

TODAY

PASSIVE & ACTIVE FILTERS

- TERMINOLOGY
- PASSIVE FILTERS
- OP AMPS -- BRIEF REVIEW
- ACTIVE FILTERS
- MORE EXAMPLES THAN I CAN DO IN CLASS!

BE SURE TO  
DO PRE-LABS  
BEFORE LAB!

TERMS

FILTER = A CIRCUIT THAT APPLIES SOME TRANSFER FUNCTION:  $V_{out} = G(\omega) V_{in}$ .

PASSBAND = RANGE OF FREQUENCIES ABOVE SOME MINIMAL USEFUL GAIN (E.G., -3 dB; -1 dB)

STOPBAND = RANGE OF FREQUENCIES THAT ARE SUPPRESSED BELOW SOME MAXIMUM ACCEPTABLE GAIN (E.G., -40 dB)

CUTOFF FREQUENCY = USUALLY, THE FREQUENCY AT WHICH GAIN = -3 dB.

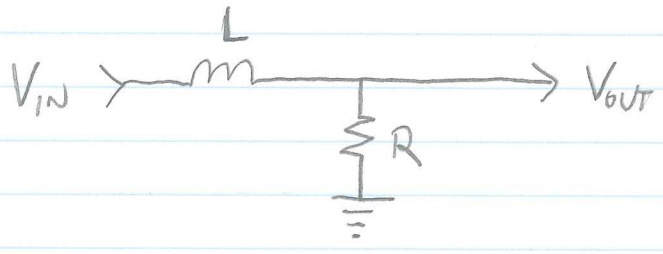
ORDER = SLOPE OF THE ASYMPTOTIC GAIN IN VOLTAGE DECADES PER FREQUENCY DECADE (1<sup>st</sup> ORDER  $\Rightarrow$   $\pm 20$  dB PER DECADE)

PASSIVE FILTER = FILTER MADE OF PASSIVE COMPONENTS (L, R, C). USES ONLY POWER FROM THE SIGNAL, SO GAIN  $\leq 1$  (0 dB POWER).

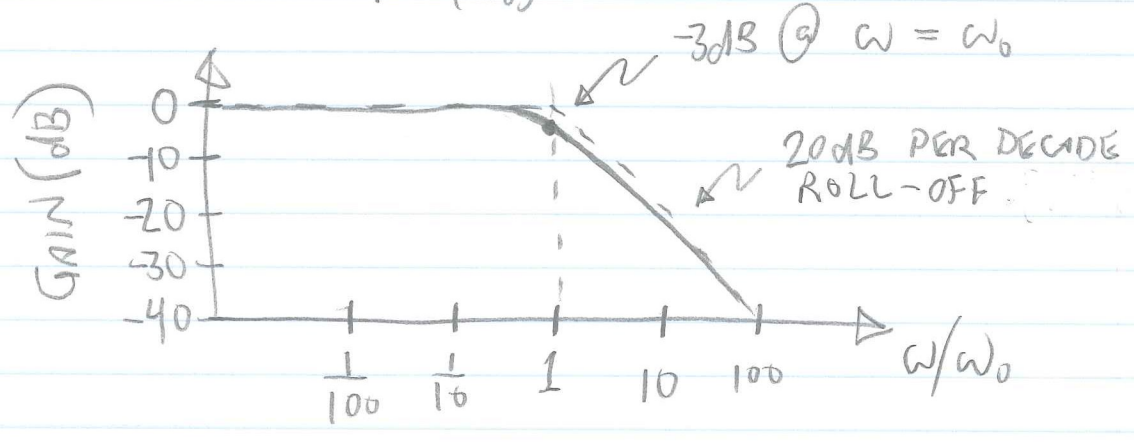
ACTIVE FILTER = FILTER USING OP AMPS WITH NEGATIVE FEEDBACK.

