On Three-Dimensional Magnetic Skeleton Elements due to Discrete Flux Sources

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Abstract. The magnetic field in the solar corona plays an important role in coronal heating, flaring activity and many other phenomena studied on the Sun. Magnetic topology is frequently used to understand complicated coronal magnetic fields. By calculating the *skeleton* of a field, it is possible to build up a sophisticated representation of the key elements of a field's configuration. This paper determines a simple relation between the numbers of *separators* (X), *coronal null points* (N_c), *flux domains* (D) and *flux sources* (S) in such a configuration: $D = X + S - N_c - 1$. This equation is used to explain the behaviour of some of the bifurcations found in Magnetic Charge Topology, and to show that a one-to-one relationship exists between the number of circuits in the domain graph and the augmented null graph. Finally, it is shown that in quiet Sun regions, the number of separators is approximately proportional to the number of flux sources.

Keywords: Sun: magnetic fields, Sun: corona, Magnetic fields: topology

1. Introduction

The solar coronal magnetic field is in a state of continual change. In the photosphere, many flux elements ranging in strength from tiny ($\sim 10^{17}$ Mx) intra-network regions to gigantic ($\sim 10^{21}$ Mx) sunspots continually emerge, move around, fragment, cancel, coalesce, and submerge; since these photospheric *flux sources* are responsible for the overlying field, these activities can lead to large changes in the corona; interactions between topologically distinct regions can lead to dynamic events including solar flares (Lau, 1993; Aulanier *et al.*, 1998; Fletcher *et al.*, 2001).

The sheer number of flux sources makes the coronal field extremely complicated. *Magnetic Topology* (Molodenskii and Syrovatskii, 1977; Gorbachev *et al.*, 1988; Priest and Titov, 1996; Longcope, 1996, e.g.) was developed as a tool to extract important features from a complicated field.

In this paper, we use the principles of magnetic topology to relate the numbers of some of these features – separators, null points, flux domains and flux sources, all of which are defined later in this section.

Magnetic topology is most commonly undertaken in the form of Magnetic Charge Topology or MCT (Longcope, 1996), in which flux sources are modelled as point charges and the field is generally assumed to be potential. Here, we make neither assumption; however, we must assume that the flux system under consideration is balanced, and that the sources are discrete and resolved. We can apply the same results to an unbalanced system by adding a balancing source at a large distance from the system.

The photosphere is usually modelled as either an infinite plane or a sphere; the results in this paper apply whenever the photosphere is a topologically spherical (genus-0) flux surface on which all of the sources lie. The vertical field is assumed to consist of isolated, unipolar regions surrounded by a connected *field-free sea*. Here, field-free is taken to mean the component of the magnetic field normal to the photosphere vanishes; the components tangent to the photosphere are generally non-zero. Point sources satisfy this assumption, although many other configurations are possible.

We also assume that the coronal field is smooth and that, with a small number of exceptions defined below, each field line is anchored to a positive and a negative photospheric source.

Given more than two sources under these conditions, there must exist a number of *null points*, locations at which the magnetic field vanishes. Due to the MCT assumptions, a population of these (*photospheric* nulls) must lie on the photospheric surface, within the field-free sea. It is also possible for nulls to exist outside of the surface; nulls outside or above are called *coronal* nulls, and those inside or below are *mirror* nulls.

Parnell *et al.* (1996) described the linear magnetic field in the vicinity of a null point; a typical scenario is illustrated in Figure 1. They found that the only stable nulls possible in any field were *force-free* – with the nearby field satisfying $\mathbf{J} \times \mathbf{B} = \mathbf{0}$. A co-ordinate system can be found such that the first-order linear field near the null can be written in the form $\mathbf{B} = \mathbf{M} \cdot \mathbf{x}$, where $\mathbf{x}$ is the displacement from the null point and $\mathbf{M}$ is a matrix of the form:

$$\mathbf{M} = \begin{pmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_x}{\partial y} & \frac{\partial B_x}{\partial z} \\ \frac{\partial B_y}{\partial x} & \frac{\partial B_y}{\partial y} & \frac{\partial B_y}{\partial z} \\ \frac{\partial B_z}{\partial x} & \frac{\partial B_z}{\partial y} & \frac{\partial B_z}{\partial z} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2}(q - j_{\parallel}) & 0 \\ \frac{1}{2}(q - j_{\parallel}) & p & 0 \\ 0 & j_{\perp} & -(p+1) \end{pmatrix}, \quad (1)$$

where $j_{\parallel}$, $j_{\perp}$, p and q are parameters of the force-free field.

