Physics 261
Homework # 12
Remember to show your work.

1. Carefully read the section entitled "Measures of Amplitude" (pp. 57-58) in Faissler. Equations 7-10 and 7-11 have incorrect limits of integration. Figure out what Faissler means, write the equations correctly, and explain in a few words what you did.

2. Do all the problems in Ch. 7 of Faissler.
   (a) 7-1
   (b) 7-2
   (c) 7-3
   (d) 7-4

3. Use Euler's formula to prove equations 7-16 and 7-17.
1. The limits of integration in Faissler 7-10 and 7-11 are in radians, but in each case the independent variable is time. Where he uses $\pi$, he intends $\frac{1}{2}$ cycle, i.e. $t \rightarrow \frac{T}{2}$. Likewise, $2\pi$ means $t \rightarrow T$.

Likewise, each average is normalized by the size of the integration interval:

$$\frac{1}{\pi} \rightarrow \frac{2}{T}, \quad \frac{1}{2\pi} \rightarrow \frac{1}{T}.$$ 

So

$$V_{AV} = \frac{2}{T} \int_{0}^{\frac{T}{2}} V_{P} \sin(\omega t) \, dt \quad (7-10)$$

$$V_{RMS} = \frac{1}{T} \int_{0}^{\frac{T}{2}} [V_{P} \sin(\omega t)]^{2} \, dt \quad (7-11)$$

Where $T = \frac{2\pi}{\omega}$.

The results Faissler gives in each case are correct. An alternative fix would be to simply change the independent variable:

$$\int dt \rightarrow \int d(\omega t)$$
(a) \text{ Faissler 7-1}
\[ f = 60 \text{ Hz} \quad T = \frac{1}{f} = \boxed{16.7 \text{ ms}} \]

(b) \text{ Faissler 7-2}
\[ \omega = 6280 \text{ rad/s} \]
\[ f = \frac{\omega}{2\pi} = \boxed{1 \text{ kHz}} \quad , \quad T = \frac{1}{f} = \boxed{1 \text{ ms}} \]

(c) \text{ Faissler 7-3 (See P. 58)}
\[ V = \frac{2.5V}{2} \sin \omega t \quad (V_o = \frac{1}{2} P_{\text{p-p}}) \]
\[ \text{AMPLITUDE: } V_o = \boxed{12.5V} \]
\[ \text{AVERAGE: } \bar{V} = 0 \]
\[ \text{AVERAGE ABSOLUTE VALUE: } <|V|> = 0.637 \quad V_o = 7.96V \]
\[ \text{RMS VALUE: } V_{\text{rms}} = \frac{V_o}{\sqrt{2}} = 0.707 \quad V_o = 8.84V \]

(d) \text{ Faissler 7-4}
\[ V_{\text{rms}} = 120V \quad \text{so} \]
\[ V_o = \sqrt{2} \quad V_{\text{rms}} = \boxed{170V} \]
\[ V_{\text{p-p}} = 2V_o = \boxed{339V} \]
(3) \[ e^{i(x \pm \beta)} = \cos(x \pm \beta) + \text{i} \sin(x \pm \beta) \] (1)  

We can also use the properties of the exponential to write:
\[ e^{i(x \pm \beta)} = e^{ix} e^{i\beta} = (\cos x + \text{i} \sin x)(\cos \beta \pm \text{i} \sin \beta) \]
\[ = (\cos x + \text{i} \sin x)(\cos \beta \mp \text{i} \sin \beta) \]
\[ = \cos x \cos \beta \mp \sin x \sin \beta + \text{i} \left[ \sin x \cos \beta \mp \cos x \sin \beta \right] \] (2)  

Now, the real part of (1) must equal the real part of (2):
\[ \cos(x \pm \beta) = \cos x \cos \beta \mp \sin x \sin \beta \]  

Likewise, the imaginary parts must match:
\[ \sin(x \pm \beta) = \sin x \cos \beta \mp \cos x \sin \beta \]  

QED.