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THE DISTRIBUTION OF DUST INSIDE THE ORION NEBULA

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ABSTRACT

By inverting the brightness integral for the dust-scattered continuum, we have determined how the dust grains are distributed inside the Orion nebula. The scattering characteristics of the Orion dust at a given wavelength is kept constant within the nebula, and the geometry of the nebula is assumed to have a hemispherical shape. The resulting radial distance dependence of the distribution of dust number density, N_d (r), shows that the dust grains are depleted at the central region of the Orion nebula and concentrated in the region 5′~6′ away from the Trapezium stars. The scattering characteristics of the Orion dust are of moderately forward throwing nature, and the Orion dust has low values of albedo.

I. INTRODUCTION

The Orion nebula is one of the most extensively observed H II regions in the Galaxy. Being the nearest H II region, this nebula becomes an ideal object to make detailed studies on the distribution and the nature of its internal dust grains. The wavelength dependence of the interstellar extinction suffered by the illuminating source, the Trapezium, is so different from that of the general interstellar extinction law (Bless and Savage 1972; Bohlin and Savage 1981) that most of the grains responsible for the peculiar extinction are certainly the ones inside the ionized region than the ones in the intervening interstellar space. As another evidence for the internal dust, the scattered continuum of the central star has been observed at the visible and the far ultraviolet wavelengths. Many authors have obtained the distribution of internal dust from the observed spatial variation of the dust-scattered continuum.

O'Dell and Hubbard (1965) determined the amount of dust grains in the Orion nebula by analyzing the observation of the scattered stellar continuum at visible wavelength. Under the assumption of single, isotropic scattering, they claimed that the dust-to-gas ratio is about twenty eight times smaller in the region close to the Trapezium than it is in the general interstellar medium: the central region of the Orion nebula is heavily depleted in dust content. Martin and Gull (1976) inferred the total optical depth of the dust in the Orion nebula from the discrepancy between the observed radio continuum profiles and the theoretical ones expected from the distribution of H_α emission. The resulting dust-to-gas ratio turned out to be one order-of-magnitude higher in the nebula than it is in general interstellar medium. They thus took a position that the twenty-eight times depletion of dust reported by O'Dell and Hubbard (1965) is probably due to their adoption of an unrealistically large nebula size.

With the aid of *IUE*, Perinotto and Patriarchi (1980b) observed the dust-scattered continuum at a number of ultraviolet wavelengths and analyzed their observation to have information on the dust-to-gas ratio in the region close to the Trapezium. Under the assumptions of isotropic, single scattering and no internal extinction, the resulting dust-to-gas ratio in the central region of the Orion nebula does not differ much from the corresponding value in general interstellar medium. This result is not in accordance with the earlier studies. Schiffer and Mathis (1974), Mathis et al.(1981), and Patriarchi and Perinotto (1985) have made serious attempts at the distribution of dust grains in the Orion nebula. Adopting various models of internal dust distribution, they have calculated the expected distributions of the scattered continuum from the adopted model dust distributions at a number of wavelengths. Their best model distribution has a shell of dust and gas whose inner radius is about 0.2~0.3 pc. And they concluded that the Trapezium is surrounded by a 'hole' which contains little dust that scatters light efficiently. They claimed their computation of the best model reproduced the observed distribution faithfully. However, there appeared to be a systematic difference between their model calculations and the observations.

Quite a different approach was taken by Kim and Hong (1982). With a non-LTE analysis for the recombination process in the ionized medium, they determined the distributions of electron density and temperature within the Orion nebula from the observed distributions of the radio recombination lines and continuum. By solving the thermal balance equation explicitly, they were able to determine the radial variation of the abundance of trace elements. Their results showed that the gas phase concentration of the cooling elements has about the solar value at the central part of the nebula, decreases outwards, and finally attains about one quarter of the solar value in the outer extended envelope. They interpreted that the severe depletion of the gas phase metal abundance in the outskirt of the Orion nebula is due to the increasing content of dust there, and that the normal metal abundance at the center is due to complete evaporation of the central dust. So most of the gas phase metals in the outskirt of the nebula must be locked in the solid phase. This interpretation is very suggestive that the Orion nebula has a central dust-hole and the Orion dust is concentrated in the outskirt.

Although many studies have been made on the distribution of the Orion dust in various wavelength regions, still inconsistencies exist between them. All of the previous studies employed parameterized functions of radial distance as models for the radial dust distribution, and examined whether the model distributions reproduce the observed distributions of the dust scattered continuum. In this kind of approach, the results are destined to be constrained within the flexibility of the chosen functions of the dust distribution. Instead, in the present study, we propose to deduce the dust distribution directly from the observed brightness distribution by inverting the brightness integral for the scattered stellar continuum. The method of inverting brightness integral has been used successfully in the analysis of zodiacal light problems by Hong (1987). An advantage of inverting the brightness integral over the usual model fitting is to be free from the constraint of the chosen model distributions.

Outlay of this paper is as follows. In section II, theoretical framework is presented of the method for deducing the dust distribution from the observed brightness distributions. In section III, observational data used in this analysis are summarized. The resulting dust distribution and its optical properties are thoroughly discussed in section IV. Conclusions are given in section V.

II. METHOD OF ANALYSIS

a) Geometry of the Nebula

Contours of the free-free radio continuum from the Orion nebula are observed to be circularly symmetric with respect to the Trapezium (Schraml and Mezger 1969). Although there are some large-scale structures (usually called the 'bar' and the 'bay') in the eastern part of the nebula and the radio contours are indeed somewhat disturbed there, the most obvious interpretation given to the circularly symmetrical contours is that the nebula is more-or-less spherically symmetric. Especially, on the western side of the nebula, there are no large-scale structures within a parsec from the Trapezium. So the regular behavior of the brightness distribution is apparent in the western part not only in the radio continuum but also in the dust-scattered continuum (Schiffer and Mathis 1974; Mathis et al. 1981). For this reason, we concentrate on the western part of the Orion nebula.

We know a massive molecular cloud is located behind the ionized region (Zuckerman 1973; Schiffer and Mathis 1974; Balick et al. 1974). Since the dense molecular cloud entirely blocks the flow of ionized gas into itself, a full sphere is obviously an unrealistic choice for the ionized nebula; we consider the ionized region to be more of a hemisphere than of a sphere. However, the flow of ionized gas to the dense molecular cloud will produce a layer of partially ionized material sandwiched between the hemisphere and the dense molecular cloud. Besides, this transition layer may not be of a planar slab, but probably of a parabolic one, since the Trapezium is known to move into the cloud with a speed of ~3 km/sec (Zuckerman 1973). Then a very large pressure gradient will be established in the slab material when the dense molecular cloud is ionized by the Trapezium. The ionized gas will rapidly flow out from the transition layer. Because of this fast flow of the ionized gas, the dust grains are also carried out and then blown away from the transition layer (Schiffer and Mathis 1974; Mathis et al. 1981; Goudis 1982). So the probability that the layer of partially ionized material contains dust grains is quite small with respect to the ionized hemisphere on the front side. Furthermore, the transition layer has been known to be of a thickness of 0.3~0.4 pc (Balick et al. 1974; Harper 1974). Therefore, the dense molecular cloud is located quite far from the ionized hemisphere. We ignore the starlight reflected from the Orion molecular cloud, for it will contribute very little to the observed intensity due to the attenuation by the dust grains at the front side of the nebula (Schiffer and Mathis 1974). Moreover, if the scattering characteristics of the particles is of the forward throwing nature, the contribution from the dense molecular cloud will be extremely small. Therefore, we will consider only the front hemisphere of dust and gas mixture, and put the center of the hemisphere at the Trapezium stars. We ignore the parabolic part and the backward reflection of the starlight by the rear molecular cloud.

b) Brightness Integral for the Scattered Continuum

Assuming that the Orion nebula is optically thin, we consider only singly scattered photons. The specific scattering emissivity, j_λ, from the dust in unit volume at radial distance r from the Trapezium is given by

$$j_\lambda = \frac{L_\lambda}{4\pi r^2} \exp\left[-\tau_{ext}(r;\lambda)\right] N_d(r)\sigma_{sca}(r;\lambda)\Phi(\theta;g), \tag{1}$$

where L_λ is the specific luminosity of the central source at wavelength λ, $\tau_{ext}(r;\lambda)$ the extinction optical depth over the distance, r, $N_d(r)$ the local dust number density, $\sigma_{sca}(r;\lambda)$ the mean scattering cross section of the dust particles at r and λ, and $\Phi(\theta;g)$ their mean scattering phase function. For the scattering phase function, $\Phi(\theta;g)$, we utilized the parametric representation by Henyey and Greenstein (1941):

$$\Phi(\theta;g) = \frac{1}{4\pi}\frac{1-g^2}{(1+g^2-2g\cos\theta)^{3/2}}. \tag{2}$$

The scattering asymmetry factor, g, is assumed to be independent of radial distance but it may depend on wavelength.

Geometrical relation between scattering angle θ, radial distance r, and line-of-sight distance l is illustrated in Figure 1. Besides the geometrical dilution, $(4\pi r^2)^{-1}$, the stellar continuum experiences further extinction due to the internal dust. Thus, $L_\lambda/4\pi r^2 \cdot \exp[-\tau_{ext}(r;\lambda)]$ becomes the stellar flux reaching the local dust particles at radial distance r, and j_λ represents the intensity scattered into unit solid angle by all the dust particles in unit volume.

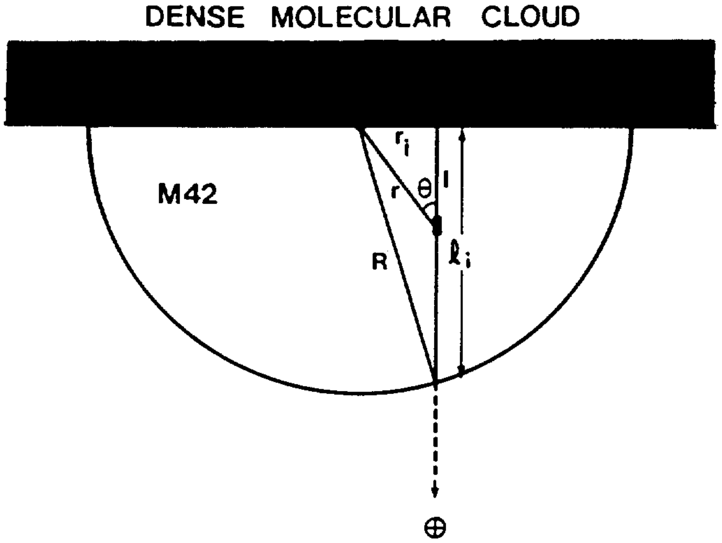


Fig. 1. Schematic illustration of the geometry involved in the scattering process in the Orion nebula M42.

