Modeling Hard X-ray Emission from Three-dimensional Magnetic Loops†

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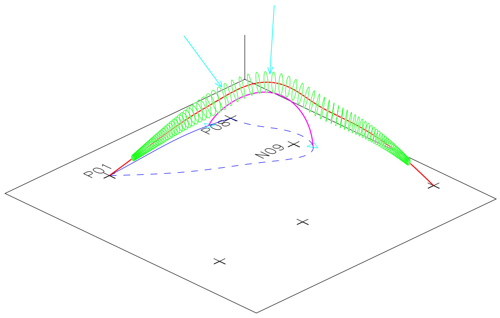
Overview

1. The Magnetic Model



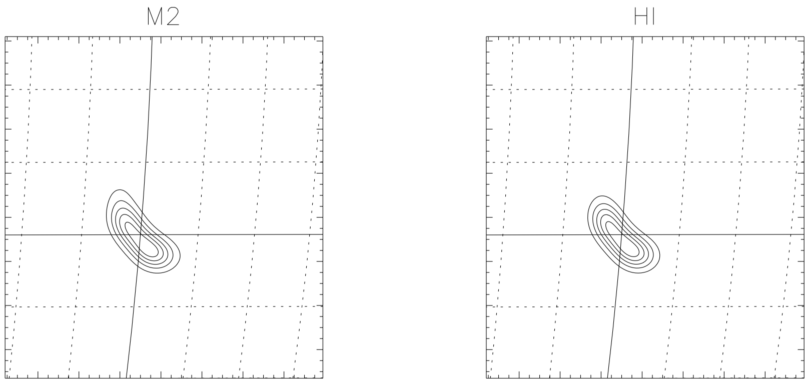
- Extrapolate from m-gram
- Identify reconnection site
- Select post-reconn. flux tube

2. The Non-thermal e^-population

- Hypothesize non-thermal source @ reconnection site
- NT e^- remain on post-reconnection flux tube
- Solve for $f_{\rm nt}(s, v, \mu; t)$ by Monte Carlo Method (MacKinnon & Craig 1991)

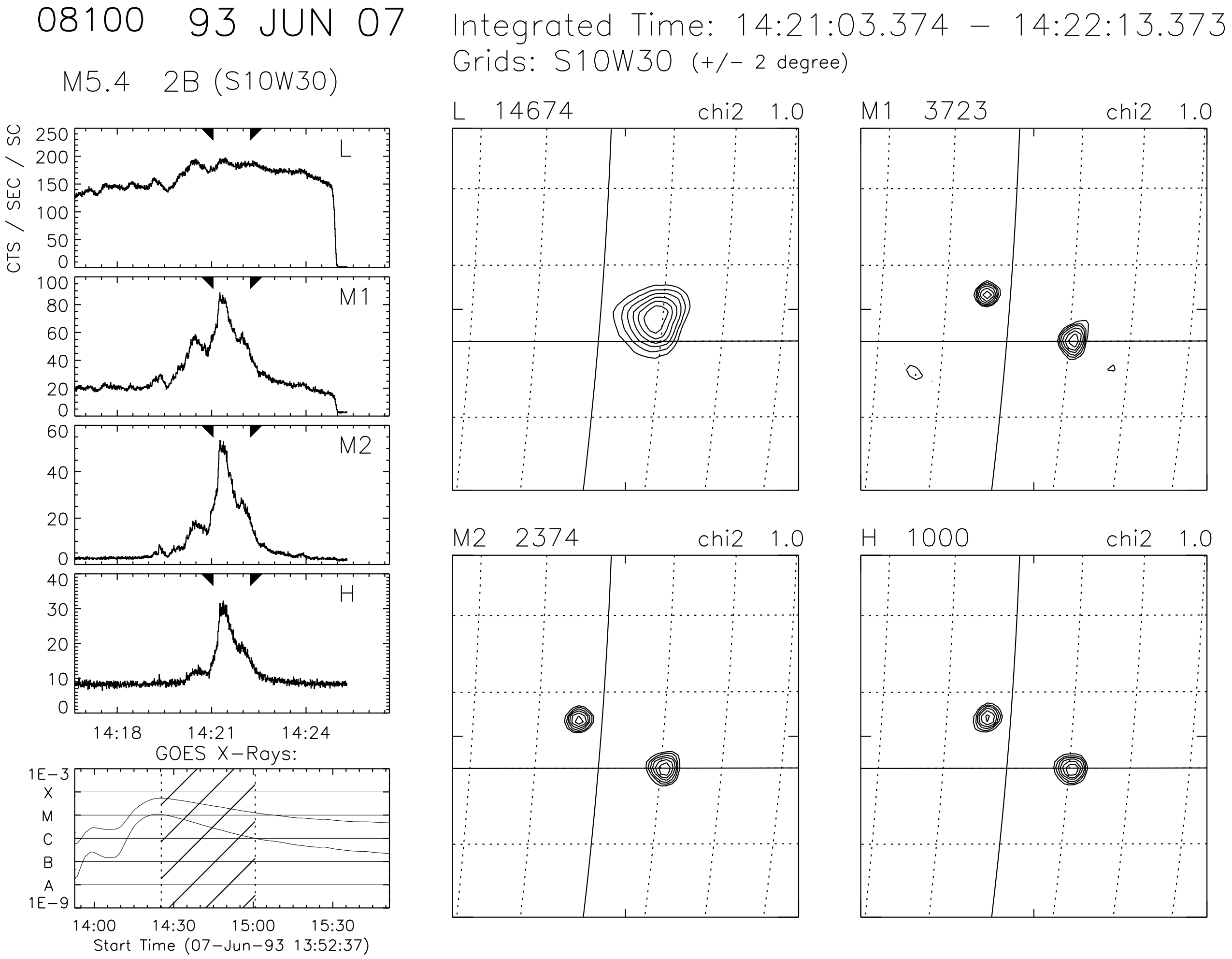
3. Synthetic image of HXRs from NT e^- population

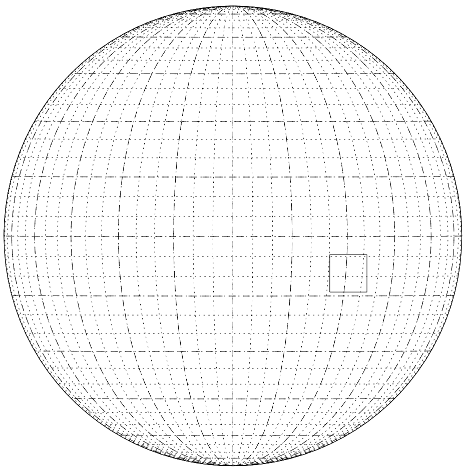
- Brehmstrahlung emission from $f_{\rm nt}(s, v, \mu; t)$
- Distribute on post-reconnection flux tube
- Pass through response for detector channel x
- Integrate along line-of-site for each pixel



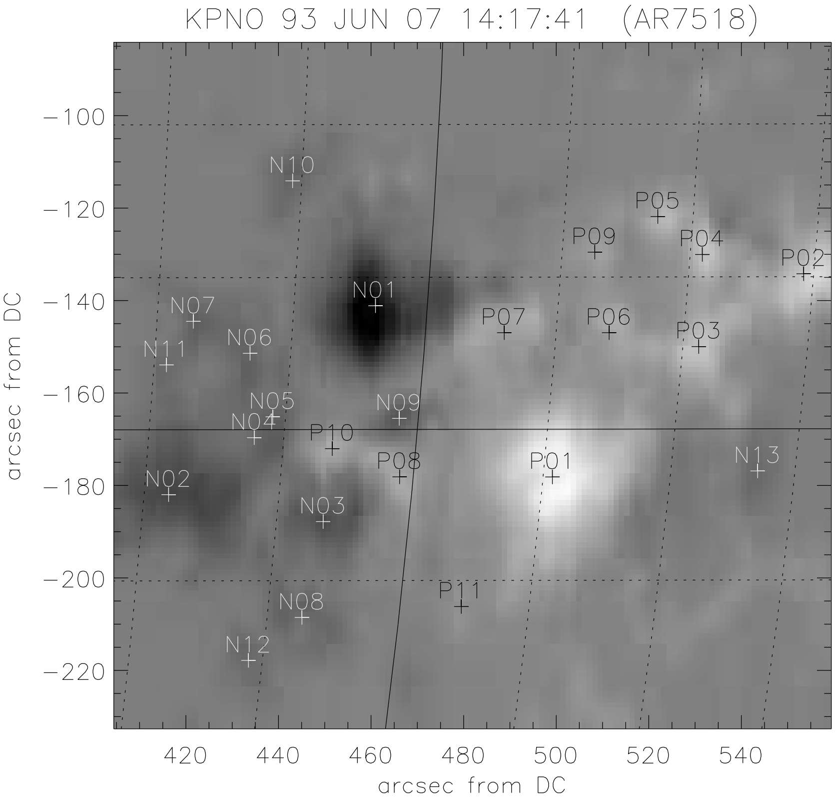
1. Magnetic modeling — an example

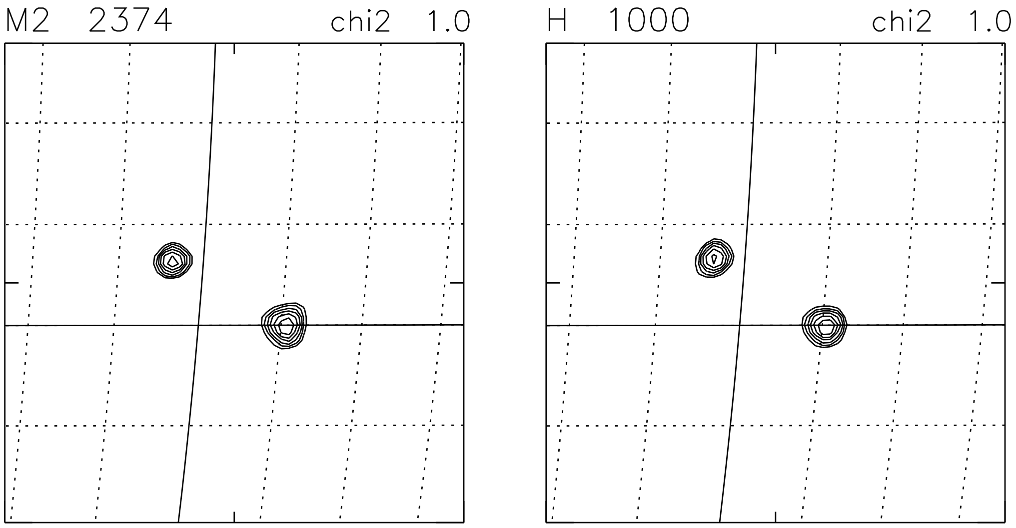
June 7, 1993 — 1 of 6 studied by Li et al. 1997



