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To begin we will start with the typical notion of Bernoulli's Principle

$$
\begin{equation*}
\text { Bern }=(1 / 2) r h o * v^{2}+p=\text { constant } \tag{1}
\end{equation*}
$$

Since, the derivative of a constant is zero we can say,

$$
\begin{equation*}
\left.\partial_{z}\left((1 / 2) r h o * v^{2}+p\right)\right)=0 \tag{2}
\end{equation*}
$$

Since we are observing the plasma in a LeGrangian frame in which the plasma flows past the object instead of a Eulerian frame in which the object moves through fluid at rest the Bernoulli Equation useful to us is,

$$
\begin{equation*}
\partial_{z}\left((1 / 2) u^{2}+1 / r h o \partial z * p=0\right. \tag{3}
\end{equation*}
$$

When we incorporate in the conservation of mass equation that is

$$
\begin{equation*}
\text { constant }=\mathbf{r h o} * \mathbf{u} * \mathbf{A} \tag{4}
\end{equation*}
$$

We get

$$
\begin{equation*}
\partial_{z} \rho=-\operatorname{grad} \cdot(\rho \mathbf{U}) \tag{5}
\end{equation*}
$$

The next equation of use to us is the Isothermal Energy Equation which we will use to eliminate the pressure variable p.

$$
\begin{equation*}
p / r h o=C_{s}^{2} \tag{6}
\end{equation*}
$$

Making this substitution gives us

$$
\begin{equation*}
\partial_{z}(1 / 2) u^{2}+1 / r h o \partial_{z} * C_{s}^{2}=0 \tag{7}
\end{equation*}
$$

Now the goal will be to illustrate that the derivative of some formula is equal to zero. Allowing us to determine that the said formula is a constant. To accomplish this we will multiply both sides by two and seperate the formula at the addition sign. Leaving the left hand side alone as,

$$
\begin{equation*}
\partial_{z}\left(2 * u^{2}\right) \tag{8}
\end{equation*}
$$

And modifying the right hand side through integration. When integrated we get,

$$
\begin{equation*}
\ln (\rho) 2 * C_{s}^{2}=0 \tag{9}
\end{equation*}
$$

Putting them back together we get,

$$
\begin{equation*}
\partial_{z}\left(2 * u^{2}\right)+\ln (\rho) 2 * C_{s}^{2}=0 \tag{10}
\end{equation*}
$$

This can be altered to look like,

$$
\begin{equation*}
\partial_{z}\left(2 * u^{2}\right)+\ln \left(\rho^{2}\right) C_{s}^{2}=0 \tag{11}
\end{equation*}
$$