The intensity, I_λ, of the scattered stellar continuum that is observed at a projected distance r_i is given by the integral of j_λ over l_i:

$$I_\lambda(r_i) = exp[-\tau_{ext}(ISM;\lambda)] \int_0^{l_i} N_d(r_i)\,\sigma_{sca}(r;\lambda)\,\Phi(\theta;g)\,\frac{L_\lambda}{4\pi r^2}\,\cdot \tag{3}$$

$$\cdot\, exp[-\tau_{ext}(r;\lambda)]\,dl,$$

where l_i represents the total geometrical thickness of the nebula at the projected radial distance r_i. Notice that the factor, $\exp[-\tau_{ext}(ISM;\lambda)]$, takes into account for the extinction by the general interstellar dust lying over the pathlength from the outer boundary of the Orion nebula to the Earth. We may obtain L_λ from an observable quantity F_λ as:

$$L_\lambda = 4\pi d^2 F_\lambda\, exp[\tau_{ext}(R;\lambda) + \tau_{ext}(ISM;\lambda)], \tag{4}$$

where d is the distance from the Earth to the nebula, F_λ the observed flux of the Trapezium stars, and $\tau_{ext}(R;\lambda)$ the internal extinction optical depth over the total radius of the nebula. For d and R, we take 500 pc and 1 pc, respectively. Since both the scattered continuum and the stellar flux are attenuated by the same amount in the general interstellar medium, equation (3) becomes

$$I_\lambda(r_i) = d^2 F_\lambda\, exp[\tau_{ext}(R;\lambda)] \int_0^{l_i} N_d(r)\,\sigma_{sca}(r;\lambda)\,\Phi(\theta;g)\,\cdot \tag{5}$$

$$\cdot\, exp[-\tau_{ext}(r;\lambda)]\,\frac{dl}{r^2}.$$

We call this as the brightness integral for the scattered stellar continuum.

We now change integral variable from l to θ by using the following geometrical relations:

$$\theta = sin^{-1}(r_i/r) \qquad \text{and} \qquad r_i\,dl = -r^2\,d\theta. \tag{6}$$

Then, equation (5) becomes as

$$I_\lambda(r_i) = d^2 F_\lambda\,\frac{exp[\tau_{ext}(R;\lambda)]}{r_i} \int_{\sin^{-1}(r_i/R)}^{\pi/2} N_d(r_i/\sin\theta)\,\sigma_{sca}(r_i/\sin\theta;\lambda)\,\cdot \tag{7}$$

$$\cdot\,\Phi(\theta;g)\, exp[-\sigma_{ext}(r_i/\sin\theta;\lambda)]\,d\theta.$$

If one replaces $I_\lambda(r_i)$ by the observed intensity of the scattered starlight at r_i and F_λ by the observed flux of the Trapezium, the brightness integral for the scattered continuum, equation (7), becomes an integral equation for the unknown $N_d(r)\,\sigma_{sca}(r;\lambda)$. By inverting the brightness integral, one may determine how the volumetric scattering cross section, $N_d(r)\,\sigma_{sca}(r;\lambda)$, varies with radial distance r inside the Orion nebula.

In this paper, we adopt an approximation of single scattering for the transfer of radiation in the Orion nebula. However, we made a partial correction for the internal extinction. The starlight is assumed to experience some extinction by the internal dust while it travels from the Trapezium to the dust in unit volume at distance r from the star, scattered once by the internal dust, and to travel inside the nebula without suffering any more internal extinction. If the total optical depth of the Orion nebula is much larger than unity, we should have taken into account for the internal extinction more seriously than this. For an optically thick nebula, the transfer of radiation should be described by the multiple scattering theory. We hope our simplified approach makes a good start for future detailed studies on the dust distribution inside the Orion nebula.

c) Method of Computations: Inversion and Iteration

Following the inversion method by Hong (1987), we replace the integral, equation (7), by a quadrature sum:

$$I_\lambda(r_i) = d_2 F_\lambda \frac{\exp[\tau_{ext}(R;\lambda)]}{r_i} \sum_{\theta_{ji}=\theta_{1i}}^{\pi/2} N_d(r_i/\sin\theta_{ji})\,\sigma_{sca}(r_i/\sin\theta_{ji};\lambda)\,\cdot \tag{8}$$

$$\cdot\,\Phi(\theta_{ji};\lambda)\exp[-\tau_{ext}(r_i/\sin\theta_{ji};\lambda)]\,\Delta\theta_{ji},$$

where $\Delta\theta_{ji}$ is the summation interval. The quardrature sum for the brightness integral can be described symbolically as

$$C_i = \sum_j A_{ji} B_{ji}, \tag{9}$$

where

$$C_i = I_\lambda(r_i)\, r_i / \left\{ d^2 F_\lambda \exp[\tau_{ext}(R;\lambda)] \right\},$$

$$A_{ji} = N_d(r_i/\sin\theta_{ji})\,\sigma_{sca}(r_i/\sin\theta_{ji};\lambda), \tag{10}$$

and

$$B_{ji} = \Phi(\theta_{ji};g)\exp[-\tau_{ext}(r_i/\sin\theta_{ji};\lambda)]\,\Delta\theta_{ji}.$$

We then divide the whole nebula, as illustrated in Figure 2, into concentric circles, and evaluate the matrix elements A_{ji} and B_{ji} at the intersections of concentric circles and lines of sight. For a fixed i, A_{ji} becomes a function of r only. For a given j, A_{ji} becomes independent of i, because A_{ji}'s are all on the circle of a same radius. We may therefore omit the index from A_{ji} for a simplicity, and equation (9) is given as

$$C_i = \sum_j A_j B_{ji}. \tag{11}$$

By substituting the observed intensity of the scattered continuum into C_i, we have a set of simultaneous linear equations for the unknown volumetric scattering cross section. The coefficients, B_{ji}, form an upper triangular matrix:

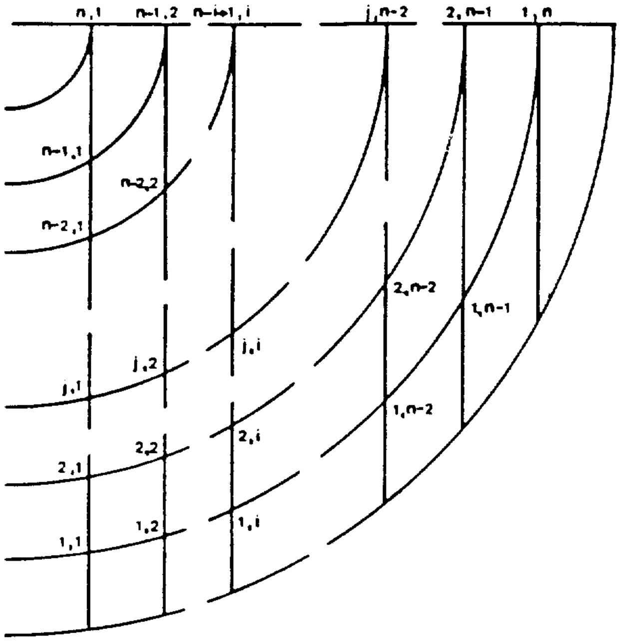


Fig. 2. Schematic illustration for the locations where the matrix elements A_{ji} and B_{ji} are to be evaluated.

$$\begin{array}{rl} A_1 B_{1,1} + A_2 B_{2,1} + \cdots\cdots + A_{n-1} B_{n-1,1} + A_n B_{n,1} & = C_1 \\ A_1 B_{1,2} + \cdots\cdots + A_{n-2} B_{n-1,2} + A_{n-1} B_{n-1,2} & = C_2 \\ \vdots & \\ A_1 B_{1,n-1} + A_2 B_{2,n-1} & = C_{n-1} \\ A_1 B_{1,n} & = C_n . \end{array} \tag{12}$$

The simultaneous linear equations are easily solved by the method of backward-substitution. The last equation involves only one unknown A_1, so we have

$$A_1 = C_n / B_{1,n} . \tag{13}$$

Since we already know A_1, the second equation from the bottom has only one unknown A_2. It follows then that

$$A_2 = (C_{n-1} - A_1 B_{1,n-1}) / B_{2,n-1} . \tag{14}$$

With A_k's for $k = 1$ to $k - 1$ being already known, A_k is given by

$$A_k = (C_{n-k+1} - \sum_{i=1}^{k-1} A_i B_{i,n-k+1}) / B_{k,n-k+1} . \tag{15}$$

In order to evaluate B_{ji} we are required to have $\tau_{ext}(r;\lambda)$ as a function of r, but it is not known *a priori*. Consequently, we cannot calculate matrix elements, B_{ji}, from the beginning. We have to rely on a some sort of iteration for the r-dependence of $\tau_{ext}(r;\lambda)$. We first suppose that there is no internal extinction, and obtain a first order approximation for the radial variation of $N_d(r)\,\sigma_{sca}(r;\lambda)$ by the inversion method. By integrating the first order result of $N_d(r)\,\sigma_{sca}(r;\lambda)$ over r, we derive the r-dependence of the scattering optical depth $\tau_{sca}(r;\lambda)$. What we wanted in evaluating B_{ji}'s is not the scattering optical depth, but the extinction optical depth; conversion should be made from $\tau_{sca}(r;\lambda)$ to $\tau_{ext}(r;\lambda)$. As we assumed that the optical properties of internal dust are homogeneous over the radial distance, the ratio of extinction to scattering optical depth, $\tau_{ext}(r;\lambda)/\tau_{sca}(r;\lambda)$, is independent of the radial distance. We know $\tau_{sca}(r;\lambda)$ from the first order approximation for the radial dependence of the volumetric scattering cross section, and we know $\tau_{ext}(R;\lambda)$ from the observation of the Trapezium stars (Bohlin and Savage 1981; Conti and Alschuler 1971). Therefore, a first order approximation for the r-dependence of $\tau_{ext}(r;\lambda)$ is obtained by simply multiplying $\tau_{sca}(r;\lambda)$ by the ratio $\tau_{ext}(R;\lambda)/\tau_{sca}(R;\lambda)$ known at the boundary $r=R$. With the first order approximation for $\tau_{ext}(r;\lambda)$, we now determine a new solution for the volumetric scattering cross section, $N_d(r)\,\sigma_{sca}(r;\lambda)$, which will give us a new r-dependence for $\tau_{ext}(r;\lambda)$. We continue the iteration until the same radial dependence of $N_d(r)\,\sigma_{sca}(r;\lambda)$ is obtained from two consecutive steps of iteration.

Applying the inversion methodology iteratively to the observed distribution of scattered stellar continuum, we derive the r-dependence of the volumetric scattering cross section at the visible and ultraviolet wavelengths. As far as the optical properties of the Orion dust do not vary with position inside the nebula, the derived volumetric scattering cross sections will provide information on how dust particles are distributed inside this ionized region.

We divide the nebula into 30 sectors, making the interval be 0.′25 in angular distance. The result obtained with 30 sectors is almost identical to the result obtained with 75 sectors. No more than 20 iterations were necessary for having same radial dependence of $N_d(r)\,\sigma_{sca}(r;\lambda)$ from two consecutive steps of iteration.