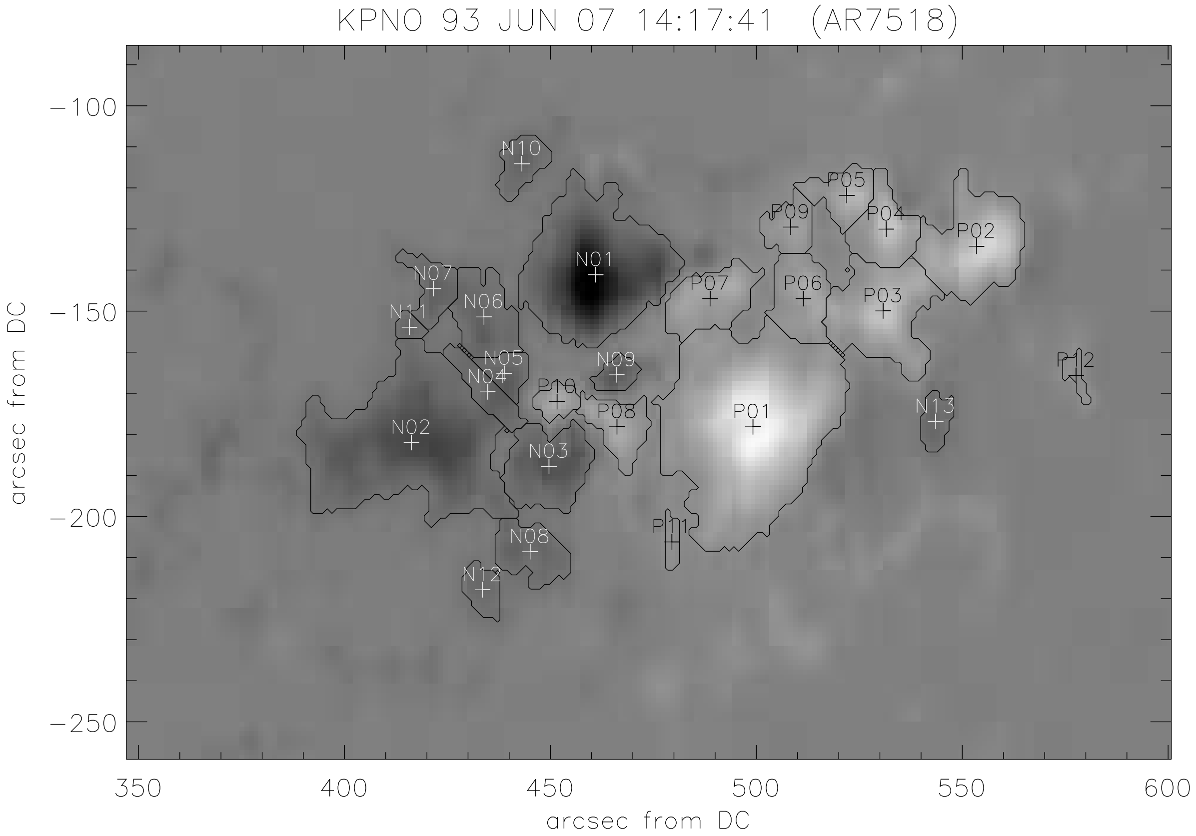


Occurs within active region AR7518





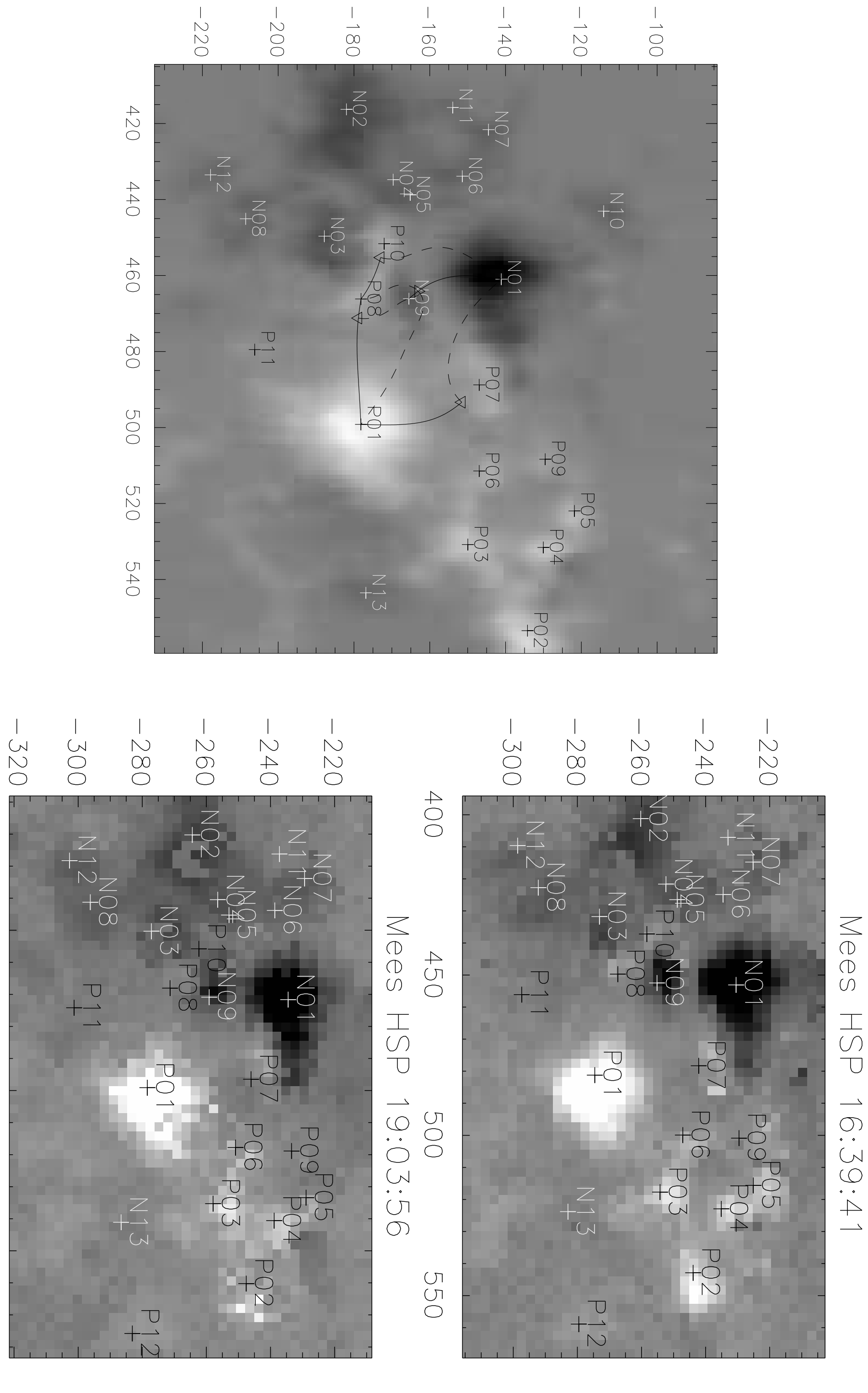
Identifying flux sources:



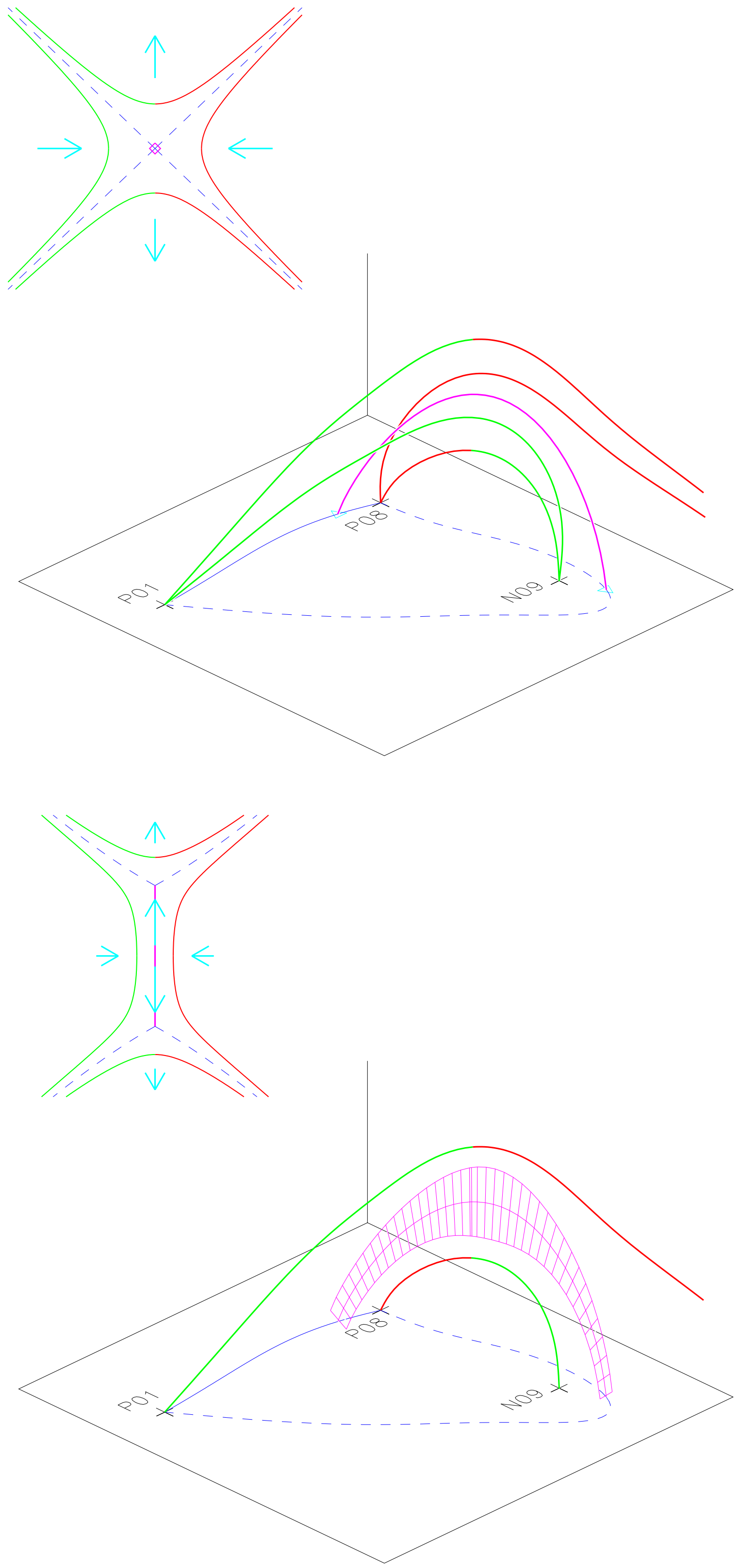
Regions ($|B| > 120$ G) $\Longrightarrow$ point sources

+	location		$\Phi(10^{20}$ Mx)	(−)	location		$\Phi(10^{20}$ Mx)
P01	10.°6 S	32.°1 W	44.66	N01	8.°4 S	29.°2 W	27.66
P02	8.°0 S	35.°8 W	11.54	N02	10.°9 S	26.°4 W	23.85
P03	8.°9 S	34.°3 W	9.45	N03	11.°2 S	28.°7 W	5.39
P04	7.°7 S	34.°2 W	4.67	N04	10.°1 S	27.°6 W	1.76
P05	7.°2 S	33.°4 W	2.66	N05	9.°8 S	27.°8 W	1.40
P06	8.°7 S	32.°8 W	3.60	N06	9.°0 S	27.°4 W	3.06
P07	8.°7 S	31.°2 W	4.18	N07	8.°6 S	26.°5 W	1.64
P08	10.°6 S	29.°8 W	2.60	N08	12.°5 S	28.°5 W	2.38
P09	7.°7 S	32.°5 W	1.58	N09	9.°9 S	29.°7 W	1.03
P10	10.°3 S	28.°7 W	1.08	N10	6.°7 S	27.°9 W	1.40
P11	12.°3 S	30.°9 W	0.40	N11	9.°1 S	26.°2 W	0.40
P12	9.°9 S	37.°9 W	0.34	N12	13.°1 S	27.°8 W	0.95
				N13	10.°6 S	35.°4 W	0.89
		Total:	86.76			Total:	71.82

