Mathematical Equations for Daily Log

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To begin we will start with the typical notion of Bernoulli's Principle

$$\mathbf{Bern} = (1/2)rho * v^2 + p = constant \tag{1}$$

Since, the derivative of a constant is zero we can say,

$$\partial_z ((1/2)rho * v^2 + p)) = 0 \tag{2}$$

Since we are observing the plasma in a LeGrangian frame in which the plasma flows past the object instead of a Eulerian frame in which the object moves through fluid at rest the Bernoulli Equation useful to us is,

$$\partial_z ((1/2)u^2 + 1/rho\partial z * p = 0 \tag{3}$$

When we incorporate in the conservation of mass equation that is

$$constant = rho * u * A$$
 (4)

We get

$$\partial_z \rho = -grad \cdot (\rho \mathbf{U}) \tag{5}$$

The next equation of use to us is the Isothermal Energy Equation which we will use to eliminate the pressure variable p.

$$p/rho = C_s^2 \tag{6}$$

Making this substitution gives us

$$\partial_z (1/2)u^2 + 1/rho\partial_z * C_s^2 = 0 \tag{7}$$

Now the goal will be to illustrate that the derivative of some formula is equal to zero. Allowing us to determine that the said formula is a constant. To accomplish this we will multiply both sides by two and separate the formula at the addition sign. Leaving the left hand side alone as,

$$\partial_z (2 * u^2)$$
 (8)

And modifying the right hand side through integration. When integrated we get,

$$\ln(\rho)2 * C_s^2 = 0 \tag{9}$$

Putting them back together we get,

$$\partial_z (2 * u^2) + \ln(\rho) 2 * C_s^2 = 0 \tag{10}$$

This can be altered to look like,

$$\partial_z (2 * u^2) + \ln(\rho^2) C_s^2 = 0 \tag{11}$$