

TODAY

- 2nd ORDER FILTERS
 - RC CASCADE
 - RLC
 - ACTIVE
- PERFORMANCE
 - PASS BAND RIPPLE
 - ROLL OFF
 - PHASE & TIME DELAY
 - STEP & IMPULSE RESPONSE

MOTIVATION FOR TODAY'S LECTURE:

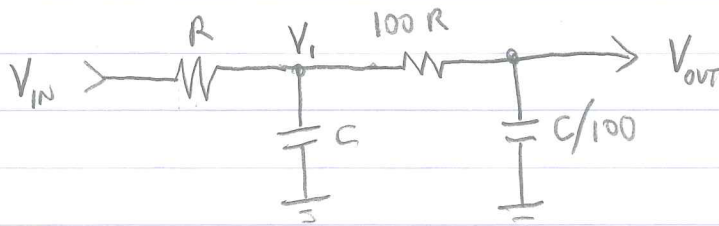
- ALL OF THE FANCY HIGH-ORDER FILTERS WE WILL LEARN ABOUT NEXT WEEK ARE MADE FROM CASCADED 2nd ORDER FILTERS.
- THE TECHNIQUES WE'LL USE TO EVALUATE PERFORMANCE OF 2nd ORDER FILTERS WORK EQUALLY WELL FOR HIGH-ORDER FILTERS.

NEXT TIME: HIGHER ORDER FILTERS

- BUTTERWORTH
 - BESSEL
 - CHEBYSHEV
- CHOOSING THE RIGHT FILTER FOR A GIVEN APPLICATION

RC > RC

FROM LAST TIME, RECALL OUR STRATEGY TO CASCADE RC FILTERS:



THE FIRST FILTER HAS A TRANSFER FUNCTION*

$$\tilde{G}_1 \equiv \frac{V_1}{V_{IN}} = \frac{(1/j\omega C)}{R + (1/j\omega C)} = \frac{1}{1 + j\omega RC}$$

THE SECOND FILTER DOES THE SAME BECAUSE RC IS THE SAME:

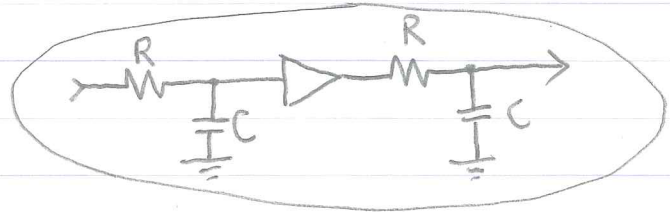
$$\tilde{G}_2 \equiv \frac{V_{OUT}}{V_1} = \frac{(100/j\omega C)}{100R + (100/j\omega C)} = \frac{1}{1 + j\omega RC}$$

THE TOTAL RESPONSE IS

$$\tilde{G} \equiv \frac{V_{OUT}}{V_{IN}} = \tilde{G}_1 \tilde{G}_2 = \frac{1}{(1 + j\omega RC)^2} = \frac{1}{1 + 2j\omega RC - (\omega RC)^2}$$

THIS SIMPLE CIRCUIT ANALYSIS WORKS BECAUSE THE IMPEDANCE OF THE SECOND FILTER IS MUCH HIGHER THAN THE FIRST, SO IT DOESN'T LOAD THE FIRST (THE CURRENT IN 100R IS NEGLIGIBLE COMPARED TO THAT IN R & C). AS A RESULT, IT IS A FAIR APPROXIMATION TO TREAT THE FIRST FILTER AS A VOLTAGE DIVIDER.

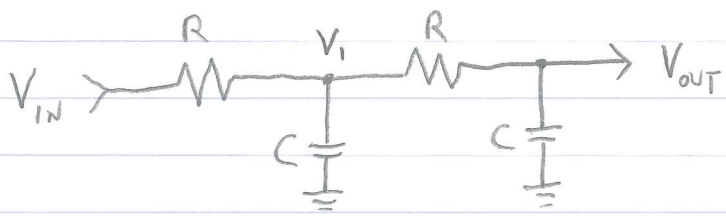
EQUIVALENT APPROACH:



*I USE FOURIER TRANSFER FUNCTIONS, WHICH ARE RESPONSES TO $e^{j\omega t}$. TRADITIONAL EEs USE LAPLACE: $s = j\omega$.

RC-RC

WHAT IF WE CASCADE 2 COPIES OF THE SAME RC LOWPASS FILTER?



NODE VOLTAGE ANALYSIS —

KCL AT V₁:

$$\frac{V_{IN} - V_1}{R} + \frac{0 - V_1}{1/j\omega C} + \frac{V_{OUT} - V_1}{R} = 0$$

$$\Rightarrow V_1 (2 + j\omega RC) = V_{IN} + V_{OUT} \quad (1)$$

KCL AT V_{OUT}: *

$$\frac{V_1 - V_{OUT}}{R} + \frac{0 - V_{OUT}}{1/j\omega C} = 0$$

$$\Rightarrow (1 + j\omega RC) V_{OUT} = V_1 \quad (2)$$

SUBSTITUTING (2) INTO (1) TO ELIMINATE V₁,

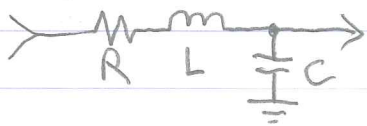
$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + 3j\omega RC - (\omega RC)^2}$$

A BIT DIFFERENT!