$\mathbf{M}$ generally has three non-zero complex eigenvalues. The sum of these, in view of $\mathrm{tr}(\mathbf{M}) = \nabla \cdot \mathbf{B} = 0$, must vanish. Let λ_1 be either the only real eigenvalue or the eigenvalue of opposite sign to the other two.

The eigenvector $\mathbf{e_1}$ associated with λ_1 defines two isolated *spine* field lines (Priest and Titov, 1996). For photospheric nulls, the spines either

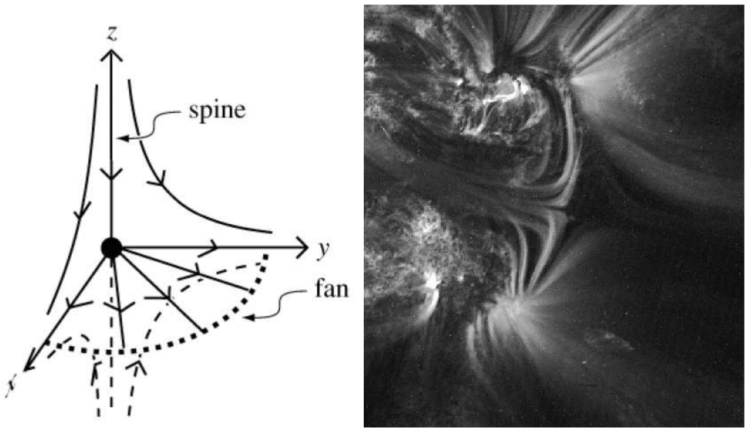

Figure 1. Left: The skeleton of a (positive) magnetic null point. The null point is represented by a heavy dot; thin solid and dashed lines are field lines. A thick, dotted curve with no physical meaning helps to visualise the fan plane. Two isolated field lines (here, above and below the null) constitute spines; another set of field lines originating at the null point form a fan, or separatrix surface. *Right*: Null-like structure observed in TRACE's 171Åpassband. The spine runs almost vertically, while the fan plane is horizontal and out of the page.

lie entirely in the photosphere, or are directed into the corona. In the first instance, the null is labelled *prone*; in the second, it is *upright*. The *spine sources* of a null are defined to be the sources at which its spine field lines end.

The numbers of prone and upright nulls (N_p and N_u) are given by the two-dimensional Euler characteristic (Molodenskii and Syrovatskii, 1977):

$$S + N_u = N_p + 2, \tag{2}$$

where S is the total number of sources, assuming flux balance.

The two remaining eigenvectors define the null's *fan plane*. If the spine sources are distinct, then field lines beginning in this plane form a *separatrix surface*.

A *flux domain* is a contiguous volume of magnetic field lines sharing the same start and end sources. The surface of a flux domain is formed by separatrix surfaces.

If both spine field lines of a null end at the same source, field lines in its fan plane form a surface which is not a flux boundary, so it is not considered a separatrix surface.

A separatrix surface in which the field lines connect to more than one source must be bounded in part by the spine field lines of a null of the opposite sign (Beveridge, 2003). In this case, at least one field line connects the two null points; such a field line is a *separator* and is a key location for reconnection, as it constitutes a boundary between four distinct flux domains (Greene, 1988; Lau and Finn, 1990; Priest and Titov, 1996; Galsgaard and Nordlund, 1997). All of these topological elements are present in Figure 2.

There is, however, one class of null point that we do neglect. It is theoretically possible, although not so far observed, for a pair of nulls each to have one spine source and one fan source, and to be connected by a separator, as shown in Figure 3. The spine of either null in this configuration surrounds the fan of the other entirely. In such a *chain-link* configuration, the surfaces formed by the fan are not separatrices; the arrangement lies entirely within a flux domain and makes no contribution to the skeleton. Thus, we ignore it.

