Physics 261
Homework # 13
Remember to show your work.

1. Do the odd-numbered problems in Ch. 8 of Faissler.
   (a) 8-1
   (b) 8-3
   (c) 8-5

2. Calculate the maximum energy that can be stored, in each of the following capacitors.\(^1\)
   (a) 1.4 F, 5.5 V maximum, weighing 12 g.
   (b) 102,000 \(\mu\)F, 130 V maximum, weighing 2.02 kg.

3. Suppose an AC voltage source is connected in series with a capacitor.
   (a) Find the phase difference between the voltage and the current.
   (b) Plot \(v(t)\) and \(i(t)\) on the same axes, as I did for the inductor in class.

4. Use KVL and complex impedance to derive the rules for combining:
   (a) Two capacitors in series.
   (b) Two inductors in series.
   (c) Two capacitors in parallel.
   (d) Two inductors in parallel.

5. Each of the following statements is true. Justify (briefly) using complex impedance.
   (a) In the limit \(\omega \rightarrow 0\) (the DC limit), the capacitor behaves like an open circuit and the inductor behaves like a short.
   (b) In the limit \(\omega \rightarrow \infty\), the capacitor behaves like a short circuit and the inductor behaves like an open.

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\(^1\) Just for fun, I worked out how fast you’d have to throw each capacitor to give it a kinetic energy equivalent to the maximum it can store. I find about 59 m/s for (a) and 29 m/s for (b).
1. \( \textbf{Faissler 8-1} \)

(a) \( C = 0.1 \mu F, \quad f = \{100 \text{ Hz}, 1000 \text{ Hz} \} \)

\[ \omega = 2\pi f = \{628 \text{ s}^{-1}, 6280 \text{ s}^{-1} \} \]

\[ Z_C = -\frac{j}{\omega C} = \left\{ -j \frac{15.9 \Omega}{\omega}, -j \frac{15.9 \Omega}{\omega} \right\} \]

(b) \( C = 4.0 \mu F; \quad f, \omega \text{ AS BEFORE} \)

\[ Z_C = -\frac{j}{\omega C} = \left\{ -j \frac{398 \Omega}{\omega}, -j \frac{398 \Omega}{\omega} \right\} \]

(c) \( L = 1 \text{ mH}; \quad f, \omega \text{ AS BEFORE} \)

\[ Z_L = j\omega L = \{j \frac{0.628 \Omega}{\omega}, j \frac{6.28 \Omega}{\omega} \} \]

2. \( \textbf{Faissler 8-3} \)

\[ f = 440 \text{ Hz}; \quad \omega = 2\pi f = 2765 \text{ s}^{-1} \]

\[ Z = 10 \Omega + \frac{-j}{(2765 \text{ s}^{-1})(0.1 \mu F)} + j(2765 \text{ s}^{-1})(3.3 \text{ H}) \]

\[ = 10 \Omega + j \left[ -3617 \Omega + 9125 \Omega \right] \]

\[ = (10 - 5.51 j) \Omega \]

3. \( \textbf{Faissler 8-5} \)

\[ \omega = 2765 \text{ s}^{-1} \]

SAME COMPONENTS AS IN 8-3, So \( Z_L = 9125 \Omega; \quad Z_C = -3617 \Omega \)

\[ Z = Z_C + \frac{1}{\sqrt{R^{\prime} + \frac{1}{Z_L}}} = -3617 \Omega + \frac{1}{10 \Omega + \frac{1}{9125 \Omega}} \]

\[ = -3617 \Omega + \frac{10^4 \Omega}{(1 + 1.0963)(1 + 1.0963)} = -3617 \Omega + \frac{10^4 \Omega}{(4.55 - 8.60 \Omega)} \]
\[ U = \frac{1}{2} CV^2 \]

(a) \( C = 1.4 \, \text{F}, \quad V = 5.5 \, \text{V} \)
\[ U = 21.2 \, \text{J} \]

(b) \( C = 0.102 \, \text{F}, \quad V = 130 \, \text{V} \)
\[ U = 862 \, \text{J} \]

3. \[ V \quad C \quad Z = \frac{1}{j \omega C} \]

**Ohm's Law:** \( V = I \cdot Z \), so \( I = \frac{V}{Z} = j \omega C V \)

(a) **The result is a positive imaginary voltage times \( V \),** so the phase is:
\[ j = e^{j \pi/2} \Rightarrow \varphi = \frac{\pi}{2} \]
That is, the current leads the voltage by 90°.

(b) [Graph showing sinusoidal waveforms for \( V(t) \) and \( I(t) \)]
(a) \[ Z_{T_0} = Z_{C_1} + Z_{C_2} = \frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} = \frac{1}{j\omega \left( \frac{1}{C_1} + \frac{1}{C_2} \right)} \]

Now, if \( Z_{T_0} \) is equivalent to some capacitance \( C_{T_0} \), then

\[ Z_{T_0} = \frac{1}{j\omega C_{T_0}} = \frac{1}{j\omega \left( \frac{1}{C_1} + \frac{1}{C_2} \right)} \]

\[ \Rightarrow \frac{1}{C_{T_0}} = \frac{1}{C_1} + \frac{1}{C_2} \]

(b) \[ Z_{T_0} = j\omega L_{T_0} = j\omega L_1 + j\omega L_2 \]

\[ \Rightarrow L_{T_0} = L_1 + L_2 \]

(c) \[ Z_{T_0} = \frac{1}{\frac{1}{\sqrt{L_1 + L_2}} + \frac{1}{j\omega C_1 + j\omega C_2}} = \frac{1}{j\omega C_{T_0}} \]

\[ \Rightarrow C_{T_0} = C_1 + C_2 \]

(d) \[ Z_{T_0} = \frac{1}{\frac{1}{j\omega L_1} + \frac{1}{j\omega L_2}} = j\omega L_{T_0} \]

\[ \Rightarrow \frac{1}{L_{T_0}} = \frac{1}{L_1} + \frac{1}{L_2} \]
\( (a) \quad Z_c = \frac{1}{j\omega C} \)

As \( \omega \to 0 \) (DC), \( Z_c \to \infty \)

INFINITE IMPEDEANCE \( \Rightarrow \) OPEN CIRCUIT \( (R \to \infty) \)

\( Z_L = j\omega L \)

As \( \omega \to 0 \), \( Z_L \to 0 \)

ZERO IMPEDEANCE \( \Rightarrow \) SHORT \( (R \to 0) \)

\( (b) \) In the limit \( \omega \to \infty \),

\[ Z_c = \frac{1}{j\omega C} \to 0 \Rightarrow \text{SHORT CIRCUIT} \]

\[ Z_L = j\omega L \to \infty \Rightarrow \text{OPEN} \]