BOTH FILTERS ARE 2nd ORDER ("2 POLES") LOW PASS — FALLS OFF AS 1/ω².

* SINCE THE SECOND STAGE IS UNLOADED, WE GET THE USUAL VOLTAGE DIVIDER RELATIONSHIP.

A MORE GENERAL 2nd ORDER FILTER: RLC



VOLTAGE DIV:

$$G = \frac{1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{1}{1 + j\omega RC - \omega^2 LC}$$

THIS FOLLOWS THE TRADITIONAL FORM

$$\frac{k}{1 + \frac{j}{Q} \frac{\omega}{\omega_0} - \left(\frac{\omega}{\omega_0}\right)^2}$$

WHERE, FOR OUR CIRCUIT,

$$\left. \begin{aligned} \omega_0 &= 1/\sqrt{LC} \\ Q &= \frac{1}{R} \sqrt{\frac{L}{C}} \end{aligned} \right\} \text{EASILY ADJUSTABLE BY CHOOSING COMPONENTS}$$

$$k = 1 \quad (\text{GAIN CAN BE INTRODUCED } \Rightarrow \triangle)$$

MANY HIGH FREQUENCY ($f > 1\text{MHz}$) FILTER DESIGNS ARE DONE WITH PASSIVES (R, L, C). IF ONLY L & C ARE USED, VERY COMPLEX, HIGH-PERFORMANCE DESIGNS ARE POSSIBLE (SEE H&H FIGURE 5.3!).

AT LOW FREQUENCIES ($f < 1\text{MHz}$), THE INDUCTORS BECOME LESS PRACTICAL:

- LARGE, LOTS OF COPPER \Rightarrow BULKY, EXPENSIVE
- SUSCEPTIBLE TO ELECTROMAGNETIC INTERFERENCE
- PARASITIC RESISTANCE (USE MORE COPPER!)
- CAPACITANCE BETWEEN ADJACENT WINDINGS

CONSTRUCTING SUCH FILTERS IS BOTH AN ART AND A SCIENCE.

TRADITIONAL FORMS OF 2nd ORDER TRANSFER FUNCTIONS:

$$\frac{k}{1 + j\frac{\omega}{\omega_0} - \left(\frac{\omega}{\omega_0}\right)^2} \quad \text{LOW PASS}$$

$$\frac{-k\left(\frac{\omega}{\omega_0}\right)^2}{1 + j\frac{\omega}{\omega_0} - \left(\frac{\omega}{\omega_0}\right)^2} \quad \text{HIGH PASS}$$

$$\frac{k\frac{j\omega}{\omega_0}}{1 + j\frac{\omega}{\omega_0} - \left(\frac{\omega}{\omega_0}\right)^2} \quad \text{BAND PASS}$$

$$\frac{k\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)}{1 + j\frac{\omega}{\omega_0} - \left(\frac{\omega}{\omega_0}\right)^2} \quad \text{BAND STOP (NOTCH)}$$

- EACH OF THESE HAS A SERIES RLC EQUIVALENT.
- EACH HAS MANY POSSIBLE ACTIVE FILTER IMPLEMENTATIONS.
- ω_0 IS NOT THE CRITICAL POINT ANYMORE.

$$\left(\text{E.G. FOR LOW PASS, } \omega_{3dB} = \omega_0 \sqrt{b + \sqrt{1+b^2}}, \quad b \equiv 1 - \frac{1}{2Q^2} \right)$$

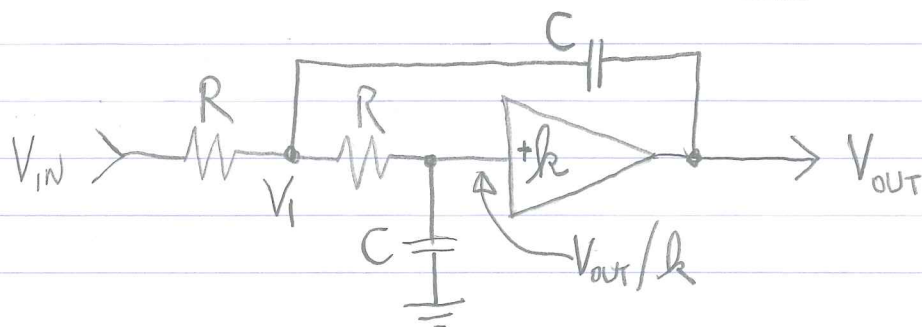
- Q CORRESPONDS TO CORNER SHARPNESS.
- k IS GAIN AT DC.

FOR THE PASSIVE VERSIONS, $k=1$. NOTE THAT WHEN $\omega = \omega_0$, GAIN = kQ .