Longcope and Klapper (2002) presented a systematic method for characterising the skeleton of an arbitrarily complicated field. They found the following relationship between flux domains (D_Σ), separators (X_Σ), null points (n_Σ) and sources (S), in a potential field over all of space, including the mirror corona:

$$D_\Sigma = X_\Sigma - n_\Sigma + S. \tag{3}$$

However, their analysis did not include nulls with unbroken fans, or upright nulls. Equation (3) can thus not be applied directly to cases where such nulls are present.

In the present paper, we are concerned merely with the photosphere and real corona. Maclean *et al.* (2004) modified Equation (3) to apply in this region; reasoning that flux domains with a *photospheric footprint* (photospheric domains) are counted only once, while those without (coronal domains) have a real and mirror presence. D_Σ can then be expressed as $D_\phi + 2D_c$ where D_ϕ is the number of photospheric and D_c the number of coronal domains.

By the same token, the number of nulls $n_\Sigma = n_\phi + 2n_c$, while the number of separators (none of which lie in the photosphere) is $X_\Sigma = 2X$, leading to the modified Longcope-Klapper equation:

$$D_\phi + 2D_c = 2X - n_\phi - 2n_c + S. \tag{4}$$

However, these equations ignore nulls unconnected to any other by separators. In this paper, we present a relationship between all of the skeletal elements, neglecting only the chain-link configurations described above.

In the following section, we use Descartes-Euler and Schläfli relationships to establish, from geometric principles, a relationship between the numbers of sources, nulls, separators and flux domains. We then discuss