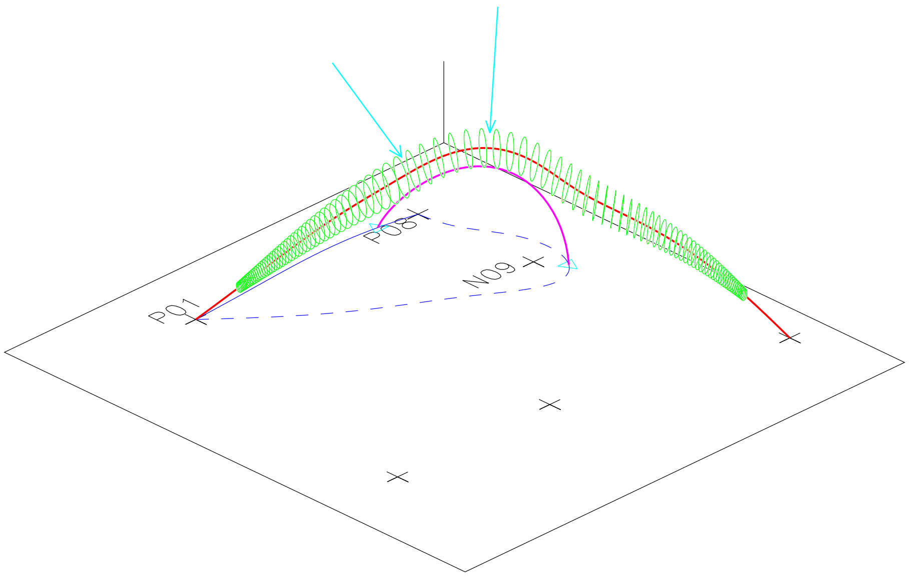
Pole P08 & P10 disappear after (& before?) flare

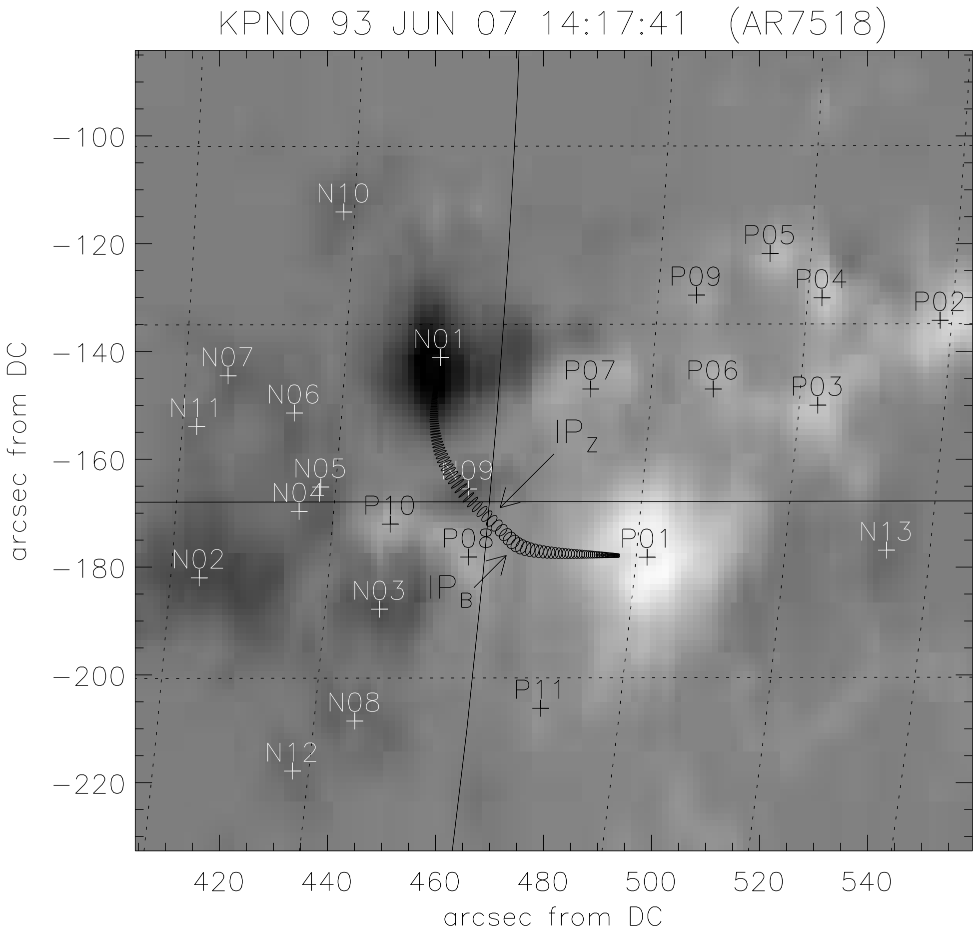


Creating subductable flux through reconnection:

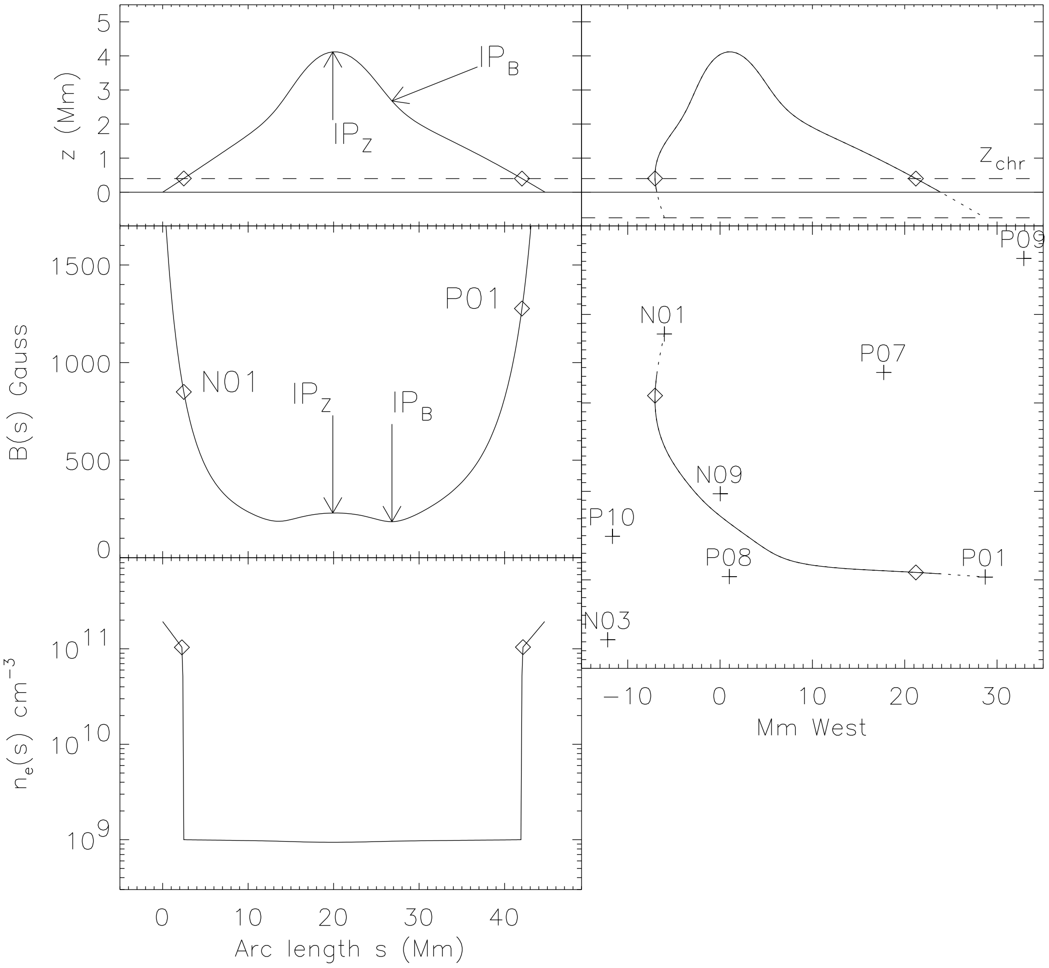


The Post-reconnection loop





The field model defines:



- Field line: $\mathbf{r}_i$ at grid-points $i = 0,\, 1,\, \ldots,\, N-1$
- Arc length s_i at grid-points $i = 0,\, 1,\, \ldots,\, N-1$

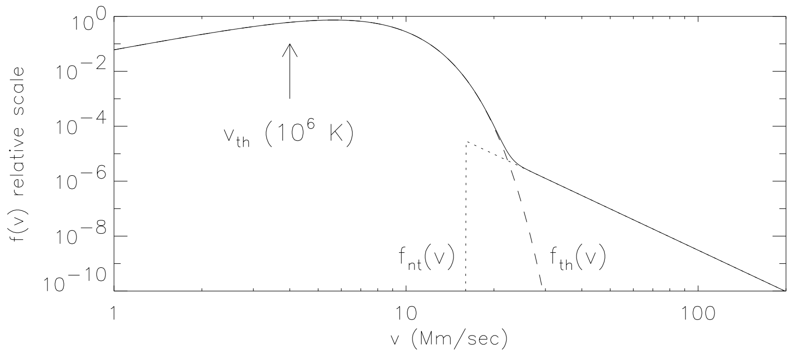
$$s_{i+1} \;=\; s_i \;+\; |\,\mathbf{r}_{i+1} - \mathbf{r}_i\,|$$

- Magnetic field strength

$$B_i \;=\; |\,\mathbf{B}(\mathbf{r}_i)\,|$$

2. The electron distribution function

$$f(s,v,\mu;t) \;=\; f_{\rm th}(s,v,\mu;t) \;+\; f_{\rm nt}(s,v,\mu;t)$$



- Thermal (collisional) population: Maxwellian

$$f_{\rm th}(s,v,\mu) \;=\; \frac{A(s)\,n_{\rm th}(s)v^2}{\sqrt{2\pi}}\left(\frac{m_e}{k_{\rm B}T(s)}\right)^{3/2}\exp\left[-\frac{v^2}{2k_{\rm B}T(s)}\right]$$

- NT e^-s (less collisional) obey Fokker-Planck equation

$$\mathcal{L}f_{\rm nt} \;\equiv\; \underbrace{\frac{\partial f_{\rm nt}}{\partial t}+\frac{\partial}{\partial s}[\mu v f_{\rm nt}]-\frac{\partial}{\partial \mu}\left[\tfrac{1}{2}v(1-\mu^2)\frac{d\ln B}{ds}f_{\rm nt}\right]}_{\text{Liouville's Equation}}$$

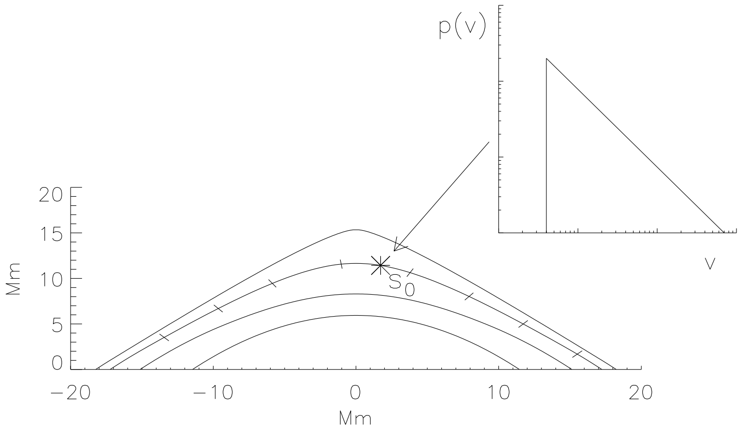
$$-\;\underbrace{\frac{\partial}{\partial v}\left[\tfrac{1}{2}\nu_e v f_{\rm nt}\right]}_{\text{slowing}}-\underbrace{\frac{\partial}{\partial \mu}\left[\tfrac{1}{2}\nu_e\,(1-\mu^2)\frac{\partial f_{\rm nt}}{\partial \mu}\right]}_{\text{phase-space diffusion}}$$

$$=\;\underbrace{S_{\rm nt}(s,v,\mu;t)}_{\text{acceleration}}\;=\;a(t)\,p_s(s)\,p_\mu(\mu)p_v(v)$$

- Thermal e^-s vastly outnumber NT e^-:

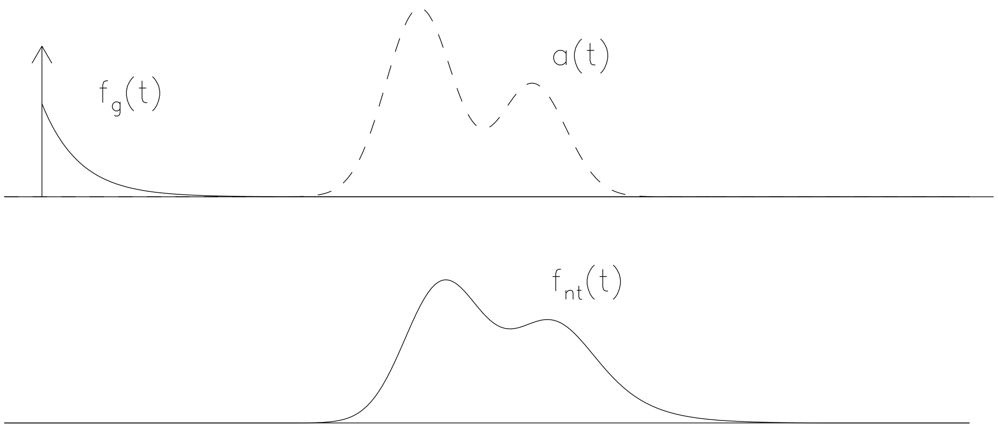
$$n_{\rm nt}(s)\;=\;\frac{1}{A(s)}\int_0^\infty\int_{-1}^{1}f_{\rm nt}(s,v,\mu)\,d\mu\,dv\;\ll\;n_{\rm th}(s)\;\simeq\;n_e(s)$$

Modeling the NT e^{-}s



Assumed properties of acceleration source:

- Spatially localized: $p_s(s) = \delta(s - s_0)$
- Isotropic: $p_\mu(\mu) \ = \ \frac{1}{2}$, $-1 \leq \mu \leq 1$
- Power-law: $p_v(v) \ = \ C\, v^{-\gamma}$, $v \geq v_{\rm c}$, typically $\gamma = 5$
- Electron production: $a(t)$ electrons per second



Solution using Green function $f_{\rm g}(s, v, \mu; t)$:

$$f_{\rm nt}(s, v, \mu; t) \ = \ \int_{-\infty}^{t} f_{\rm g}(s, v, \mu; t - t')\, a(t')\, dt'$$

$$\mathcal{L} f_{\rm g} \ = \ 0 \quad , \quad f_{\rm g}(s, v, \mu; t{=}0) = p_s(s)\, p_\mu(\mu) p_v(v)$$

Numerical solution *via* Monte-Carlo Method
(MacKinnon & Craig 1991)

- Represent nonthermal pop. by $N \simeq 10{,}000$ *sample* particles

$$[\, s^{(r)}(t),\, v^{(r)}(t),\, \mu^{(r)}(t)\,] \quad , \quad r = 1,\, 2,\, \ldots,\, N$$

- Initialize particles randomly using $p_s(s)$, $p_\mu(\mu)$, $p_v(v)$
- Time-integrate using Euler's method:* $s_n = s^{(r)}(n\Delta t)$

$$\begin{aligned}
\Delta s_n &= s_{n+1} - s_n \;=\; \mu_n\, v_n\, \Delta t \\
\Delta v_n &= -\tfrac{1}{2}\nu_{e,n}\, v_n\, \Delta t \\
\Delta \mu_n &= -\tfrac{1}{2} v_n (1-\mu_n^2)\; \frac{d\ln B}{ds}\bigg|_{s=s_n} \Delta t \\
&\quad -\; \nu_{e,n}\, \mu_n\, \Delta t \;+\; \sqrt{\nu_{e,n}(1-\mu_n^2)}\; \underbrace{\sqrt{\Delta t} R_n}_{\text{noise}}
\end{aligned}$$

$$\text{where} \quad \nu_{e,n} \;\equiv\; \frac{8\pi e^4 \Lambda}{m_e^2}\, \frac{n_e(s_n)}{v_n^3}$$

- Noise from *normal deviate* R_n:

$$\langle\, R_n \,\rangle \;=\; 0 \quad , \quad \langle\, R_m\, R_n \,\rangle \;=\; \delta_{m,n} \quad .$$

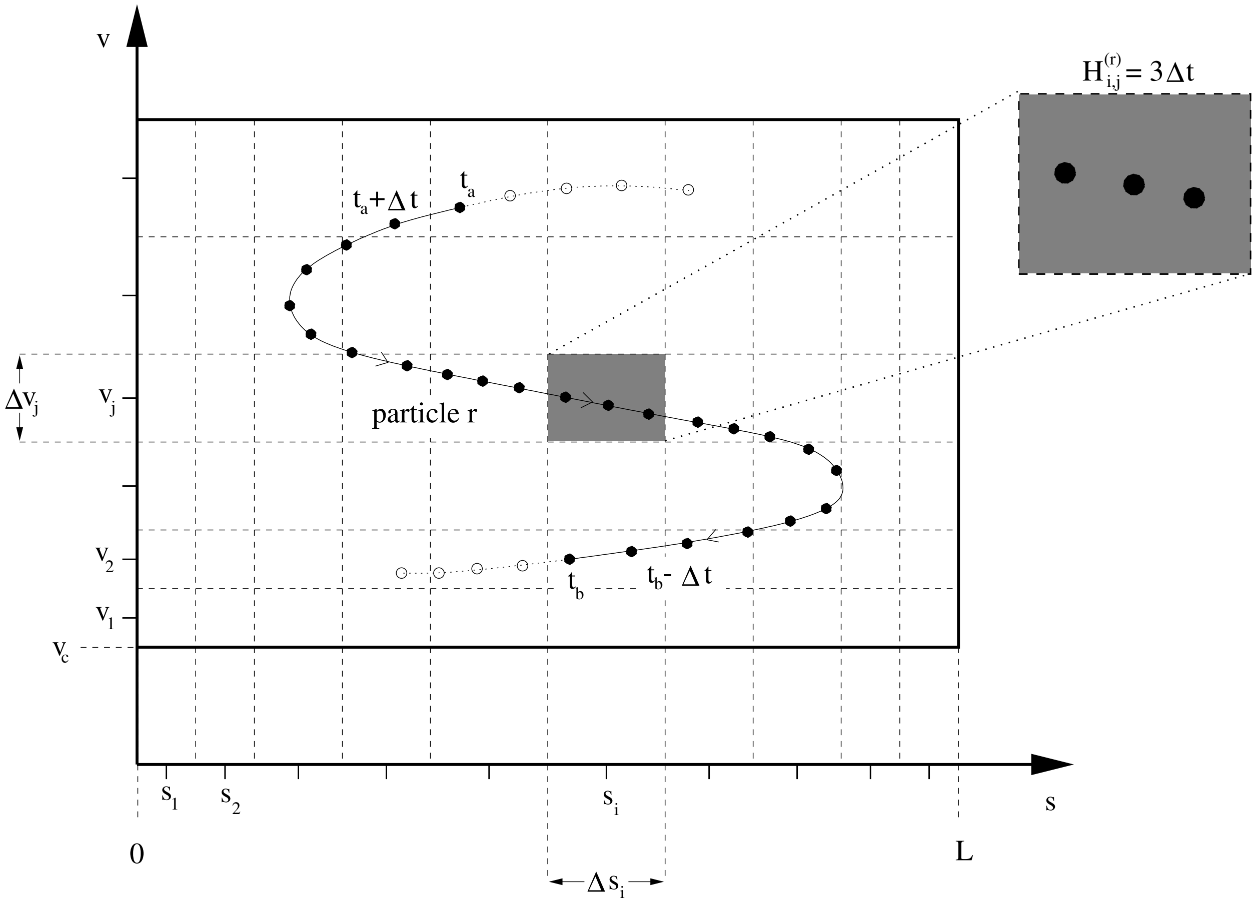
$$\int\limits_{t_n}^{t_n+\Delta t} \eta(t)\, dt \;=\; \sqrt{\Delta t} R_n$$

- End integration once particle "thermalizes"
(or becomes too slow to produce detectable photons)

$$v_n < v_{\rm cut} \;=\; 30\,{\rm Mm/sec}$$

$*$ In this work we use a 2nd order method

Approximating $f_{\mathrm{g}}(s, v, t)$



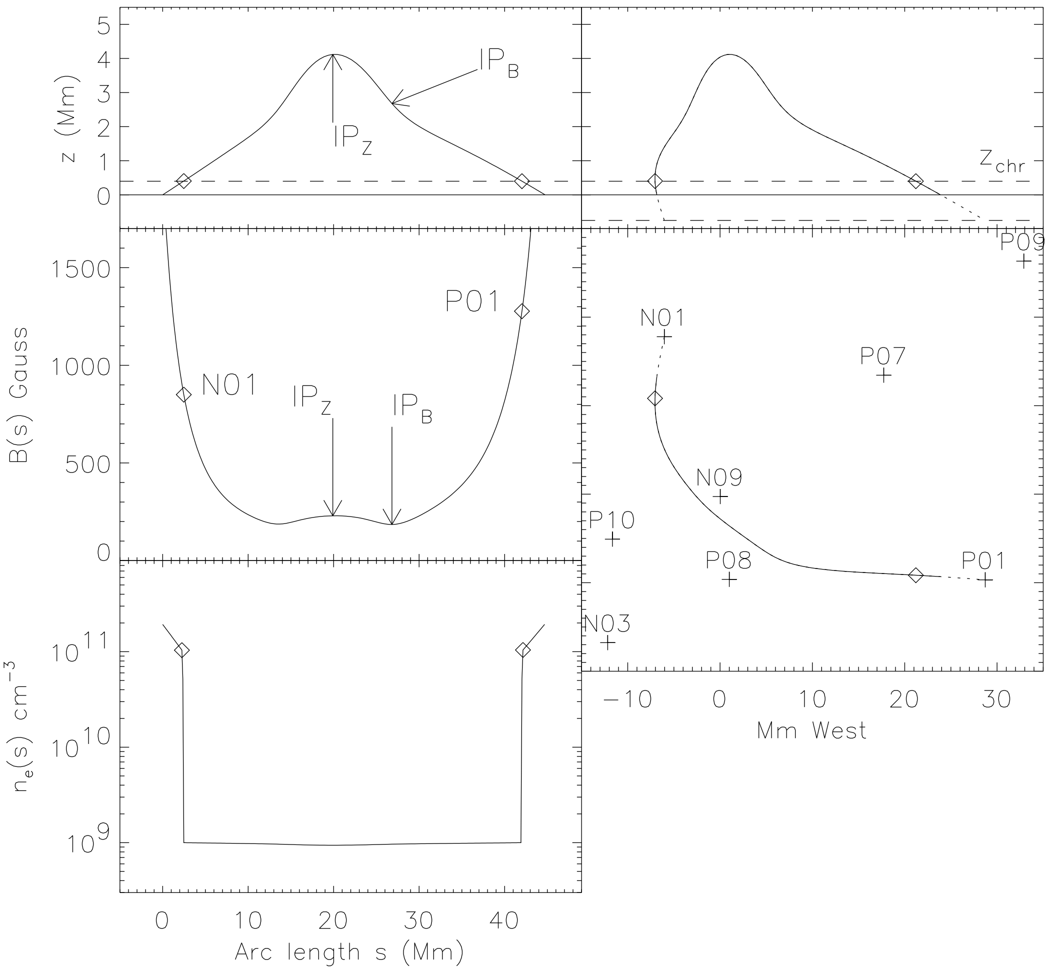
- $H^{(r)}_{i,j}$ is total time spent spent in bin i, j (during $t_a \le t \le t_b$)
  - Accumulate for all N particles $\qquad H_{i,j} \; = \; \dfrac{1}{N} \sum\limits_{r=1}^{N} H^{(r)}_{i,j}$
- Approximate distribution function

$$\int\limits_{-1}^{1} f_{\mathrm{g}}(s_i, \mu, v_j, \bar{t}) \, d\mu \;\simeq\; \frac{H_{i,j}}{(t_b - t_a)\Delta s_i \, \Delta v_j}$$