ACTIVE FILTERS — USE OP AMPS, RESISTORS AND CAPACITORS TO IMPLEMENT A 2nd ORDER FILTER

- NO INDUCTORS

SALLEN-KEY (VCVS) LOW PASS FILTER



KCL @ V_1 :

$$\frac{V_{IN} - V_1}{R} + \frac{V_{OUT} - V_1}{1/j\omega C} + \frac{V_{OUT}/k - V_1}{R} = 0$$

$$\Rightarrow V_{IN} + (j\omega RC + \frac{1}{k})V_{OUT} = (2 + j\omega RC)V_1 \quad (1)$$

KCL @ V_{OUT}/k :

$$\frac{V_1 - V_{OUT}/k}{R} + \frac{0 - V_{OUT}/k}{1/j\omega C} = 0$$

$$\Rightarrow V_1 = \frac{V_{OUT}}{k} (1 + j\omega RC) \quad (2)$$

COMBINE (1) & (2) TO ELIMINATE V_1 :

$$\underline{\underline{G}} = \frac{\underline{\underline{V_{OUT}}}}{\underline{\underline{V_{IN}}}} = \frac{k}{1 + j(3-k)\omega RC - (\omega RC)^2}$$

WHICH LOOKS LIKE THE FAMILIAR, GENERIC 2nd ORDER LOW PASS.

2nd ORDER Sallen-Key (VCVS) FILTERS: IN TERMS OF THE STANDARD RESPONSE FUNCTIONS (P. 5, ABOVE):

LOW PASS (PREVIOUS PAGE)

k = AMPLIFIER GAIN, USED TO ADJUST Q .

$$Q = \frac{1}{3-k}$$

$$\omega_0 = 1/RC$$

HIGH PASS — SAME AS LOW PASS, WITH R & C SWAPPED.
SAME RELATIONS HOLD.

NARROW BAND PASS AND BAND STOP — THERE ARE NUMEROUS VARIATIONS. EVERY TEXTBOOK SEEMS TO OFFER A DIFFERENT BP FILTER!

USING k TO SET Q IS A POPULAR SIMPLIFICATION. IF WE ALLOW THE OTHER COMPONENT VALUES TO VARY, THEN WE COULD CHOOSE ANY Q AND ANY k .

FILTER PERFORMANCE IS FULLY SPECIFIED BY $G(\omega)$.

- FOR 2nd ORDER FILTERS, THE QUANTITIES k , ω_0 , AND Q ARE USEFUL.

BODE
PLOT

GAIN

$$G(\omega) = |G(\omega)| = \frac{|V_{out}|}{|V_{in}|}$$

FROM THIS, WE DEFINE THINGS LIKE...

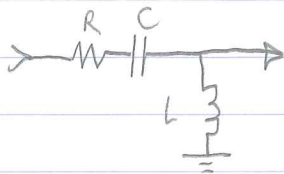
- PASS BAND
- STOP BAND
- SHARPNESS OF TRANSITION BETWEEN THE TWO.
- k , Q , ω_0

PHASE:

$$\tan(\varphi(\omega)) = \frac{\text{Im}(G)}{\text{Re}(G)}$$

$$\varphi(\omega) = \tan^{-1} \frac{\text{Im}(G)}{\text{Re}(G)} \pm n\pi \quad (n=0,1)$$

EXAMPLE: CHARACTERIZE THIS FILTER:



$$G = \frac{j\omega L}{1/j\omega C + R + j\omega L} = \frac{-\omega^2 LC}{1 + j\omega RC - \omega^2 LC}$$

Q: HIGH PASS, LOW PASS, BAND PASS OR NOTCH?

Q: DC GAIN $k = ?$

$\omega_0 = ?$

$Q = ?$

A: HELPS TO COMPARE TO STANDARD FORM:

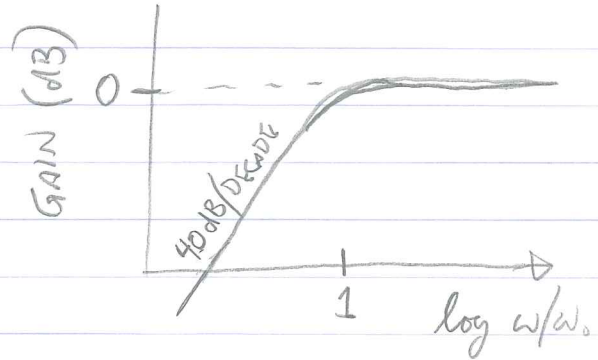
$$\underline{G} = \frac{-\omega^2 LC}{1 + j\omega RC - \omega^2 LC} = \frac{-k \left(\frac{\omega}{\omega_0}\right)^2}{1 + \frac{j}{Q} \left(\frac{\omega}{\omega_0}\right)^2 - \left(\frac{\omega}{\omega_0}\right)^2}$$

HIGH PASS,

$k = 1$

$\omega_0 = 1/\sqrt{LC}$

$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$



Q: WHAT IS THE PHASE?

