Modeling and Measuring the Flux Reconnected by the Two-ribbon Flare on 2004-11-07

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I. Two ribbon flares

 \circ Classical (2d) CSHKP model — (a)

(Carmichael 1964; Sturrock 1968; Hirayama 1974; Kopp and Pneuman 1976)



- 3d generalization (b) (Gosling 1990; Gosling et al. 1995)
- Recon'n flux: magnetic flux swept by flare ribbons (R) (Forbes & Priest 1984; Poletto & Kopp 1986; Fletcher *et al.* 2001; Qiu *et al.* 2002)
- Ejected flux rope (FR) → Magnetic Cloud (MC) at 1 AU (Burlaga et al. 1981; Lepping et al. 1990)

Outstanding Questions/Challenges

- \star Predicting ribbon flux from photospheric shear
- \star Predicting energy storage/release from photospheric shear
- \star Relating ribbon flux to MC flux (Leamon *et al.* 2004)
- \star How many sites (X) reconnect in what order?
- \star Why do ribbon features move || to ribbon? (Fletcher et al. 2004)

II. The Magnetic Model of AR 10696



- 40 Hourly MDI FD magnetograms (Scherrer et al. 1995)
- $\circ \times 1.56$ calibration factor (Berger & Lites 2003)
- \circ P-spheric **v** from LCT (November & Simon 1988)
- Partition into unipolar regions (Barnes et al. 2005; Longcope et al. 2006)
 - $\cdot B_z > 35$ G; Label downhill from local maxes
 - \cdot Merge saddles < 300 G or $\Phi < 7.6 \times 10^{19} \ {\rm Mx}$
 - \cdot Label regions from reference advected with ${\bf v}$



Histograms summarizing the mask (left) and partitioning (right) LEFT: $pos/neg = solid/dashed; +/\times$ amount in partition RIGHT: Bars: flux in regions; stair-steps: cummulative flux

Magnetic Model (cont'd)

• Characterize region \mathcal{R}_a by net flux & centroid

$$\Phi_a = \int_{\mathcal{R}_a} B_z(x, y) \, dx \, dy \quad , \quad \bar{\mathbf{x}}_a = \frac{1}{\Phi_a} \int_{\mathcal{R}_a} \mathbf{x} \, B_z(x, y) \, dx \, dy \quad .$$

• Project onto co-rotating tangent plane (correct Φ_a for LOS)

- Φ_a s change very little \implies simplify by making them const.
- Smooth $\bar{\mathbf{x}}_a(t)$ (five-hour box-car) reduce centroiding noise



Fluxes & centroids (left); centroid trajectories (right)

Helicity Injection

 $\Delta H/\Phi^2 = -1/3$ turn Full fields B_z & **v**:

$$\dot{H} = -2\int (\mathbf{v} \cdot \mathbf{A}_P) B_z \, dx \, dy$$

Centroids only:

$$\dot{H}_{br} = -\frac{1}{2\pi} \sum_{a} \sum_{b \neq a} \Phi_a \Phi_b \frac{d\theta_{ab}}{dt}$$
$$\tan \theta_{ab} = \frac{\bar{y}_b - \bar{y}_a}{\bar{x}_b - \bar{x}_a}$$



The Coronal Field: Connectivity

• Field lines $Pa \rightarrow Nb$: Domain Pa-Nb a.k.a. a/b

• Total flux in domain:

$$\int \mathbf{B} \cdot d\mathbf{a} = \psi_{a/b}$$

• For potential (vacuum) field: $\psi_{a/b} = \psi_{a/b}^{(v)}$

• Footprint (below) outlines domains at z = 0



· +s, & –s: centroids; $\bigtriangleup s$ & $\bigtriangledown s$: null points

 \cdot Dashed lines: separatrices; solid lines: spines

 \cdot Shaded domains: P02–N02 and P03–N03

A Sheared Arcade?