- Approximate any moments of distribution function

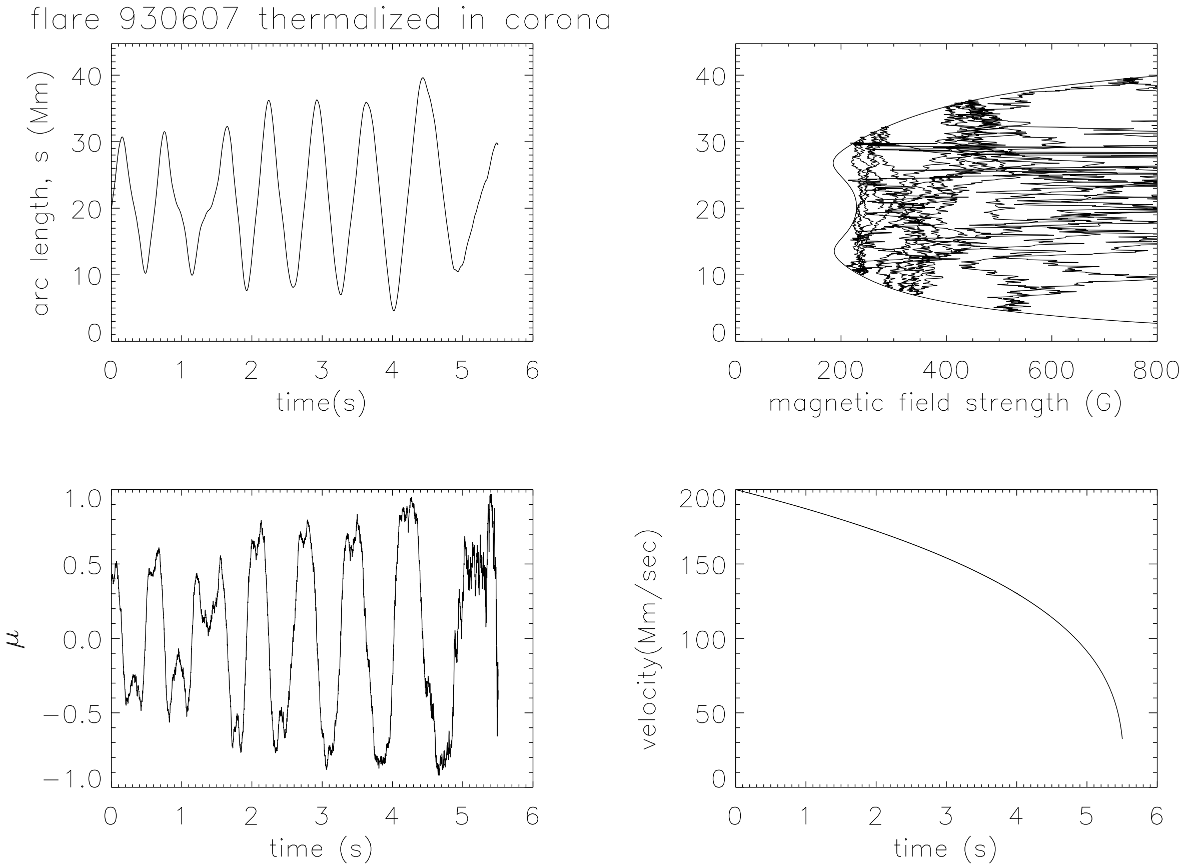
$$\int K(v) \, f_{\mathrm{g}}(s_i, v, \mu; \bar{t}) \, d\mu \, dv \;\simeq\; \frac{1}{(t_b - t_a)\Delta s_i} \sum_j K(v_j) \, H_{i,j}$$

The 1d world of the NT e^-

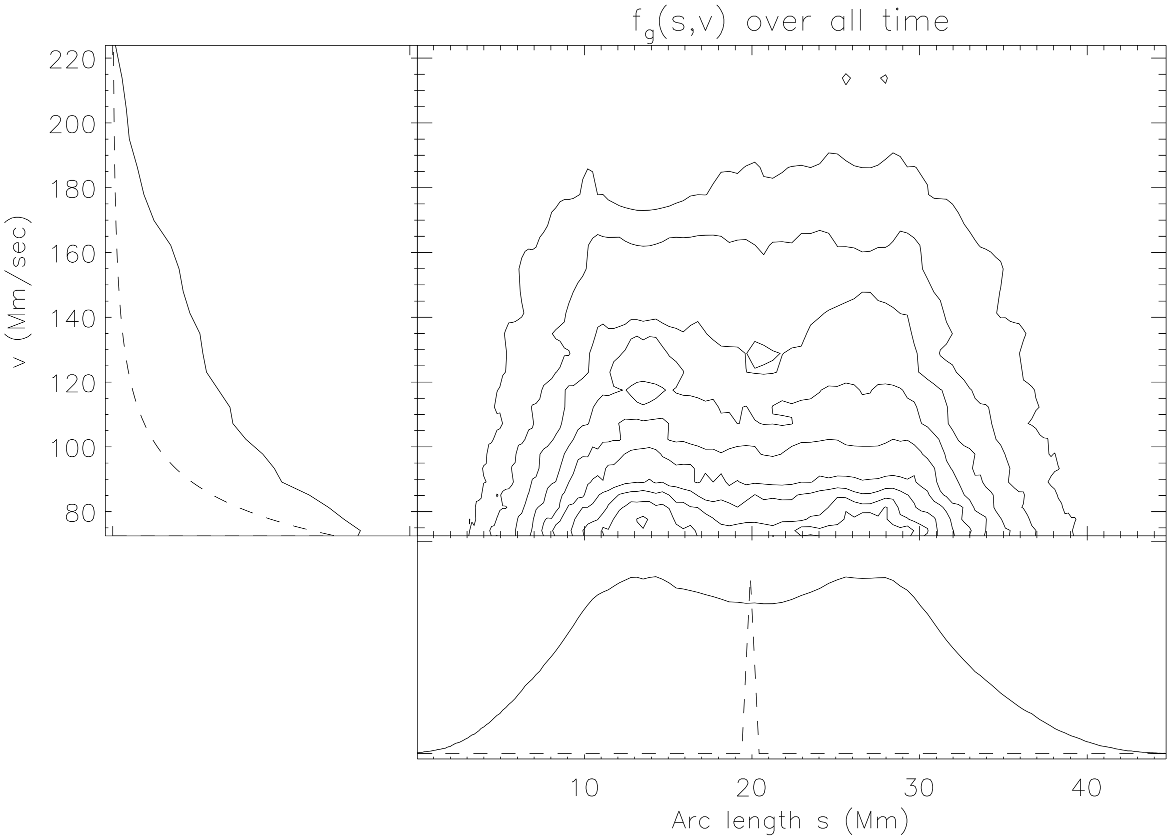


- Low-lying loop: 4 Mm high, 40 Mm long
- "Model surface": ($z = -750$ km) where point sources are
- Chromosphere: $z_{\rm chr} = 400$ km where density increases
- $B_{N01} = 850$ G, $\quad B_{P01} = 1280$
- Two injection points: IP_Z IP_B
- $B(IP_Z) = 230$ G, $\quad B(IP_B) = 185$ G
- Lose 15% (IP_Z) and 12% (IP_B) of NT e^- on first bounce

The life of one test electron:



The distribution function:



- Injected at loop apex
- Initial power law $f(v) \sim v^{-5} \quad \Longleftrightarrow \quad f(E) \sim E^{-3}$
- Normalizations:

$$\Delta s \, \Delta v \int_0^\infty f_{\rm g}(s, v; t) \, dt \;=\; \text{time spent in } \Delta s \times \Delta v \text{ cell by } e^-$$

$$\int f_{\rm g}(s, v; t) \, ds \, dv \;=\; P(t) \;=\; \text{prob. of surviving to time } t$$

$$\int f_{\rm g}(s, v; t) \, ds \, dv \, dt \;=\; \int_0^\infty P(t) \, dt \;=\; -\int_0^\infty t \frac{dP}{dt} \, dt \;=\; \langle \, t_{\rm life} \, \rangle$$

3. Imaging emitted X-rays (Fletcher 1995)

- Brehmstrahlung emission of photons, energy ϵ

$$I(\epsilon, s; t) \;=\; \int n_e(s)\, v\, \sigma_{\rm bh}(\epsilon, v)\, f_{\rm nt}(s, v, \mu; t)\, d\mu\, dv$$

- Bethe-Heitler cross-section ($v \ll c$):

$$\sigma_{\rm bh}(\epsilon, v) \;=\; 3 \times 10^{-27}\,{\rm cm}^2 \left(\frac{c}{v}\right)^2 \epsilon^{-1} \ln\left[\frac{1+\sqrt{1-2\epsilon/m_e v^2}}{1-\sqrt{1-2\epsilon/m_e v^2}}\right]$$

- Approximate integral w/ sum over *sample* particles

$$\int_{t_a}^{t_b} I(\epsilon, s_i; t)\, dt \;\simeq\; \frac{1}{N \Delta s_i} \sum_{r=1}^{N} \sum_j n_e(s_i)\, v_j\, \sigma_{\rm bh}(\epsilon, v_j)\, H_{i,j}^{(r)}$$

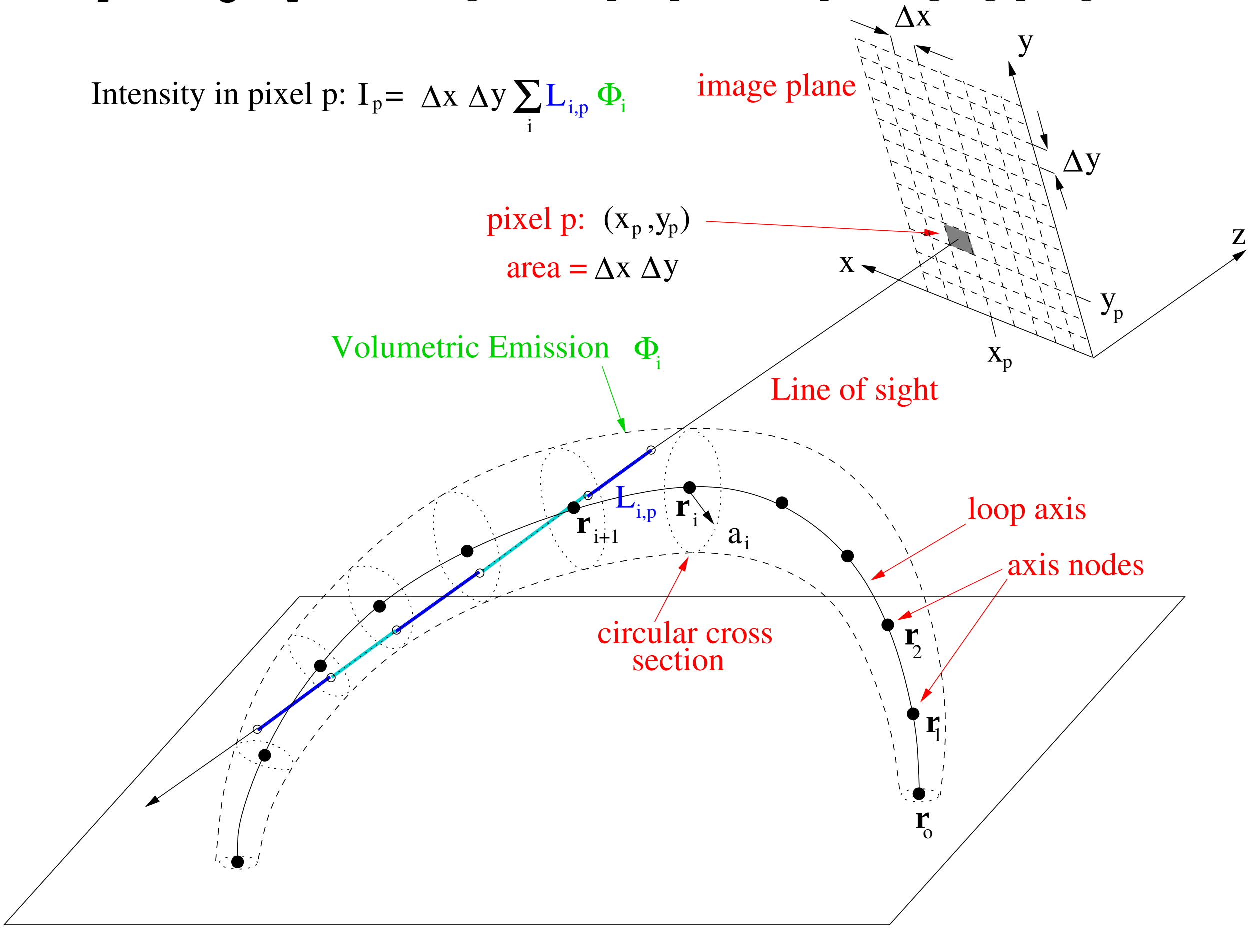
- Counts (per length) in channel x: $\epsilon_x^{\rm lo} \le \epsilon \le \epsilon_x^{\rm hi}$

$$C_x(s) \;=\; \int_{\epsilon_x^{\rm lo}}^{\epsilon_x^{\rm hi}} \underbrace{\frac{A_x(\epsilon)}{4\pi R^2}}_{\text{aperture}} \left\{ \int_{t_a}^{t_b} I(\epsilon, s; t)\, dt \right\} d\epsilon$$

