On magnetic reconnection in compact flares: How can the models apply to real flares?

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1. Introduction

Solar flares are understood to be the rapid release of magnetic energy which has been stored in the corona. Investigators have focused on a mechanism called "fast magnetic reconnection" to explain this release. Models of magnetic reconnection have tended to focus on either the transfer of magnetic flux, or on the release of the magnetic energy. In order to explain observations of a solar flare these two effects must be unified in a single model of reconnection.

2. Two dimensions

Most reconnection modeling has used two-dimensional MHD where magnetic fields are written in terms of a flux function $\psi(x,y)$

$$\mathbf{B}(x,y) \;=\; \hat{\mathbf{z}}\times\nabla\psi \;+\; B_z(x,y)\,\hat{\mathbf{z}} \quad , \tag{1}$$

the contours of which represent complete field lines. Whether or not a third vector component B_z is included (making it a so-called two-and-a-half-dimensional model) reconnection occurs at an X-point – that point where $\nabla\psi(x,y) = 0$ (Dungey 1958). The X-point is dynamically significant because characteristics of Alfvén waves asymptote there. It is also topologically significant since it lies at the intersection of separatrices which divide topologically distinct classes of field lines.

The process of two-dimensional reconnection can be studied in the absence of other MHD effects, and free of complicated boundary conditions, using a simple scenario proposed by Craig (1994). A highly-conducting or perfectly ideal plasma is confined inside a conducting, cylindrical boundary penetrated by magnetic flux (see fig. 1). The boundary flux is consistent with a vacuum X-point

$$\psi_X(x,y) \;=\; \tfrac{1}{2}B'(x^2-y^2) \quad , \tag{2}$$

called equilibrium X. The boundaries are moved to distort the magnetic field into configuration X^*, which is not an equilibrium. The new boundary flux is defined to be consistent with a vacuum X-point field

$$\psi_{X'}(x,y) \;=\; \tfrac{1}{2}B'(x^2-y^2) \;+\; \Delta\psi \quad , \tag{3}$$

called equilibrium X'. Indeed, equilibrium X' is the magnetic field with the *minimum possible* energy, given the boundary flux.

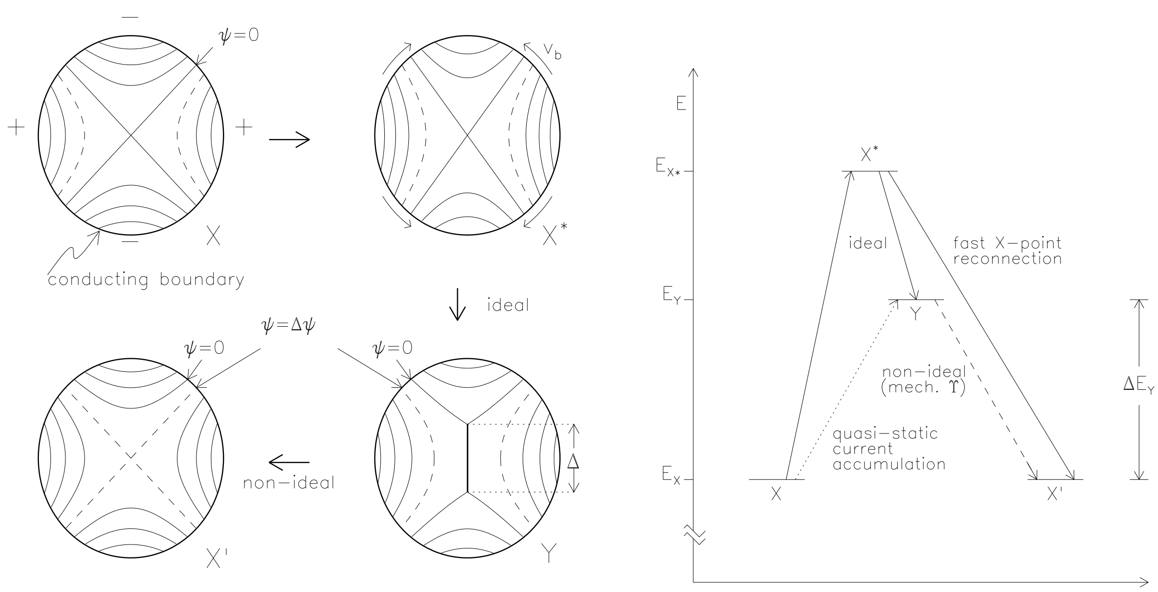

Fig. 1.—. The magnetic fields which form landmarks in the reconnection scenario of Craig (1994). Contours of the flux function $\psi(x, y)$ in states X, X^*, Y and X' are shown on the left. The energies of each state are shown on the right.

