

# Today

## 2/1/11 Lecture 3

- **Higher Order Active Filters**
  - Cascaded Filters and Higher Order Filter Response
  - VCVS filter design
  - Butterworth Active Filters
    - Design
    - Performance
- **Homework**
  - See next slide
- **Reading**
  - H&H Ed 2 268-276
- **Lab this week**
  - Lab 2
  - Do pre-lab of lab 2 BEFORE lab on Thursday
  - Lab 1 and 2a due Friday at 10am
- **Quiz**

# Homework

## Due 2/8/11 HW3

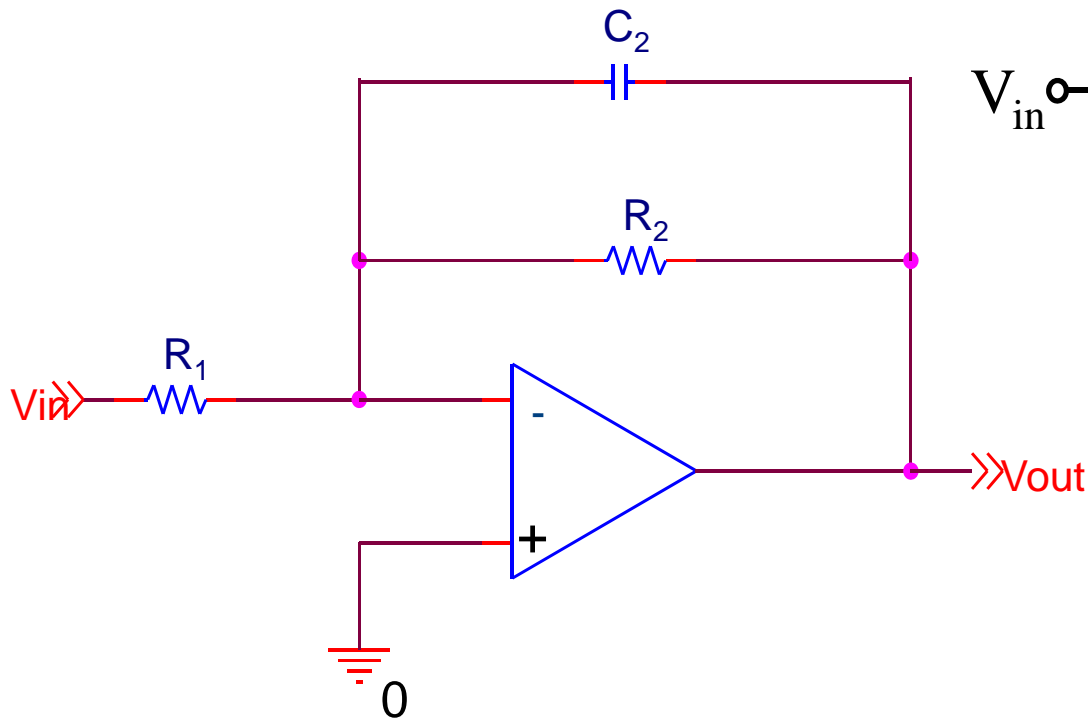
1. Calculate the complex transfer functions for the two active filters on the next slide. Write expression for the magnitudes of their gains and phases as functions of frequency. What are the cut-off (3dB) frequencies of the two filters? What are their gains at zero frequency? What are the phase shifts of their outputs (relative to the inputs) at very low and very high frequencies?
2. Design a 2- pole Butterworth low pass filter with cut-off frequency  $\sim 30\text{kHz}$ .