A:
$$\underline{G} = \frac{+\omega^2 LC (-1 + \omega^2 LC + j\omega RC)}{(1 - \omega^2 LC)^2 + (\omega RC)^2}$$

$$\tan \phi = \frac{\text{Im}(\underline{G})}{\text{Re}(\underline{G})} = \frac{\omega RC}{\omega^2 LC - 1}$$

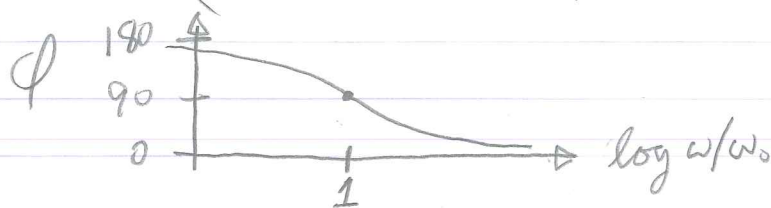
$\text{Im}(\underline{G}) > 0$

$\text{Re}(\underline{G}) > 0$ IF $\omega^2 LC > 1 \Rightarrow$ QUADRANT I

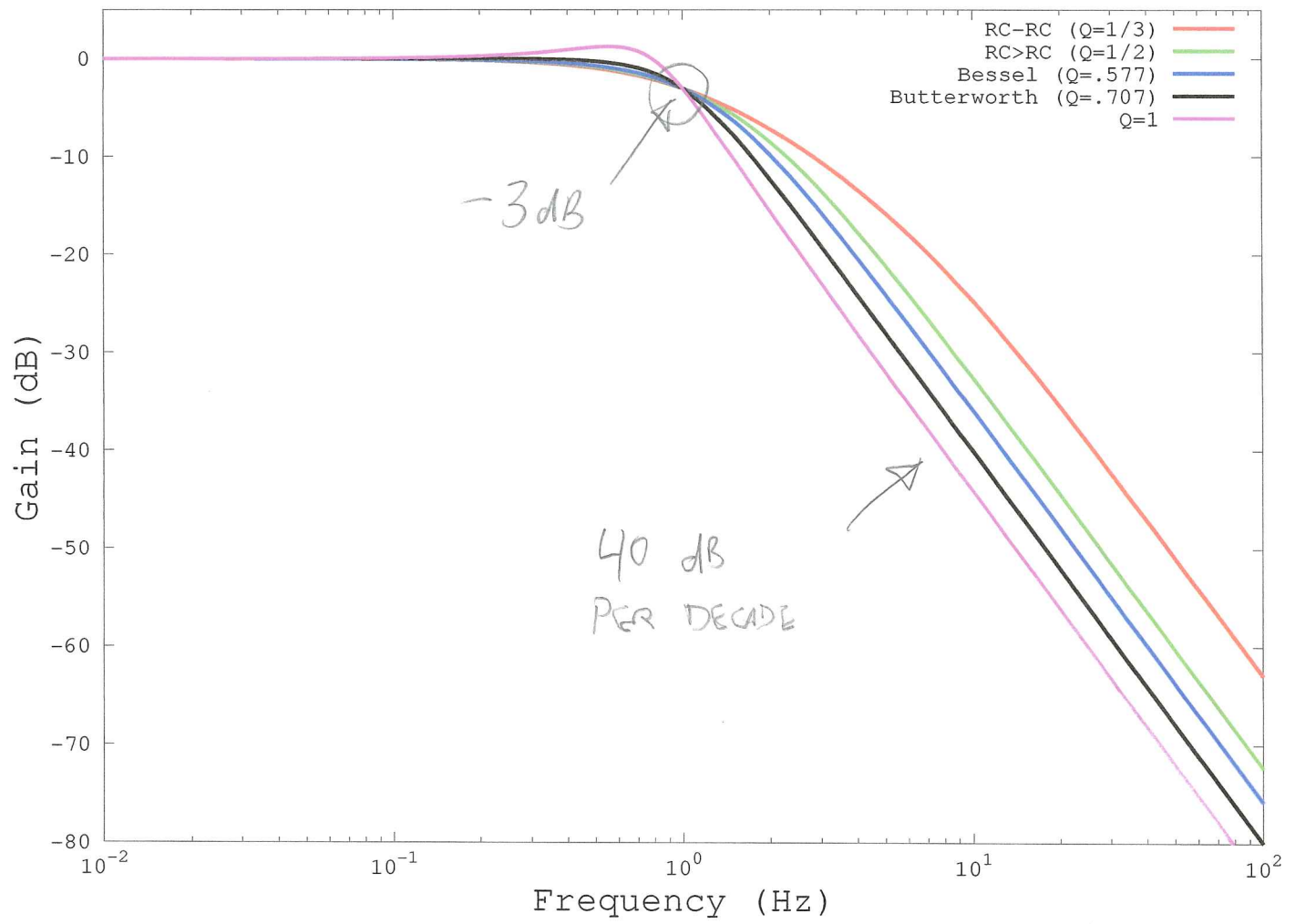
< 0 IF $\omega^2 LC < 1 \Rightarrow$ QUADRANT II

