4\kappa_0}\right)^{2/7} = 5\times10^{7}\,\mathrm{K}$$

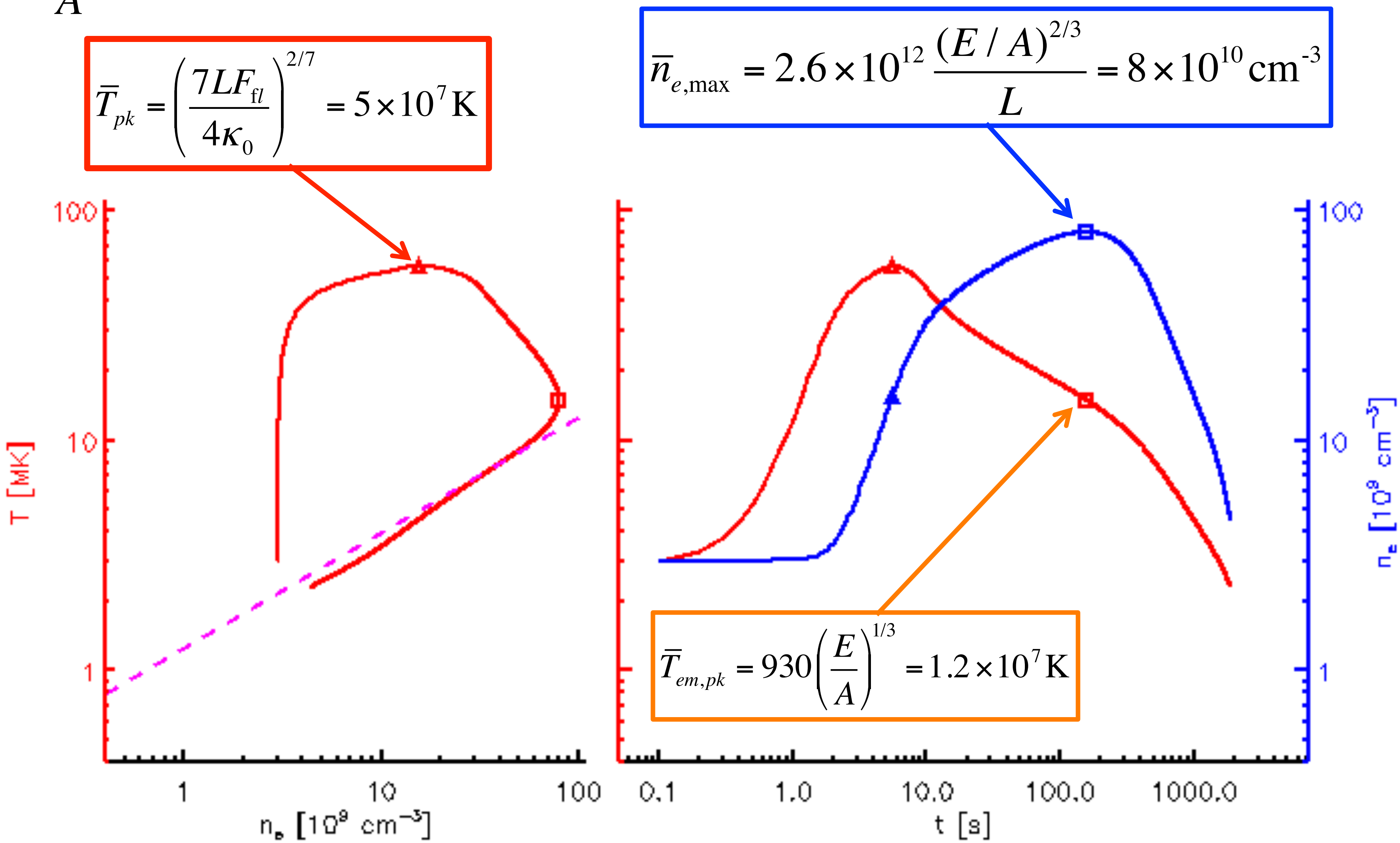
$$\bar{n}_{e,\max} = 2.6\times10^{12}\,\frac{(E/A)^{2/3}}{L} = 3\times10^{10}\,\mathrm{cm^{-3}}$$



$$\bar{T}_{em,pk} = 930\left(\frac{E}{A}\right)^{1/3} = 7\times10^{6}\,\mathrm{K}$$