## Rest of HW delayed until next time

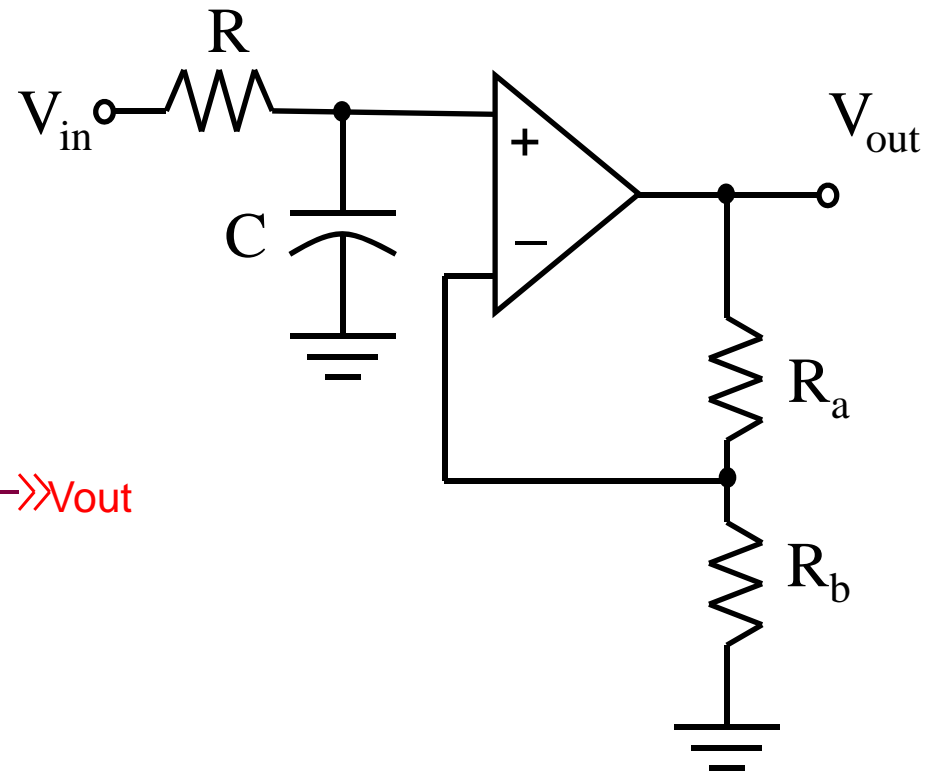
1. What is the formula for its gain as a function of frequency? What is its attenuation (in dB) at  $f=3f_c$ ?
2. Design a 4- pole Butterworth high pass filter with cut-off frequency  $\sim 60\text{kHz}$ . What is the formula for its gain as a function of frequency? What is its attenuation (in dB) at  $f=f_c/2$ ?

# 1<sup>st</sup> Order Active Filters

INVERTING



NON-INVERTING



# Cascaded Filters

Cascaded filters multiply to give total transfer function

$$T(f) = T_1(f)T_2(f)T_3(f) \dots$$

Cascaded passive filters increase the order of the total filter.

Number of cascaded FIRST order filters = filter order

$$|\tilde{T}_{total\_n}(f)| = \left( \frac{1}{\sqrt{1 + (f/f_c)^2}} \right)^n \approx \left( \frac{1}{(f/f_c)^n} \right) \text{ for } f \gg f_c$$

***/Voltage Slope in stopband/ = order of filter***

Cascaded first order filters have weak knees.

$$|\tilde{T}_{total\_n}(f_c)| = \left( \frac{1}{\sqrt{2}} \right)^n$$

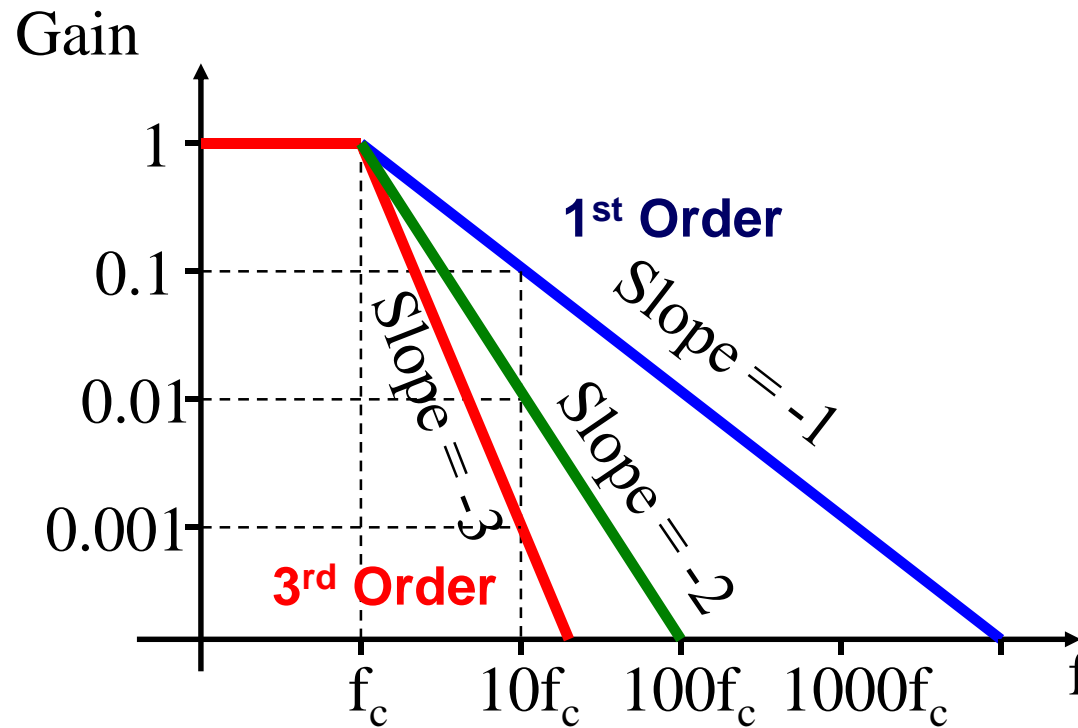
Each cascaded passive filter's loads previous section.