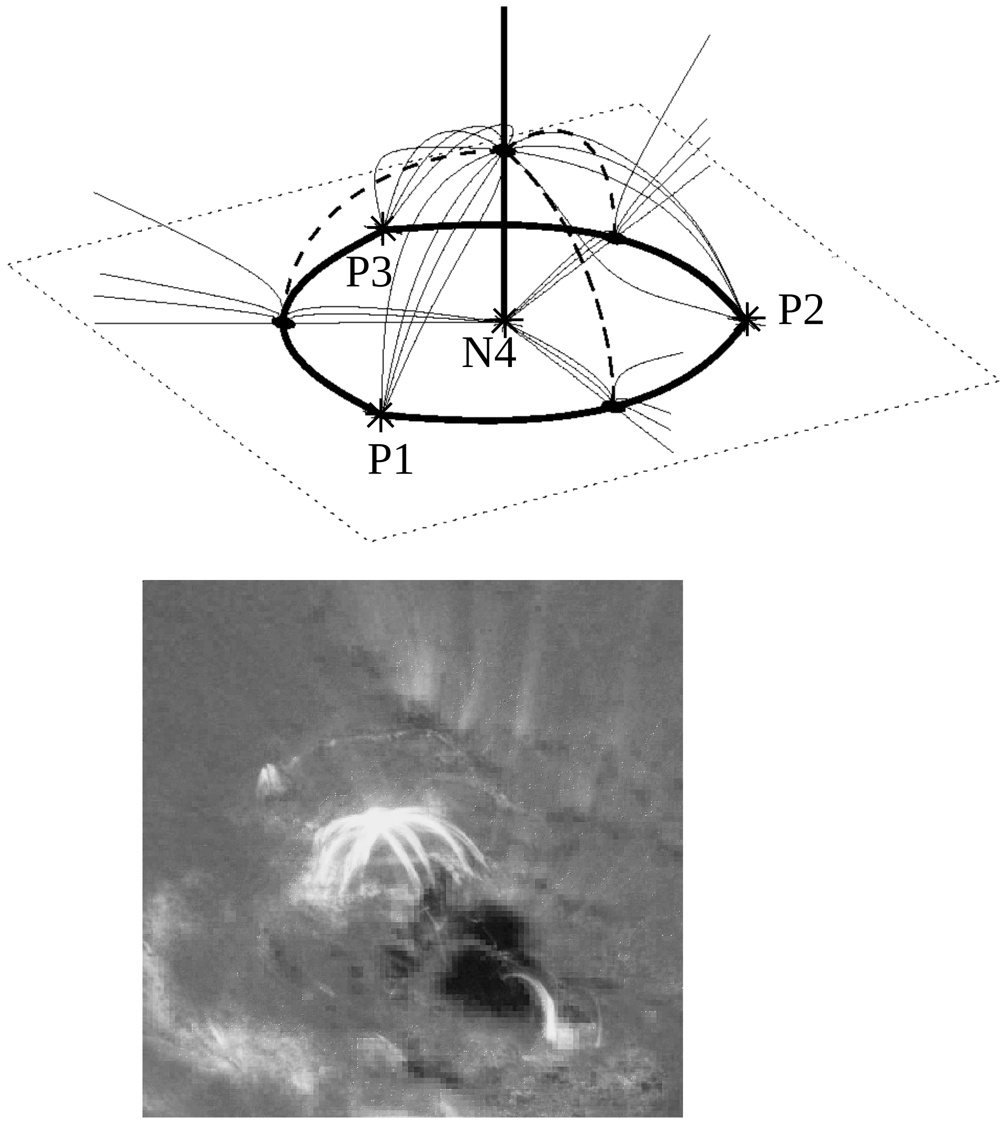

Figure 2. Above: A schematic view of a topological configuration with four null points (heavy dots), one of which is coronal (Brown and Priest, 2001). Three of the separatrix surfaces (thin lines) form walls, while that of the coronal null forms a dome. There are three separators (dashed lines). The stars are flux sources and the heavy lines denote spine field lines. There are six flux domains, of which three are inside and three outside the dome. All six are photospheric. There is an implicit fifth, balancing source at infinity. *Below*: A TRACE image with an apparently similar configuration.

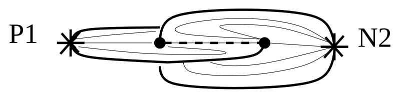

Figure 3. A chain-link configuration. The fan of the left-hand null lies horizontally, surrounded by the spines of the right-hand null. The right-hand null's fan is vertical and bounded by the spines of the left-hand null. A separator connects the two nulls.

the implications of our result for the study of solar magnetic fields in general.

2. Derivation

2.1. Motivation

It has long been suspected that a simple equation links the numbers of flux domains, separators, null points and sources. Empirical studies (Close, 2004; Schrijver and Title, 2002; Beveridge, Longcope and Priest, 2003, e.g.) have found the number of separators (X) and number of flux domains (D) both scale linearly with the number of sources (S). This latter relation can be written as

$$D \sim \frac{\eta}{2} S,$$

where η is the average number of flux domains connected to a source. The factor of one half comes about because each domain connects to two sources. The value of η is disputed: Beveridge, Longcope and Priest (2003) found η to be a little over 20, while Schrijver and Title (2002) placed it between five and seven. Close (2004) obtained a value of $\eta \sim 13$.

2.2. Overview of derivation

We begin by defining our terms, and introducing a transformation to remove some of the complications involving upright null points. We note that the magnetic field under consideration can be divided into *isolated regions* that can be treated separately; two null points belong to the same region if and only if there exists a path along separators connecting the nulls. The spine sources of such nulls are part of the region, as are the flux domains connecting them.

We then examine the photospheric footprint of an isolated region, and treat it as a surface comprising polyhedra, to which we apply the Descartes-Euler formula to relate the numbers of faces (flux domains), edges (skeleton field lines) and vertices (nulls and sources).

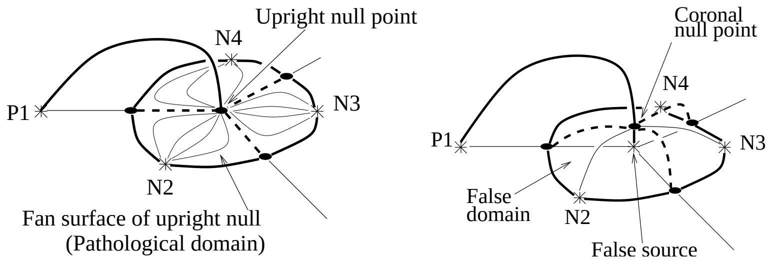

Figure 4. To avoid the problems associated with a pathological, two-dimensional domain due to an upright null point (*left*), we transform it into a null-source pair (*right*). This creates new, *false* flux domains which can be easily extracted later. The separatrix surface of the false null point forms a dome, surrounding as many false domains as there are separators (here, three). For clarity, the mirror corona is excluded. Dots represent null points; thick solid lines denote their spine field lines. Thin solid lines are field lines in the fan plane, and thick dashed lines represent separators.