$F_{fl} = 10^{11}$ erg/s/cm^2 $\quad$ $L = 5 \times 10^9$ cm = 50 Mm

$$\frac{E}{A} = 2\int F_{fl}\,dt = 2\times10^{12}\ \mathrm{erg\,cm^{-2}}$$



$$\boxed{\bar{n}_{e,\max} = 2.6\times10^{12}\,\frac{E^{2/3}}{V^{2/3}L^{1/3}}} \qquad \boxed{\bar{T}_{em,pk} = 930\left(\frac{EL}{V}\right)^{1/3}}$$

IF flare were a single loop

$$\max(EM) = V\,\bar{n}_{e,\max}^{2} = 7\times10^{24}\,\frac{E^{4/3}}{V^{1/3}L^{2/3}} = 7\times10^{49}\,\mathrm{cm}^3\,\frac{E_{30}^{4/3}}{V_{27}^{1/3}L_9^{2/3}}$$

$$\bar{T}_{em,pk} = 930\left(\frac{EL}{V}\right)^{1/3} = 9\times10^{6}\,K\left(\frac{E_{30}L_9}{V_{27}}\right)^{1/3}$$

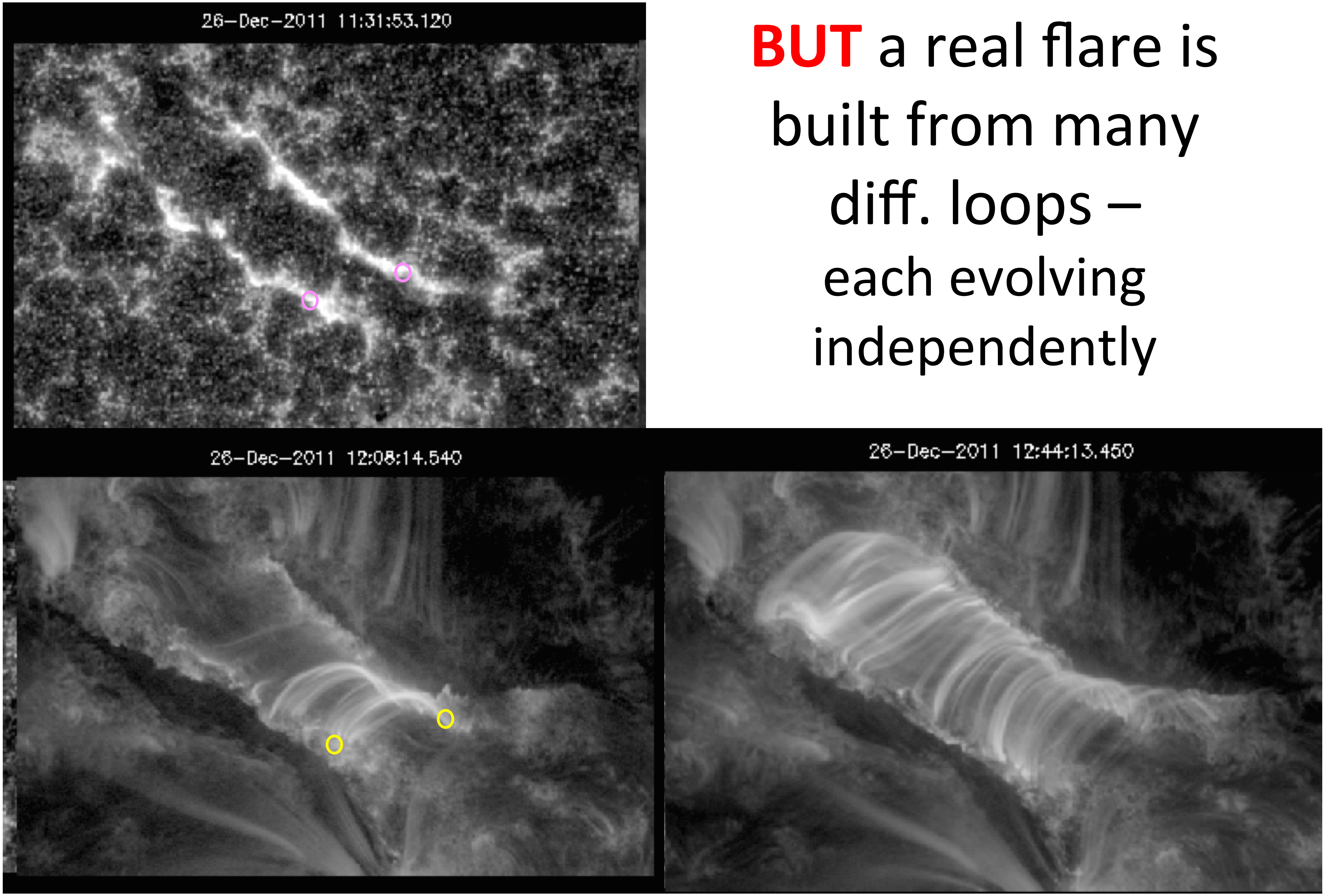
$E_{30} = E/10^{30}$ ergs

$\sim E^{7/4}$

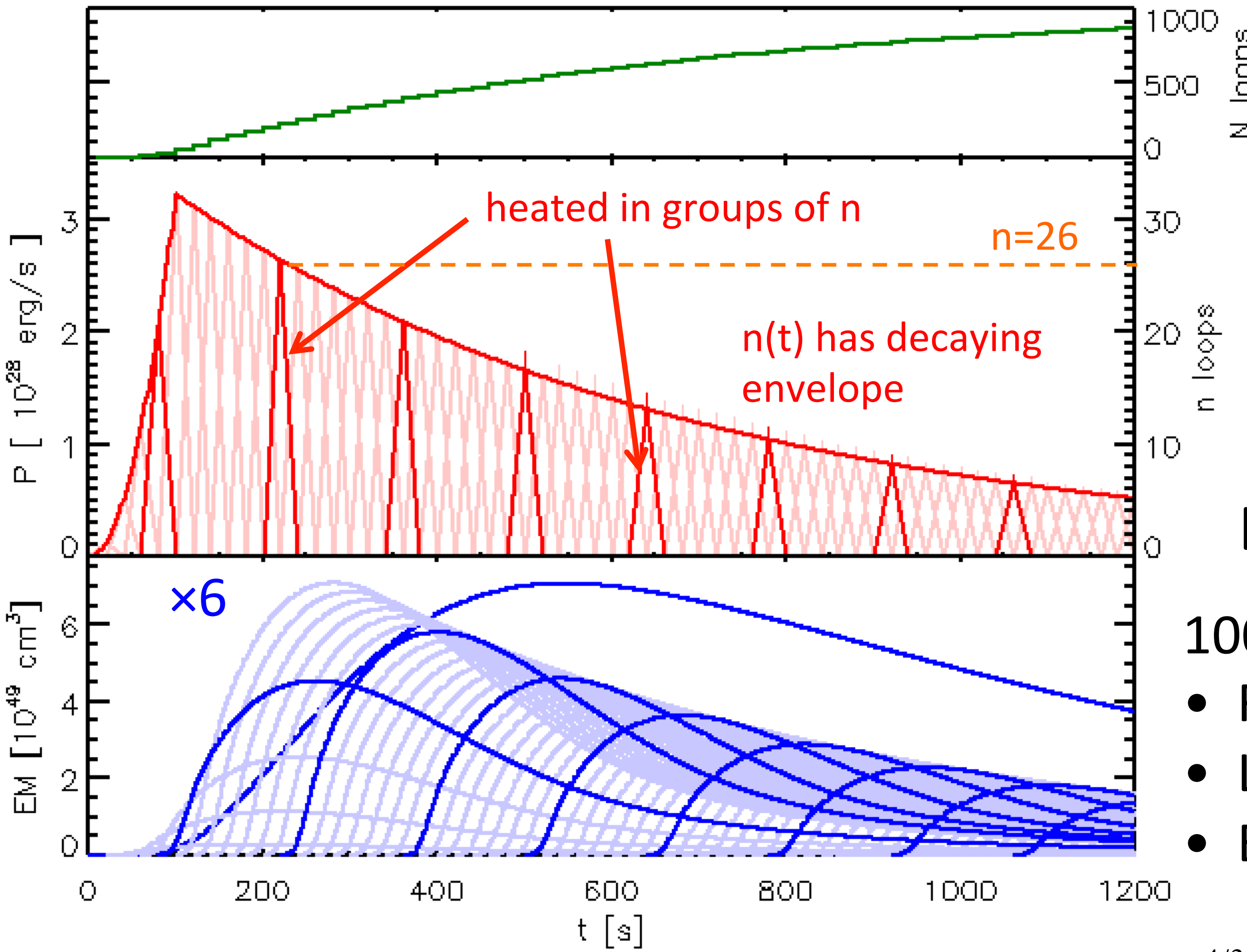
GOES peak:

$$\boxed{F_{1-8} \approx 10^{-63}\,\frac{\mathrm{W}}{\mathrm{m}^2}\,EM\;T^{5/4} = \underbrace{4\times10^{-5}\,\frac{\mathrm{W}}{\mathrm{m}^2}}_{\text{M4 flare}}\cdot\frac{E_{30}^{7/4}}{L_9^{1/4}V_{27}^{3/4}}}$$

Warren & Antiochos 2004



BUT a real flare is built from many diff. loops – each evolving independently



Example model:

Hori et al. 1997, Warren et al. 2002, Reeves & Warren 2002, Warren 2006, Qiu et al. 2012, 2013, …

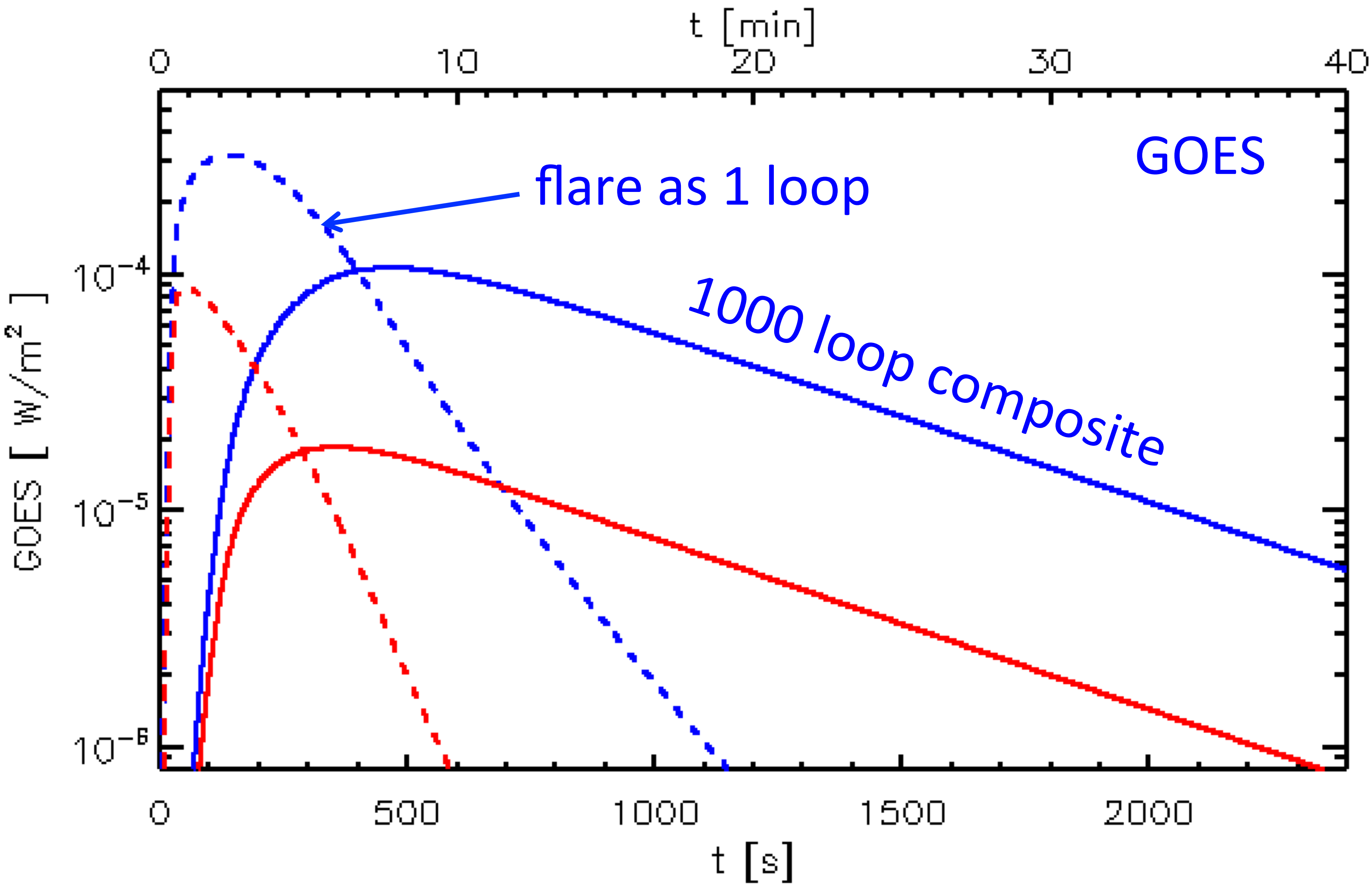
$E = 2 \times 10^{31}$ erg

1000 loops:

- $F_{fl} = 10^{11}$ erg/s/cm^2
- $L = 5 \times 10^9$ cm
- $E_i = 2 \times 10^{28}$ erg