***– Degrades response***

# Desired Low-Pass Filter Response

Order = | Slope in log-log plot |

Near Ideal Low Pass Filters  
(with Gain = 1)

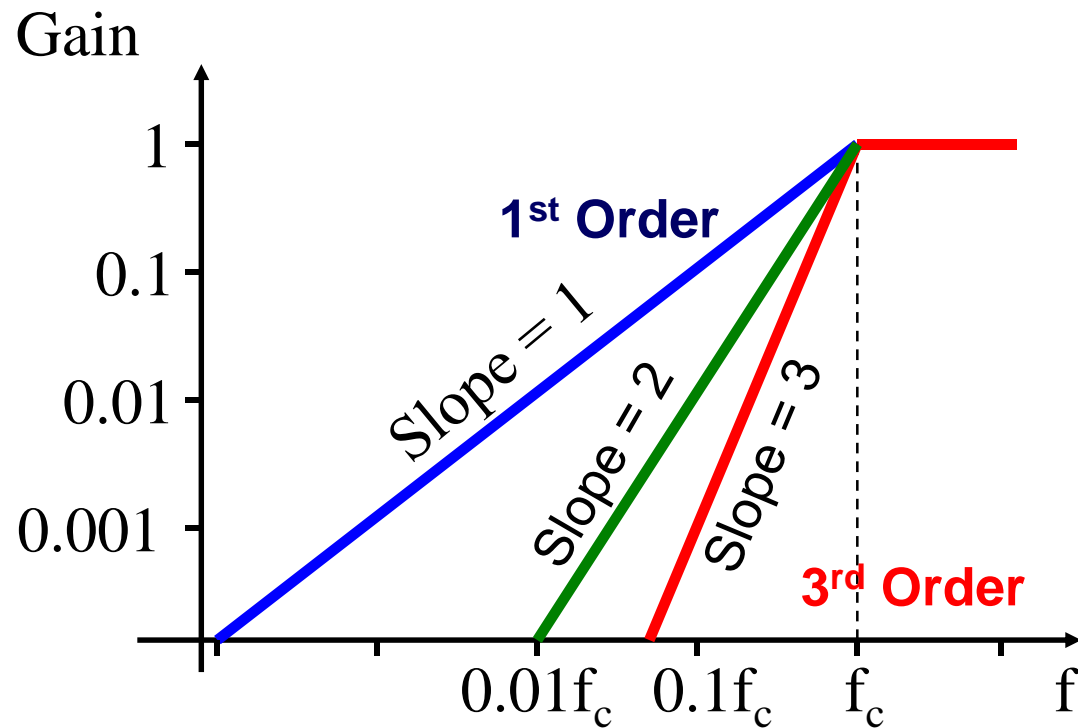


LPF with different roll-off rates

# Desired High-Pass Filter Response

**Order = | Slope in log-log plot |**

Near Ideal High Pass Filters



HPF with different roll-off rates

# Cascaded Active Filters

Cascaded passive filters increase the order of the total filter.

*Same is true for active filters*

Each cascaded passive filter's loads previous section.

*Not true with active filters*

- *OpAmp has low current input => High input impedance*
- *OpAmp provides lots of current => Low output impedance*

Cascaded first order filters have weak knees.