III. OBSERVATIONAL DATA

The continuum scattered by the Orion dust has been observed by Mathis et al. (1981) and by Patriarchi and Perinotto (1985) at 8 ultraviolet wavelengths ranging from 1300Å to 3000Å. With the help of *IUE*, Mathis et al. (1981) observed the western part of the Orion nebula at 16 different positions located between 30″ to 5′ from the Trapezium. And Patriarchi and Perinotto (1985) observed the nebula at 4 more positions closer than 1′ to the Trapezium. These 20 points are all located in the western part of the Orion nebula, where no large-scale structures are apparent.

The continuum intensity observed from the Orion nebula has two components: one is due to the dust grains, the other is due to the atomic continuum of free-free transition. The atomic component has to be removed from the observed intensity. The correction procedure for the atomic continuum is quite complicated. Mathis et al. (1981) and Patriarchi and Perinotto (1985) estimated the intensity of atomic continuum from the observed intensity of H_β emission. From the given emissivities of the atomic continuum and H_β emission, they calculated the expected intensity of the atomic continuum. The atomic continuum amounts to only 10% of the total continuum intensity for most of the wavelengths we are considering. At λ =3000Å, the atomic continuum makes somewhat larger contribution (~20%) to the total intensity than it does at other wavelengths.

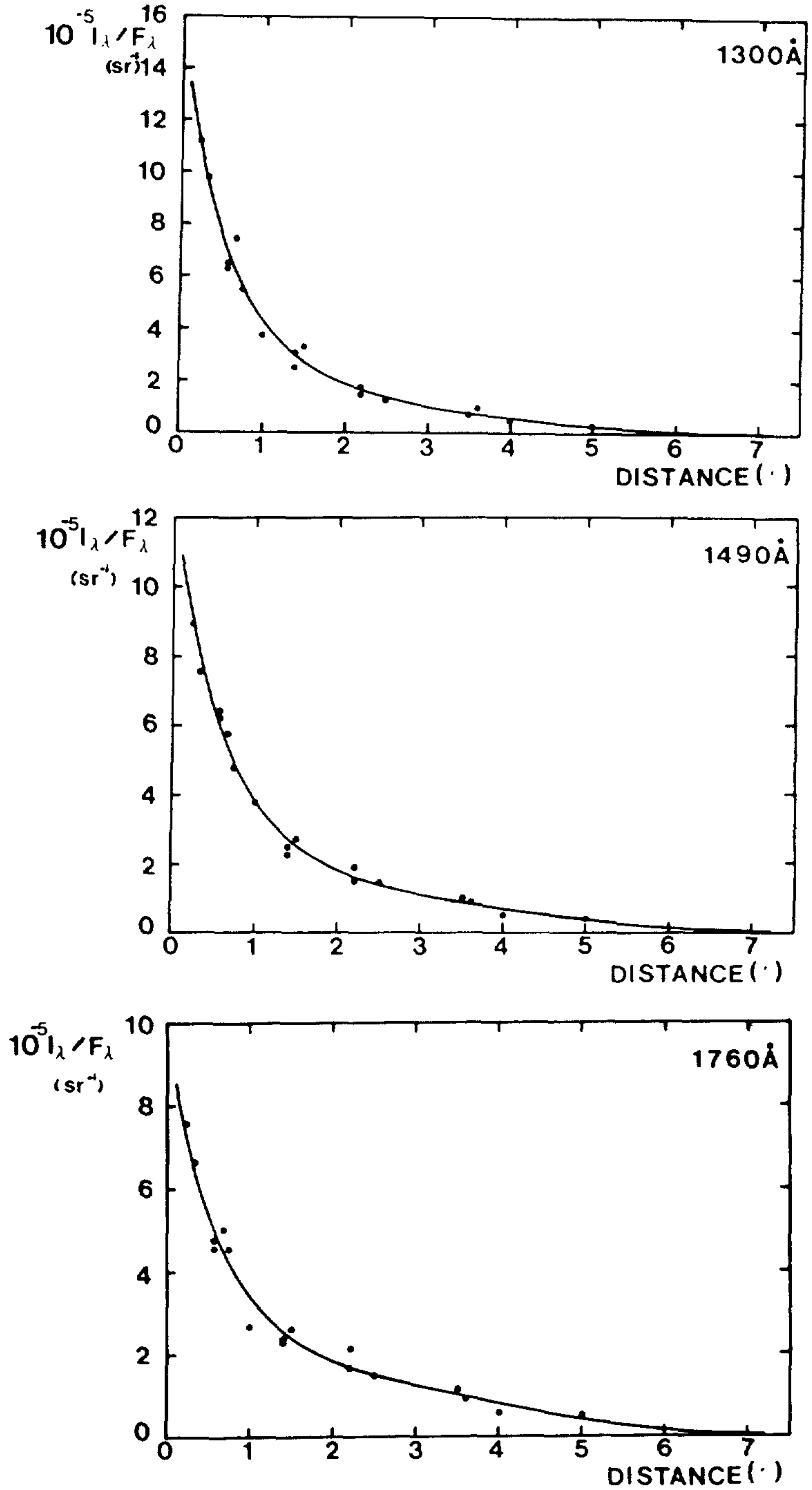
For the dust-scattered continuum at visual wavelength, we adopted the observations by Schiffer and Mathis (1974). The regions observed by them are almost identical to those observed by Mathis et al. (1981). They subtracted the expected intensity of the atomic continuum from the total observed continuum intensity using the same method as the one just explained.

The observed intensities of the dust-scattered continuum at the ultraviolet and visual wavelength are shown by dots in Figure 3. The scattered continuum at a wavelength is given in the units of the observed total stellar flux at the corresponding wavelength. The total flux is obtained by simply summing fluxes of the Trapezium stars. For the ultraviolet fluxes, we adopted the observations by Bohlin and Savage (1981), and for the visual flux we used the observations by Anderson (1970). To execute the inversion methodology, we need a continuous distribution of the brightness rather than the brightness values at discrete points. We thus fit the observed data points to a smooth line for each wavelength. The smoothed curves are shown in Figure 3 by solid lines. As the slope of brightness distribution becomes very steep towards the central star, fitting the data to a smooth curve becomes increasingly difficult towards the center. We thus performed the curve fitting on the log-log plane, and used these smoothed-out distribtuions in the inversion.

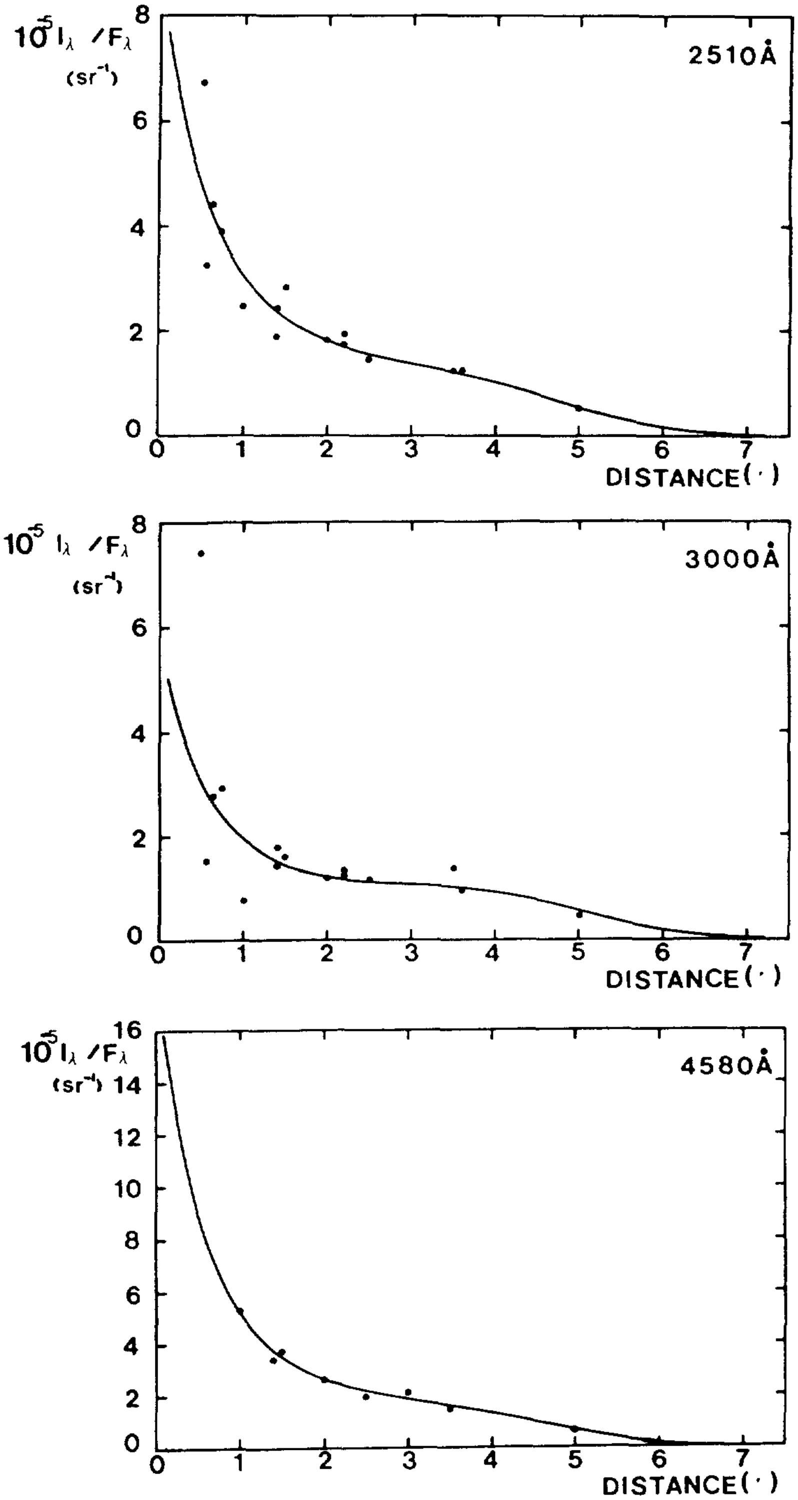
For the luminosity of the central stars, we have to correct the observed stellar flux for the internal and external extinctions. The internal extinction optical depth, $\tau_{ext}(R;\lambda)$, due to the Orion dust is obtained by subtracting the external extinction optical depth $\tau_{ext}(ISM;\lambda)$ from the observed total extinction optical depth $\tau_{ext}(OBS;\lambda)$ at a given wavelength:

$$\begin{aligned}\tau_{ext}(R;\lambda) &= \tau_{ext}(OBS;\lambda) - \tau_{ext}(ISM;\lambda)\\ &= \{A_\lambda(OBS) - A_\lambda(ISM)\}/1.086 \qquad (16)\\ &= 0.92\{(E_\lambda + R_v)_{OBS}\ E(B-V)_{OBS} - (E_\lambda + R_v)_{ISM}\ E(B-V)_{ISM}\}.\end{aligned}$$