# PASSIVE FILTERS

LAST WEEK, WE SOLVED AN LR LOW-PASS FILTER:

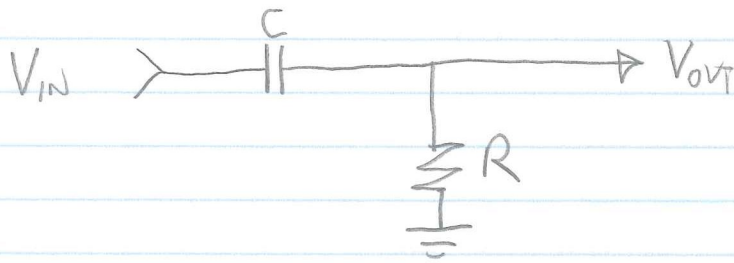


$$\text{GAIN} = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}, \quad \omega_0 \equiv \frac{R}{L}$$



Q: WHAT "ORDER" IS THIS FILTER?

ANOTHER EXAMPLE —



$$\frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{1}{1 - \frac{j}{\omega RC}} = \frac{1}{1 - j \frac{\omega_0}{\omega}}$$

$$\boxed{\text{TAN } \phi = \frac{+\omega_0}{\omega}}$$

$$\text{GAIN} = \frac{|V_{out}|}{|V_{in}|} = \boxed{\frac{1}{\sqrt{1 + \left(\frac{\omega_0}{\omega}\right)^2}}}$$

$$\omega \ll \omega_0 : G \rightarrow \omega/\omega_0 \quad (\text{FIRST ORDER})$$

$$\omega \gg \omega_0 : G \rightarrow 1$$

SIMPLE RC OR LR FILTERS ARE ALWAYS FIRST-ORDER — NOT VERY SHARP!

IDEA: "CASCADED" FILTERS:

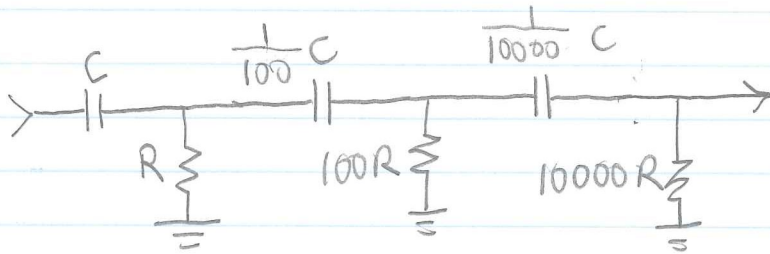


E.G., IF ALL ARE HIGH PASS AS ABOVE,

$$\omega \ll \omega_0 : G \rightarrow \left(\frac{\omega}{\omega_0}\right)^n \quad (n^{\text{TH}} \text{ ORDER})$$

$$\omega \gg \omega_0 : G \rightarrow 1^n = 1$$

## CASCADED FILTERS IN PRACTICE :



Q: WHY THE INCREASING COMPONENT VALUES?  
 DISCUSS! (HINT: WHAT HAPPENS WHEN THERE IS MUCH CURRENT AT THE OUTPUT OF A VOLTAGE DIVIDER?)

ANOTHER ISSUE WITH CASCADED FILTERS IS THE "SOFT KNEE": CONSIDER  $n$  CASCADED HIGH-PASS FILTERS:

$$G = \frac{1}{\left(1 + \left(\frac{\omega_0}{\omega}\right)^2\right)^{n/2}} \quad \left. \vphantom{G} \right\} \begin{array}{l} \text{GAIN IS SAME FOR} \\ \text{LR OR RC. (BUT} \\ \text{THE PHASE WOULD DIFFER)} \end{array}$$

FIX  $-3\text{dB}$  POINT AT  $\omega = \omega_3$  :

$$\left(1 + \left(\frac{\omega_0}{\omega_3}\right)^2\right)^{n/2} = \sqrt{2} \Rightarrow \left(\frac{\omega_0}{\omega}\right)^2 = 2^{1/n} - 1$$