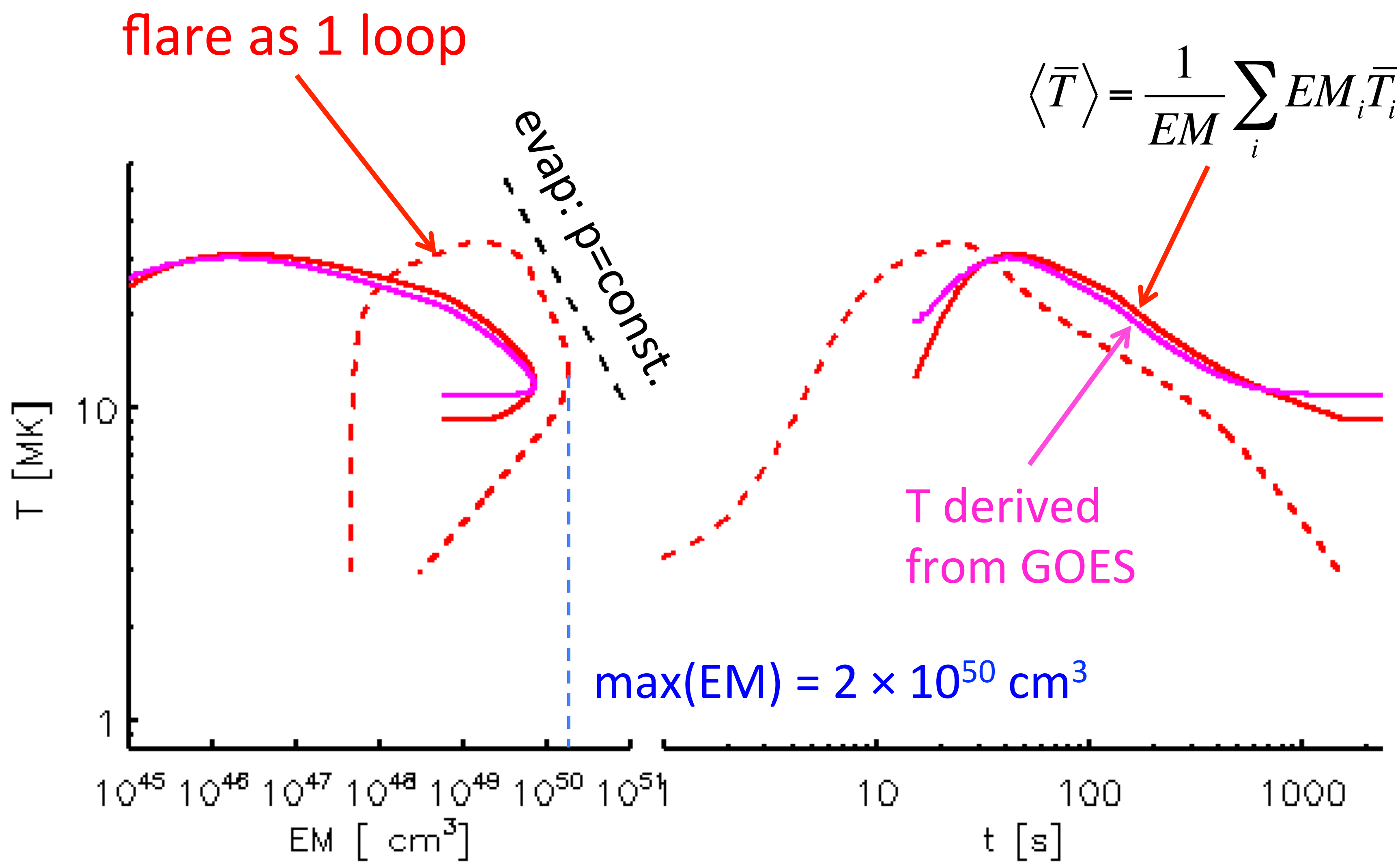
Each loop: $$\max(EM) = 7\times 10^{49}\,\mathrm{cm}^3 \frac{E_{30}^{4/3}}{V_{27}^{1/3} L_9^{2/3}} = 0.03\times 10^{49}\,\mathrm{cm}^3$$

Flare as 1 loop: $$\max(EM) = 7\times 10^{49}\,\mathrm{cm}^3 \frac{E_{30}^{4/3}}{V_{27}^{1/3} L_9^{2/3}} = 30\times 10^{49}\,\mathrm{cm}^3$$