• Begin after M-flare Nov-6 00:11

 \circ End before X2 flare Nov-7 15:42

• Sources move

- \implies potential field changes
- \implies connections $\psi_{a/b}^{(v)}$ change

	N01	N02	N03	N07	N10	Φ_a
P01	$0.73 + 0.64_{1.37}$	$0.00 + 0.05_{0.05}$	$0.09 - 0.09_{0.00}$	$0.72 + 0.09_{0.81}$	$0.68 - 0.34_{0.34}$	6.82
P02	$^{0.03} + 1.01_{1.04}$	$2.15 - 0.77_{1.38}$	—	—	—	2.50
P03	$0.95 - 0.84_{0.12}$	—	$^{0.67} + 0.90_{1.57}$	—	$0.04 - 0.04_{0.00}$	1.71
P04	$1.85 - 0.36_{1.49}$	$^{0.21} + 0.18_{0.40}$	$0.03 + 0.27_{0.30}$	—	$0.12 - 0.12_{0.00}$	2.24
P05	—	—	$1.08 - 0.88_{0.20}$	$0.02 + 0.58_{0.61}$	$0.00 + 0.30_{0.30}$	1.10
P15	$0.00 + 0.00_{0.00}$	$0.00 + 0.00_{0.00}$	$0.60 - 0.24_{0.36}$	$^{0.18} + 0.00_{0.18}$	$0.02 + 0.23_{0.25}$	0.78
Φ_b	5.33	3.08	2.46	1.75	0.88	

Domain fluxes $\psi_{a/b}^{(v)}$ and their changes $\Delta \psi_{a/b}^{(v)}$ in units of 10²¹ Mx. Each entry gives the fluxes at $t_1 = \text{Nov-6 } 00:03$ (upper left) and $t_2 = \text{Nov-7}$ 16:03 (lower right) and the net change (center); a dash indicates that no connection exists between those sources.



Clockwise shearing: $\dot{\theta}_{+-} < 0 \Longrightarrow \dot{H} < 0$ -'ve sources break connections Eastward ($\Delta \psi^{(v)} < 0$ dashed) form new connections Westward ($\Delta \psi^{(v)} > 0$ solid)



III. The Flare and its 2 Ribbons

- GOES class X2 flare: Nov-7 15:42
- Ribbons observed in 1600 Å by TRACE (Handy et al. 1999)
- \circ 15:36 16:40 at cadence of 2–4 seconds
- Sequence co-aligned to reference via cross-correlation to nonflaring reference region \implies sub-pixel accuracy



• 2 flare phases in total count rate (bottom)

The Ribbons



 Suggests reconnection between domains interconnecting Positive (North) line: P02, P04, P07, P03, P15, P05 Negative (South) line: N02, N01, N03, N10, N07



IV. Measuring the Reconnected Flux

- \circ 1600 Å seq. aligned to avg'd MDI m-gram centered @ 15:11
- \circ Extrapolate m-gram (pot'l field) 2,000 km to chromosphere
- Identify "ribbon pixels" in TRACE images: brightens* in running average of 10 1600Å frames
 *: exceeds intensity threshold

• Tally flux in corresponding MDI pixels —



Ribbon pixels (grey) during initial phase (left) & entire flare (right)

• Repeat, varying alignment & ribbon-intensity threshold

Initial (Eastern) phase of flare (a) reconnects

$$\Psi_{+} = 2.3 \pm 0.2 \times 10^{21} \,\text{Mx} \quad , \quad \Psi_{-} = 2.6 \pm 0.5 \times 10^{21} \,\text{Mx}$$
Entire flare (b) reconnects

$$\Psi_{+} = 4.8 \pm 0.3 \times 10^{21} \,\text{Mx} \quad , \quad \Psi_{-} = 4.6 \pm 0.6 \times 10^{21} \,\text{Mx}$$

Modeling the Flux Transfer

• Assume potential field after M-flare $t_1 =$ Nov-6 00:11

$$\psi_{a/b}(t_1) = \psi_{a/b}^{(\mathbf{v})}(t_1)$$

• Fluxes do not change thereafter: $\psi_{a/b}(t) = \psi_{a/b}^{(v)}(t_1)$

• Build up discrepancy from pot'l field

$$\Delta \psi_{a/b} = \psi_{a/b} - \psi_{a/b}^{(v)}(t) = -\Delta \psi_{a/b}^{(v)} \neq 0$$

• Discrepancies in intra-ribbon domains:



Light: $\Delta \psi_{a/b} > 0$; Dark: $\Delta \psi_{a/b} < 0$

• Complete relaxation:* $\psi_{a/b} \to \psi_{a/b}^{(v)} \quad (\Delta \psi_{a/b} \to 0)$

