Physics 261
Homework #2

1. Faissler, problem 3-2.

2. Faissler, problem 3-4.

3. The textbook unfortunately puts two different currents simultaneously through $R_2$ in problem 3-5.\footnote{The example given in the chapter (fig. 3-11) is set up the same way. This is confusing but not strictly wrong: Faissler wants the total current through $R_2$ to be thought of as $I_2 - I_3$. However, no clear motivation is supplied for this peculiar choice. Faissler is probably thinking of Mesh Current Analysis (Chapter 5), in which the current in a circuit element may be broken out into two components associated with adjoining loops. But that concept belongs in a later chapter.} Please rectify Faissler's blunder by crossing out $I_3$, and let $I_2$ represent the total current through $R_2$.

   (a) Now write the KVL equations for loops 1 and 2 accordingly.

   (b) Also write the KVL equation for "loop 3", which Faissler has not drawn; it runs around the outside of the entire circuit. Is this really a new equation, or could you have obtained it by combining the equations from loops 1 and 2?

   (c) Write KCL for nodes 1 and 2. Are these two independent equations, or are they equivalent?

   (d) Assuming that the voltages and resistances are known, how many unique equations and how many unknowns are there in this problem?

4. A light bulb of resistance $R$ is placed in series with a 3 V battery and a 1 $\Omega$ resistor.

   (a) Show that the power dissipated in the bulb is

   \[ P = \frac{V^2 R}{(R + 1\Omega)^2} \]

   (b) Choose the resistance, $R$, of the bulb to maximize $P$. 
1. FAISSLER # 3-2

\[ i_1 = 4.0 \, A \]
\[ i_2 = i_3 = 2.0 \, A \]
\[ i_6 = 3.0 \, A \]
\[ i_4 = ? \]

KCL: \[ i_1 - i_2 - i_3 - i_4 - i_5 = 0 \]
\[ \Rightarrow i_4 = i_1 - i_2 - i_3 - i_5 = 4 - 2 - 2 - 3 = -3.0 \, A \]

2. FAISSLER # 3-4

Moving clockwise around the loop,

\[ \text{KVL: } -V_1 + iR_1 + V_2 + iR_2 - V_3 + iR_3 = 0 \]

Always pay attention to the direction of voltage sources.

Note that the labeled direction of current flow in the loop implies a voltage rise as we move clockwise through the resistors.
(a) KVL 1: \( V_1 - i_1 R_1 - i_2 R_2 + V_2 = 0 \)

2: \(-V_2 + i_2 R_2 + i_4 R_3 = 0\)

(b) KVL 3: \( V_1 - i_1 R_1 + i_4 R_3 = 0 \)

This is the sum of 1 and 2, so it adds no new information!

(c) KCL 0: \( i_1 - i_2 + i_4 = 0 \)

KCL 2: \(-i_1 + i_2 - i_4 = 0 \) -> Same equation.

(d) 3 UNIQUE EQUATIONS: KVL 1, KVL 2, KCL 1
3 UNKNOWNS: \( i_1, i_2, i_4 \)
(a) \( P = I^2R \)
\[ I = \frac{V}{R+1Ω} \]
\[ \therefore P = \frac{V^2R}{(R+1Ω)^2} \]

(b) \( P \) is maximized when \( \frac{dP}{dR} = 0 \).
\[ 0 = \frac{V^2}{(R+1Ω)^2} - \frac{2V^2R}{(R+1Ω)^3} \]
\[ \Rightarrow (R+1Ω) - 2R = 0 \]
\[ \Rightarrow R = 1Ω \]