- Volumetric emission — distribute uniformly over cross-section

$$\Phi_i \;\simeq\; \frac{C_x(s_i)}{\frac{1}{2}(A_{i+1} + A_i)}$$

where $A_i = \Psi / B_i$ is area of flux tube `loop_image.pro` — a general purpose loop-imaging program

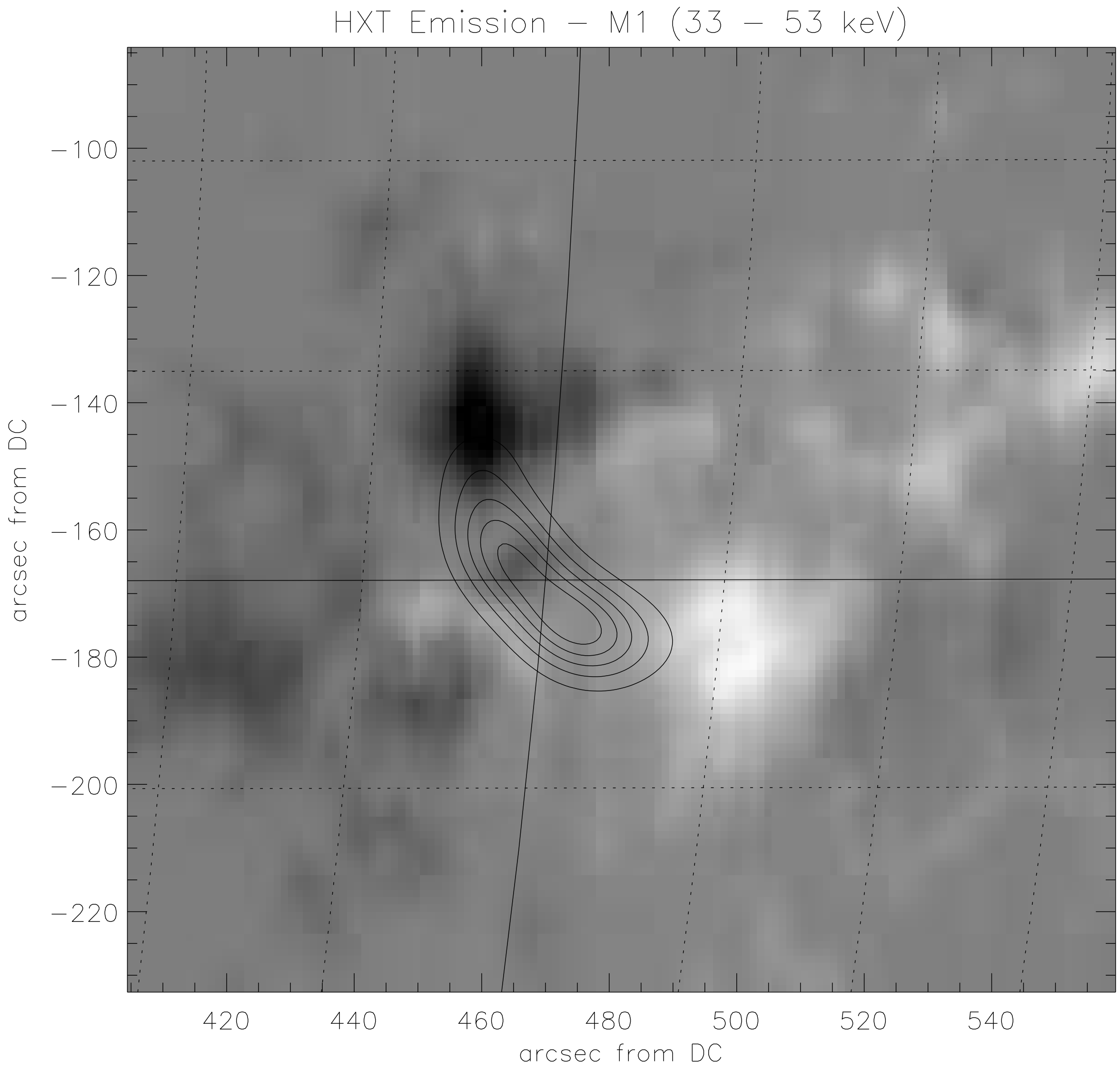


- Axis $\mathbf{r}_i$ from magnetic model – `r(3,N)`
- Cross section radius: $a_i = \sqrt{\Psi/\pi B_i}$ – `a(N)`
- Φ_i from Brehmstrahlung through channel x – `Phi(N-1)`
- x_p and y_p define $N_x \times N_y$ detector "pixels"

```
img = loop_image( r, a, Phi, xp, yp )
```

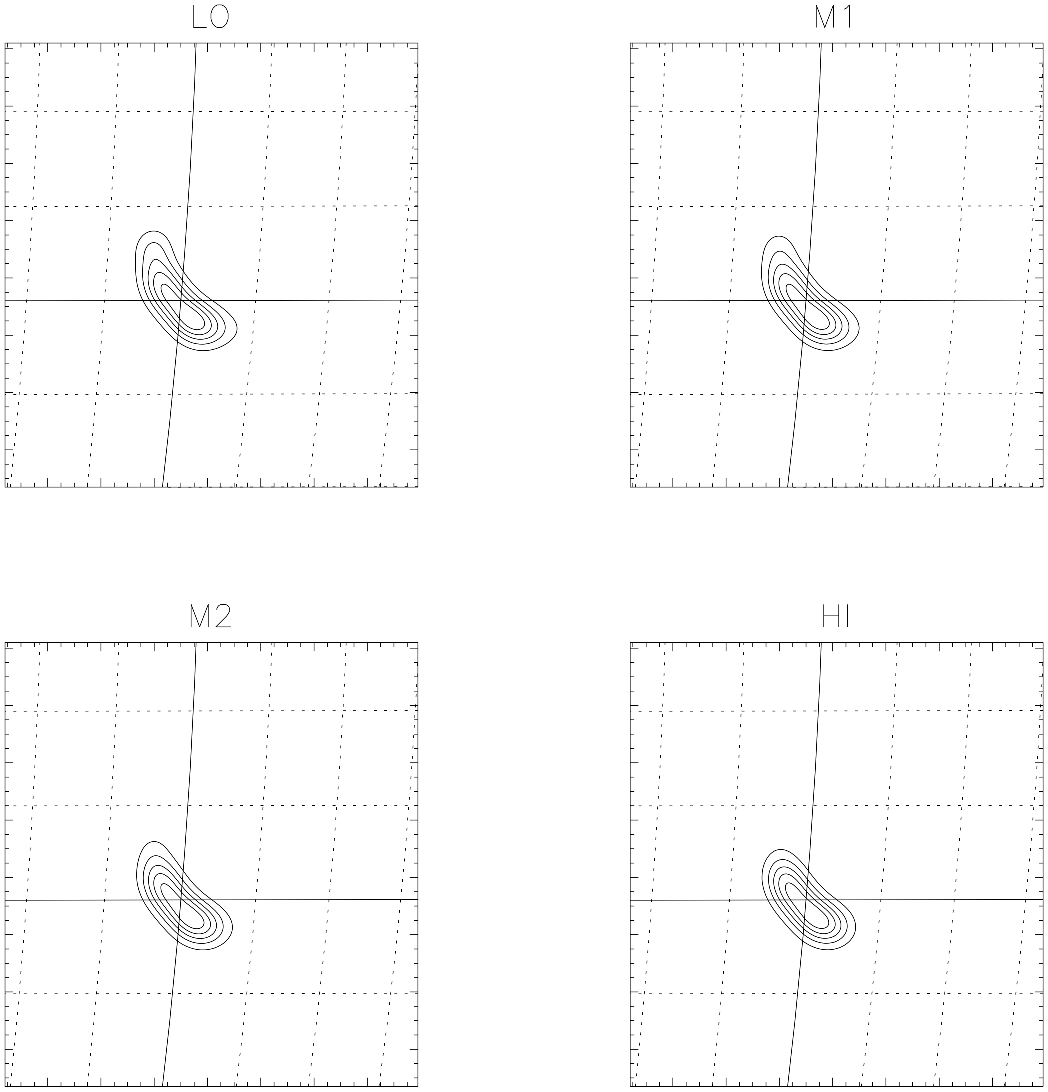
- `img` is $N_x \times N_y$ image array
- Blur to actual resolution with Gaussian

Synthesized HXT emission



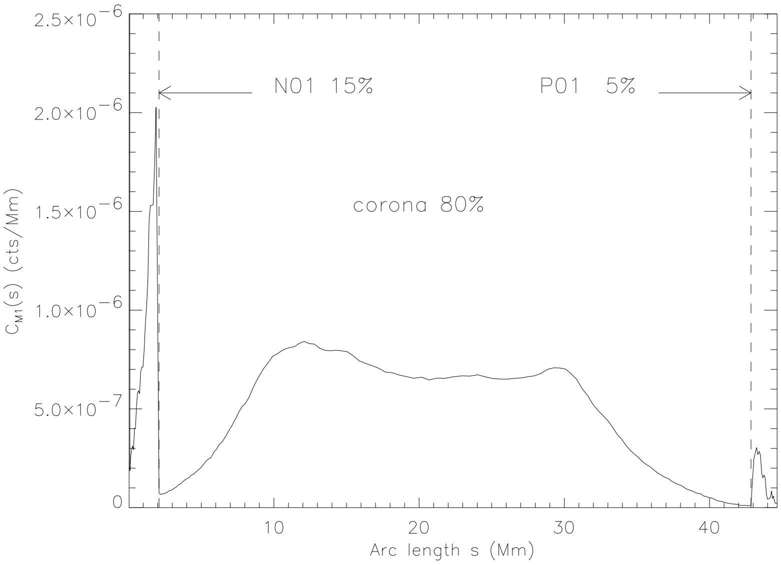
- Injection point: IP_Z
- Emission integrated over all time
- image blurred to $\sigma = 5''$