 \circ Transfer flux across separators \implies reconnection

- ↑: X-fer **in** to make $\psi_{a/b}$ ↑ (when $\Delta \psi_{a/b} < 0$)
- ↓: X-fer **out** to make $\psi_{a/b}$ ↓ (when $\Delta \psi_{a/b} > 0$) • Would require total x-fer of:

$$\Delta \Psi_{\downarrow} = \sum_{\Delta \psi > 0} \Delta \psi_{a/b} = 3.36 \times 10^{21} \,\mathrm{Mx} \ ,$$

$$\Delta \Psi_{\uparrow} = \sum_{\Delta \psi < 0} |\Delta \psi_{a/b}| = 3.57 \times 10^{21} \,\mathrm{Mx} \ .$$

$\Delta \psi_{a/b}$	N02	N01	N03	N10	N07	Φ_a
P02	0.77	-1.01	_	_	_	2.50
P07	0.10	-0.09	—	—	—	0.31
P04	-0.18	0.36	-0.27	0.12	—	2.24
P03	—	0.84	-0.90	0.04	—	1.71
P15	-0.00	-0.00	0.24	-0.23	-0.00	0.78
P05	_	_	0.88	-0.30	-0.58	1.10
Φ_b	3.08	5.33	2.46	0.88	1.75	

• Compare to

 \cdot Total of fluxes in affected sources

$$\sum_{+} \Phi_a = 8.3 \times 10^{21} \,\mathrm{Mx} \;, \; \sum_{-} \Phi_b = 13.5 \times 10^{21} \,\mathrm{Mx}$$

 \cdot Observed ribbon fluxes in intitial phase

$$\Psi_{+} = 2.3 \pm 0.2 \times 10^{21} \,\mathrm{Mx}$$
, $\Psi_{-} = 2.6 \pm 0.5 \times 10^{21} \,\mathrm{Mx}$

* Incomplete relaxation $\implies \Psi_{\pm} < \Delta \Psi_{\uparrow}$ — as observed

Flux Transfer in the Full Flare

Include Western sources: P01, P06, P09, N05, N06



$\Delta \psi_{a/b}$	N02	N01	N03	N10	N07	N05	N06	\sum
P02	0.77	-1.01	—	_	—	_	—	-0.24
P07	0.10	-0.09	_	_	_	—	_	0.01
P04	-0.18	0.36	-0.27	0.12	—	—	—	0.03
P03	_	0.84	-0.90	0.04	_	_	_	-0.02
P15	-0.00	-0.00	0.24	-0.23	-0.00	—	—	0.00
P05	—	—	0.88	-0.30	-0.58	—	—	0.00
P01	-0.05	-0.64	0.09	0.34	-0.09	0.85	-0.07	0.42
P06	—	—	—	0.00	0.34	-0.55	0.21	-0.00
P09	—	—	—	_	0.33	-0.18	-0.15	-0.00
Σ	0.63	-0.54	0.05	-0.03	-0.01	0.12	-0.01	

$$\Delta \Psi_{\downarrow} = 5.52 \times 10^{21} \,\mathrm{Mx} \ , \ \Delta \Psi_{\uparrow} = 5.32 \times 10^{21} \,\mathrm{Mx}$$

Observed ribbon fluxes for entire flare

$$\Psi_{+} = 4.6 \times 10^{21} \,\mathrm{Mx}$$
, $\Psi_{-} = 4.5 \times 10^{21} \,\mathrm{Mx}$

Once again $\Psi_{\pm} < \Delta \Psi_{\uparrow}$ — incomplete relaxation (?)

V. Energy/Helicity Storage & Release

Minimum Current Corona: constrain domain fluxes

• Minimize magnetic energy

$$W = \frac{1}{8\pi} \int |\mathbf{B}|^2 d\mathbf{x}$$

 \circ Constrain all domain fluxes $\psi_{a/b}$

- \circ Equivalent to constraining fluxes through separators, ψ_i
- Equivalent to fixing discrepancies $\Delta \psi_i \equiv \psi_i \psi_i^{(v)}$
- Minimum = FCE*: $\nabla \times \mathbf{B} = 0$ except for ...
- ... Singular current ribbons along separators
- Estimate current I_i on each separator

$$\Delta \psi_i \simeq I_i L_i \ln(eI_i^*/|I_i|) + \sum_{j \neq i} M_{ij} I_j$$

 $L_i =$ length of (pot'l-field) separator

 $I_i^{\star} = \text{mean} \perp \text{magnetic shear about (pot'l-field) separator}$ $M_{ij} = \text{mutual inductance}$