The total extinction optical depth, $\tau_{ext}(OBS;\lambda)$, at an arbitrary wavelength is obtained from the normalized extinction law, $E(\lambda-V)/E(B-V)$, for the Trapezium stars (Bohlin and Savage 1981) with R_v =5.2 (Johnson, 1977) and $E(B-V)$=0.38 (Conti and Alschuler 1971). For the wavelength dependence of the external extinction optical depth, $\tau_{ext}(ISM;\lambda)$, we use the general interstellar extinction law given by Savage and Mathis (1979) with R_v =3.1. Lee (1968) performed an extensive photometric study for the 196 stars in the Orion association. Minimum value of the measured extinction of the stars is $E(B-V)$ =0.05, and sixteen stars have this minimum value. Assuming that the minimum color excess has been resulted from the external dust alone, we adopted $E(B-V)$=0.05 for the external color excess. This minimum color excess was used for setting up an absolute scale of the external extinction curve. Figure 4 represents the resulting



$10^{-5} I_\lambda / F_\lambda$ (sr^{-1})
1300Å
DISTANCE (′)
$10^{-5} I_\lambda / F_\lambda$ (sr^{-1})
1490Å
DISTANCE (′)
$10^{-5} I_\lambda / F_\lambda$ (sr^{-1})
1760Å
DISTANCE (′)


$10^{-5} I_\lambda / F_\lambda$
(sr^{-1})
2510Å
DISTANCE(′)
$10^{-5} I_\lambda / F_\lambda$
(sr^{-1})
3000Å
DISTANCE(′)
$10^{-5} I_\lambda / F_\lambda$
(sr^{-1})
4580Å
DISTANCE(′)

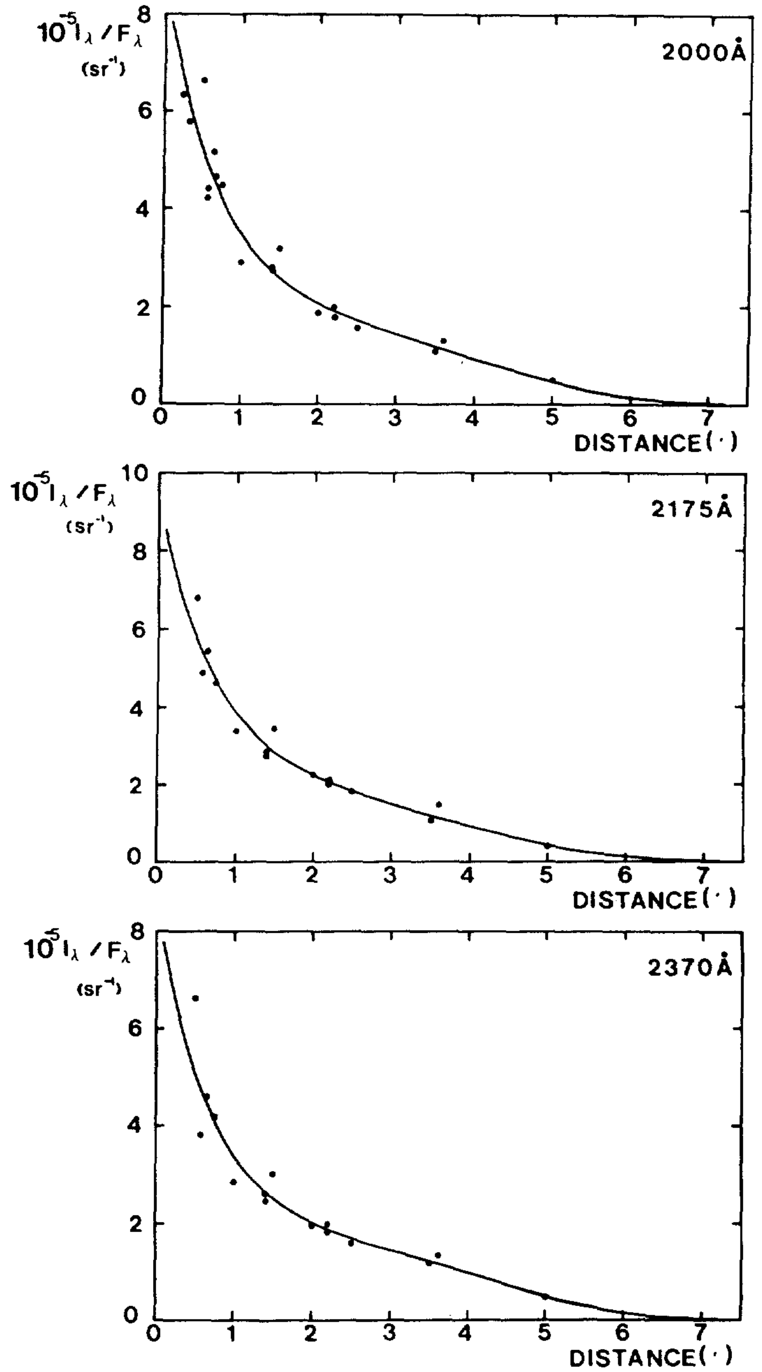


Fig. 3. The radial distributions of the scattered continuum at 8 ultraviolet wavelengths from 1300 to 3000Å and one visual wavelength 4580Å. The ordinate represents the intensity of the dust scattered continuum in units of the observed stellar flux at the corresponding wavelength and abscissa represents the projected radial distance in unit of arc minutes. One arc minute corresponds to 0.131 pc at the distance of 500 pc from the Earth. Dots are the observed values and the solid lines are our best-fitted curves.

Table 1. The critical values of the asymmetry factor

λ (Å)	1300	1490	1760	2000	2175	2370	2510	3000	4580
g_c	0.56	0.55	0.53	0.52	0.51	0.49	0.45	0.35	0.49

variations of the internal, external, and observed extinction optical depth over the wavelength.

IV. RESULTS AND DISCUSSIONS

a) Distribution of the Volumetric Scattering Cross Section

From the observed brightness distributions of the dust-scattered continuum at nine different wavelengths, we have determined how the volumetric scattering cross section, $N_d(r)\sigma_{sca}(r;\lambda)$, varies with the distance from the center of the Orion nebula. For a given wavelength, the resulting $N_d(r)\sigma_{sca}(r;\lambda)$ depends, as illustrated in Figure 5, upon the asymmetry factor for the scattering function. If the chosen scattering asymmetry factor is greater than a certain value, the volumetric scattering cross section becomes negative in the cental region of the nebula. Table 1 lists these critical values of the scattering asymmetry factor for the wavelengths considered in this study.

Although the details of the resulting $N_d(r)\sigma_{sca}(r;\lambda)$ depends on g, the following characteristics