$$\Rightarrow \omega_0 = \omega_3 \sqrt{2^{1/n} - 1}$$

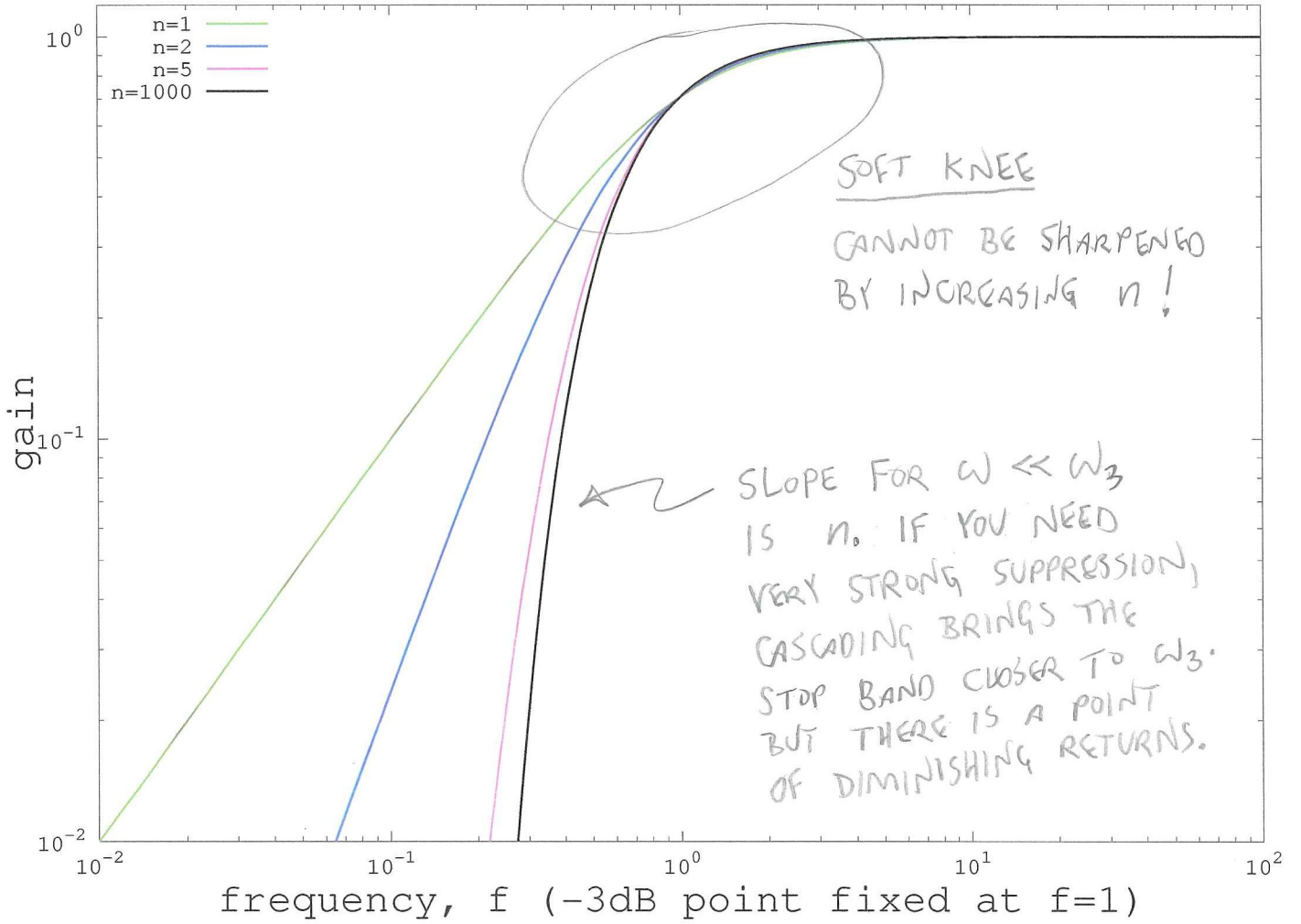
SINCE  $\omega_0 \equiv 1/RC$ , CHOOSE COMPONENT VALUES ACCORDINGLY.