After this, we can examine the region in three-dimensions using a Schläfli equation to relate faces (fan sectors), edges (spines and separators), vertices (nulls and sources) and volumes (flux domains).

We then combine these two results to obtain a simple relation. We can then reconcile the results from each region to show that the relation holds for the entire magnetic field.

2.3. Transformation and isolation

The critical distinction will concern flux domains which are split in two by the photosphere (*photospheric domains*) and those which contact the photosphere only at points or curves (*coronal domains*). There is one class of domain which defies this categorisation. The fan surface of an upright null point lies in the photosphere, forming a pathological, two-dimensional domain. In order to avoid this complication, it is useful to make a simplifying transformation to eliminate upright nulls, replacing the pathological domains with conventional photospheric domains.

To do this, we replace each upright null point by a photospheric source, a coronal null point and a mirror null point (Figure 4). Each photospheric separator is transformed into two: one coronal and one mirror. The transformation also creates as many new *false* flux domains as there were photospheric separators; each of these is a photospheric domain.

There are also issues involving nulls with unbroken fans, and indeed any two flux systems in which no path exists along separators linking two nulls in different systems.

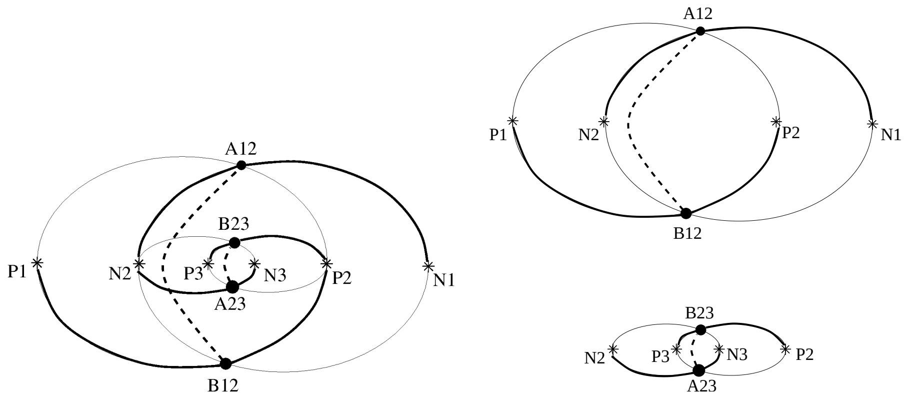

Figure 5. Left: A complex topology with six sources (stars), four null points (dots) and two separators (dashed lines). The thick and thin solid lines are spine field lines and separatrix traces, respectively. All field lines plotted lie in the photosphere except the separators, which are in the corona. *Right*: This configuration may be decomposed into two separate isolated regions in which each pair of null points is linked by a separator path. Note that the sources P2 and N2 and the domain connecting them appear in both isolated regions. This apparent anomaly is accounted for in Section 2.8.

To deal with this, we define an *isolated region* as a flux system in which each null point is linked by a path along separators to each other null (see Figure 5), or a null with an unbroken fan. The system includes all spine and fan sources of null points in the region. If more than one isolated region exists, then exactly two sources (one of either polarity) in a region also belong to at least one other region. We label these *external* sources.

To see this, we note that each isolated region or combination thereof may stand alone as a topology satisfying the Euler equations. Since we have eliminated upright nulls, the two-dimensional version becomes:

$$S_{ir} = N_{p,ir} + 2$$

$$S_{or} = N_{p,or} + 2$$

$$S = N_p + 2,$$

where the suffices *ir* and *or* refer to the isolated region and the outer region, and the last line refers to the total number of sources and prone nulls for the entire topology.

Adding the first and noting that $N_{p,ir} + N_{p,or} = N_p$ (since each null belongs to exactly one region) gives:

$$S_{ir} + S_{or} = N_p + 4.$$

Substituting $N_p = S - 2$, we get:

$$S_{ir} + S_{or} = S + 2,$$

and so exactly two external sources exist. The *surrounding domain* connecting these sources is considered to be part of the region.

We show later in this section that isolated regions can be pieced back together recursively to reconstruct the entire field. In so doing, we find that the relationships between numbers of elements apply to the reconstructed field.

2.4. Nomenclature

Throughout this section, the numbers of each topological element in a single isolated region will be represented by a capital letter.

