Converting E&M formulae from MKS to cgs

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Maxwell's equations assume famously different forms in the two systems of units.

	MKS	cgs
Faraday	$\frac{\partial}{\partial t}\mathbf{B}_{\rm mks} = -\nabla\times\mathbf{E}_{\rm mks}$	$\frac{\partial}{\partial t}\mathbf{B}_{\rm cgs} = -c\nabla\times\mathbf{E}_{\rm cgs}$
Ampère	$\nabla\times\mathbf{B}_{\rm mks} = \mu_0\mathbf{J}_{\rm mks}$	$\nabla\times\mathbf{B}_{\rm cgs} = \frac{4\pi}{c}\mathbf{J}_{\rm cgs}$
Gauss	$\nabla\cdot\mathbf{E}_{\rm mks} = \frac{1}{\epsilon_0}\rho_{\rm mks}$	$\nabla\cdot\mathbf{E}_{\rm cgs} = 4\pi\rho_{\rm cgs}$

Each formula on the left (MKS) can be converted to the corresponding formula on the right (cgs) using the substitutions in the left column of the following table

convert	cgs/MKS factor	note
$\mathbf{B}_{\rm mks} \to \mathbf{B}_{\rm cgs}$	10^4	Tesla $= 10^4$ Gauss
$\mu_0 \to 4\pi$	10^7	$\mu_0 = 4\pi\times 10^{-7}$
$\epsilon_0 \to \frac{1}{4\pi c^2}$	10^{-11}	$\epsilon_0\mu_0 = 1/c^2$
$\mathbf{E}_{\rm mks} \to c\,\mathbf{E}_{\rm cgs}$	10^6	from Faraday
$\mathbf{J}_{\rm mks} \to \frac{1}{c}\mathbf{J}_{\rm cgs}$	10^{-5}	from Ampère
$\rho_{\rm mks} \to \frac{1}{c}\rho_{\rm cgs}$	10^{-7}	from Gauss and/or $\frac{\partial\rho}{\partial t} = -\nabla\cdot\mathbf{J}$
$q_{\rm mks} \to \frac{1}{c}q_{\rm cgs}$	10^{-1}	$(q\mathbf{E})_{\rm mks} \to (q\mathbf{E})_{\rm cgs}$
$I_{\rm mks} \to \frac{1}{c}I_{\rm cgs}$	10^{-1}	$I = \int \mathbf{J}\cdot d\mathbf{a}$
$\eta_{\rm mks} \to c^2\,\eta_{\rm cgs}$	10^{11}	Ohm's law: $\mathbf{E} = \eta\,\mathbf{J}$

The first three are fundamental relations between the unit systems. The next three are obtained using the equations listed, and the last three are related variables. These same substitutions can be used to convert any E&M formula or expression from MKS to cgs. Some common examples are

$$(q\,\mathbf{v}\times\mathbf{B})_{\rm mks} \to \left(q\,\frac{\mathbf{v}}{c}\times\mathbf{B}\right)_{\rm cgs} \quad , \quad (\mathbf{J}\times\mathbf{B})_{\rm mks} \to \left(\frac{1}{c}\,\mathbf{J}\times\mathbf{B}\right)_{\rm cgs}$$

$$\left(\frac{q_1q_2}{4\pi\epsilon_0}\right)_{\rm mks} \to (q_1q_2)_{\rm cgs} \quad , \quad \left(\frac{B}{\sqrt{\mu_0\rho}}\right)_{\rm mks} \to \left(\frac{B}{\sqrt{4\pi\rho}}\right)_{\rm cgs}$$

$$\left(\frac{1}{2\mu_0}\,B^2\right)_{\rm mks} \to \left(\frac{1}{8\pi}B^2\right)_{\rm cgs} \quad , \quad \left(\frac{\epsilon_0}{2}\,E^2\right)_{\rm mks} \to \left(\frac{1}{8\pi}E^2\right)_{\rm cgs}$$

$$\omega_p^2 = \left(\frac{q^2 n}{\epsilon_0 m}\right)_{\rm mks} = \left(\frac{4\pi q^2 n}{m}\right)_{\rm cgs} \quad , \quad \Omega_c = \left(\frac{qB}{m}\right)_{\rm mks} = \left(\frac{qB}{mc}\right)_{\rm cgs}$$

In the last row the substitutions yield equalities since the unit of time is the same in both systems.