$0 \leq \phi \leq \pi$

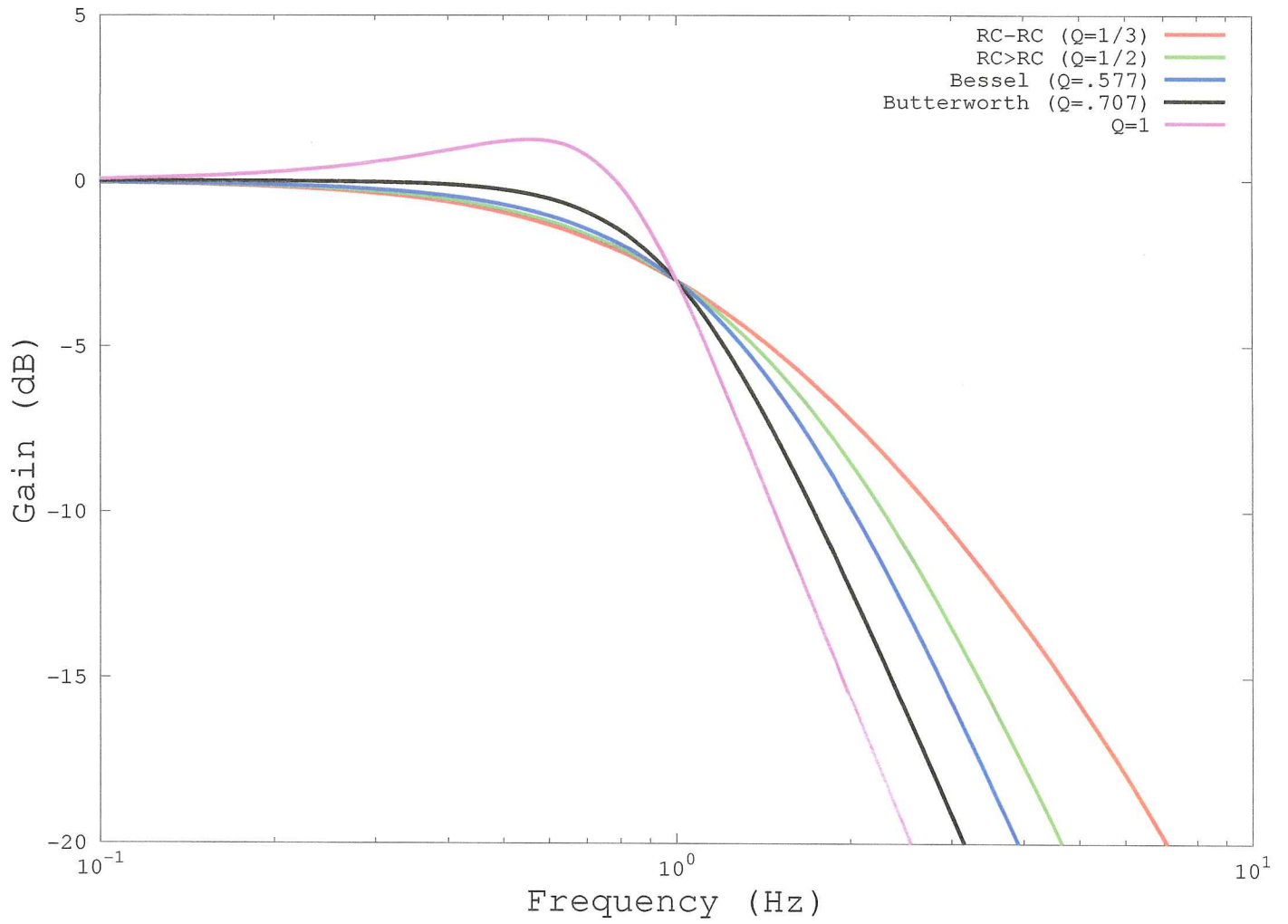
$$\phi = \begin{cases} \tan^{-1} \left(\frac{\omega RC}{\omega^2 LC - 1} \right), & \omega > \omega_0, \quad 0 \leq \phi \leq \frac{\pi}{2} \\ \pi + \tan^{-1} \left(\frac{\omega RC}{\omega^2 LC - 1} \right), & \omega < \omega_0, \quad \frac{\pi}{2} \leq \phi \leq \pi \end{cases}$$