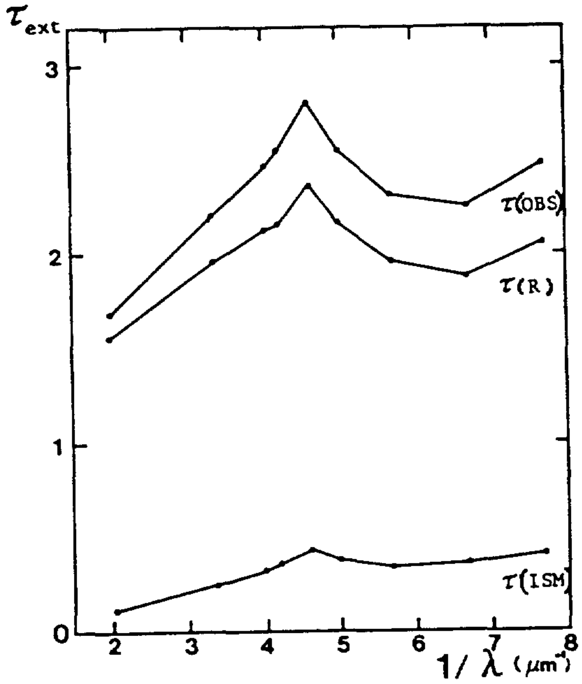
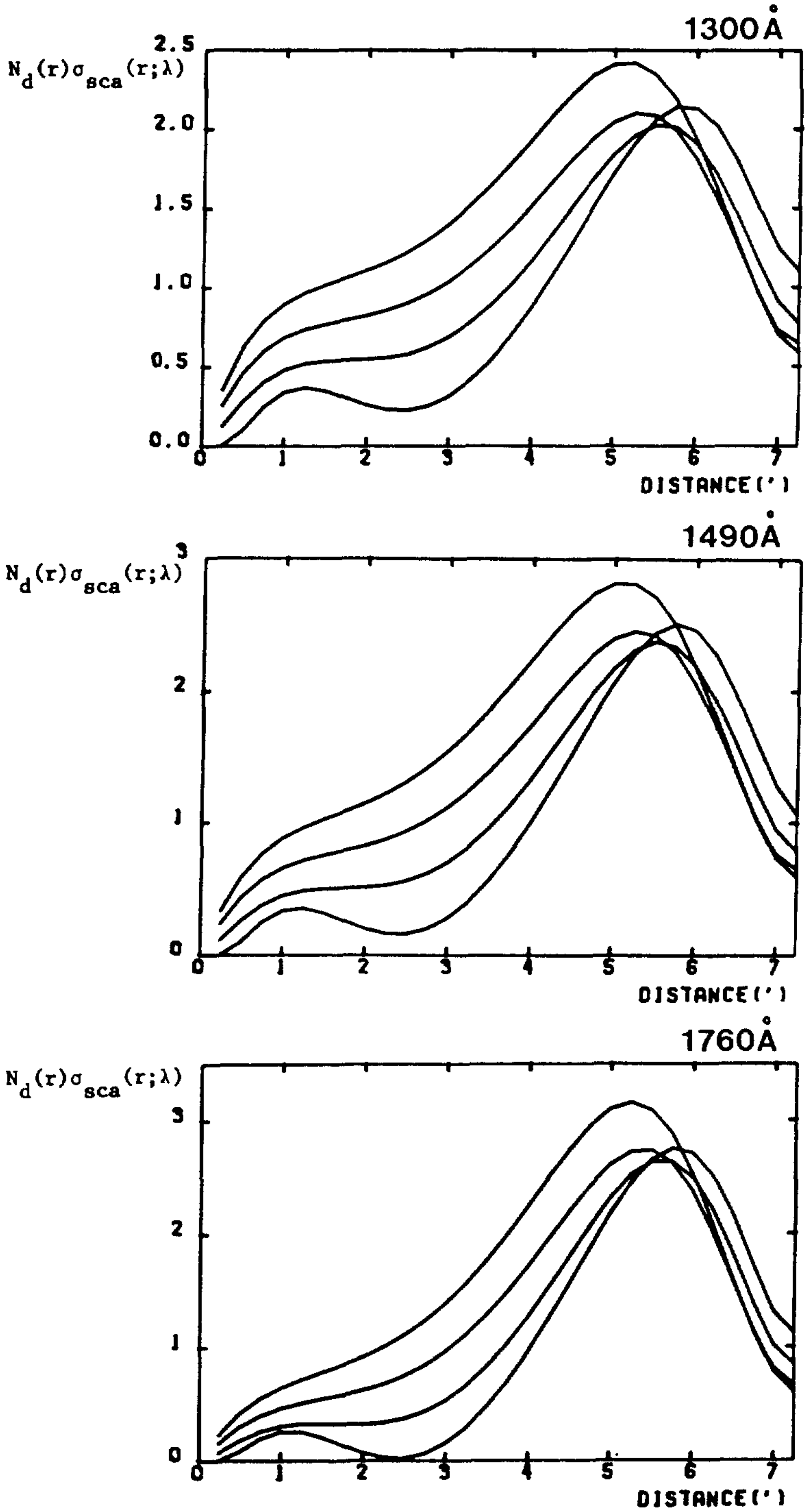
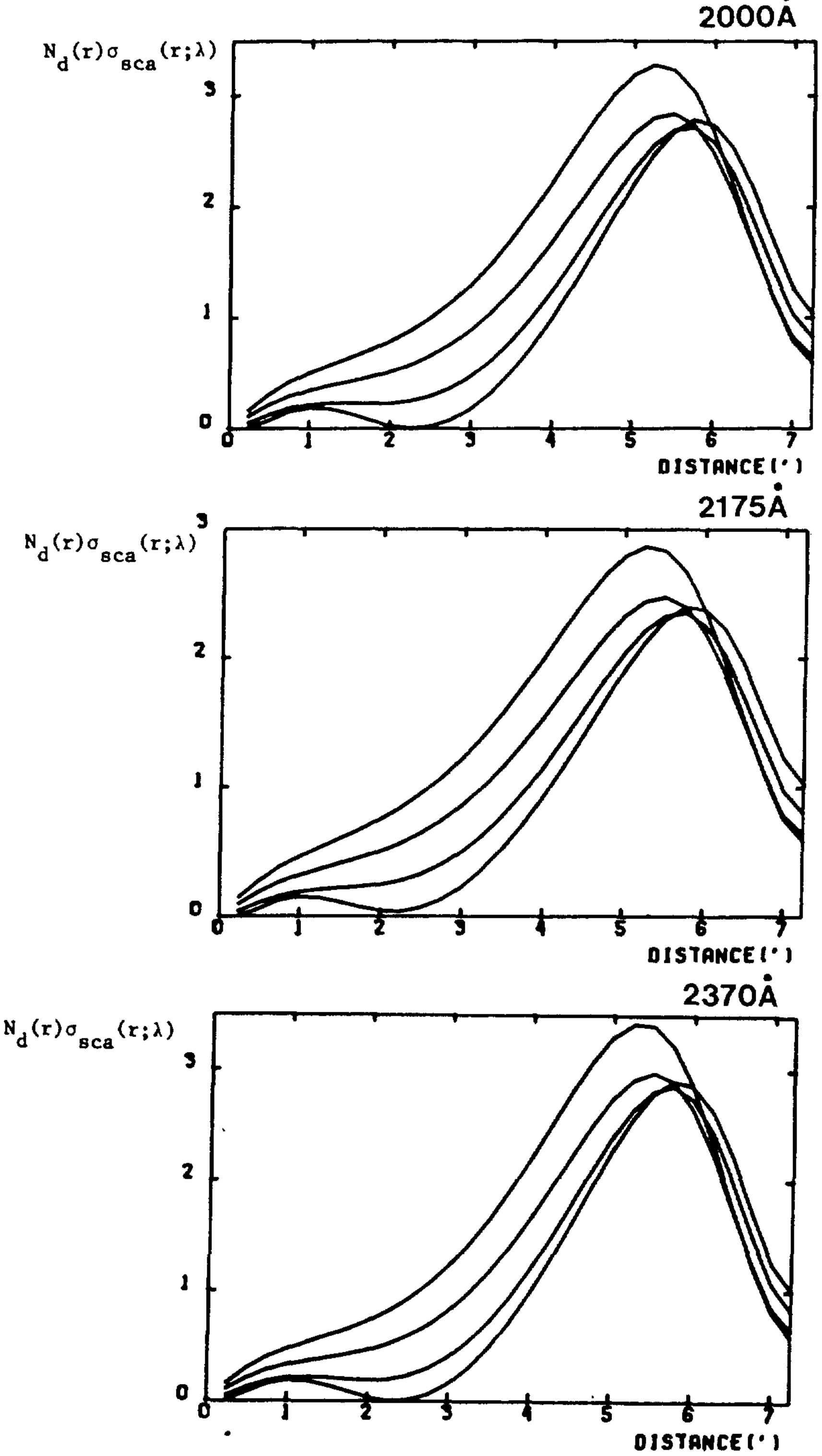


Fig. 4. The wavelength dependence of the total, $\tau_{ext}(OBS;\lambda)$, the internal, $\tau_{ext}(R;\lambda)$, and the external, $\tau_{ext}(ISM;\lambda)$, optical depths of the Orion nebula.


1300Å
$N_d(r)\sigma_{sca}(r;\lambda)$
2.5
2.0
1.5
1.0
0.5
0.0
0 1 2 3 4 5 6 7
DISTANCE(')
1490Å
$N_d(r)\sigma_{sca}(r;\lambda)$
3
2
1
0
0 1 2 3 4 5 6 7
DISTANCE(')
1760Å
$N_d(r)\sigma_{sca}(r;\lambda)$
3
2
1
0
0 1 2 3 4 5 6 7
DISTANCE(')


2000Å
$N_d(r)\sigma_{sca}(r;\lambda)$
0
1
2
3
0 1 2 3 4 5 6 7
DISTANCE(')
2175Å
$N_d(r)\sigma_{sca}(r;\lambda)$
0
1
2
3
0 1 2 3 4 5 6 7
DISTANCE(')
2370Å
$N_d(r)\sigma_{sca}(r;\lambda)$
0
1
2
3
0 1 2 3 4 5 6 7
DISTANCE(')

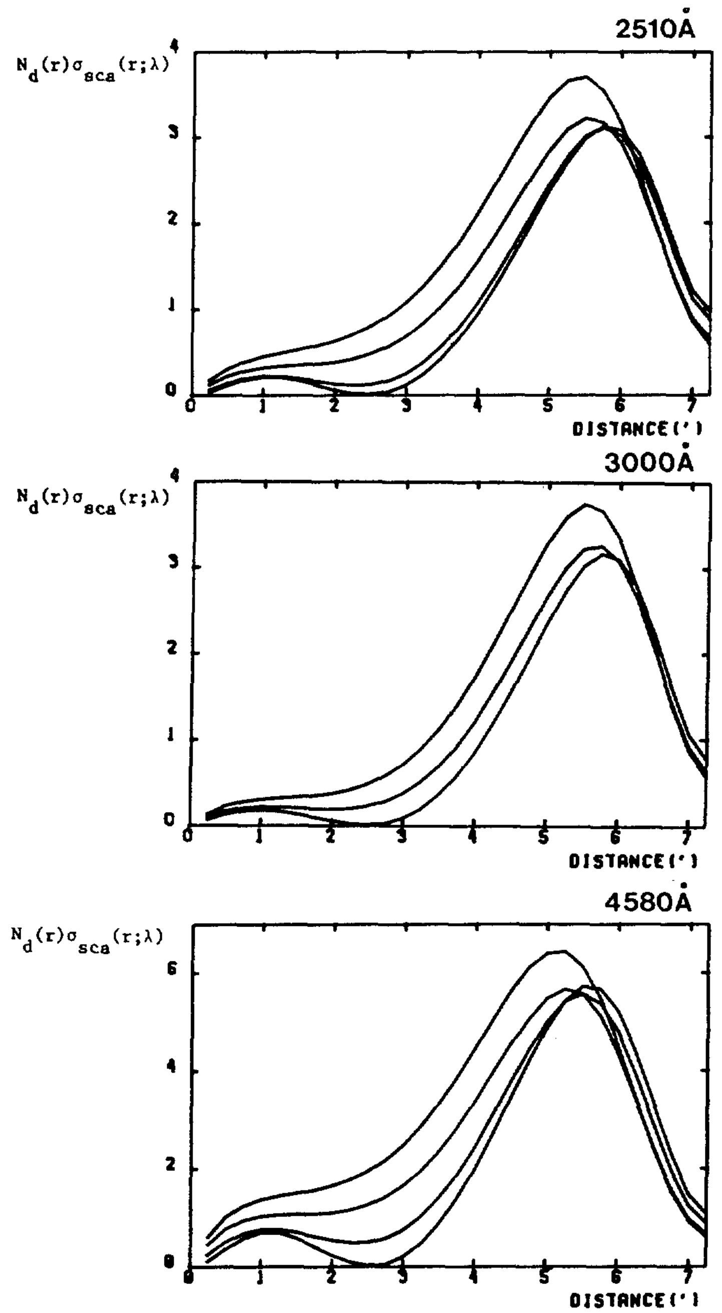


Fig. 5. The radial distribution of the volumetric scattering cross section, $N_d(r)\,\sigma_{sca}(r;\lambda)$, is shown for the 8 ultraviolet wavelengths and one visual wavelength. The ordinate is given in units of 10^{-19} cm^{-1}, and the abscissa in minutes of arc. The top curve in each frame is for the case of isotropic scattering, i.e., $g=0.0$, and we let g increase in steps of 0.2. Notice that the depletion of dust at the central part becomes severe as g increases. At each wavelength there is an upper limit for g, beyond which no meaningful result can be obtained for the distribution of $N_d(r)\,\sigma_{sca}(r;\lambda)$. The values of the limiting g's are given in Table 2. As $g_c=0.35$ for the case of 3000Å, we draw only three lines for $g=0.0, 0.2$, and $g=0.35$.

are certain: Very near the center $N_d(r)\,\sigma_{sca}(r;\lambda)$ is almost zero, and it increases slowly with r. As r approaches $3'\sim4'$, it increases abruptly, making a peak at r around $5'\sim6'$. After the peak, it decreases to the general interstellar value. Almost all the dust grains are concentrated at the outer region of the Orion nebula, i.e., the distribution of dust inside the Orion nebula has a shell-like structure with its inner radius being about 0.4~0.5 pc. Regardless of the wavelength considered, it is certain that the Orion nebula has a dust 'hole' in the central region.

As far as the characteristics in the resulting distributions of the volumetric scattering cross section are concerned, our results are in agreement with the results by Schiffer and Mathis (1974), especially with their models SM5 and SM6. Our results are also consistent with the results by Kim and Hong (1982) in the sense that the outward decrease of gas phase metal abundance is probably due to the outward increase of dust grains in the Orion nebula. It is very remarkable that our results are consistent with the studies by Kim and Hong (1982) because not only the method of analysis but also the wavelength range is totally different from each other.