The constants μ_0 and and ϵ_0 go over to combinations of c and 4π, making the inverse conversion, from cgs to MKS, more problematic: when does 4π become μ_0, and when is it just a number that does not change?

The "factor" in the table is the ratio of the cgs expression to the corresponding MKS expression. It is a power of 10 accounting for the changes from meters to centimeters and kilograms to grams. The scaling of Faraday's law, for instance, in each system can be written

$$\frac{B_{\rm mks}}{t_{\rm s}} \propto \frac{E_{\rm mks}}{\ell_{\rm m}} \quad , \quad \frac{B_{\rm cgs}}{t_{\rm s}} \propto \frac{c\,E_{\rm cgs}}{\ell_{\rm cm}}$$

where $\ell_{\rm m}$ is the value of some length scale expressed in meters and $\ell_{\rm cm}$ is the same length scale, but expressed in centimeters. The dimensionless constant of proportionality depends on the structure of the functions and is the same for both versions. That constant is eliminated by taking the ratio of the two expressions to obtain the factor listed in the table

$$\frac{c\,E_{\rm cgs}}{E_{\rm mks}} = \frac{\ell_{\rm cm}}{\ell_{\rm m}} \cdot \frac{B_{\rm cgs}}{B_{\rm mks}} = 10^2 \cdot 10^4 = 10^6 \quad . \tag{1}$$

Similarly, the scaling of Ampère's law yields

$$\frac{B_{\rm mks}}{\ell_{\rm m}} \propto \mu_0\, J_{\rm mks} \quad , \quad \frac{B_{\rm cgs}}{\ell_{\rm cm}} \propto \frac{4\pi\, J_{\rm cgs}}{c} \quad .$$

Taking the ratio of these two expressions we find the corresponding factor

$$\frac{J_{\rm cgs}/c}{J_{\rm mks}} = \frac{\ell_{\rm m}}{\ell_{\rm cm}} \cdot \frac{\mu_0}{4\pi} \cdot \frac{B_{\rm cgs}}{B_{\rm mks}} = 10^{-2} \cdot 10^{-7} \cdot 10^4 = 10^{-5} \quad . \tag{2}$$

These factors are used for unit-conversions

	: cgs unit		: MKS unit	= conversion
$\mathbf{B}_{\rm cgs}$	: Gauss	$\mathbf{B}_{\rm mks}$	: Tesla	$= 10^4$ Gauss
$\mathbf{E}_{\rm cgs}$	: statvolt/cm	$\mathbf{E}_{\rm mks}$	: V/m	$= 10^6/c = 3.33 \times 10^{-5}$ statvolt/cm
$\mathbf{J}_{\rm cgs}$	: statamp/cm^2	$\mathbf{J}_{\rm mks}$	: A/m^2	$= {\rm c}\,10^{-5} = 3 \times 10^5$ statamp/cm^2
$\Phi_{\rm cgs} = \int \mathbf{B}\cdot d\mathbf{a}$	: Maxwell	$\Phi_{\rm mks}$	: Weber	$= 10^8$ Mx
$V_{\rm cgs} = \int \mathbf{E}\cdot d\mathbf{l}$	: statvolt	$V_{\rm mks}$	: Volt	$= 10^8/c = 3.33 \times 10^{-3}$ statvolt
$q_{\rm cgs}$	: statcoulomb	$q_{\rm mks}$	: Coulomb	$= c/10 = 3 \times 10^9$ statcoulomb
$I_{\rm cgs}$	: statamp	$I_{\rm mks}$	: Amp	$= c/10 = 3 \times 10^9$ statamp