The number of flux domains is represented by D; the numbers of true photospheric and coronal domains are given by D_ϕ and D_c respectively. D^f_ϕ is the number of false domains created by the previous transformation.

The number of coronal separators, excluding false separators, is given by X; because of symmetry, the number of mirror separators is the same. X_ϕ is the number of photospheric separators in the untransformed configuration.

There are S flux sources. N_c of the nulls are coronal (and, again due to symmetry, as many are mirror nulls). There are N_p prone nulls and N_u upright nulls in the original setup.

After the transformation described above, there will be $S + N_u$ sources, N_p prone nulls, no upright nulls and $N_c + N_u$ coronal nulls. Once more due to symmetry, there are as many mirror nulls.

2.5. Photospheric flux domains

By examining the *photospheric footprint* - the layout of spine and separatrix field lines in the photospheric surface - we can determine the relative numbers of the skeletal elements found there.

We treat the network as a set of polyhedra in the photosphere, which is either a two-dimensional, topologically spherical surface or an infinite plane. In either case, the Descartes-Euler relation applies:

$$f = e - v + 2,$$

where f, e and v are the numbers of faces, edges and vertices, respectively.

As shown in Figure 6, each face in the footprint represents a photospheric flux domain. In the case where we consider a bounded region,

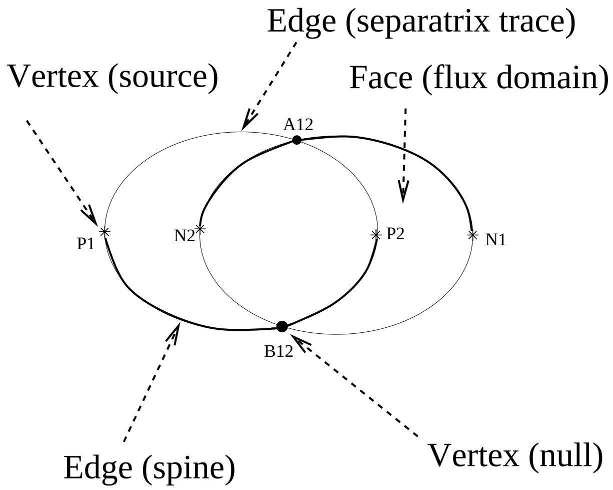

Figure 6. The network decomposition of an isolated region's footprint. Each null point (dot) and source (star) constitutes a vertex, while each spine field line (thick line) and separatrix trace (thin line) is represented as an edge. Flux domains (empty spaces) correspond to faces.

the external face is also counted as a flux domain of the region (the overlap will be accounted for later). The false domains due to the transformation are also counted, giving:

$$f = D_\phi + D_\phi^f.$$

Each edge is the analogue of a photospheric skeleton field line - either a spine or a separatrix trace. Each prone null gives rise to two of each, so that:

$$e = 4N_p.$$

Each vertex is either a source (including any due to the transformation) or a prone null point, giving:

$$v = (S + N_u) + N_p.$$

Combining these four equations gives:

$$D_\phi + D_\phi^f = 4N_p - (S + N_u) - N_p + 2 = 3N_p - S - N_u + 2. \qquad (5)$$