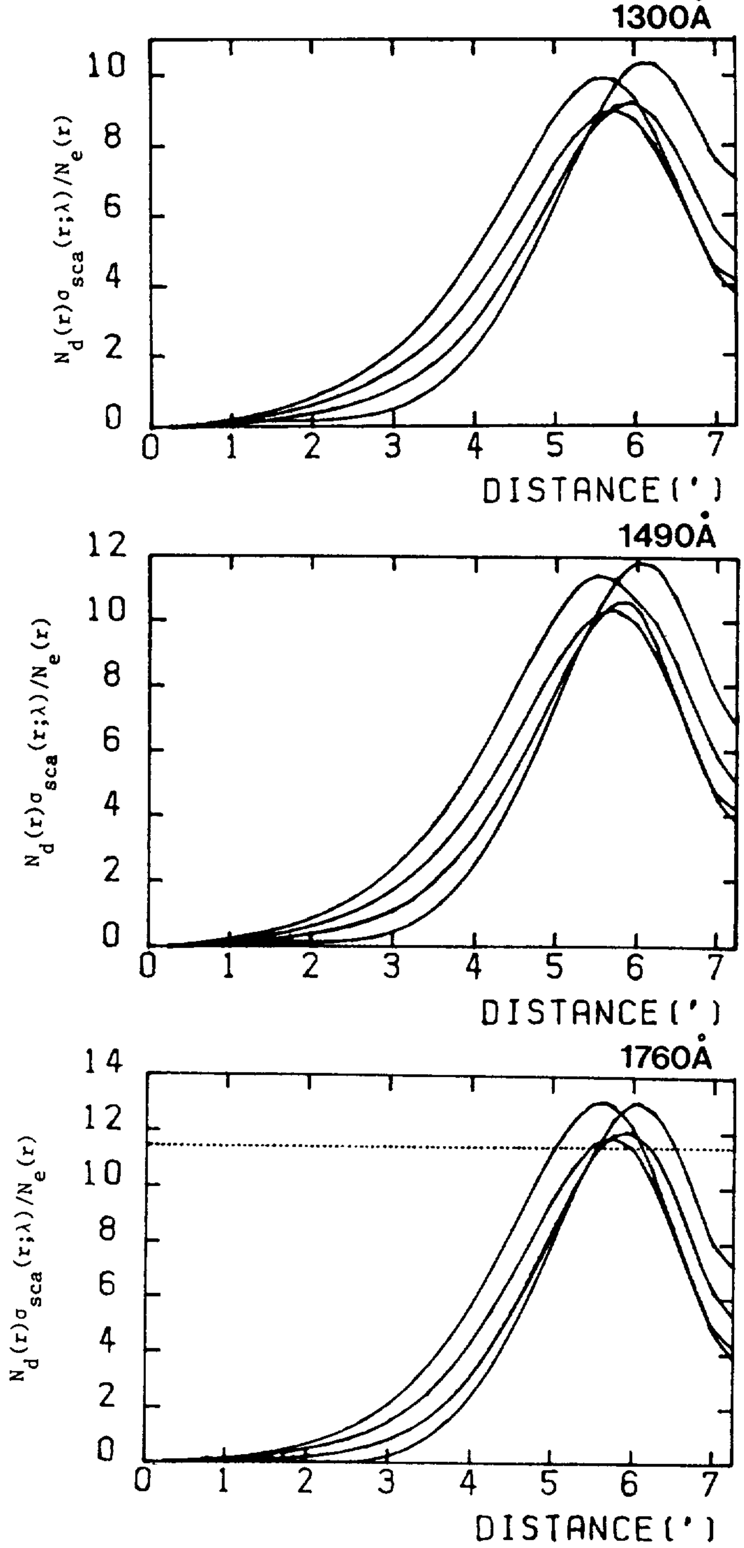
Since the details of the resulting distribution of the volumetric scattering cross section depends rather sensitively on the scattering asymmetry factor g, it is difficult to pin-point a unique solution for the distribution of dust number density. And there is a small bump at $r\sim1'$. The bump becomes conspicuous, as the chosen asymmetry factor becomes large. This is probably due to the observational contamination of the instrumentally scattered light near the Trapezium.

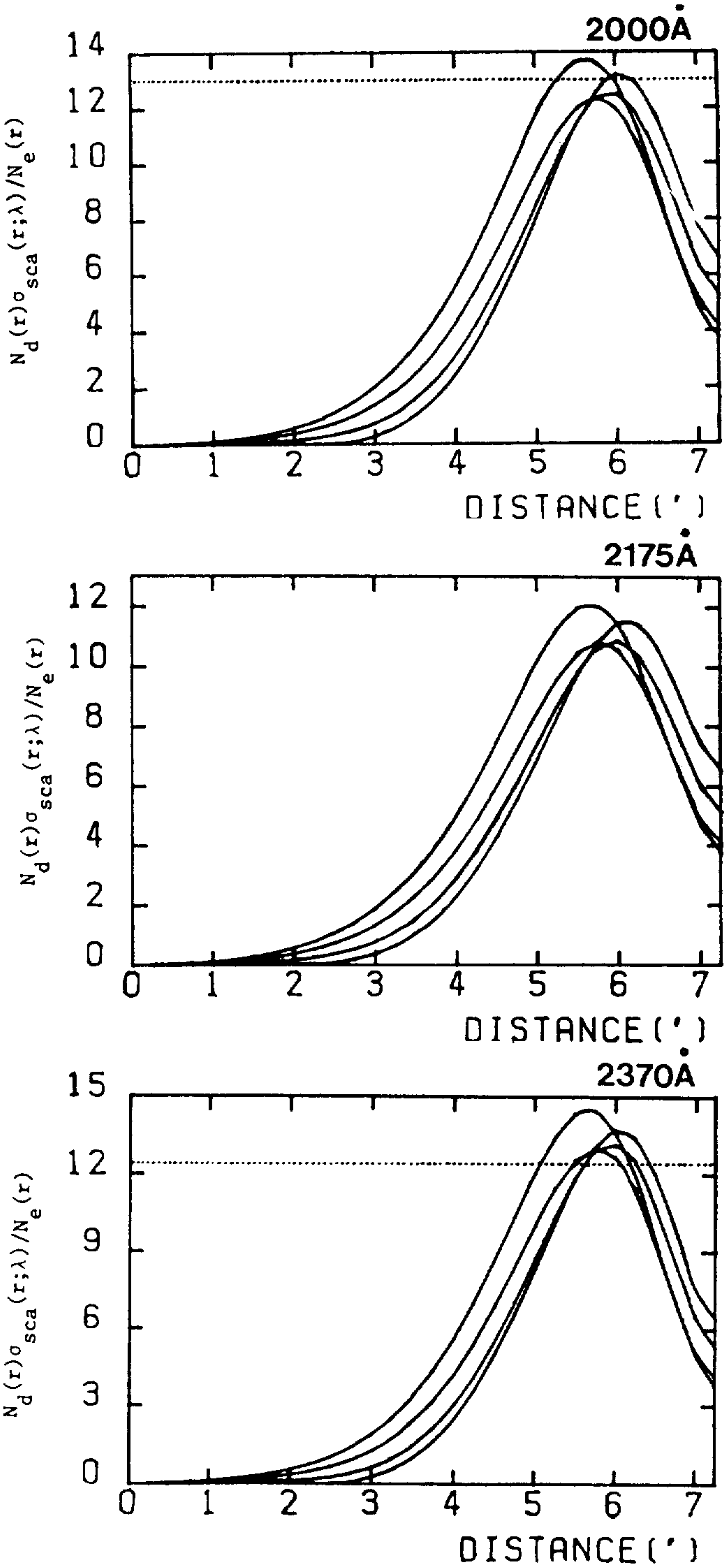
b) Distribution of the Dust-to-Gas Ratio

The real indicator for the dust amount in an H II region is the dust-to-gas ratio rather than the volumetric scattering cross section. To get the dust-to-gas ratio, we divided our result of $N_d(r)\,\sigma_{sca}(r;\lambda)$ by the electron density, $N_e(r)$, given by Kim and Hong (1982). The resulting radial variations of the dust-to-gas ratio are shown in Figure 6 for the eight ultraviolet wavelengths and one optical wavelength. From these figures, it is clear that the dust is severely depleted in the central region of the Orion nebula, and that the dust is concentrated in a shell between $5'$ and $6'$ from the central star. It should be pointed out that, if the Orion dust scatters light isotropically, our value of dust-to-gas ratio at $1'$ from the trapezium is in agreement with the estimate by O'Dell and Hubbard (1965). If the scattering characteristics of the Orion dust is of forward throwing nature, the central region of the Orion nebula is depleted in dust content more severely than their estimate.

c) Distribution of the Number Density of Dust Grains

Inversion of the observed brightness distribution of the dust scattered continuum gives us $N_d(r)\,\sigma_{sca}(r;\lambda)$, not $N_d(r)$ and $\sigma_{sca}(r;\lambda)$ separately. In order to have an idea on the distribution of the number density of dust grains in the Orion nebula, we suppose that the mean scattering cross section of the Orion dust is constant over r, i.e., $\sigma_{sca}(r;\lambda)=\sigma_{sca}(\lambda)$. Then, the radial distance dependence of the volumetric scattering cross section should have the same characteristics for different wavelengths. However, as seen in part **a**) of this section, the radial distance dependence of the volumetric scattering cross section has different characteristics depending on the scattering asymmetry factor. Therefore, by changing the scattering cross section and the asymmetry


1300Å
$N_d(r)\sigma_{sca}(r;\lambda)/N_e(r)$
0 2 4 6 8 10
0 1 2 3 4 5 6 7
DISTANCE(')
1490Å
$N_d(r)\sigma_{sca}(r;\lambda)/N_e(r)$
0 2 4 6 8 10 12
0 1 2 3 4 5 6 7
DISTANCE(')
1760Å
$N_d(r)\sigma_{sca}(r;\lambda)/N_e(r)$
0 2 4 6 8 10 12 14
0 1 2 3 4 5 6 7
DISTANCE(')


2000Å
14
12
10
8
6
4
2
0
0 1 2 3 4 5 6 7
DISTANCE(′)
$N_d(r)\sigma_{sca}(r;\lambda)/N_e(r)$
2175Å
12
10
8
6
4
2
0
0 1 2 3 4 5 6 7
DISTANCE(′)
$N_d(r)\sigma_{sca}(r;\lambda)/N_e(r)$
2370Å
15
12
9
6
3
0
0 1 2 3 4 5 6 7
DISTANCE(′)
$N_d(r)\sigma_{sca}(r;\lambda)/N_e(r)$