The magnetic flux functions for equilibria X and X' differ only by a constant, which can be defined as the value of $\psi(x, y)$ on the separatrix. The transition $X \rightarrow X'$ is forbidden by ideal MHD, since the ideal induction equation ($\dot{\psi} = 0$) does not permit ψ to change on any material point, including the X-point.

Postulating that ideal MHD will conserve at least the value of $\psi(x, y)$ on the separatrices, the lowest energy magnetic field, called equilibrium Y, has a discontinuous magnetic field (Syrovatskii 1971). The discontinuity is a current sheet carrying $I_Y \simeq \Delta\psi$ in the ignorable direction (cgs-emu is used here). The discontinuity occurs at the interface between four topologically distinct domains, and stretches a distance $\Delta \simeq \sqrt{\Delta\psi/B'}$ (the width), between Y-type magnetic nulls (see fig. 1). Equilibrium Y has a magnetic energy greater than equilibrium X' by $\Delta\mathcal{E}_Y \simeq \Delta\psi^2$; this is the *topological energy* stored by the single constraint on the value of the flux function on the separatrix.

A non-ideal effect, such as resistivity η, will resolve the mathematical discontinuity into a layer of finite thickness δ, and allow the value of ψ in the separatices to change. In steady-state resistive reconnection, called Sweet-Parker reconnection, the thickness $\delta \sim S_{\rm cs}^{-1/2}\,\Delta$ (Sweet 1958, Parker 1957), where the sheet Lundquist number is

$$S_{\rm cs} \;\equiv\; \frac{B_{\perp,{\rm cs}}\,\Delta}{\eta\sqrt{4\pi\rho}} \;\simeq\; \frac{\sqrt{\pi}}{4\pi\eta\sqrt{\rho}}\oint \mathbf{B}\cdot d\mathbf{l} \;=\; \frac{I_Y}{I_S} \quad , \tag{4}$$

and $I_S \equiv \eta\sqrt{\rho/\pi}$ is a characteristic current ($I_S \simeq 10^{-2}$ Amps in the solar corona). The most significant consequence of this thickness is the *flux transfer time*, $\tau_{\rm rx} \equiv \Delta\psi/\dot{\psi} = \tau_{\rm A}\sqrt{I_Y/I_S}$, required to change ψ on the separatix enough to permit $Y \rightarrow X'$.

The Sweet-Parker flux transfer time is extraordinarily long given that typical coronal currents are 10^{10} Amps. Thus the steady state current sheet is actually more appropriately viewed as a means of storing the energy $\Delta\mathcal{E}_Y$. slow boundary distortion $X \rightarrow Y$ will cause quasi-statical evolution through a sequence of current-sheet equilibria; a condition called *quasi-static current accumulation.* The slowly changing boundaries are characterized by a slowly changing value $\Delta\psi$, which in turn leads to slowly changing current sheet characteristics Δ, I_Y and $\Delta\mathcal{E}_Y$.

The process of magnetic reconnection might then occur as an instability of the steady current sheet. The unspecified instability, called mechanism-Υ here, must transfer flux through the separatrices on Alfvénic time scales. By eliminating the single constraint, this will give access to state X', thereby liberating $\Delta\mathcal{E}_Y$ in comparable time. Mechanism-Υ need not *dissipate* the energy it liberates; it acts as a trip-wire by releasing energy stored in the large-scale magnetic field. Finally, the instability must remain inactive for long periods of time to permit the quasi-static accumulation of $\Delta\mathcal{E}_Y$. It is somehow triggered to cause a flare.

3. Three dimensions

Magnetic reconnection in compact flares can be understood by a direct three-dimensional analogy of the scenario above (Longcope 1996, Longcope & Silva 1998). Motion of flux from the boundary (photosphere) builds stress at the interface of four flux domains. The stress takes the form of a narrow current layer which is undergoing steady reconnection – quasi-static current accumulation. This continues until an instability (mechanism-Υ) facilitates rapid topological flux transfer. This flux transfer is naturally linked to a release of magnetic energy.