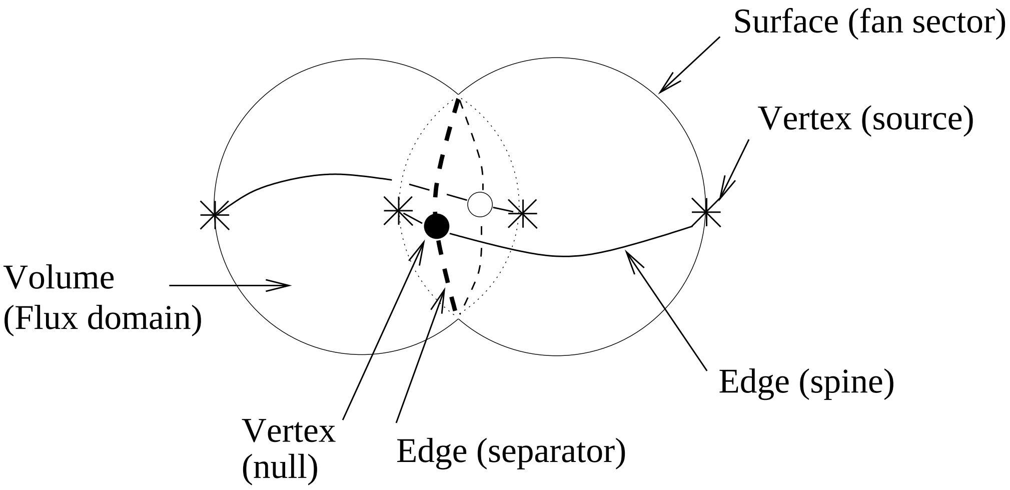

Figure 7. An isolated region in three dimensions. This can be envisaged as the intersection of two spheres, delimited by thin solid lines. The dashed lines at their intersection are separators, connecting the two null points (dots). The thick solid lines are spine field lines.

2.6. Total flux domains

We can apply a similar approach to the three-dimensional structure using the Schläfli equation (Poincaré, 1893):

$$V' = v' - e' + f'.$$

where v', e', f' and V' are the numbers of vertices, edges, faces and volumes, respectively, as shown in Figure 7.

Each volume corresponds to a flux domain:

$$V' = D_\phi + 2D_c + D_\phi^f.$$

Each vertex represents either a null point or a source, as usual including those due to the transformation:

$$v' = (S + N_u) + 2N_c + N_p + 2N_u.$$

Each edge corresponds to a spine field line or a separator (including those due to the transformation); each null point has two spines.

$$e' = 2(N_p + 2N_u + 2N_c) + 2(X + X_\phi).$$

Lastly, every face corresponds to a fan sector. There are twice as many of these as there are separators (including ones due to the transformation):

$$f' = 4(X + X_\phi).$$

Combining these equations gives:

$$D_\phi + 2D_c + D_\phi^f = S - (N_u + N_p + 2N_c) + 2(X_\phi + X). \tag{6}$$

2.7. Consolidation for one region

Adding together Equation (5) and Equation (6) gives:

$$2(D_\phi + D_\phi^f + D_c) = 2N_p - 2N_u - 2N_c + 2X_\phi + 2X + 2.$$

We also know that $D_\phi^f = X_\phi$, so these terms cancel, meaning that the false domains disappear. Furthermore, Equation (2) gives $S - 2 = N_p - N_u$. We also let $D = D_\phi + D_c$. Making these substitutions and halving:

$$D = X + S - N_c - 1, \tag{7}$$

in any isolated region.

2.8. Consolidation for all domains

The previous subsection demonstrated that for each such region, Equation (7) holds. This subsection will show that piecing the regions back together does not disrupt the equation.

The number of domains in each region includes one which connects the two sources on its boundary. This domain and these sources necessarily belong in two of the regions.

We know that Equation (7) holds in both regions:

$$D^a = X^a + S^a - N_c^a - 1;$$
$$D^b = X^b + S^b - N_c^b - 1.$$

Adding these together gives:

$$(D^a + D^b) = (S^a + S^b) + (X^a + X^b) - (N_c^a + N_c^b) - 2.$$

Note that $D^a + D^b = D + 1$, because the external domain is counted twice. Equally, $S^a + S^b = S + 2$, because of the double-counting of the two external sources. $X^a + X^b = X$ since there is no separator belonging to both regions; the same reasoning gives $N_c^a + N_c^b = N_c$. Thus, Equation (7) holds over the two regions considered together.

Applying this argument recursively shows that Equation (7) holds for the whole space under consideration.

3. Discussion

Although much of the literature concerned with Magnetic Topology deals with point sources and potential fields, we find that even without these restrictions, the number of flux domains, separators, sources, coronal nulls and discrete flux sources are simply related.

Equation (7) can be used to determine the number of separators, given the number of sources, flux domains and coronal nulls. Beveridge, Longcope and Priest (2003), Schrijver and Title (2002) and Close (2004) all found that the number of flux domains in a configuration can be written in the form

$$D \sim \frac{\eta}{2} S,$$

where η is the number of flux domains associated with an average source. This constant has been found experimentally to be somewhere between 5 and 7 by Schrijver and Title (2002), and around 18 by Beveridge, Longcope and Priest (2003). Further, it has been found by Inverarity and Priest (1999) and Longcope, Brown and Priest (2002) that the number of coronal nulls in a complicated domain is generally much smaller than the number of sources, although it is possible to construct a seven-source case with as many as six coronal nulls (Beveridge, 2003).