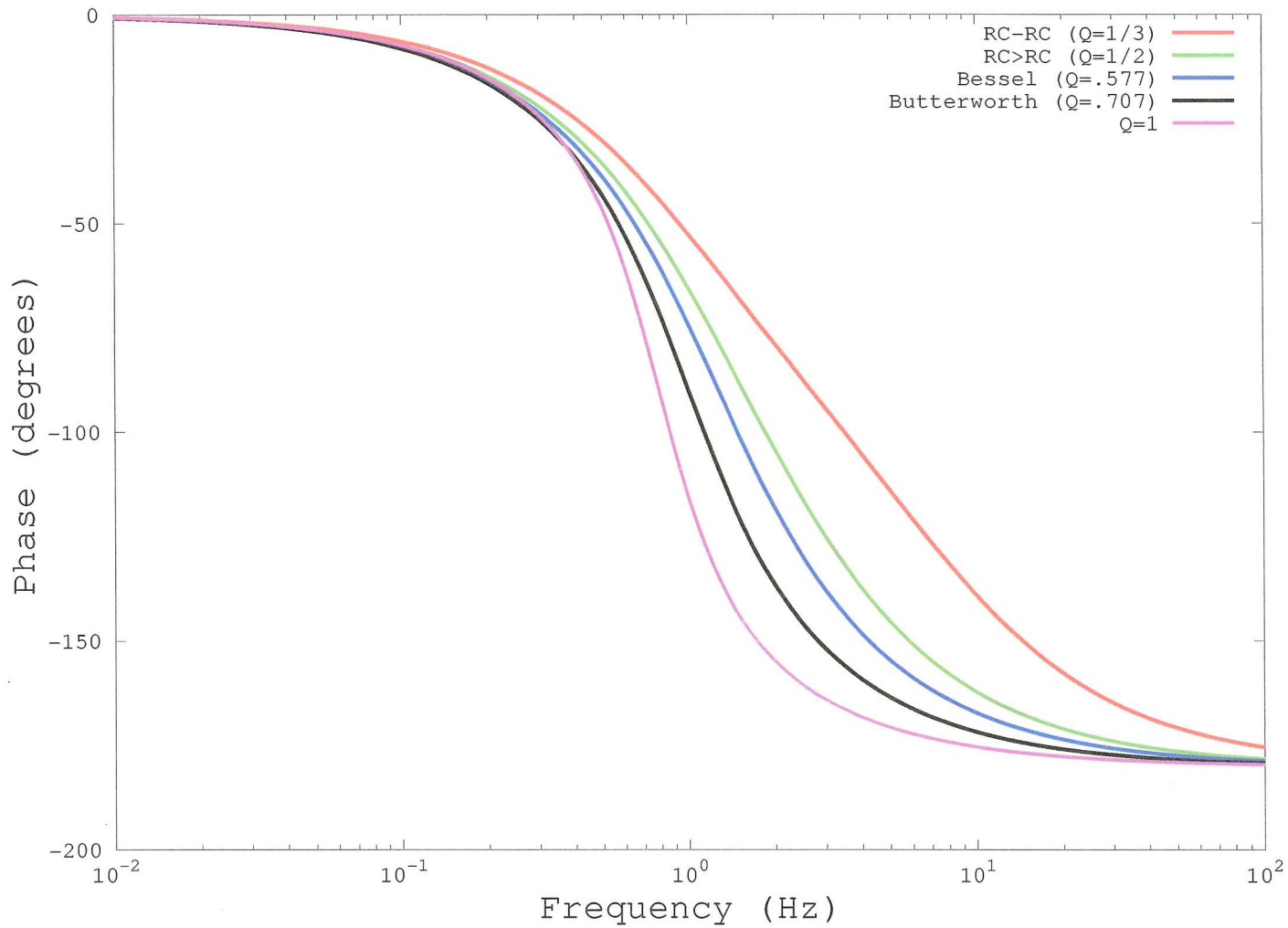
Gain of 2nd-order filters



Gain of 2nd-order filters



Phase of 2nd order filters



Q: BASED ON THE PHASE PLOT, WHICH FILTER SHOULD DISTORT THE PHASE LEAST?

Q: SUPPOSE THE SIGNAL IS DELAYED BY A FIXED TIME. WHAT IS THE RESULTING $\phi(\omega)$?