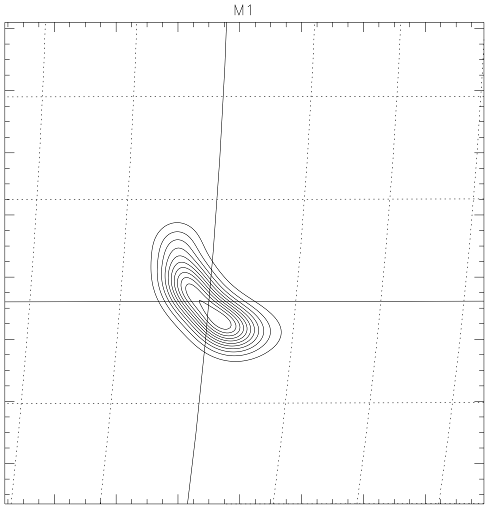
Images from 4 HXT channels — IP_Z

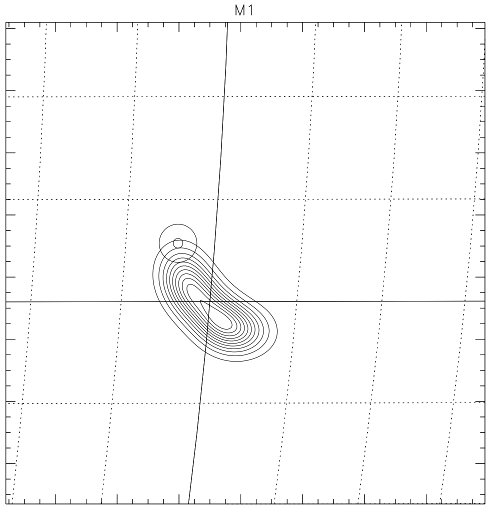


j	$\epsilon_x^{\rm lo}$ (keV)		$\epsilon_x^{\rm hi}$ (keV)
LO	14	—	23
M1	23	—	33
M2	33	—	53
HI	53	—	93

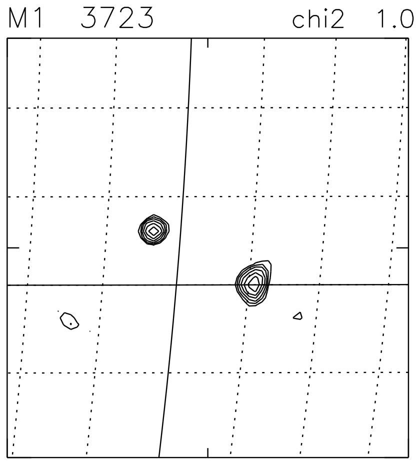
Whither the footpoints?



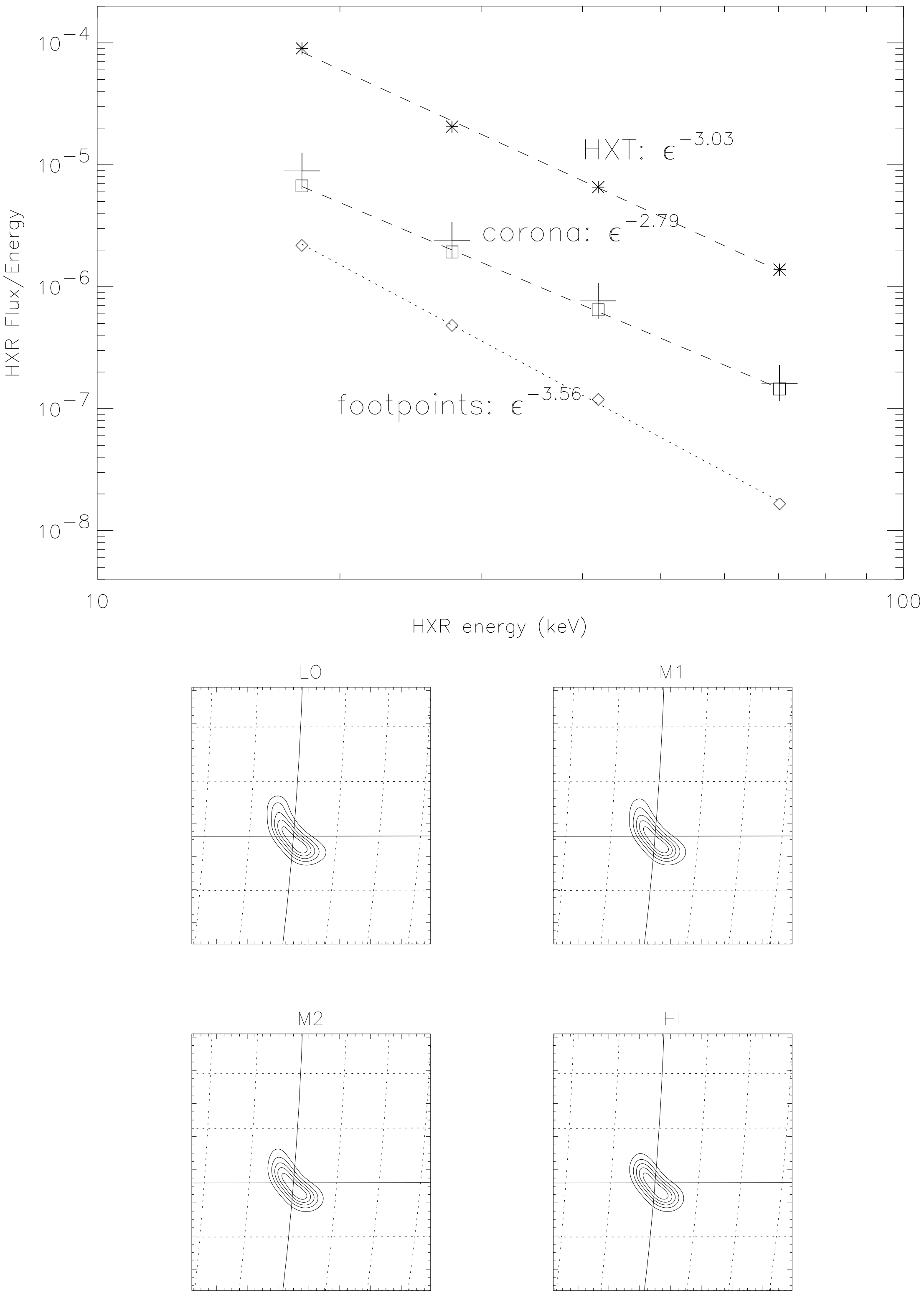




Observations:
All footpoints



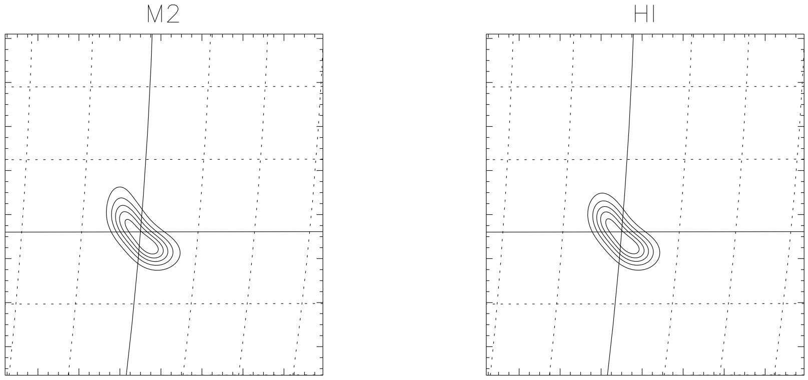
X-ray spectra



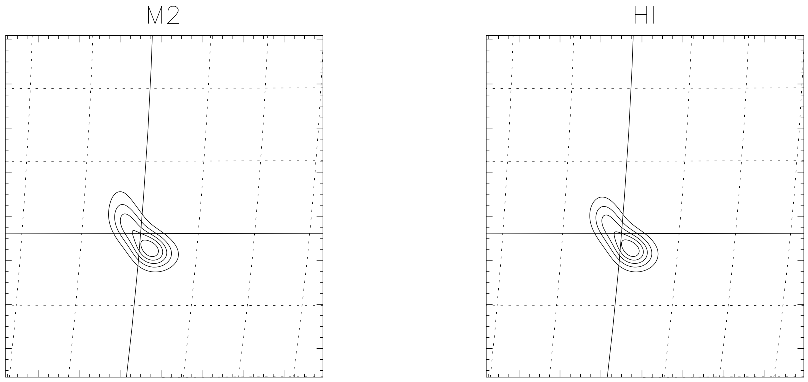
4. Parameters of the Model

A. Injection point

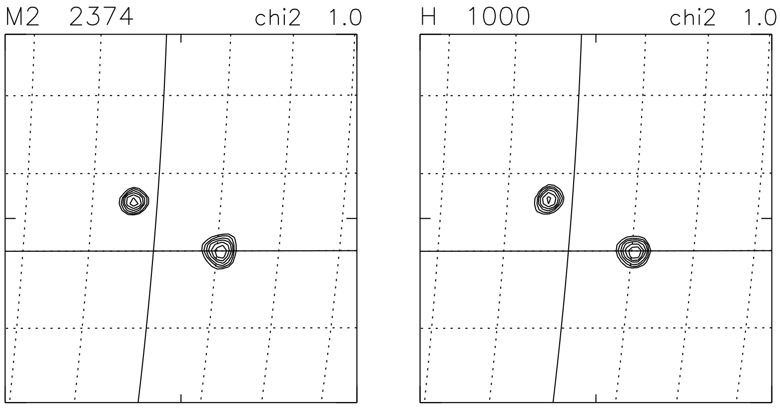
IP_Z (injected at loop apex)



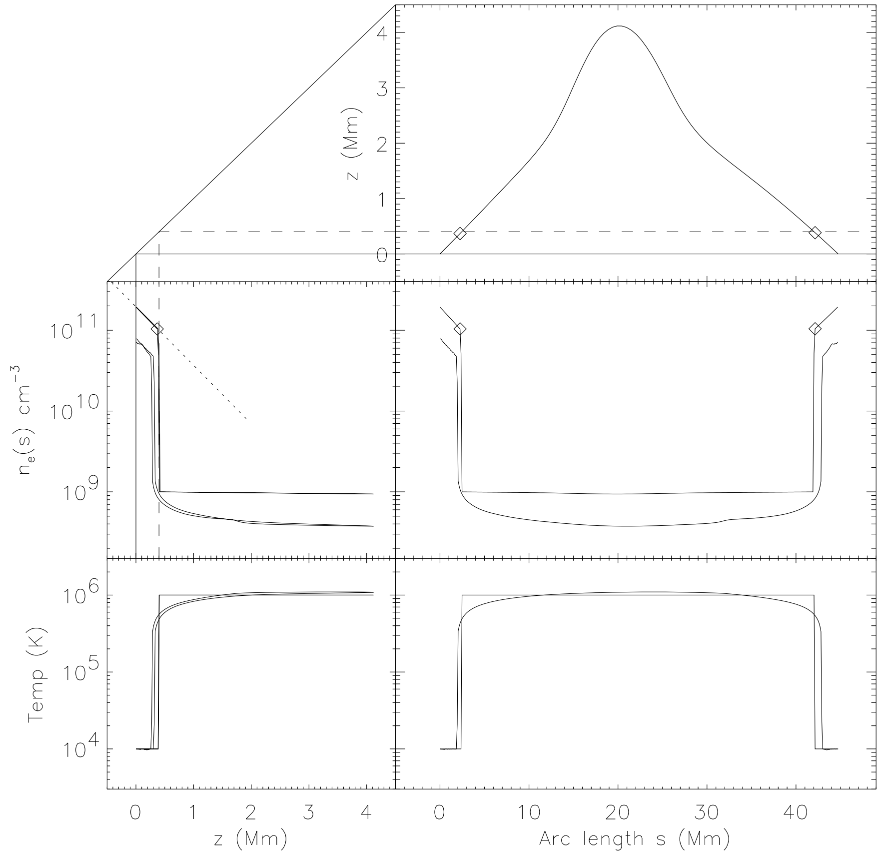
IP_B (injected at weakest field)