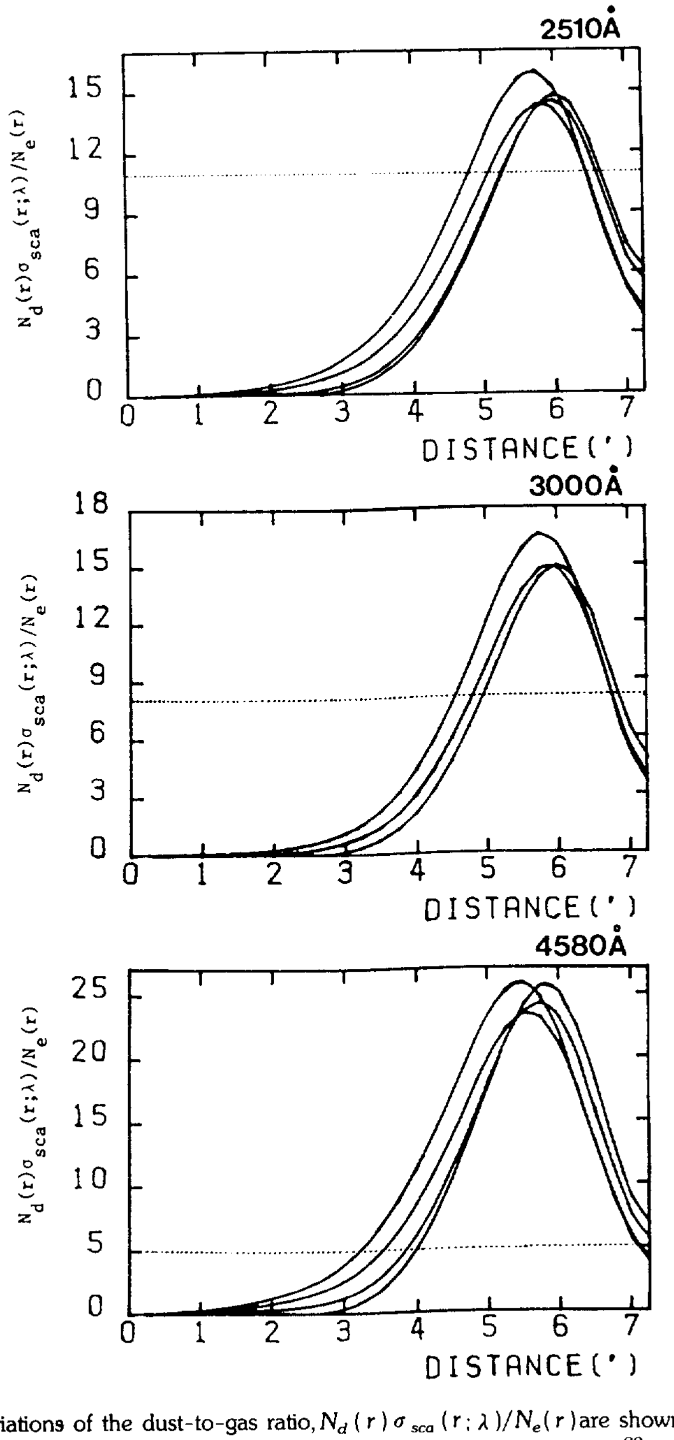


Fig. 6. The radial variations of the dust-to-gas ratio, $N_d(r)\sigma_{sca}(r;\lambda)/N_e(r)$ are shown for the 8 ultraviolet wavelengths and one visual wavelength. The ordinate is given in units of 10^{-22} cm. As g increases from 0.0 in steps of 0.2, the central depletion becomes increasingly severe. The ratio of extinction to hydrogen column density, $A(\lambda)/N_H$, in the general interstellar medium is 1.4 x 10^{-21}, 1.2 x 10^{-21}, and 1.5 x 10^{-21} mag · cm^2 for λ = 1300, 1490, and 2175Å, respectively. Corresponding values at other wavelengths are marked by horizontal lines of dots.

factor with wavelength in some way, we may obtain the same r-dependence of $N_d(r)$ from the observations of different wavelengths.

The λ-dependence of the scattering cross section can be obtained from that of the total scattering optical depth over the whole nebula. Since we neglect the r-dependence of the scattering cross section, the wavelength dependence of the total scattering optical depth is a simple manifestation of the wavelength dependence of the mean scattering cross section $\sigma_{sca}(\lambda)$ of the Orion dust:

$$\tau_{sca}(R;\lambda) = \int_0^R N_d(r)\sigma_{sca}(r;\lambda)\,dr \cong \sigma_{sca}(\lambda)\int_0^R N_d(r)\,dr\,, \tag{17}$$

or

$$\tau_{sca}(R;\lambda)/\sigma_{sca}(\lambda) = \int_0^R N_d(r)\,dr\ . \tag{18}$$

In other words, the λ-dependence of the mean scattering cross section of the Orion dust can be inferred from the λ-dependence of the total scattering optical depth. The latter is already known from our analysis. Table 2 lists the total scattering optical depths that were calculated from the distribution of the volumetric scattering cross section at each wavelength and with varying asymmetry factor. To assign a scale to the number density of dust grains, we assumed that the mean scattering cross section of the Orion dust at $\lambda = 2000$Å is equal to the geometrical cross section of a shpere of radius $0.1\,\mu m$. To obtain the dust column density, $\int_0^R N_d(r)\,dr$, we divide the total optical depth $\tau_{sca}(R;\lambda=2000\text{Å})$ by the assumed scattering cross section. We then divide $\tau_{sca}(R;\lambda)$ at various wavelengths by the resulting column density, which gives the wavelength dependence of the scattering cross section $\tau_{sca}(\lambda)$. In this way, we have obtained the distribution of the number density of dust grains as derived from the observations at each wavelength. Figure 7 illustrates the deduced distributions of the dust number density for a fixed asymmetry factor g.

Table 2. The total scattering optical depth, $\tau_{sca}(R;\lambda)$, for each observed wavelength and the assumed scattering asymmetry factor.

λ (Å)	$\tau_{sca}(R;\lambda)$						
	g=0.0	g=0.1	g=0.2	g=0.3	g=0.4	g=0.5	$g=g_c$
1300	0.42	0.38	0.35	0.32	0.30	0.28	0.27
1490	0.47	0.42	0.39	0.36	0.33	0.31	0.30
1760	0.47	0.42	0.38	0.35	0.33	0.31	0.30
2000	0.46	0.41	0.38	0.35	0.32	0.30	0.30
2175	0.42	0.37	0.34	0.31	0.29	0.27	0.27
2370	0.47	0.42	0.38	0.35	0.32	*	0.30
2510	0.47	0.42	0.39	0.36	0.33	*	0.32
3000	0.43	0.38	0.35	0.32	*	*	0.31
4580	0.74	0.67	0.61	0.56	0.52	*	0.49

Note: Asterisks mean the cases where the assumed value of asymmetry factor is larger than the critical value for the given wavelength.

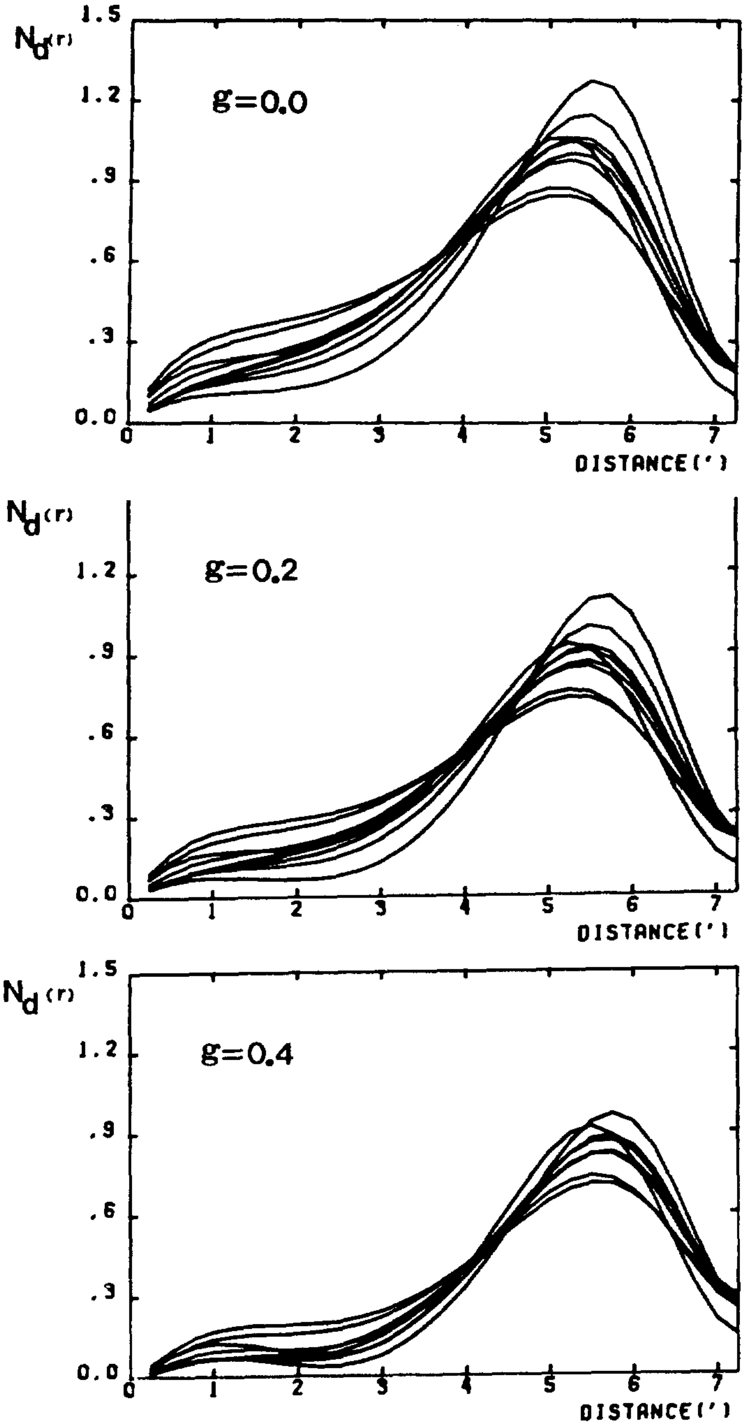


Fig. 7. The distribution of the number density of dust grains in the Orion nebula for a fixed g and varying wavelength. The longer the wavelength, the more severe the depletion is in the central part of the nebula. The unit of ordinate becomes 10^{-9} cm^{-3}, if we take the geometrical cross section of a $0.1\,\mu$m radius sphere for the mean scattering cross section at 2000Å.

The shape of deduced dust distributions depends on wavelength. This is due to the fact that the wavelength dependence of the scattering asymmetry factor has not yet been taken into account.

In order to take into account the λ -dependence of the scattering asymmetry factor, we normalize all the distributions of the volumetric scattering cross section to their peak values. As a result, we know that there is a boundary, as shown in Figure 8 by two curves, beyond which the distribution of dust number density cannot be defined. The reason for selecting this boundary is that the distribution of dust number density must be of an identical shape for all the wavelengths. However, the range represents the uncertainties due to the two assumptions we took in this analysis: the homogeneity condition for the dust scattering properties, and the single scattering approximation for the radiative transfer process. Of course, some portion of the uncertainties is related to the simplified geometry we took for the Orion nebula. Nevertheless, it may be safe to conclude that the range bounded by the two curves describes the radial dependence of dust distribution reasonably well.

