

### LOGARITHMIC GAIN, BODE PLOTS, AND DECIBELS

THE SIGNALS PROCESSED BY A FILTER OR OTHER CIRCUIT OFTEN CONTAIN FREQUENCIES THAT RANGE OVER SEVERAL ORDERS OF MAGNITUDE. IN THE AUDIO RANGE, FOR EXAMPLE, THE REPRODUCTION OF SOUND MUST BE ACCURATE FROM 20 HZ TO 20 KHZ.

WITH FREQUENCY SPREAD ACROSS MANY ORDERS OF MAGNITUDE, CIRCUIT GAIN MAY ALSO RANGE OVER ORDERS OF MAGNITUDE. FOR THIS REASON, LOG-LOG PLOTS OF  $G(\omega)$  ARE COMMONLY USED.

LOGARITHMIC GAIN IS MEASURED IN DECIBELS, WHERE ONE "BELL" IS A POWER OF 10:

$$\text{GAIN (dB)} = 10 \log_{10} \left( \frac{P_{\text{OUT}}}{P_{\text{IN}}} \right); \quad P_{\text{OUT}} = 10^{\left(\frac{\text{dB}}{10}\right)} P_{\text{IN}}$$

YOU CAN CONVERT TO AND FROM DB IN YOUR HEAD IF YOU REMEMBER TWO THINGS:

- 3dB IS A FACTOR OF ABOUT 2.
- $\sqrt{10} \approx \pi$

TO WITHIN 1%:

dB	$P_{\text{OUT}} / P_{\text{IN}}$
0	1 = $10^0$
1	5/4 = $10^{0.1}$
2	$\pi/2$ = $10^{0.2}$
3	2
4	5/2
5	$\pi$
6	4
7	5
8	$2\pi$
9	8
10	10 = $10^{1.0}$

VOLTAGE AND CURRENT GAINS IN dB:

SINCE, WITH A PURE RESISTIVE LOAD,

$$P = I^2 R = V^2 / R,$$

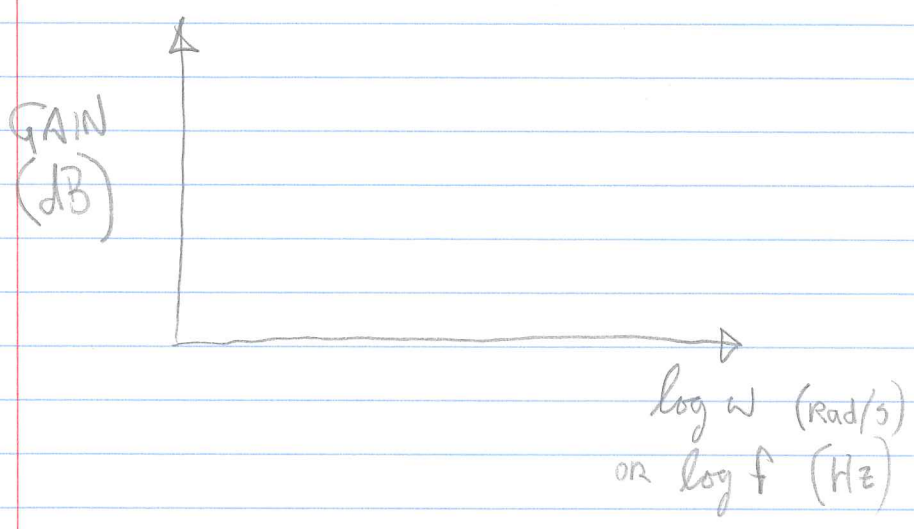
WE DEFINE

$$\text{GAIN (dB)} = 20 \log_{10} \left( \frac{V_{OUT}}{V_{IN}} \right); \quad V_{OUT} = 10^{(\text{dB}/20)} V_{IN}$$

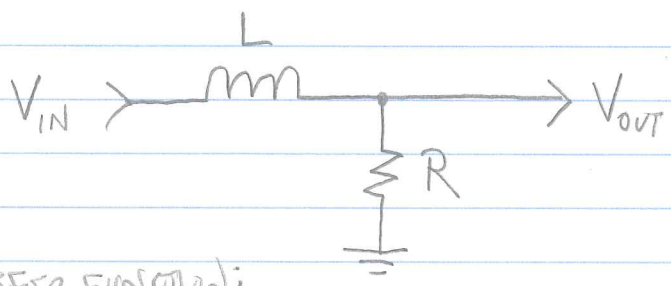
(AND SIMILARLY FOR CURRENT).

