

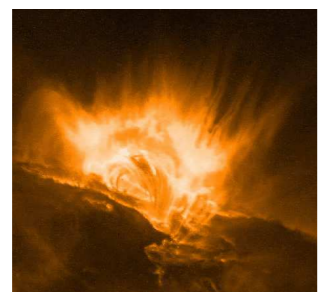
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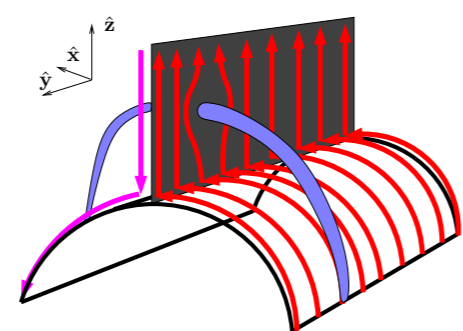
Abstract

One branch of supra-arcade downflows research attempts to describe low-emission features in terms of a process known as peristaltic pumping. The model uses a circular intrusion to deform the magnetic field. The field aligned fluid cross-section is used to parametrize plasma flow along the field lines. This investigation builds on previous results, using conformal mapping to transform the circular intrusion into an elliptical intrusion. Ultimately, we intend to explore how the eccentricity of the elliptical intrusion affects the critical velocity at which shock begin to form.

Motivation



(a)



(b)

Figure 1: (1a) This EUV image from Trace displays regions of low emission called voids, which form above post-CME arcades. (1b) The low emission voids have been interpreted as retracting coronal loops; this figure illustrates how the retracting coronal loop pierces the supra-arcade as it descends toward the photosphere.

We are motivated to investigate the formation of voids because improving our knowledge of the supra-arcade could potentially provide insight into the bigger questions, such as the coronal heating problem.

The Model

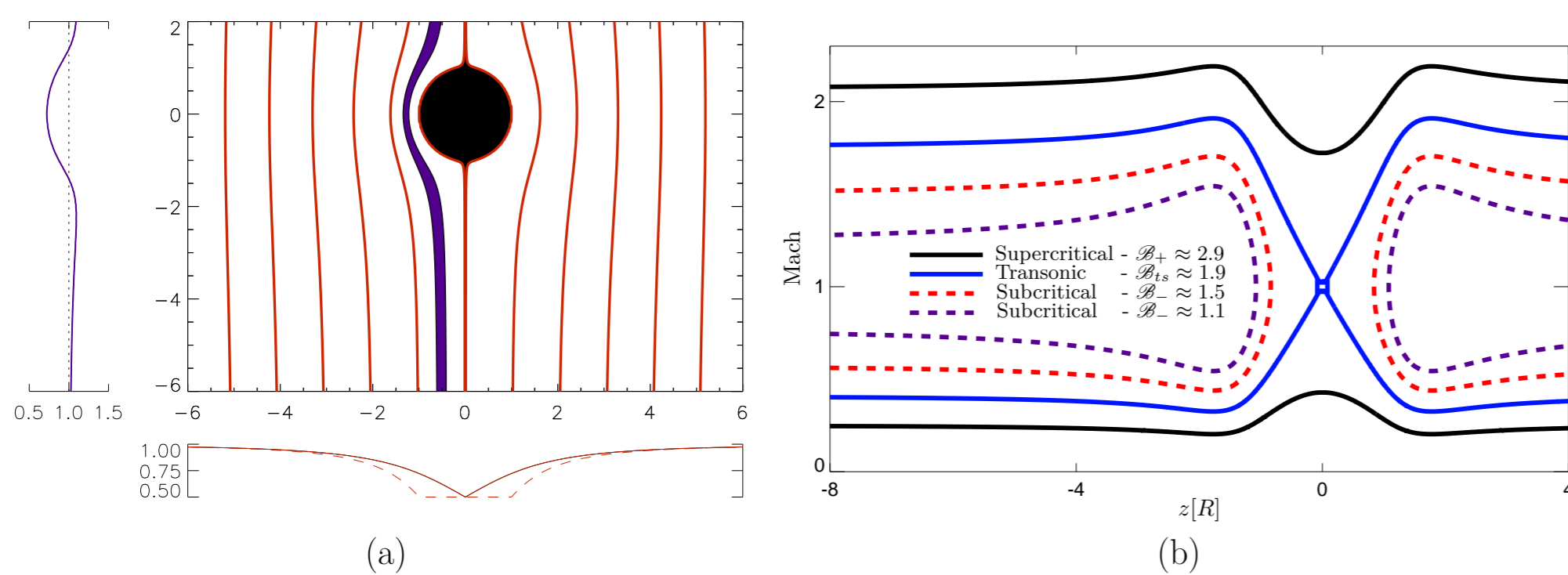


Figure 2: (2a) This figure displays a cross-section of the supra-arcade structure, demonstrating that the intruding element of reconnecting flux deforms the surrounding field. (2b) Models of peristaltic pumping use the cross sectional area along an individual field line, which varies inversely with the field strength, to develop solutions to de Laval's nozzle equation. Solutions that fall between the upper and lower transonic flow lines correspond to inflow Mach values that cannot satisfy the conditions of plasma flow unless shocks are introduced.