The variation of the scattering asymmetry factor with wavelength is illustrated in Figure 9, where our results (filled circles) are compared with the previous results (open circles) by Schiffer and Mathis (1974) and Mathis et al. (1981). If the distribution of dust number density in the Orion nebula follows the upper curve in Figure 8, the variation of asymmetry factor with wavelength follows the lower curve of filled-circles in Figure 9. The upper filled-circles in the figure corresponds to the lower curve in Figure 8. Except at λ =3000Å, our results are quite similar to the model distributions of SM5 and SM6 by Schiffer and Mathis (1974), and the one by Mathis et al.

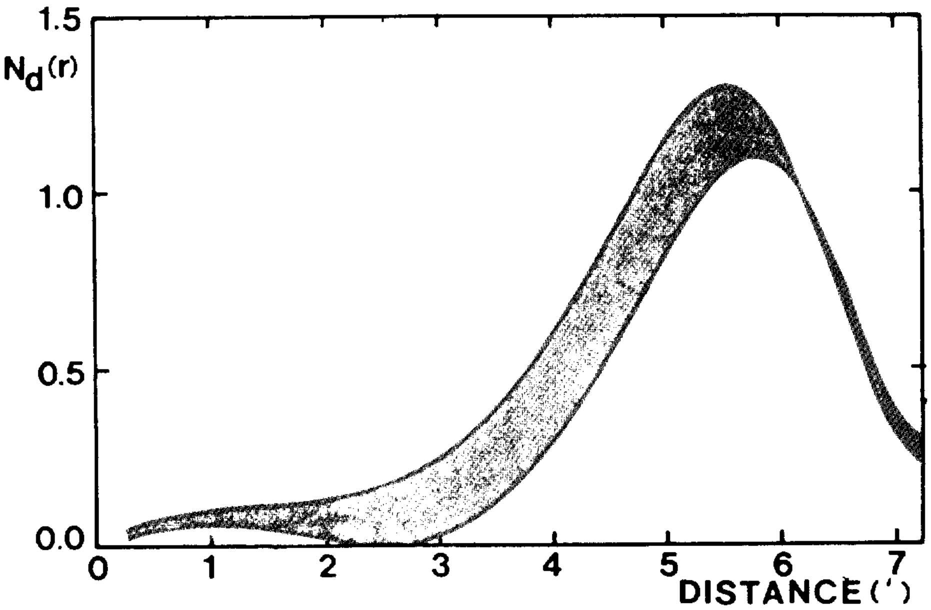


Fig. 8. The distribution of the number density of dust grains in the Orion nebula. The ordinate is given in units of 10^{-9} cm^{-3}. The actual distribution of the dust number density inside the Orion nebula is expected to lie somewhere in the hatched area.

(1981). Mathis et al. (1981) considered that their value of the scattering asymmetry factor at 3000Å is uncertain. In fact, as seen in the Figure 3, the distribution of the scattered continuum at 3000Å has large scatters. Probably the scatters are the reason for the uncertainties in thier g-values. Our results show that the asymmetry factor is between 0.2 and 0.6 for the ultraviolet wavelengths, and for the visual wavelength it is between 0.3 and 0.5.

We may have some idea on the wavelength dependence of the albedo ϖ for the Orion dust. If the dust properties are more-or-less uniform in the nebula, we have

$$\varpi = \sigma_{sca}(\lambda) / \sigma_{ext}(\lambda) = \tau_{sca}(R;\ \lambda) / \tau_{ext}(R;\ \lambda)\ . \tag{19}$$

The variation of the albedo is shown in Figure 10 as a function of wavelength. Also, in this figure, we compare our results (circles) with the ones (triangles) by Schiffer and Mathis (1974), and by Mathis et al. (1981). The upper curve of our results is obtained from the maximum distribution of the number density of dust grains in Figure 8, and the lower curve of our results is from the minimum distribution. Except at 3000Å, the albedo is nearly constant over wavelength, and stays in the range 0.1 to 0.2. Our values of ϖ are less than the results by Schiffer and Mathis (1974) and Mathis et al. (1981) by a factor of three.

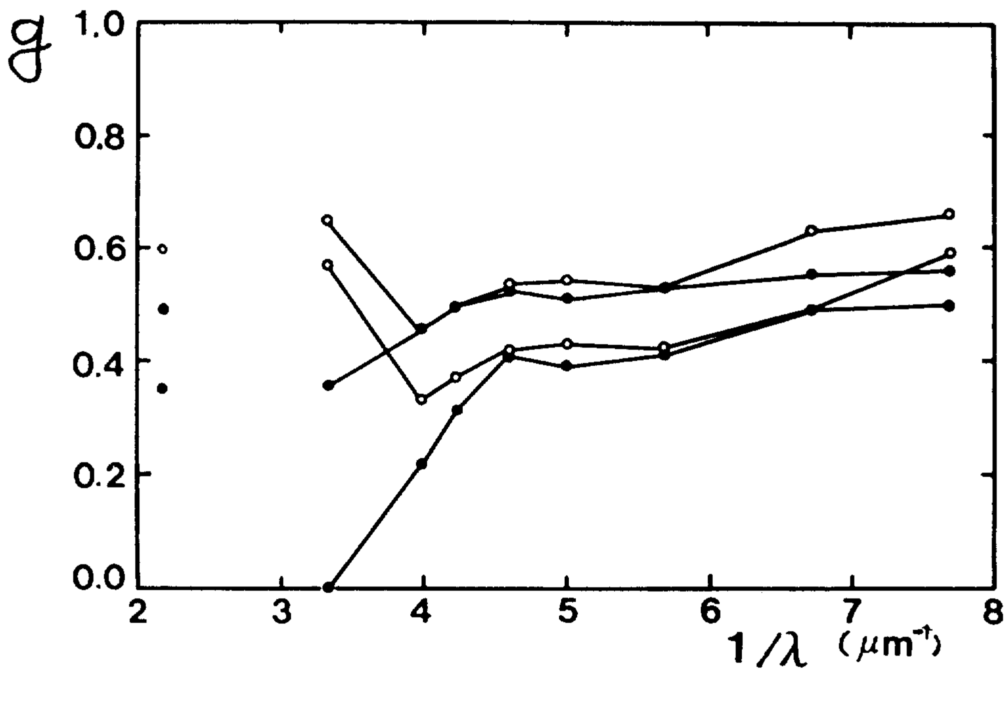


Fig. 9. The variation of the scattering asymmetry factor g with wavelength. The filled circles are our results. The open circles are the results given by Mathis et al. (1981) at ultraviolet wavelengths and by Schiffer and Mathis (1974) at visual wavelength.

V. CONCLUSIONS

With the inversion methodology, we have deduced the distribution of the volumetric scattering cross section inside the Orion nebula directly from the observed brightness distribution of dust-scattered stellar continuum. It is found that the dust grains are concentrated in the region 5′∼6′ away from the Trapezium, forming a shell-like structure. Our result for the dust-to-gas ratio near the nebula center is in agreement with the one by O'Dell and Hubbard (1965). It is also found that the dust grains are heavily depleted near the Trapezium. The wavelength dependences of the asymmetry factor and the albedo of the Orion dust are also derived. The scattering characteristics of the Orion dust grains are of a moderately forward throwing nature at the ultraviolet and visual wavelengths, and they absorb light rather strongly in the optical range of wavelength.

The inversion methodology is based on the single scattering approximation for the radiative transfer in the scattering medium. As far as the scattering optical depth is concerned (Table 2), use of the single scattering approximation is well justified. The total extinction optical depth over the radius of the nebula is, however, somewhat larger than unity; hence, we should have considered

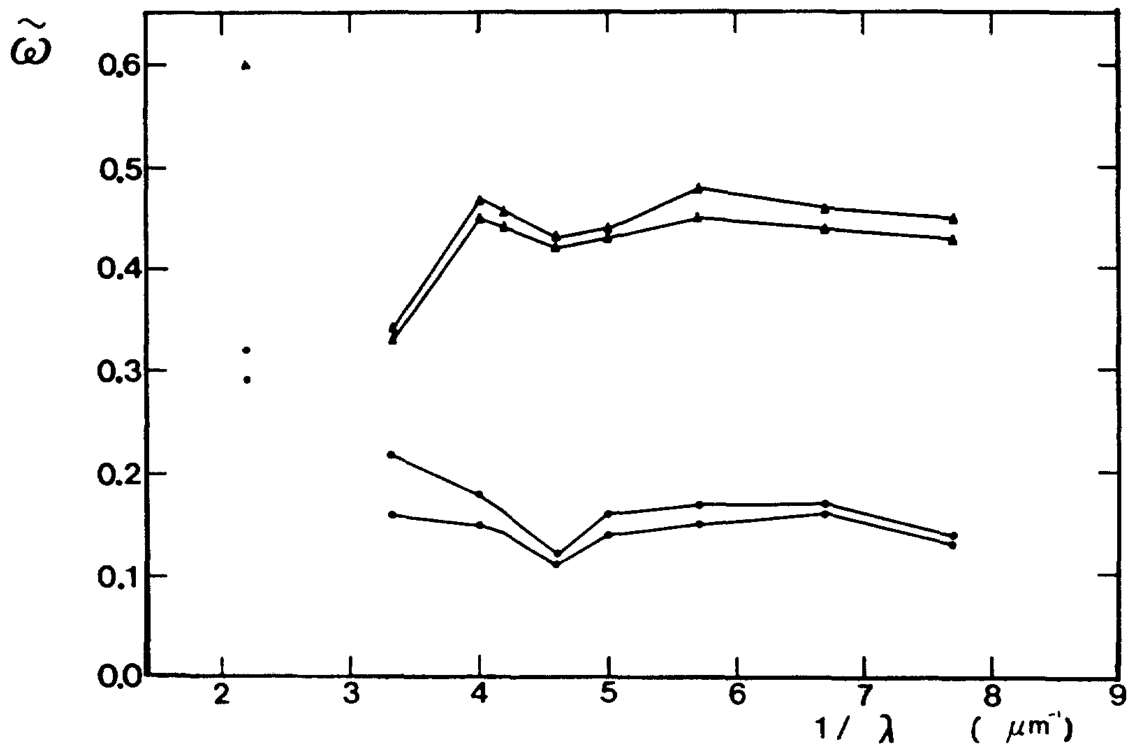


Fig. 10. The variation of the albedo $\tilde{\omega}$ with wavelength. Triangles are from Mathis et al. (1981) at ultraviolet wavelengths and from Schiffer and Mathis (1974) at visual wavelength. Their upper curve is for model SM6 and the lower one for SM5. Filled circles are our result.

the internal extinction a little more seriously than what we have done here. Refinements on the correction of internal extinction can be made in future, as we now know an overall distribution of the dust inside the Orion nebula. We have not studied anything about the physical reasons why the Orion nebula has a shell-like structure in her dust distribution. This is an interesting problem to be pursued in the next step.

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