• Free energy stored (prior to flux x-fer)

$$\Delta W \simeq \sum_{i} \frac{1}{2} L_i I_i^2 \ln(e^{1/2} I_i^* / |I_i|) + \sum_{i} \sum_{j \neq i} \frac{1}{2} M_{ij} I_i I_j$$

• Helicity of non-potential field (mutual helicity)

$$H \simeq \sum_{i} I_{i} \oint_{\mathcal{C}_{i}} \mathbf{Z} \cdot d\mathbf{l} = \sum_{i} H_{i}$$

 $\nabla \times \mathbf{Z} = \mathbf{A}_P$ and $\nabla \times \mathbf{A}_P = \mathbf{B}_P$ potential field

* Flux Constrained Equilibrium (Longcope 2001)



Separators of intra-ribbon domains



i	nulls		encloses	L_i	$z_{\rm max}$	$\Delta \psi_i$	I_i	ΔW_i	H_i
	—	+		Mm	Mm	$10^{21} \mathrm{Mx}$	GAmps	10^{30} ergs	$10^{42}\mathrm{Mx}^2$
1	A01	B06	P02-N08	43.9	8.7	-0.23	-24.2	0.06	-0.05
2	A02	B06	P02-N01	55.0	18.6	-1.01	-465.4	17.79	-2.92
3	A02	B08	P04-N02*	127.3	42.6	-0.79	2.7	0.00	0.04
4	A03	B08	P04-N03*	91.2	28.7	-1.11	-65.4	0.73	-0.72
5	A03	B07	P03-N01	35.3	10.8	-0.84	-562.6	14.98	-2.29
6	A04	B10	$P15-N03^{*}$	97.2	32.3	-1.13	-97.7	1.45	-1.17
7	A04	B11	P05-N03	36.1	9.8	-0.88	-948.4	29.06	-3.16
8	A05	B11	$P05-N10^*$	39.0	11.7	-0.59	57.3	0.31	0.19
to	tal					-6.57		64.38	-10.09

Including Mutual Inductances

$$\Delta W = 8.2 \times 10^{31} \,\mathrm{ergs} = 0.06 \,W_{\mathrm{pot}}$$

* One of several domains enclosed by separator

VI. Flare Energetics

- Most significant* separators: $\sigma_2, \sigma_5 \& \sigma_7$
- Each reconn'n: 2 **donor** domains \rightarrow 2 **recipient** domains



- Energy ΔW_i deposited into recipient domains 50:50
- Each domain: volume $V_{a/b}$ and mean field line length, $\langle L \rangle$
- Assume e^- accel. efficiency of 50% (guess)
- Flare volume V_f = smaller of $V_{a/b}$ or frac. w/ recon'd flux
- Use Warren & Antiochos (2004) peak GOES fluxes:

	$F_{1-8} = 3.7$	7×10^{-1}	$^{-35}\left(\frac{E}{V}\right)$	$\left(\frac{L}{L}\right)^{1.75}$	$\frac{V}{L^2} ,$	$F_{0.5-4}$	= 4.4 >	$< 10^{-42}$	$\left(\frac{EL}{V}\right)^2$	$\frac{.24}{L^2}$
i	recipient	ψ_i	$\psi_{a/b}^{(\mathrm{v})}$	$\langle L \rangle$	$V_{a/b}$	V_f	E_d	E_f	F_{1-8}	$F_{0.5-4}$
	domain	10^{21}	Mx	Mm	10^{27}	cm^3	10^{30}	erg	10^{-6} V	W/m^2
2	P02-N01	1.01	1.05	33.8	4.27	4.12	4.45	4.45	127.9	22.0
2	P04-N02	1.01	0.37	87.9	30.29	30.29	4.45	1.61	3.8	0.2
5	P04-N01	0.84	1.53	64.2	46.27	25.34	3.74	3.74	20.6	1.8
5	P03-N03	0.84	1.58	20.9	3.18	1.69	3.74	3.74	208.3	40.3
7	N10-P05	0.88	0.31	25.9	1.26	1.26	7.26	2.53	123.7	25.3
7	P15-N03	0.88	0.33	53.4	8.45	8.45	7.26	2.76	29.0	3.5
	total				93.72	71.14	30.92	18.84	513.4	93.2