Under these conditions (a negligible number of coronal nulls and a linear dependence between the numbers of sources and flux domains), Equation (7) reduces to:

$$\frac{\eta}{2} S \sim X + S - 1,$$

or, setting $\mu = \frac{\eta}{2} - 1$ and assuming $S \gg 1$:

$$X \sim \mu S. \tag{8}$$

That is to say, the number of separators scales linearly with the number of sources.

Equation (7) also has applications to domain graphs (Longcope, 2001; Longcope and Klapper, 2002). In a domain graph, each source is represented by a vertex and each flux domain by an edge connecting the corresponding vertices. Such a graph which excludes the mirror corona is denoted $\mathcal{D}^+$.

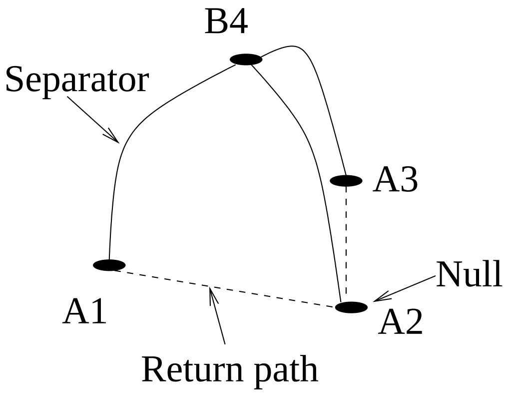

Figure 8. Augmented null graph $\mathcal{N}^*$ of the configuration shown in Figure 2. The separators (solid lines) and sub-photospheric return paths (dashed lines) are edges and the null points (heavy dots) are vertices. There are two distinct separator paths in this configuration.

The number of independent circuits in a graph is defined as the minimum number of edges that need to be removed in order to leave a graph without a circuit. The number of independent circuits is related to the elements by:

$$\mathcal{C} = e - v + 1$$

In the domain graph, $e = D$ and $v = S$, so that

$$\mathcal{C}_{\mathcal{D}^+} = D - S + 1 = X - N_c. \tag{9}$$

Longcope (2001) finds that a complete specification of all D domain fluxes requires that all S source fluxes be given, along with $\mathcal{C}_{\mathcal{D}^+}$ additional contraints. Equation (9) shows that this number of extra constraints matches the number of distinct separator paths connecting one photospheric null point to another, including those that pass through coronal nulls (see Figure 8).

By constraining the flux passing through an *isolated circuit* (Longcope, 2001), which may be formed by closing a separator path beneath the photosphere, we can provide restrictions on the domain fluxes.

To see this, consider a *null graph* in which edges represent separators and vertices null points. We augment this graph by adding edges beneath the photosphere from one of the prone nulls to each of the others. This *augmented null graph* is denoted $\mathcal{N}^*$. The number of vertices in this graph is then $v = N_c + N_p$, where N_p is the number of prone nulls. The number of edges is $e = X + (N_p - 1)$, so that the number of circuits is

$$\mathcal{C}_{\mathcal{N}^*} = e - v + 1 = X - N_c = \mathcal{C}_{\mathcal{D}^+}.$$

This circuit number $\mathcal{C}_{\mathcal{N}^*}$ can be understood as the number of independent separator paths between prone nulls. That a one-to-one correspondence exists between circuits in $\mathcal{N}^*$ and $\mathcal{D}^+$ in a potential field has been shown previously (Longcope and Klapper, 2002), but not in such a general case.

It is also interesting to consider the application of Equation (7) to MCT bifurcations such as those enumerated in Beveridge, Priest and Brown (2004), Beveridge (2003) and Maclean *et al.* (2004). Since global bifurcations have no effect on the number and type of nulls, it is clear that any bifurcation changing the number of flux domains is associated with a corresponding change in either the number of separators or the number of sources (or both). Equally, since a local bifurcation has no effect on the domain structure, the creation of a coronal null corresponds directly to the creation of a separator.

The magnetic topology of the solar corona is an intricate subject. Our results show that certain topological concepts can be related by simple equations, opening up possibilities of more detailed and accurate studies of the coronal magnetic field configuration and its links to solar activity.

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