

# PHYSICS 262 — LECTURE 0

- BRIEFLY DISCUSS SYLLABUS
- REVIEW OF AC CIRCUITS & IMPEDANCE
- MIGHT DO DECIBELS (dB) + BODE PLOTS.

THE FIRST HALF OF THE WIKIPEDIA ARTICLE ON COMPLEX NUMBERS IS A GOOD REFERENCE FOR THIS CLASS.

# IMPEDANCE REVIEW

OHM'S LAW:  $V = iR$

CAPACITOR:  $q = CV \Rightarrow \frac{dV}{dt} = i/C$

INDUCTOR:  $V = L \frac{di}{dt}$

IF WE USE  $\underline{V} = V_0 e^{j\omega t} = V_0 [\cos \omega t + j \sin \omega t]$ ,

THEN THE RELATIONS FOR CAPACITOR & INDUCTOR LOOK A WHOLE LOT LIKE OHM'S LAW:

$$\frac{dV}{dt} = j\omega \underline{V} = \underline{i}/C$$

$$\Rightarrow \underline{V} = \underline{i} \underbrace{\left(\frac{-j}{\omega C}\right)}_{Z_C}$$

$$\underline{V} = L \underbrace{\left(j\omega \underline{i}\right)}_{Z_L} = \underline{i} \underbrace{\left(j\omega L\right)}_{Z_L}$$

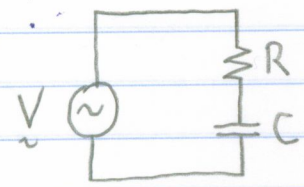
IMPEDANCE  
(LIKE RESISTANCE,  
BUT  $Z \in \mathbb{C}$ )

- AT A GIVEN FREQUENCY, A CIRCUIT INVOLVING R, C, L CAN BE ANALYZED USING GENERALIZED OHM'S LAW,

$$\underline{V} = \underline{i} Z_0$$

- LINEAR CIRCUITS (R, C, L, AND OPAMPS THAT OBEY THE GOLDEN RULES) OBEY THE PRINCIPLE OF SUPERPOSITION.

EXAMPLE



- (a) FIND THE TOTAL IMPEDANCE.
- (b) FIND  $i(t)$ .
- (c) FIND MEAN POWER DISSIPATED.

(a)  $Z_{TOT} = R + \frac{1}{j\omega C}$ .

(b) START WITH  $\underline{v} = V_0 e^{j\omega t}$ ; FIND  $\underline{i}$ , THEN  $i(t)$ .

(THE PHYSICAL PART IS THE REAL PART:  $v(t) = V_0 \cos \omega t$ )

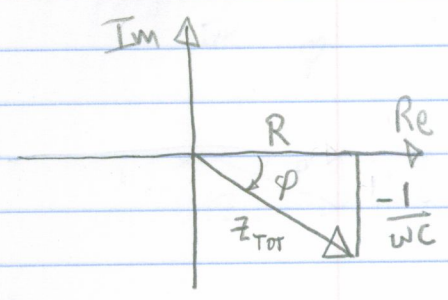
NOW,  $\underline{i} = \underline{v} / Z_{TOT} = \frac{V_0 e^{j\omega t}}{R + 1/j\omega C}$

$i(t) = \text{Re}(\underline{i})$  — HOW TO FIND ??

LET'S PUT THE DENOMINATOR OF  $\underline{i}$  IN POLAR FORM:

$$R + \frac{1}{j\omega C} = \left| R + \frac{1}{j\omega C} \right| e^{j\phi}$$

$$= \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} e^{j\phi}$$



$\tan \phi = -1/\omega RC$

$\phi = -\tan^{-1}\left(\frac{1}{\omega RC}\right)$

SO  $\underline{i} = \frac{V_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} e^{j(\omega t - \phi)}$

$i(t) = \frac{V_0}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \cos(\omega t - \phi)$

EXAMPLE, CONT.

$$(c) P_R(t) = i^2 R = \frac{V_0^2 R}{R^2 + (\frac{1}{\omega C})^2} \cos^2(\omega t - \phi)$$

$$\bar{P}_R = \frac{V_0^2 R}{R^2 + (\frac{1}{\omega C})^2} \langle \cos^2(\omega t - \phi) \rangle$$

$$\bar{P}_R = \frac{\frac{1}{2} V_0^2 R}{R^2 + (\frac{1}{\omega C})^2}$$

POWER  
DISSIPATED  
IN  
RESISTOR

HOW ABOUT POWER OUTPUT BY VOLTAGE SOURCE?

$$P_V(t) = i(t) V(t) = \frac{V_0^2}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \cos \omega t \cos(\omega t - \phi)$$

Now,  $\cos(\omega t - \phi) = \cos \omega t \cos \phi + \sin \omega t \sin \phi$

$$P_V(t) = \frac{V_0^2}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \left[ \underbrace{\cos^2 \omega t \cos \phi}_{\text{AVERAGE } \frac{1}{2}} + \underbrace{\sin \omega t \cos \omega t \sin \phi}_{\text{AVERAGES TO ZERO}} \right]$$

AND  $\cos \phi = \frac{R}{|Z_{TOT}|} = \frac{R}{\sqrt{R^2 + (\frac{1}{\omega C})^2}}$

SO  $\bar{P}_V = \frac{\frac{1}{2} V_0^2 R}{R^2 + (\frac{1}{\omega C})^2} = \bar{P}_R$ . DOES THIS RESULT MAKE SENSE?

NOTE:  $P_R(t) \neq P_V(t)$  — WHY?