$$e^{i\omega t} \rightarrow e^{i\omega(t-\Delta t)} = e^{i(\omega t + \phi)}$$

$$\Rightarrow \phi = -\omega \Delta t$$

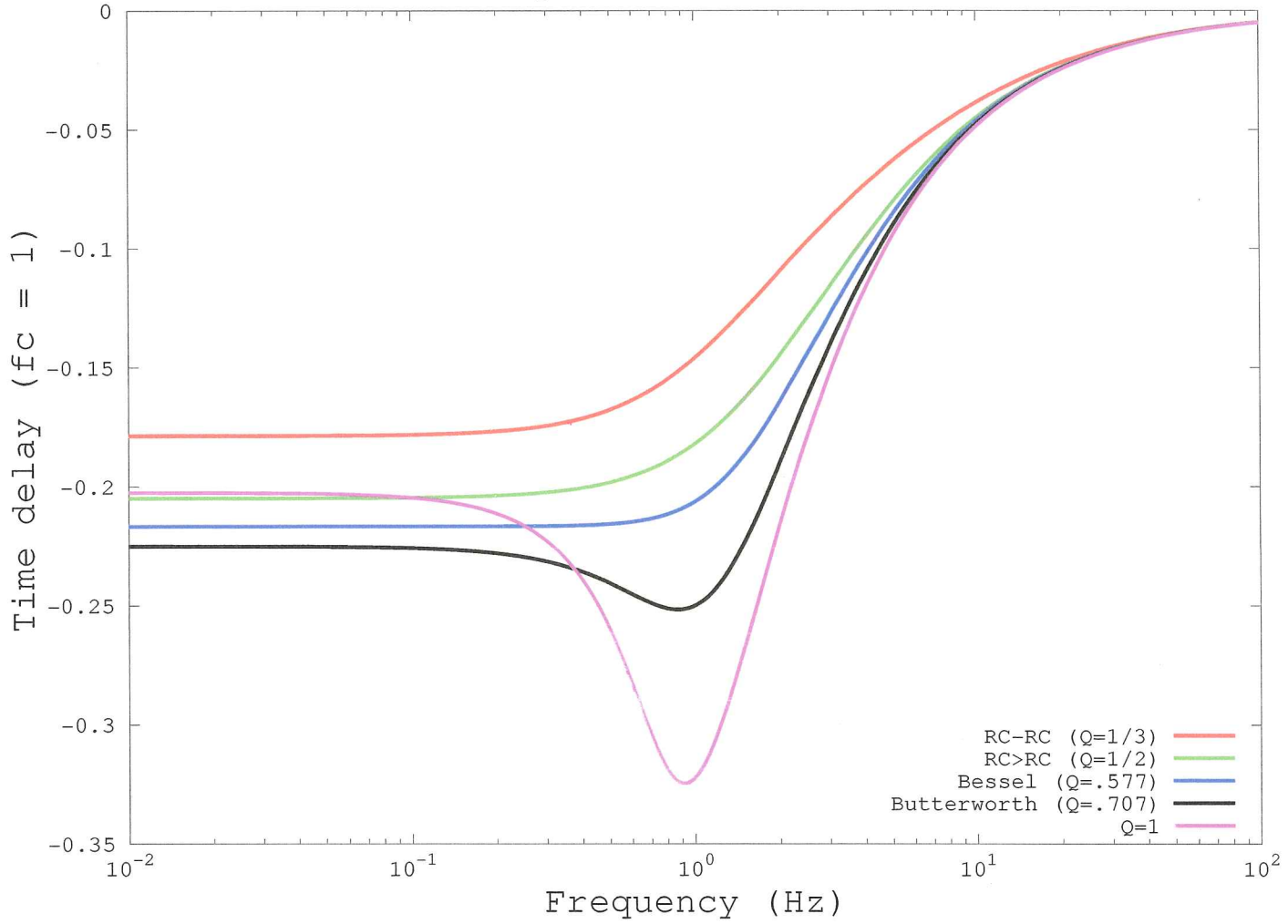
TURNING THIS AROUND, THE TIME DELAY ASSOCIATED WITH PHASE ϕ AT FREQUENCY ω IS:

$$\boxed{\Delta t_{\omega} = -\phi(\omega)/\omega}$$

IT IS INTERESTING TO PLOT Δt_{ω} . IF IT IS FLAT OVER THE PASSBAND, THEN THE FILTER LEAVES ITS OUTPUT UNDISTORTED BUT DELAYED.

- THE "BESSEL FILTER" IS A PRESCRIPTION FOR VERY FLAT TIME DELAY OVER THE PASSBAND.

Time delay of 2nd order filters

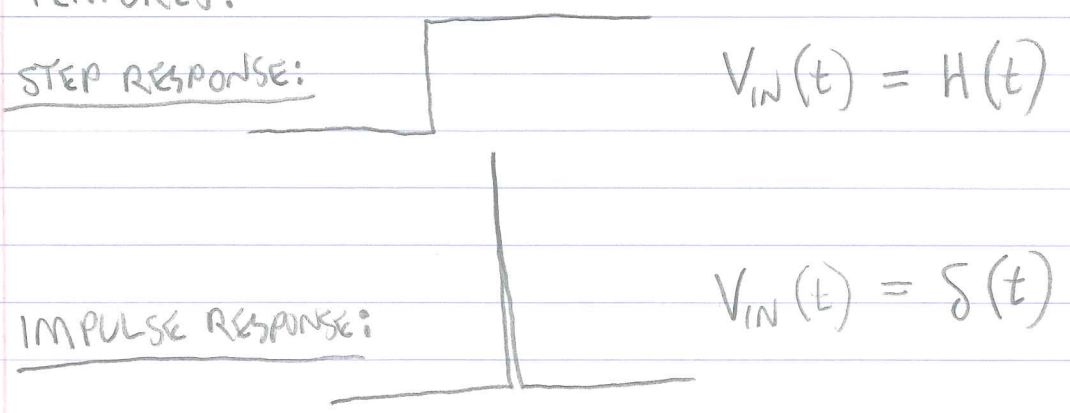


TIME DOMAIN

SO FAR, WE'VE BEEN LOOKING AT WHAT A FILTER DOES AS A FUNCTION OF FREQUENCY, ω (THE "FREQUENCY DOMAIN"). IT IS ALSO WORTH TESTING FILTERS IN THE "TIME DOMAIN," I.E. COMPARE