The magnetic field, which must be tangent to the surface of the intrusion, is determined according to the flux function,

$$f = x \left(1 - \frac{1}{R^2} \right), \quad (1)$$

with $|\mathbf{B}| = |\nabla f|$. The cross-sectional area is inversely proportional to the field strength, thus knowing field strength de Laval's equation is found by combining Bernoulli's Principle:

$$\partial_z \left(\frac{1}{2} u^2 + \frac{1}{\rho} \partial_z p \right) = 0, \quad (2)$$

with the Continuity equation:

$$\partial_z \rho u A = 0, \quad (3)$$

under the Isothermal Energy conditions :

$$p/\rho = C_s^2. \quad (4)$$

Resulting in de Laval's equation:

$$M^2 - \ln(M^2) - \ln(A^2) = \mathcal{B}. \quad (5)$$

Conformal Mapping

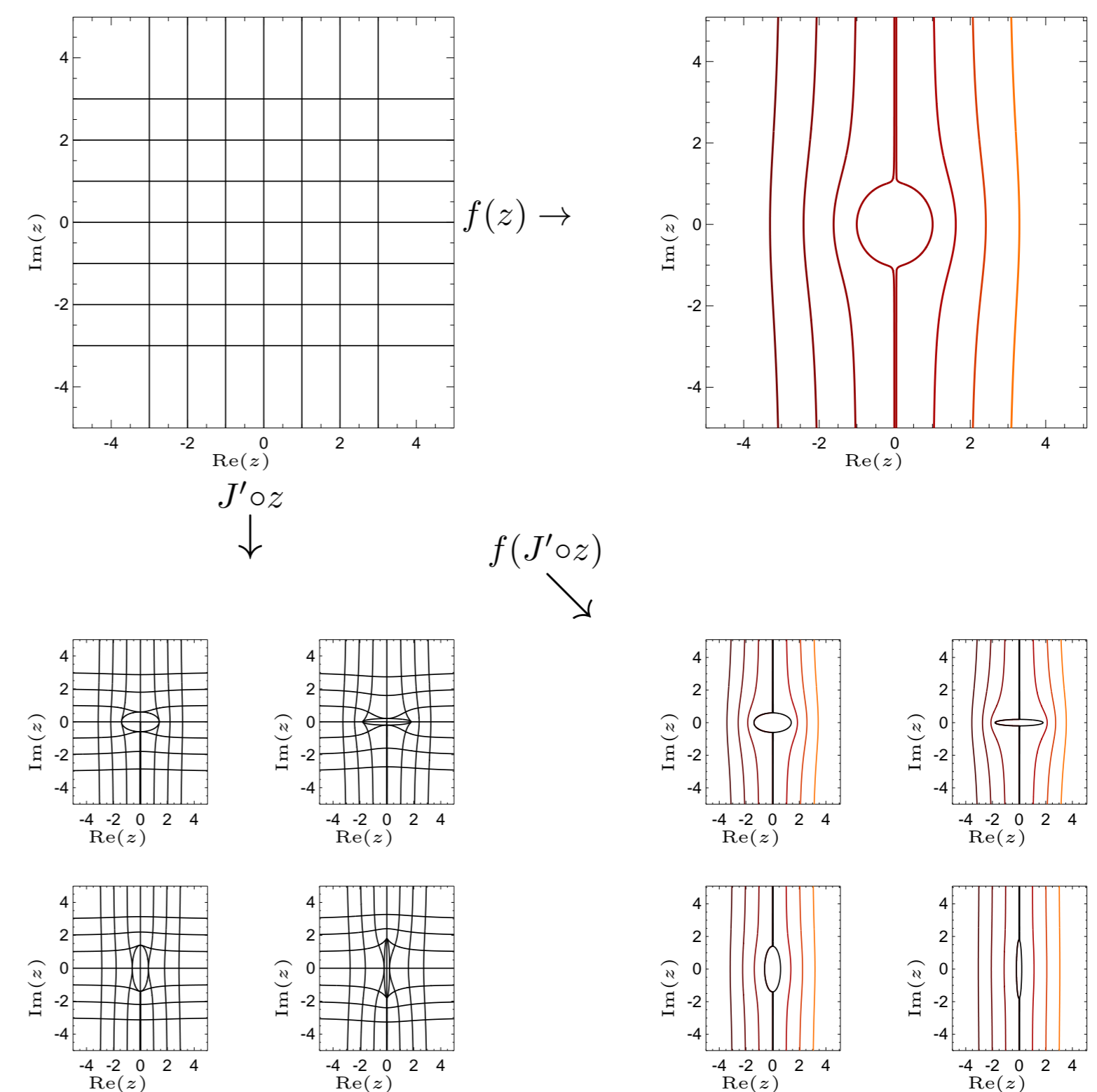


Figure 3: This figure demonstrates how conformal mapping may be used to generate complex magnetic fields with elliptical boundary conditions from a simple dipole solution.

If $f(z)$ is the flux function for the simple dipole magnetic field,

$$f = \text{Re}(z) \left(1 - \frac{1}{|z|^2} \right), \quad (6)$$

then an entire class of flux functions, $g(z)$, can be generated as,

$$g(z) = f(J' \circ z), \quad (7)$$

where

$$J' \circ z = \frac{1}{2A} \left(z \pm \sqrt{z^2 - 4AB} \right). \quad (8)$$

Here J' is the Inverse Joukowski Transformation, which maps an ellipse, defined in terms of A and B, to the unit circle.

Implications

The phenomenon of peristaltic pumping offers one explanation for how energy can be deposited into the unreconnected field by nearby reconnection events. Previous work on peristaltic pumping predicts an upper limit on the speed at which downflows can produce shocks ($M_{crit} \approx 2$), which may have ramifications for the rate at which the energy deposition can occur. Through this work we intend to investigate how the critical velocity changes when the shape of the intrusion is modified. The formation of shocks along an individual field line is directly related to the shape of the transonic solution. The critical speed at which shocks form is reliant on the minimum cross-section of the most severely constricted field line. Through conformal mapping we are able to determine how this cross-section is related to the eccentricity of the intrusion. Future research will concentrate on isolating how the eccentricity affects both the critical Mach number for shock formation and the characteristic width of the shocked column.