NOW, HOLDING  $\omega_3$  FIXED,

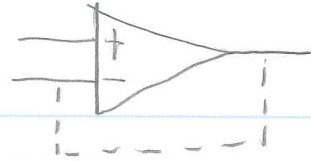
$$G = \frac{1}{\left(1 + (2^{1/n} - 1) \left[\frac{\omega_3}{\omega}\right]^2\right)^{n/2}}$$

SEE ATTACHED PLOT!

### Cascaded nth-order LR high-pass filter



OP AMPS — "GOLDEN RULES"

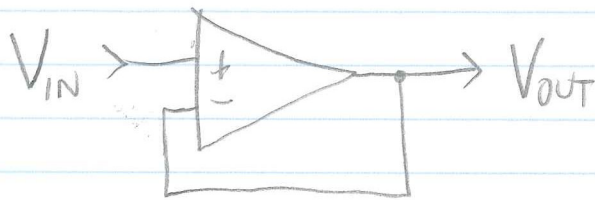


ASSUMING NEGATIVE FEEDBACK AND INFINITE GAIN  
(NOT SATURATED),

①  $i_+ = i_- = 0$

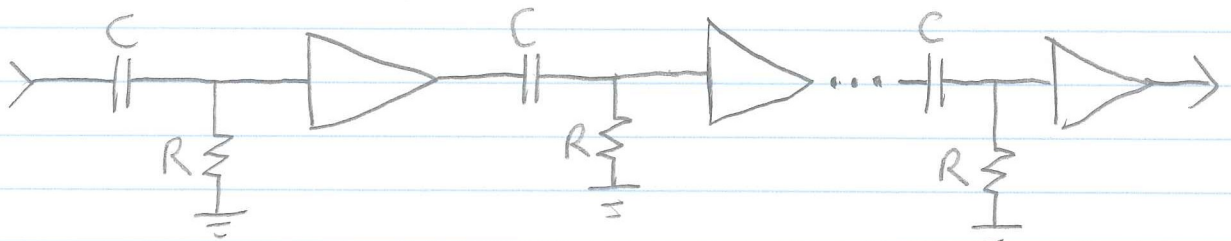
②  $V_+ = V_- = 0$

EXAMPLE: VOLTAGE FOLLOWER (BUFFER)



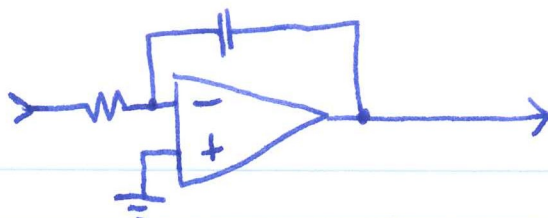
$V_{IN} = V_+ = V_- = V_{OUT} \Rightarrow G = 1$

WE COULD USE BUFFERS TO CASCADE FILTERS:



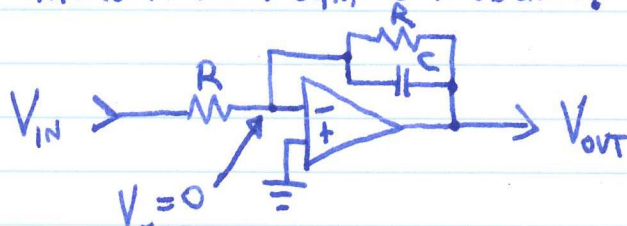
THIS IS THE SIMPLEST SORT OF ACTIVE FILTER,  
BUT STILL RATHER LIMITED BECAUSE OF THE  
"SOFT KNEE" PROBLEM.

HOW ABOUT:



DOOPS!

AT DC, THERE IS NO NEGATIVE FEEDBACK! INSTEAD:



KCL:

~~$$\frac{V_{OUT} - 0}{R - \frac{j}{\omega C}} + \frac{V_{IN} - 0}{R} = 0$$~~

$$(V_{OUT} - 0) \left( \frac{1}{R} + \frac{j\omega C}{1} \right) + \frac{V_{IN} - 0}{R} = 0$$

