Physics 261
Homework # 7
Remember to show your work.

1. Faissler, problem 28-1.

2. Faissler, problem 28-2. The inverting amp is covered in the text even though we didn’t discuss it in lecture.


5. You need to produce a very stable instrumentation amplifier that will operate on battery power in the field, recharged daily by solar cells, for several years. Your research student has proposed two options: a common-emitter amp using a BJT, or an inverting op-amp design such as that in figure 28-5. Which of these two designs is least sensitive to fluctuations in the power supply? Justify your answer, and explain what aspect of the design makes it insensitive to its power supply.
WE WILL ASSUME INFINITE INPUT IMPEDANCE ($i_+ = i_- = 0$), BUT $V_+ \neq V_-$. INSTEAD, THE VOLTAGES WILL BE RELATED USING THE OPEN LOOP GAIN EQUATION:

$$V_{out} = A(V_+ - V_-) = -AV_- \Rightarrow V_- = -V_{out}/A \ (a)$$

KCL:

$$\frac{V_{in} - V_-}{R} + \frac{V_{out} - V_-}{R_f} = 0$$

$$\Rightarrow \ V_- (R + R_f) = R V_{out} + R_f V_{in} \ (b)$$

COMBINING (a) AND (b) TO ELIMINATE $V_-:

$$R V_{out} + V_{out} \frac{R + R_f}{A} = -R_f V_{in}$$

$$V_{out} = \frac{-R_f V_{in}}{R + (\frac{R + R_f}{A})}$$

**THE CIRCUIT GAIN** $G = \frac{-R_f}{R + (\frac{R + R_f}{A})} = -\frac{100}{1 + \frac{10^1}{10^6}} = -99.90$

**THE NOMINAL GAIN** IS IN THE CASE $A \rightarrow \infty$,

$$G_{nominal} = \frac{-R_f}{R} = -100$$
### Problem 28-2

NOMINAL GAIN: \( G_{\text{nominal}} = \frac{-R_f}{R} = -100 \)

ACTUAL GAIN: \( G = \frac{-R_f}{R + R^2 R_f} \) as in #1.

| \( A \) | \( G_{\text{nominal}} \) | \( G \) | \% ERROR \( \left( \frac{G_{\text{nominal}} - G}{G} \right) \times 100 \)%
<table>
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<th></th>
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<tbody>
<tr>
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<tr>
<td>( \infty )</td>
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*X* FAISSLER ASKS FOR "ERROR", I'VE CHosen TO EXPRESS IT AS A PERCENTAGE.
3. FAISSLER #28-3

\[ A = 10^5, \ R = 1 \Omega \]. USING SAME EXPRESSIONS AS BEFORE,

<table>
<thead>
<tr>
<th>( R_f )</th>
<th>( G_{\text{nominal}} )</th>
<th>( G )</th>
<th>( % \text{ERROR} ) = ( \frac{(G_{\text{nominal}} - G)}{G} \times 100% )</th>
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4. FAISSLER #28-4

EMPIRICALLY, THE RULE SEEMS TO BE CLOSE TO:

\[ \text{ERROR} = \frac{G_{\text{nominal}} - G}{G} \approx \frac{-G_{\text{nominal}}}{A} \]

\( (\% \text{ERROR} \approx -100\% \frac{G_{\text{nominal}}}{A}) \)

WE CAN DERIVE IT EXACTLY OF COURSE:

\[ E = \frac{G_{\text{nominal}} - G}{G} = \frac{-R_f}{R} + \frac{R_f}{R + (R + R_f)/A} = \frac{-R_f}{R + (R + R_f)/A} = \frac{-R_f}{R} \]

\[ = \frac{R + (R + R_f)/A}{R} - R = \frac{(R + R_f)/A}{R} \]

\[ = \frac{1 - G_{\text{nominal}}}{A} \]

OR

\[ \% \text{ERR} = \frac{1 - G_{\text{nominal}}}{A} \times 100\% \]

IF \( |G_{\text{nominal}}| >> 1 \), WE RECOVER MY FIRST GUESS.
RECALL THAT THE TRANSFER FUNCTION FOR THE COMMON Emitteramp (LECTURE 6) WAS:

\[ V_{\text{out}} = V_s + \beta \frac{R_c}{R_b} V_T - \beta \frac{R_c}{R_b} V_{\text{in}} \]

**VOLTAGE OFFSET** **AMPLIFIED SIGNAL**

SO, THE OUTPUT HAS A VOLTAGE OFFSET THAT VARIES WITH THE POWER SUPPLY, \( V_s \). *

WITH THE CIRCUIT USED IN PROBS. 1-4,

\[ V_{\text{out}} = \frac{-R_f V_{\text{in}}}{R_f \left( \frac{R_f + R_t}{A} \right)} \]

THE BIGGEST UNCERTAINTY IS \( A \), BUT IF WE HAVE LARGE \( A \) AND MODEST GAIN, ANY ERROR INTRODUCED BY A DEPENDENCE OF \( A \) ON SUPPLY VOLTAGE WILL BE SMALL,

\[ \% \text{ERR} \approx -\frac{G}{A} \].

THE DEEPER REASON FOR THE STABILITY OF THE OP AMPLIFIER DESIGN IS **NEGATIVE FEEDBACK**, WHICH LEADS TO A CIRCUIT GAIN \( G \) THAT IS ALMOST INDEPENDENT OF THE INTRINSIC (OPEN LOOP) GAIN OF THE DEVICE. IT IS SIMILARLY POSSIBLE TO EMPLOY NEGATIVE FEEDBACK IN A BJT AMP TO GET A GAIN THAT IS ALMOST INDEPENDENT OF \( \beta \).]

*[YOU COULD DEFINE THE OUTPUT SIGNAL AS THE VOLUME DROP ACROSS \( R_c \), THOUGH THE GROUND THEN DIFFERS FROM THE INPUT GROUND. IN THAT CASE, THERE ARE MORE SUBLIME REASONS WHY THE CE AMP IS INCONSISTENT; \( \beta \) AND \( V_T \) ARE NOT REALLY CONSTANT.*]