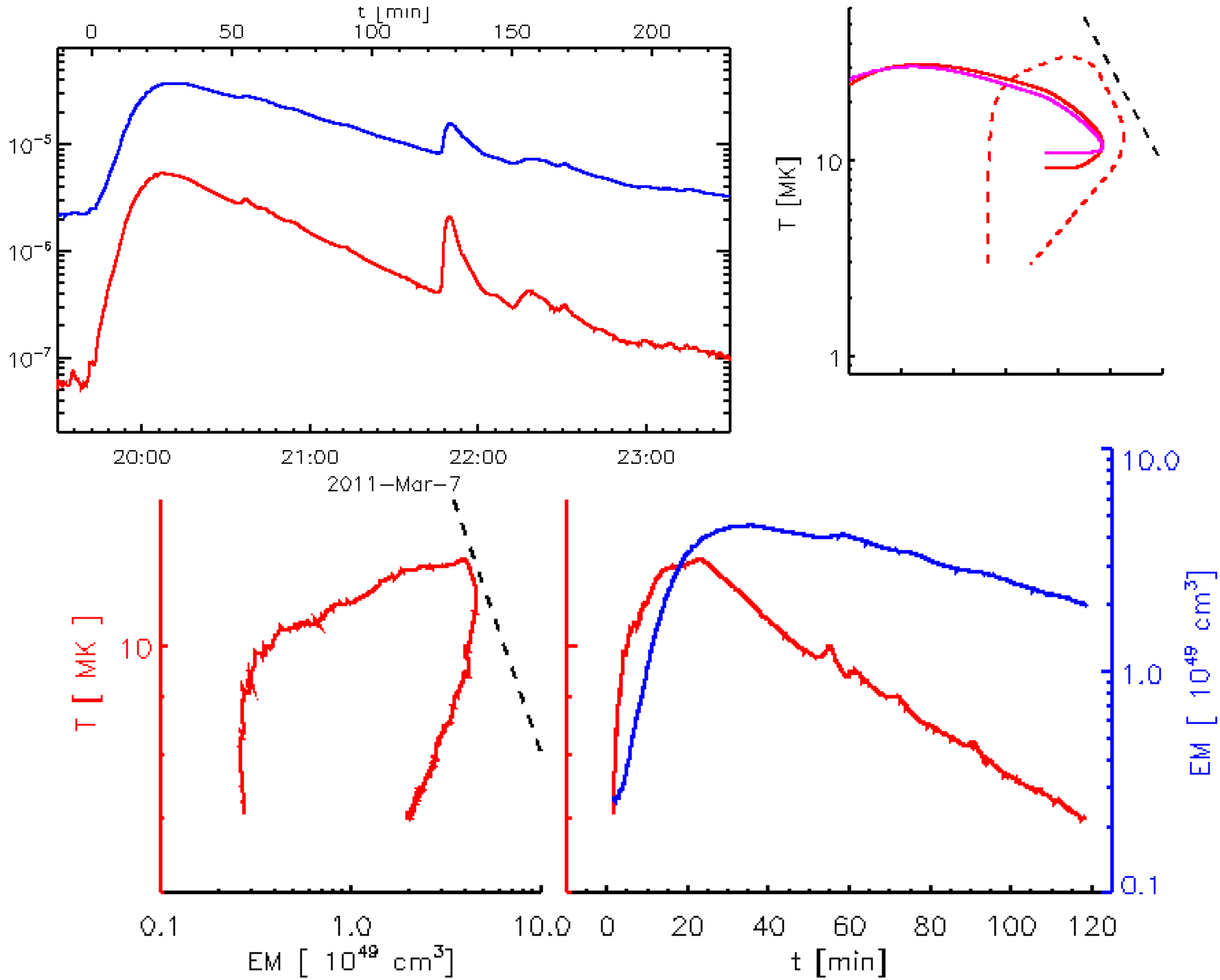


Flare as 1 loop:

$$F_{1-8} \approx 4\times10^{-5}\,\frac{\mathrm{W}}{\mathrm{m}^2}\cdot\frac{E_{30}^{7/4}}{L_9^{1/4}V_{27}^{3/4}} = 3\times10^{-4}\,\frac{\mathrm{W}}{\mathrm{m}^2}$$



flare as 1 loop
evap: p=const.
$\langle \bar{T} \rangle = \frac{1}{EM} \sum_i EM_i \bar{T}_i$
T derived from GOES
max(EM) = 2 × 10^50 cm^3
T [MK]
10
1
10^45 10^46 10^47 10^48 10^49 10^50 10^51
EM [cm^3]
1
10
100
1000
t [s]



t [min]
0
50
100
150
200
10^{-5}
10^{-6}
10^{-7}
20:00
21:00
22:00
23:00
2011−Mar−7
T [MK]
10
1
T [MK]
10
0.1
1.0
10.0
EM [10^{49} cm^3]
0
20
40
60
80
100
120
t [min]
10.0
1.0
0.1
EM [10^{49} cm^3]

Summary

- Evolution following energy release – governed by 1d gas dynamics
- Single loop experiences 3 phases:
  - Heating ($T \uparrow$ @ ~ const. n_e)
  - Evaporation ($T \downarrow$, $n_e \uparrow$, p ~ const. or $\downarrow$)
  - Radiative cooling ($T \downarrow$, $n_e \downarrow$)
- Real flare may be composite of many loops

Next time:

Evolution in one dimension