HXT data



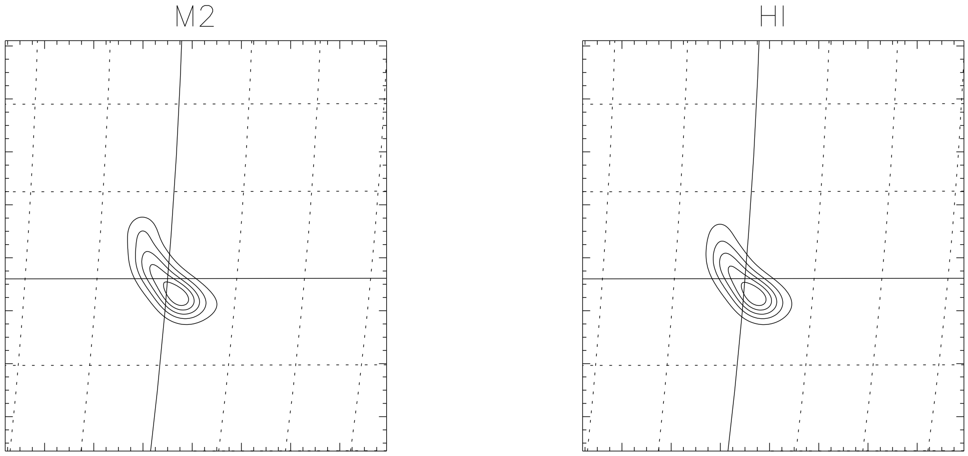
B. Models of density — simple and realistic



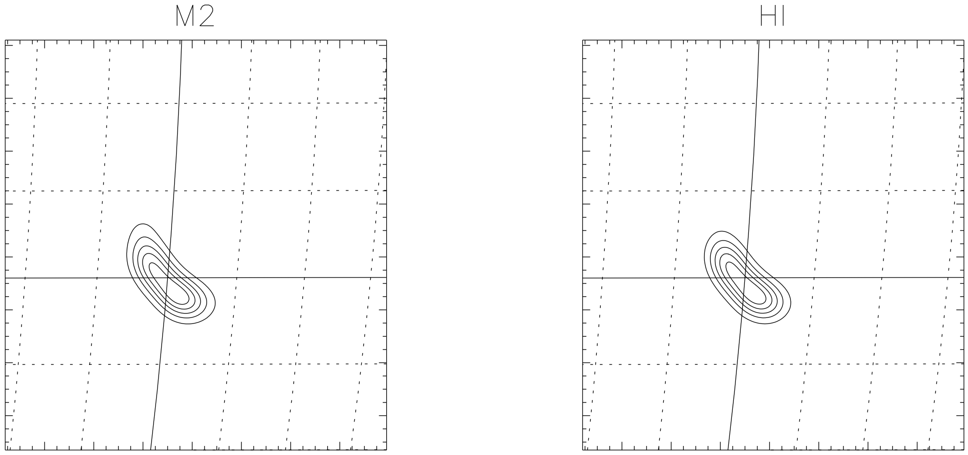
1. Simple 2 Temperature model:
   - $T_{\rm chr} = 10^4$ K, $\quad T_{\rm cor} = 10^6$ K
   - $n_e = 10^9\,{\rm cm}^{-3}$ at base of corona
   - pressure is continuous
2. Gas dynamic equilibrium:
   - Evolved 1d gas dynamic equations to equilibrium (Kankelborg & Longcope 1999)
   - $q = 2.3 \times 10^{-4}\,{\rm erg/cm}^3 \quad \Longrightarrow \quad \max(T) = 1.1$ MK
   - $\Longrightarrow \quad \min(n_e) = 0.37 \times 10^9\,{\rm cm}^{-3}$

Comparisons of density models

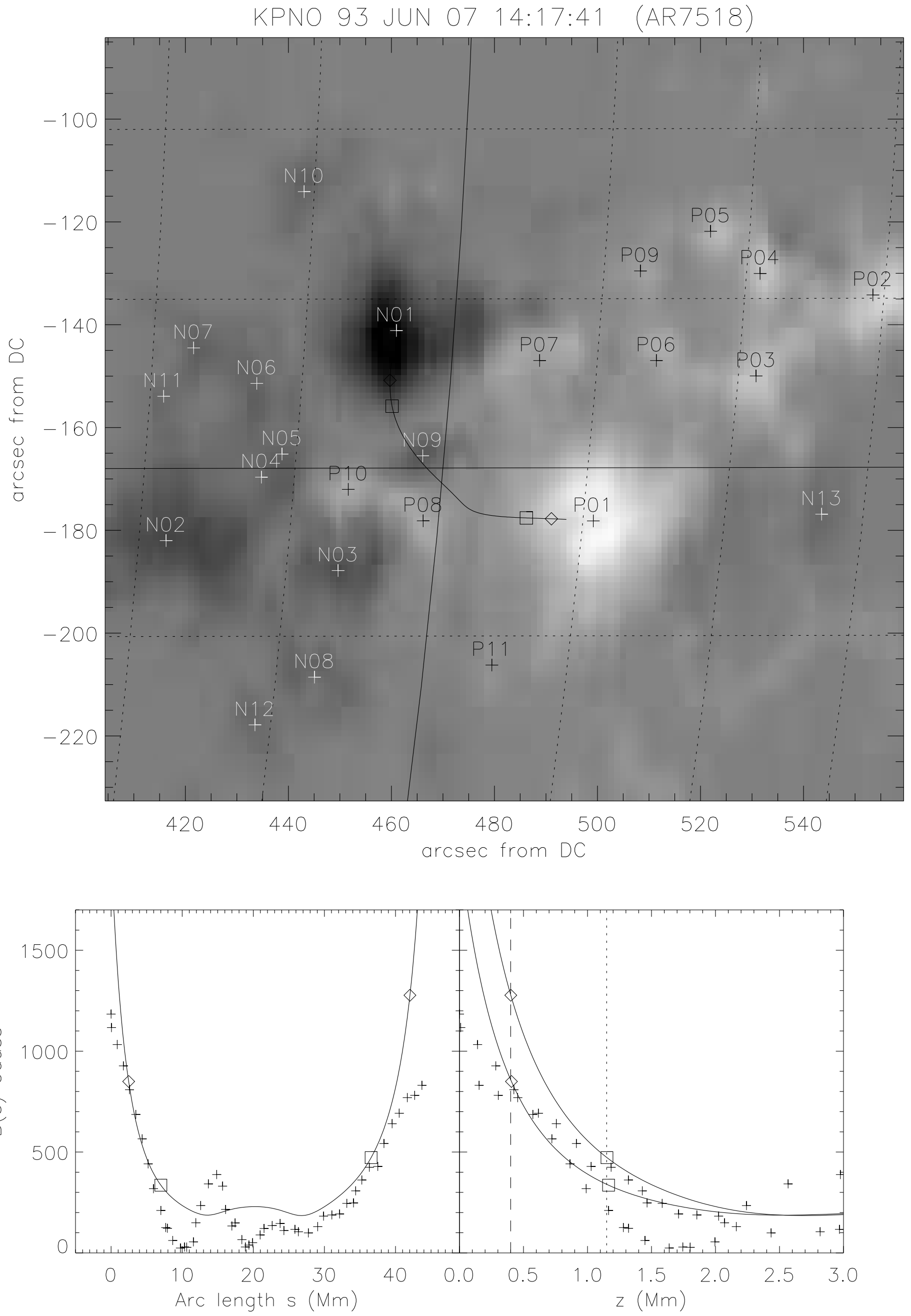
Simple 2 Temperature model: $\langle t_{\mathrm{life}} \rangle = 15.5$ sec



Gas dynamic equilibrium: $\langle t_{\mathrm{life}} \rangle = 35.3$ sec

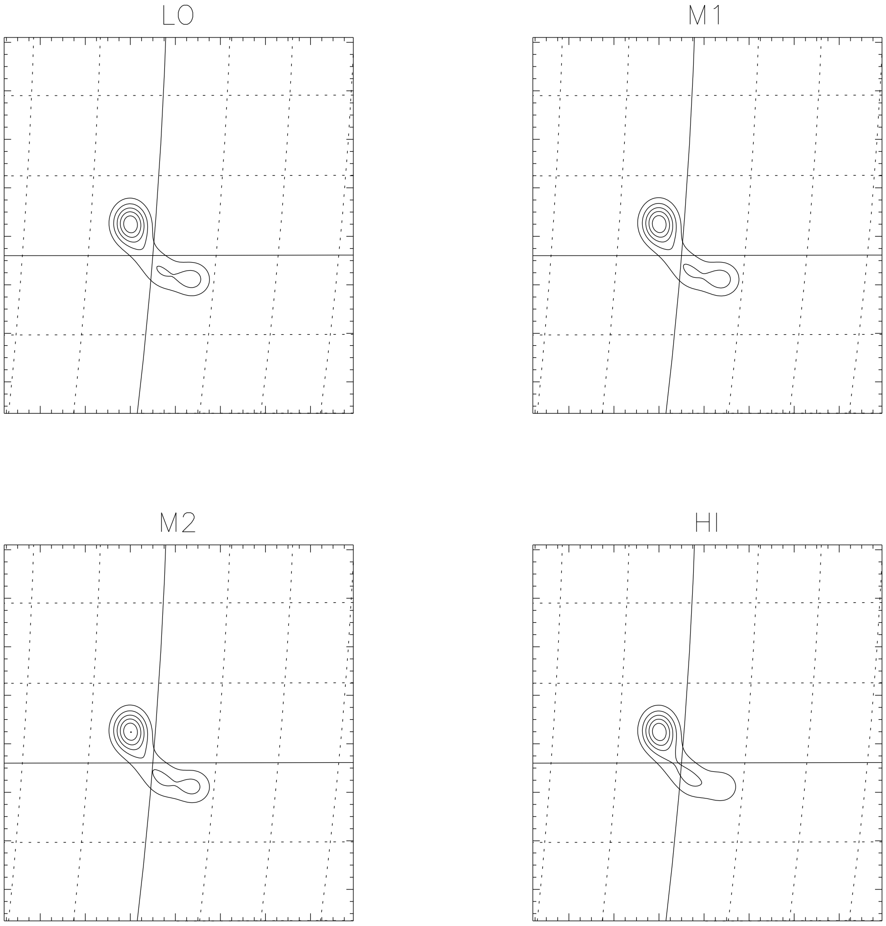


C. Choosing $z_{\rm chr}$ — some considerations

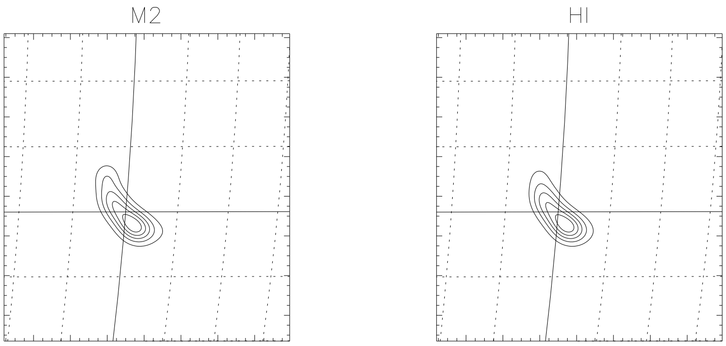


$z_{\rm chr}$	$B(N01)$	$B(P01)$	$r(IP_Z)$	frac. lost	N:P
0.4 Mm	850	1280	0.27	15%	2:1
1.13 Mm	335	472	0.68	44%	2:1

Higher chromosphere $\Longrightarrow$ footpoint emission



Compare to $z_{\rm chr} = 0.4$ M



- ◦ Corona : Footpoints = 1:2
- ◦ N01:P01 = 8:3

Summary

- Can model population of NT e^{-}s in 3d
- Magnetic geometry predicts reconnection site
- *Very different from standard 2d picture*
- Assume CS creates NT e^{-}s on post-reconnection loop
- Find qualitative agreement w/ observation (93-06-07)
- Depends on choice of z_{chr}

Future Directions

- Model other flares
- Quantify dependence on z_{chr}
- Improve magnetic field model