* Largest stored energy ΔW_i — released by reconnection

Flare Energetics (cont'd)

 \circ Synthesize GOES light-curves from peaks: $F_{\rm 1-8}\ \&\ F_{\rm 0.5-4}$

- · empirical decay curve: $\sim (t t_0)^{-p}$
- \cdot stagger reconnection episodes by 3 minutes.



 \circ GOES data alone $\implies \Delta E_{\rm rad} \simeq 1.5 \times 10^{31} {\rm ~ergs}$



 \circ 3^rd estimate from GOES data:

 $\cdot EM = 2 \times 10^{50} \,\mathrm{cm}^{-3} \quad V_f = 7 \times 10^{28} \,\mathrm{cm}^3 \text{ (recip. domains)}$ $\Longrightarrow \quad n_e = 5 \times 10^{10} \,\mathrm{cm}^{-3}$ $\cdot T = 20 \,\mathrm{MK} \quad \Longrightarrow \quad E_{\mathrm{th}} = 3n_e k_b T V_f = 3 \times 10^{31} \,\mathrm{ergs}$



VII. Flux Rope Creation

- Bubbles = domains; vertices = separators (\rightarrow to recipient)
- \circ Fluxes listed inside in $10^{19}~{\rm Mx}$ see key
- Some domains share a bubble divided by diagonals
- Shaded domains: $\Delta \psi_{a/b} < 0$
- Dashed outline: domain outside ribbons

Sequence of cascading reconnection

- Cascade: recipient from σ_i is donor for σ_j
- Separator sequence: $\sigma_7 \rightarrow \sigma_6 \rightarrow \sigma_{11} \rightarrow \sigma_4$
- \circ First donors: 15/10 and 5/3 $\Delta\psi_{5/3}=+0.88\times10^{21}~{\rm Mx}$







- Ends transferring $\Delta \psi_4 = 1.06 \times 10^{21}$ Mx.
- \circ Final recipient: P08–N01 (shown) $\psi_{8/1} = 10^{21} \text{ Mx}$
- Observed: MC (Nov-9 20:30) w/ $\Phi_{MC} = 0.72 \times 10^{21}$ Mx (Longcope, Beveridge, Qiu, Ravindra, Barnes & Dasso, 2007)
- Mutual helicity $(H_i) \Longrightarrow$ self helicity of recipient domains

$$\Delta H = \frac{1}{2}(H_7 + H_6 + H_{11} + H_4) = -2.7 \times 10^{42} \,\mathrm{Mx}^2$$

o $\Delta H/\Phi^2 = -2.7 = \#$ of RH turns in flux rope

• Observed MC: $H = -11 \times 10^{42} \,\mathrm{Mx}^2$ (assuming 2 AU rope length)

Summary: Build-up & flare fits quantitative CSHKP model

• Shearing:
$$\Delta H = -14 \times 10^{42} \,\text{Mx}^2$$

 $\Delta H/\Phi^2 = -1/3 = \text{avg.} \# \text{ of RH turns in AR}$

- Energy storage: $\Delta W = 8 \times 10^{31} \text{ ergs} = 0.06 W_{\text{pot}}$ consistent w/ 2d arcade sheared by $\operatorname{atan}(1/3) = 18^{\circ}$ (Klimchuk *et al.* 1988; Mikic *et al.* 1988; Choe & Cheng 2000) GOES light-curves suggest ~ 3×10^{31} ergs released
- MODEL: Relaxation to pot'l field requires reconnection:
 - $\cdot \ \Delta \Psi_{\uparrow} \simeq 3.5 \times 10^{21} \ \mathrm{Mx}$ initial phase
 - $\cdot \Delta \Psi_{\uparrow} \simeq 5.4 \times 10^{21} \text{ Mx} \text{total}$
- OBSERVATION: Fluxes swept by 1600Å flare ribbons:
 - $\cdot \Psi_{\pm} \simeq 2.4 \times 10^{21} \,\mathrm{Mx}$ initial phase
 - $\cdot \Psi_{\pm} \sim 4.5 \times 10^{21} \,\mathrm{Mx} \mathrm{total}$
- * COMPARISON: $\Psi_{\pm} < \Delta \Psi_{\uparrow} \iff$ incomplete relaxation
- Sequence of recon'n produces flux rope (P08–N01) with $\Phi_{\rm ft} = 1.1 \times 10^{21} \text{ Mx} - \text{c.f.} \quad \Phi_{MC} = 0.7 \times 10^{21} \text{ Mx}$
- 4 reconnections $\implies -2.7$ twists in flux rope: $H_{\rm ft} = -2.7 \times 10^{42} \,\mathrm{Mx}^2 - \mathrm{c.f.} \ H_{MC} = -11 \times 10^{42} \,\mathrm{Mx}^2$ (subsequent reconnection after flare?)