dB	$P_{OUT}/P_{IN}$	$V_{OUT}/V_{IN}$ ( $I_{OUT}/I_{IN}$ )
-40	1/10000	1/100
-30	1/1000	1/10π
-20	1/100	1/10
-10	1/10	1/π
-6	1/4	1/2
0	1	1
2	π/2	5/4
4	5/2	π/2
6	4	2
8	2π	5/2
10	10	π
12	5π	4
14	25	5
16	40	2π
18	20π	8
20	100	10
26	400	20
30	1000	10π
40	10000	100

### BODE PLOT:



### EXAMPLE —



TRANSFER FUNCTION:

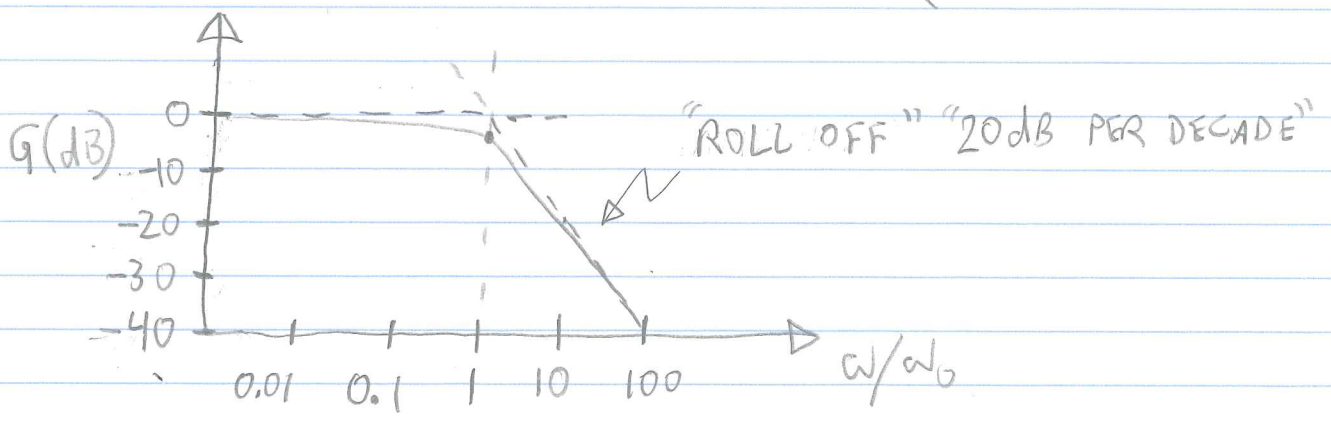
$$\underline{G} = \frac{V_{out}}{V_{in}} = \frac{R}{Z_L + R} = \frac{R}{j\omega L + R}$$

GAIN:

$$G = |\underline{G}| = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{1}{\sqrt{1 + \omega^2 / \omega_0^2}}$$

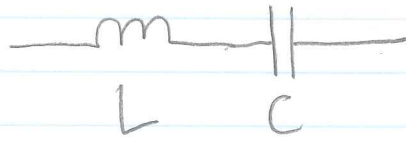
WHERE  $\omega_0 = R/L$ .

$$\begin{cases} \omega \ll \omega_0 : G = 1 \\ \omega = \omega_0 : G = -3 \text{ dB} \\ \omega \gg \omega_0 : G = \frac{\omega_0}{\omega} \end{cases}$$



# RESONANCE

SERIES LC:



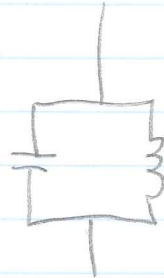
$$Z_{TOT} = j\omega L - \frac{j}{\omega C}$$

BOTH PURE IMAGINARY;  
OPPOSITE SIGNS

$Z_{TOT} = 0$  WHEN  $\omega = \frac{1}{\sqrt{LC}}$ .

