

Derivation

Jade R. Ream

The circle of unit radius is given by:

$$f = x(1 - 1/r^2) \quad (1)$$

Where z and ζ are complex numbers

$$z = x + iy \quad \text{and} \quad \zeta = \chi + i\gamma \quad (2)$$

$$J' \circ z = \frac{1}{2A} \left(z \pm \sqrt{z^2 - 4AB} \right). \quad (3)$$

This equation is special because it satisfies the Laplacian Equation:

$$\nabla^2 f(z) = 0 \quad (4)$$

This implies,

$$\text{grad}^2[f\zeta] = 0 \quad (5)$$

So,

$$g = f(\zeta) = f(J' \circ z) \quad (6)$$

$$\text{grad}g = \text{grad}f(J' \circ z) \quad (7)$$

Since z is an element of the complex number set, we must go back to this definition using real numbers. So let us redefine f as,

$$f = x(1 - 1/x^2 + y^2) \quad (8)$$

Now we will require useful facts that simplify the problem:

At the line $y = 0$

$$\text{grad}g = \text{partial}_x g \quad (9)$$

This implies that on this line

$$\text{Im}(z) = \text{Im}(\zeta) = 0 \quad (10)$$

This allows us to assume

$$f(x) = x(1 - 1/x^2) \quad (11)$$

and that

$$(J' \circ z) = (J' \circ x) = \frac{1}{2A} \left(x \pm \sqrt{x^2 - 4AB} \right). \quad (12)$$

This all leads us to the conclusion that,

$$|B| = \partial_x [f(J' \circ x)] \quad (13)$$