HANDY TRICK:

$$\bar{P}_V = \frac{1}{2} \text{Re}(\tilde{i}^* \tilde{V}) = \frac{1}{2} \text{Re}(\tilde{i} \tilde{V}^*)$$

### LOGARITHMIC GAIN, BODE PLOTS, AND DECIBELS

THE SIGNALS PROCESSED BY A FILTER OR OTHER CIRCUIT OFTEN CONTAIN FREQUENCIES THAT RANGE OVER SEVERAL ORDERS OF MAGNITUDE. IN THE AUDIO RANGE, FOR EXAMPLE, THE REPRODUCTION OF SOUND MUST BE ACCURATE FROM 20 Hz TO 20 kHz.

WITH FREQUENCY SPREAD ACROSS MANY ORDERS OF MAGNITUDE, CIRCUIT GAIN MAY ALSO RANGE OVER ORDERS OF MAGNITUDE. FOR THIS REASON, LOG-LOG PLOTS OF  $G(\omega)$  ARE COMMONLY USED.

LOGARITHMIC GAIN IS MEASURED IN DECIBELS, WHERE ONE "BELL" IS A POWER OF 10:

$$\text{GAIN (dB)} = 10 \log_{10} \left( \frac{P_{\text{OUT}}}{P_{\text{IN}}} \right); \quad P_{\text{OUT}} = 10^{\left(\frac{\text{dB}}{10}\right)} P_{\text{IN}}$$

YOU CAN CONVERT TO AND FROM dB IN YOUR HEAD IF YOU REMEMBER TWO THINGS:

- 3dB IS A FACTOR OF ABOUT 2.
- $\sqrt{10} \approx \pi$

TO WITHIN 1%:

dB	$P_{\text{OUT}} / P_{\text{IN}}$
0	1 = $10^0$
1	5/4 = $10^{0.1}$
2	$\pi/2$ = $10^{0.2}$
3	2
4	5/2
5	$\pi$
6	4
7	5
8	2 $\pi$
9	8
10	10 = $10^{1.0}$

VOLTAGE AND CURRENT GAINS IN dB:

SINCE, WITH A PURE RESISTIVE LOAD,

$$P = I^2 R = V^2 / R,$$

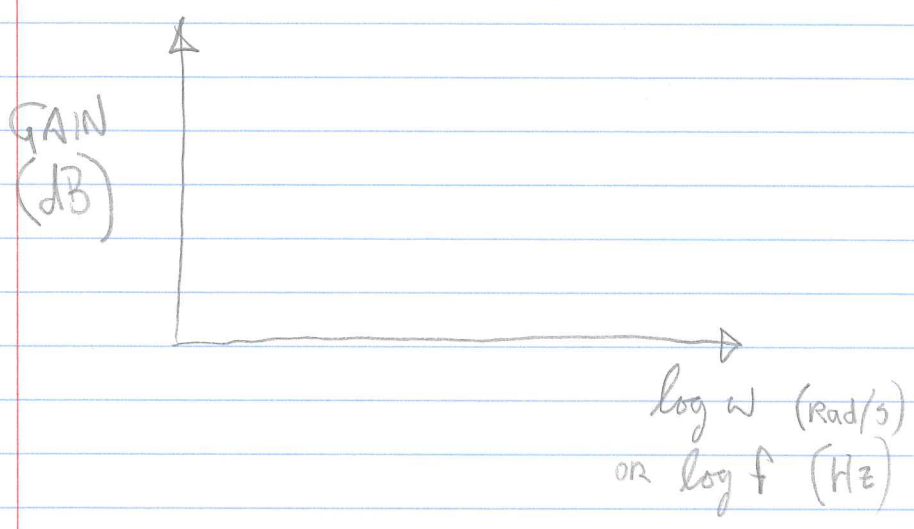
WE DEFINE

$$\text{GAIN (dB)} = 20 \log_{10} \left( \frac{V_{OUT}}{V_{IN}} \right); \quad V_{OUT} = 10^{(dB/20)} V_{IN}$$

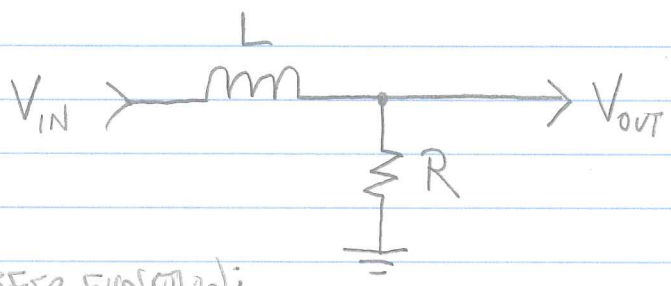
(AND SIMILARLY FOR CURRENT).

dB	$P_{OUT} / P_{IN}$	$V_{OUT} / V_{IN}$ ( $I_{OUT} / I_{IN}$ )
-40	1/10000	1/100
-30	1/1000	1/10π
-20	1/100	1/10
-10	1/10	1/π
-6	1/4	1/2
0	1	1
2	π/2	5/4
4	5/2	π/2
6	4	2
8	2π	5/2
10	10	π
12	5π	4
14	25	5
16	40	2π
18	20π	8
20	100	10
26	400	20
30	1000	10π
40	10000	100

### BODE PLOT:



### EXAMPLE —



TRANSFER FUNCTION:

$$\underline{G} = \frac{V_{out}}{V_{in}} = \frac{R}{Z_L + R} = \frac{R}{j\omega L + R}$$

GAIN:

$$G = |\underline{G}| = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{1}{\sqrt{1 + \omega^2 / \omega_0^2}}$$

WHERE  $\omega_0 = R/L$ .

$$\begin{cases} \omega \ll \omega_0 : G = 1 \\ \omega = \omega_0 : G = -3 \text{ dB} \\ \omega \gg \omega_0 : G = \frac{\omega_0}{\omega} \end{cases}$$