THIS IS CALLED RESONANCE

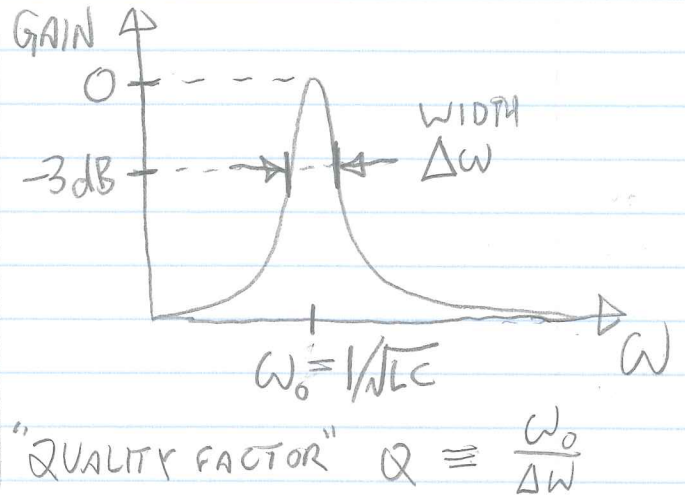
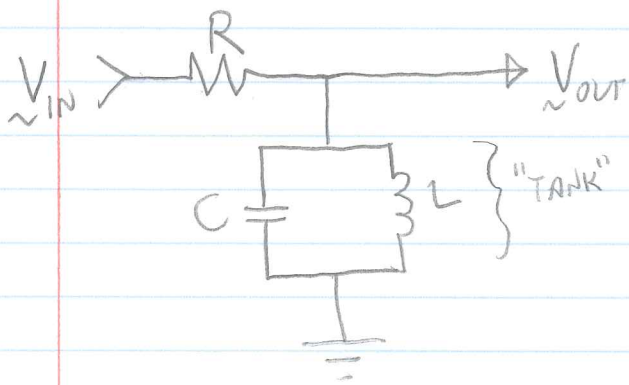
PARALLEL LC:



$$\frac{1}{Z_{TOT}} = \frac{1}{j\omega L} - \frac{\omega C}{j}$$

$Z_{TOT} \rightarrow \infty$  WHEN  $\omega = \frac{1}{\sqrt{LC}}$ .

EXAMPLE: "TANK CIRCUIT" RF FILTER



$$GAIN = \left| \frac{V_{OUT}}{V_{IN}} \right| = \frac{|V_{OVER}|}{|V_{IN}|} = \left| \frac{Z_{TANK}}{Z_{TANK} + R} \right| = \frac{1}{\left| 1 + \frac{R}{Z_{TANK}} \right|}$$

$$\frac{1}{Z_{TANK}} = \frac{1}{j\omega L} - \frac{j\omega C}{j} = -j\sqrt{\frac{C}{L}} \left( \frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)$$

$$\therefore GAIN = \frac{1}{\sqrt{1 + \frac{R^2 C}{L} \left( \frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)^2}}$$

THIS IS THE RESONANCE CURVE PLOTTED ABOVE.

-3dB POINTS: VOLTAGE GAIN =  $\frac{1}{\sqrt{2}}$

$$\Rightarrow \frac{R^2 C}{L} \left( \frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} \right)^2 = 1$$

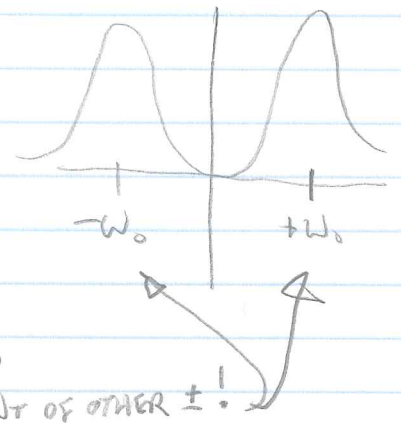
$$\Leftrightarrow \frac{\omega_0}{\omega} - \frac{\omega}{\omega_0} = \pm \sqrt{\frac{L}{R^2 C}}$$

$$\omega^2 \pm \frac{L}{R^2 C} \omega_0 \omega - \omega_0^2 = 0$$

INDEPENDENT OF OTHER ±!

$$\omega = \frac{\pm 1 \sqrt{\frac{L}{R^2 C}} \omega_0 \pm \frac{1}{2\sqrt{\frac{L}{R^2 C}}} \left( \frac{L}{R^2 C} \omega_0^2 + 4\omega_0^2 \right)}{2}$$

THIS IS  $\pm \Delta\omega/2$



WE CONCLUDE THAT

$$\Delta\omega = \sqrt{\frac{L}{R^2C}} \omega_0 = \frac{1}{RC}$$

$$Q = \frac{\omega_0}{\Delta\omega} = R\sqrt{\frac{C}{L}} = RC\omega_0$$

(AS IN H&H § 1.22)

NOTE THAT AS  $R \rightarrow \infty$   $\Delta\omega \rightarrow 0$  AND  $Q \rightarrow \infty$ .

DOES THIS MAKE SENSE?