*True with active filters*

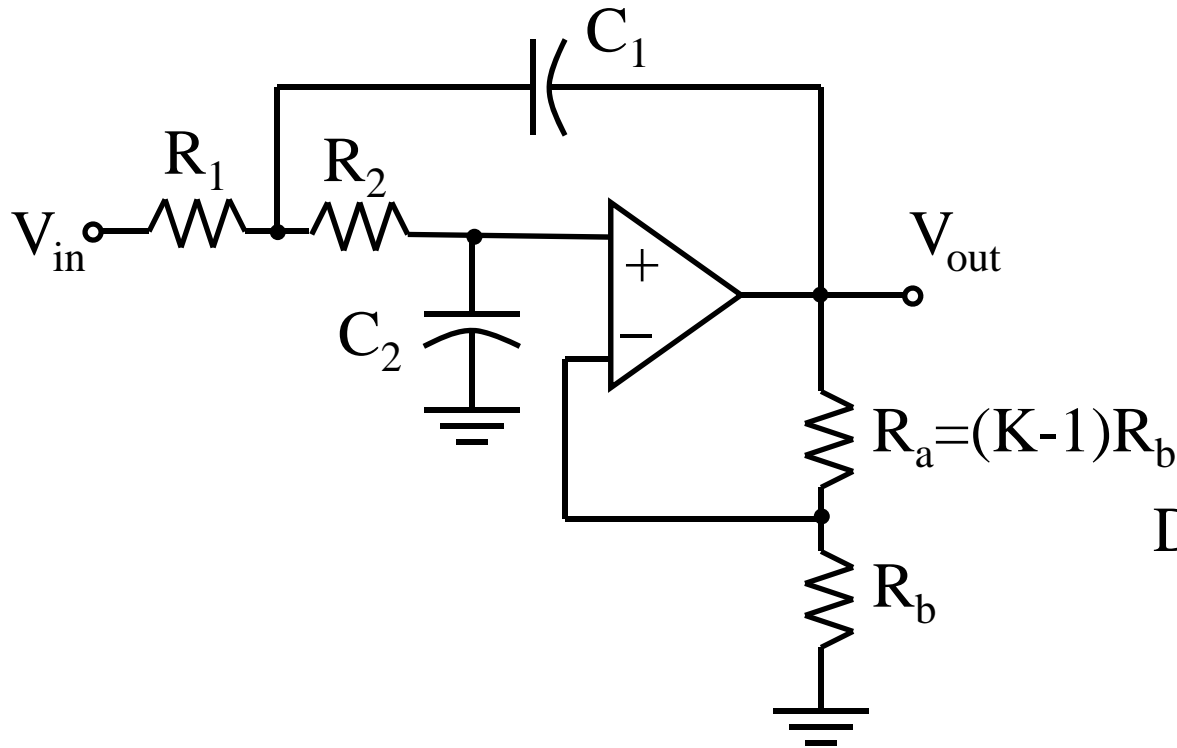
***But . . .***

*By crafting filter designs in cascaded active filters*

- *Sharper knees, better time response, OR flatter phase response*

# Sallen-Key 2<sup>nd</sup> Order Low-Pass Filter

A.K.A. Low Pass VCVS (voltage-controlled voltage-source)