$$V_{out}(t) \text{ vs. } V_{in}(t).$$

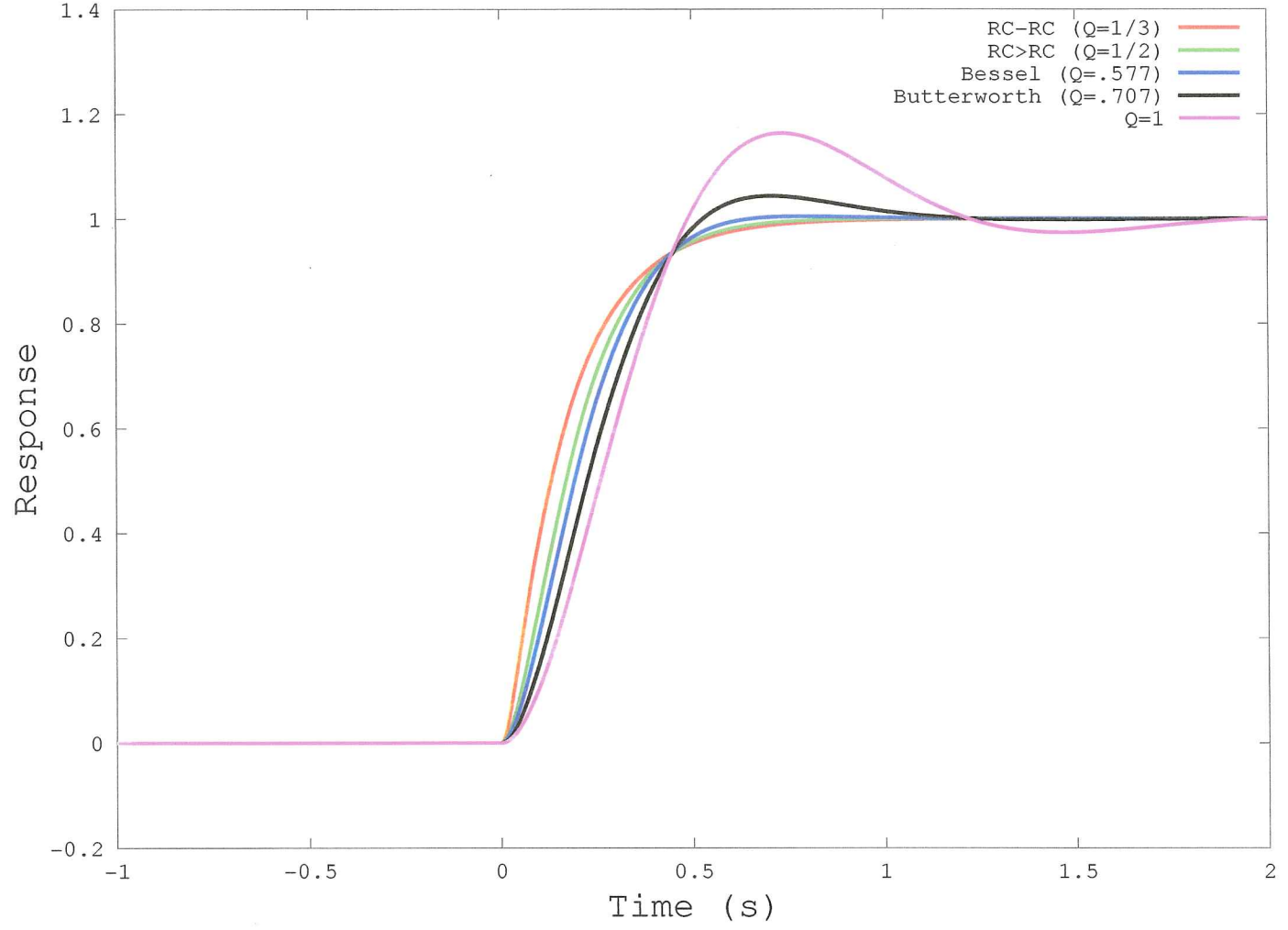
- WE EXPECT A LOW PASS FILTER TO SMOOTH OUT RAPID VARIATIONS IN THE INPUT WAVEFORM.
- IT IS USEFUL TO TRY $V_{in}(t)$ WITH VERY SHARP FEATURES:



A TYPICAL PROBLEM IS "RINGING," WHICH IN AN AUDIO SIGNAL SOUNDS LIKE A TONE FROM A TIMPANI, OR WORSE YET A BELL, WHEN IT IS STRUCK.

TIME DOMAIN ANALYSIS REQUIRES FOURIER TRANSFORMS. ALAS, THE CALCULATIONS ARE BEYOND THE SCOPE OF THIS COURSE. NEVERTHELESS, IT IS WORTH LOOKING AT SOME RESULTS.

Step response of 2nd order filters



Impulse response of 2nd order filters