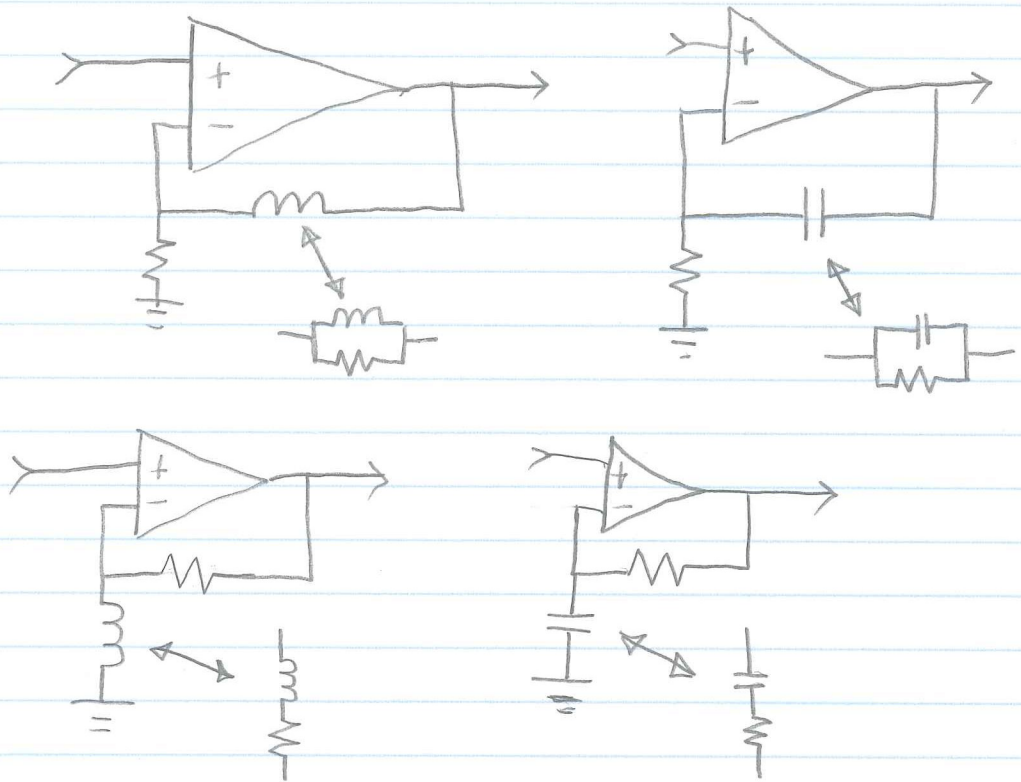
$$\Rightarrow \underline{V_{OUT}} = \frac{-\underline{V_{IN}}}{1 + j\omega RC} \Rightarrow \boxed{G = \frac{-1}{\sqrt{1 + \left(\frac{\omega}{\omega_0}\right)^2}}}$$

WHERE  $\omega_0 \equiv 1/RC$ .

SO THIS IS A FIRST-ORDER LOWPASS, BUT  
IT IS INVERTING.

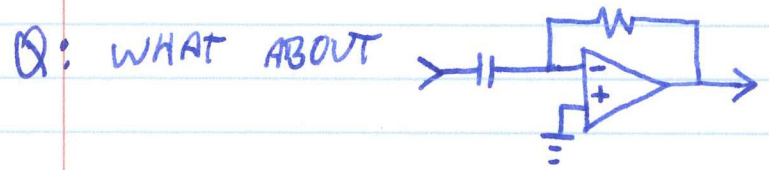
### A HINT ABOUT ACTIVE FILTERS ...

GENERALLY, WE TRY TO AVOID INFINITE OR ZERO IMPEDANCE IN PLACES THAT MIGHT LEAD TO INFINITE GAIN:



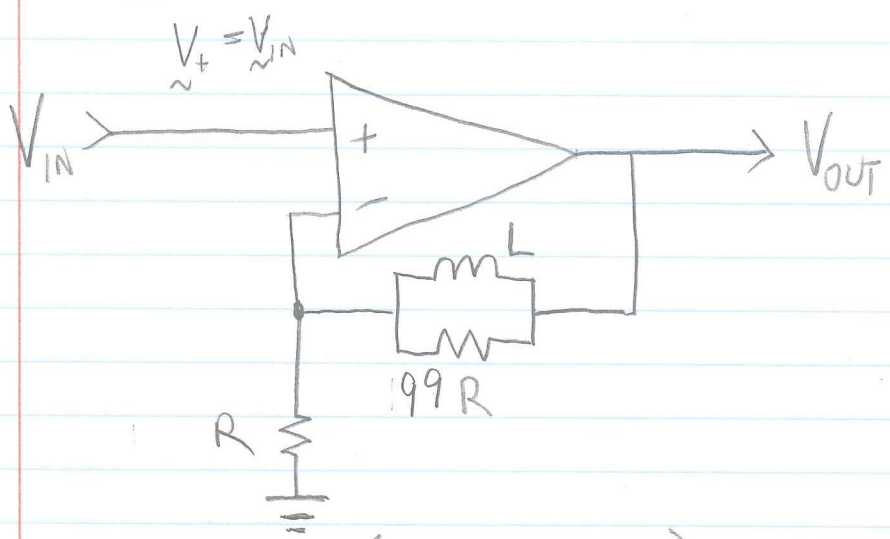
Q: HOW DO THE ALTERNATIVES I'VE SKETCHED SOLVE A PROBLEM FOR EACH CIRCUIT?

ALWAYS MAKE SURE THAT YOU HAVE NEGATIVE FEEDBACK IN THE LIMITS  $\omega \rightarrow 0$  AND  $\omega \rightarrow \infty$  !





# ACTIVE FILTER EXAMPLE

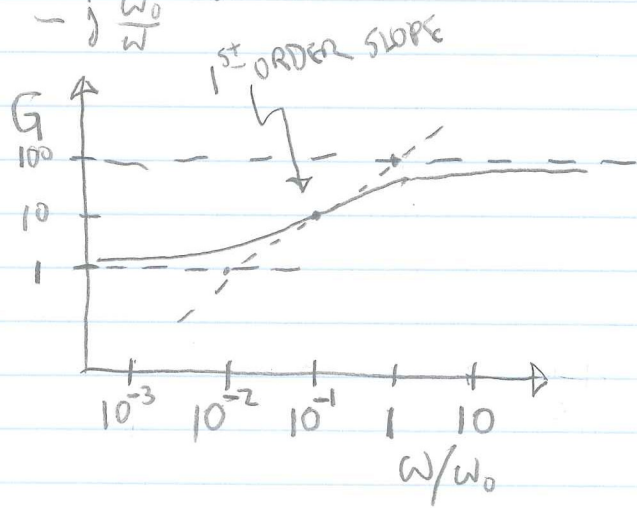


$$\underline{V}_- = \underline{V}_{out} \left( \frac{R}{R + \left( \frac{1}{99R} + \frac{1}{j\omega L} \right)^{-1}} \right)$$

$$\Rightarrow \frac{\underline{V}_{out}}{\underline{V}_{in}} = \frac{R + \left( \frac{1}{99R} + \frac{1}{j\omega L} \right)^{-1}}{R} = 1 + \frac{99}{1 + \frac{99R}{j\omega L}}$$

$$= \frac{100 + \frac{\omega_0}{j\omega}}{1 + \frac{\omega_0}{j\omega}} = \frac{100 - j \frac{\omega_0}{\omega}}{1 - j \frac{\omega_0}{\omega}} ; \omega_0 \equiv 99R/L$$

$$G = \frac{|\underline{V}_{out}|}{|\underline{V}_{in}|} = \sqrt{\frac{10^4 + \left( \frac{\omega_0}{\omega} \right)^2}{1 + \left( \frac{\omega_0}{\omega} \right)^2}}$$



$$\frac{\omega}{\omega_0} \gg 1 : G \rightarrow 100$$

$$\frac{\omega}{\omega_0} \ll \frac{1}{100} : G \rightarrow 1$$

$$\frac{1}{100} \ll \frac{\omega}{\omega_0} \ll 1 : G \approx 100 \omega / \omega_0$$

( $\omega \approx 0.1 \omega_0$ )