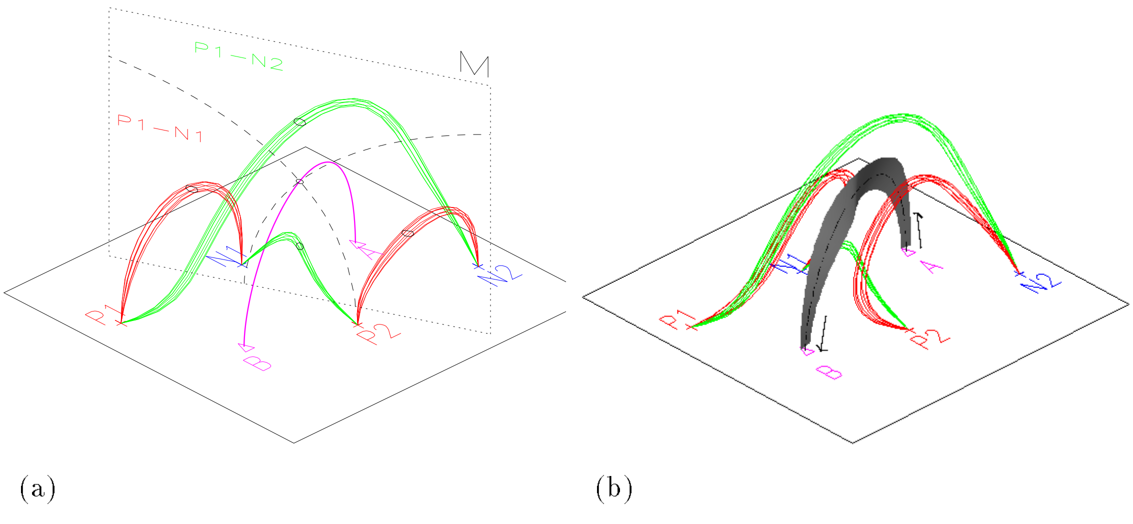

Fig. 2.—. The three dimensional analogs of states X and Y. (a) Field lines in each of the four flux domains, and the separator lying at the intersection of all four. Vertical plane M is used to demonstrate the analogy with two-dimensions. (b) The current sheet in equilibrium Y. Arrows show the direction of current.

Four localized photospheric sources, $P1$, $P2$, $N1$ and $N2$, give rise to a field with four types of magnetic field lines: $P1$–$N1$, $P1$–$N2$, etc. At the interface of these four domains is a single field line called the *separator* (see fig. 2). To see this consider the vertical plane M through which all field lines pass. The boundaries between domains, separatrices, intersect M in two curves similar to the separatrices in two-dimensional models. These curves intersect at a single point, which is the intersection of the separator with plane M. Note that the magnetic field does not vanish at the separator, nor is the magnetic field normal to M there. Unlike the two-dimensional case, the separator cannot be defined by any local property; it is a strictly topological feature.

Motion of the sources will change the form of the vacuum field, analogous to the transition $X \to X'$. These vacuum states are characterized by fluxes in domain $P2$–$N1$ which differ by $\Delta\Phi$. Barring non-ideal effects state X' is therefore inaccessible from state X, since the amount of flux in a given topological domain, such as $P2$–$N1$, cannot be changed. Equilibrium state Y has a current singularity near the original separator, and the same flux in $P2$–$N1$ as state X; it has the minimum magnetic energy given the photospheric sources.

For a medium active region each of the four flux sources will have $\sim 10^{22}$ Mx, and the separator will be $L \sim 10^{10}$ cm long. Sources moving at 100 m/s will change the vacuum flux by $\Delta\Phi \simeq 3 \times 10^{20}$ Mx in 8 hours — much longer than the Alfvén transit time. Quasi-static current accumulation occurring for this time will store $\Delta\mathcal{E}_Y \simeq 3 \times 10^{30}$ ergs in the form of $I_Y \simeq 10^{11}$ Amps. To release this energy mechanism-Υ must transfer $\Delta\Phi$ across some portion of the current sheet. Doing this during the ~ 300 seconds of a flare's impulsive phase requires 10^{10} Volts between the footpoints of the separator. While mechanism-Υ releases this energy it need not dissipate it; it is a trip-wire which eliminates a single constraint to liberate a large energy.

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