References

- Barnes, G., Longcope, D. W., and Leka, K. D.: 2005, ApJ 629, 561.
- Berger, T. E. and Lites, B. W.: 2003, Solar Phys. 213, 213.
- Burlaga, L., Sittler, E., Mariani, F., and Schwenn, R.: 1981, JGR 86, 6673.
- Carmichael, H.: 1964, in Hess, W. N. (ed.), AAS-NASA Symposium on the Physics of Solar Flares, p. 451, Washington, DC. NASA.
- Choe, G. S. and Cheng, C. Z.: 2000, ApJ 541, 449.
- Fletcher, L., Metcalf, T. R., Alexander, D., Brown, D. S., and Ryder, L. A.: 2001, *ApJ* 554, 451.
- Fletcher, L., Pollock, J. A., and Potts, H. E.: 2004, Solar Phys. 222, 279.
- Forbes, T. G. and Priest, E. R.: 1984, in Butler, D. and Papadopoulos, K. (eds), Solar Terrestrial Physics: Present and Future, pp. 35–39. NASA.
- Gosling, J. T.: 1990, in Russel, C. T., Priest, E. R., and Lee, L. C. (eds), *Physics of Magnetic Flux Ropes*, volume 58 of *Geophys. Monographs*, pp. 343–364. AGU.
- Gosling, J. T., Birn, J., and Hesse, M.: 1995, *GRL* 22, 869.
- Handy, B. N., Acton, L. W., Kankelborg, C. C., Wolfson, C. J., et al.: 1999, Solar Phys. 187, 229.
- Hirayama, T.: 1974, Solar Phys. 34, 323.
- Klimchuk, J. A., Sturrock, P. A., and Yang, W.-H.: 1988, ApJ 335, 456.
- Kopp, R. A. and Pneuman, G. W.: 1976, *Solar Phys.* 50, 85.
- Leamon, R. J., Canfield, R. C., Jones, S. L., Lambkin, K., Lundberg, B. J., and Pevtsov, A. A.: 2004, *JGR* **109**, 5106.
- Lepping, R. P., Burlaga, L. F., and Jones, J. A.: 1990, JGR 95, 11957.
- Longcope, D., Beveridge, C., Qiu, J., Ravindra, B., Barnes, G., and Dasso, S.: 2007, *Solar Phys.*, (In preparation).
- Longcope, D. W.: 2001, Phys. Plasmas 8, 5277.
- Longcope, D. W., Barnes, G., and Ravindra, B.: 2006, ApJ, In preparation.
- Mikic, Z., Barnes, D. C., and Schnack, D. D.: 1988, *ApJ* 328, 830.
- November, L. J. and Simon, G. W.: 1988, *ApJ* 333, 427.
- Poletto, G. and Kopp, R. A.: 1986, in Neidig, D. F. (ed.), The Lower Atmospheres of Solar Flares, pp. 453–465. National Solar Observatory.
- Qiu, J., Lee, J., Gary, D. E., and Wang, H.: 2002, *ApJ* 565, 1335.
- Scherrer, P. H., Bogart, R. S., Bush, R. I., Hoeksema, J. T., et al.: 1995, Solar Phys. 162, 129.
- Sturrock, P. A.: 1968, in IAU Symp. 35: Structure and Development of Solar Active Regions, pp. 471–479.
- Warren, H. P. and Antiochos, S. K.: 2004, *ApJ* **611**, L49.

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