Passband Gain =  $K$

Dual filtering at high  $f$

Different filter designs have

- Different  $K$  values

- Different  $RC$  values

- $RC$  determines  $f_c$ , *BUT*

- $f_c$  not always  $1/(2\pi RC)$

The roll-off rate for a two-pole (2<sup>nd</sup> order) filter is

**20 decades/decade in Voltage OR -40 dB/decade in POWER.**

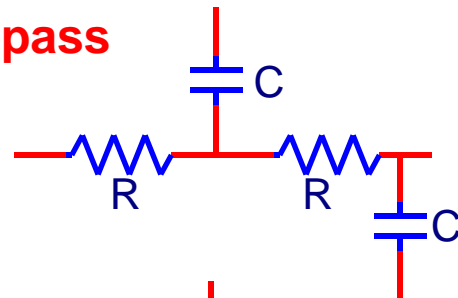


# Design of 2<sup>nd</sup> Order Active Filters

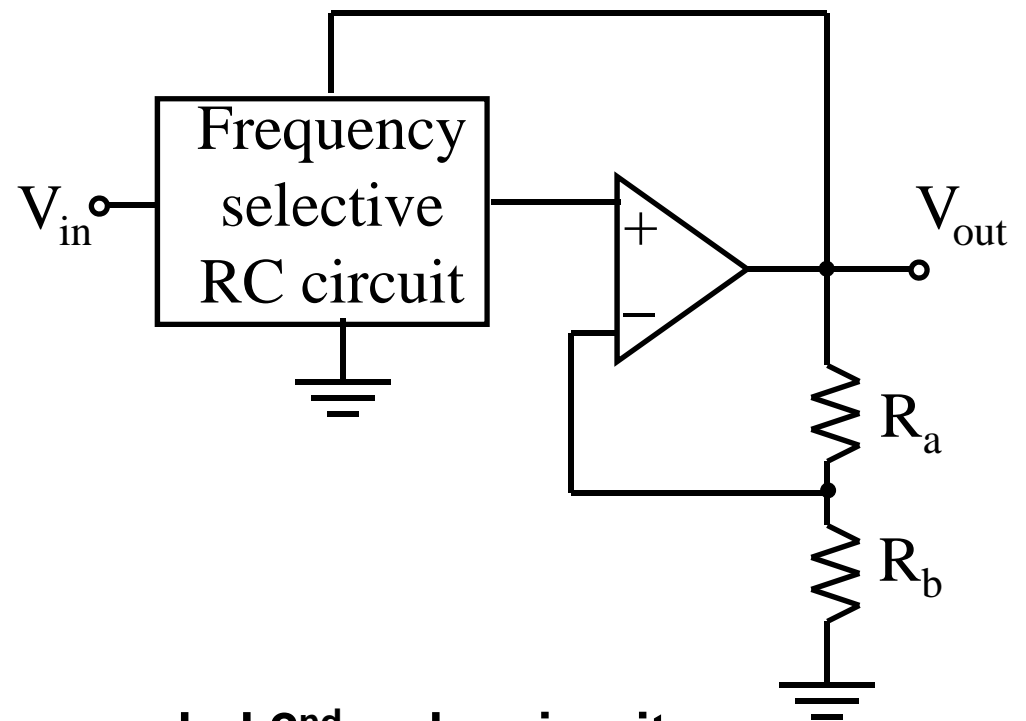
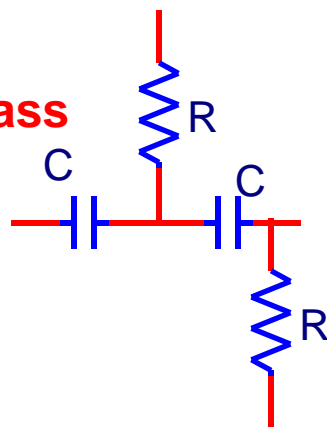
All the 2<sup>nd</sup> order active filter circuits have the same basic design

– Frequency selective RC circuit can be

- Band-pass (see H&H Figure 5.16)
- Low-pass



- High-pass



Higher order (>2) active filters are cascaded 2<sup>nd</sup> order circuits

- Built up by cascading basic filter circuits:  $V_{out\_previous} \Rightarrow V_{in\_next}$
- Only one VCVS and one op-amp is needed per every two orders

# 2<sup>nd</sup> Order Butterworth Design

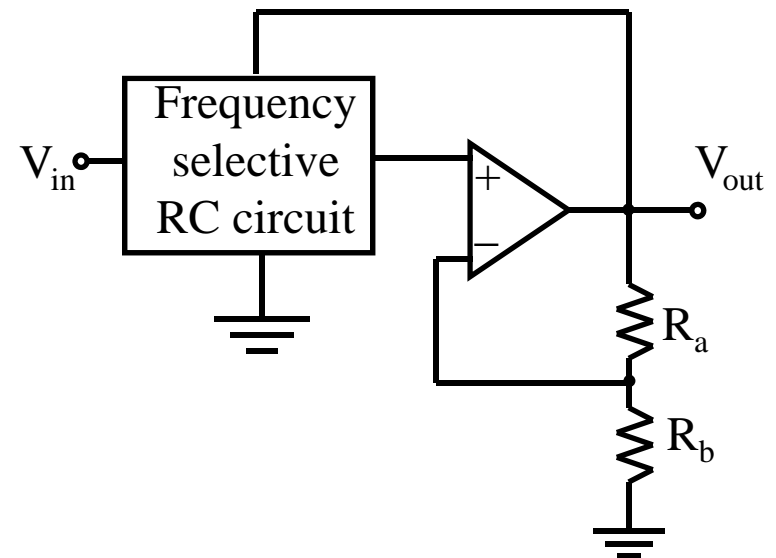
1-stage (2-pole) filter design:

	Butterworth	Bessel
Poles	K	$f_n$ K
2	1.59	1.27    1.27

Start with desire  $f_c$

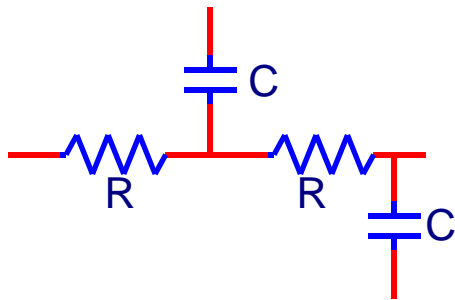
Butterworth:  $RC=1/(2\pi f_c)$  and  $R_a=(K-1)R_b$

Typically  $R_b=R$  in RC  
R is typically 10-100K ohm.  
(not hard rule)



# Higher Order Butterworth Design

VCVS Low-Pass Filter Design:



$f_c$  is desired 3dB frequency  
of total n-pole filter

Poles	Stage(n)	Butterworth	Bessel	
		$K_n$	$f_n$	$K_n$
2	1	1.59	1.27	1.27
4	1	1.15	1.43	1.08
	2	2.24	1.61	1.76
6	1	1.07	1.61	1.04
	2	1.59	1.69	1.36
	3	2.48	1.91	2.02

Butterworth:

RC circuit is the same for all stages (Determined by desired  $f_c$ )

Only the gain changes for each stage

$$RC = 1/(2\pi f_c) \quad \text{and} \quad R_a = (K_n - 1)R_b$$

Typically gains increase down the line to avoid dynamic range issues

Total Gain of multi-stage filter = product of the  $K_n$ 's

For high pass filter: Same design table except:

Use high pass VCVS

Use  $1/f_n$  to determine RC

# 4<sup>th</sup> Order Butterworth

2-stage (4-pole) Filter designs:

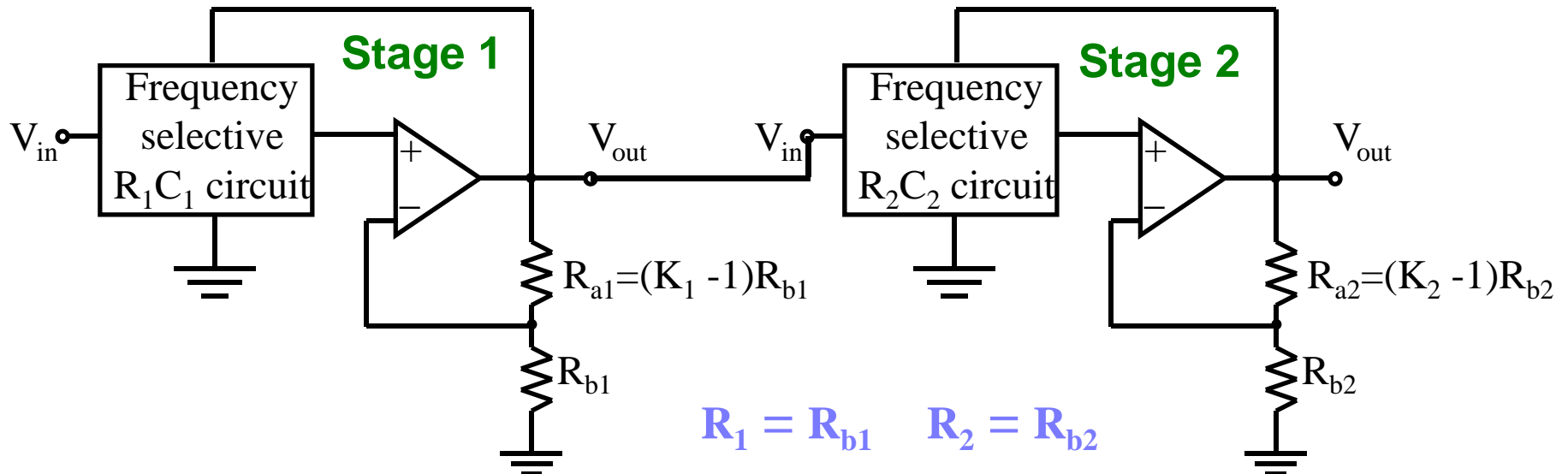
Poles	Stage(n)	Butterworth	Bessel
		$K_n$	$f_n$ $K_n$
4	1	1.15	1.43   1.08
	2	2.24	1.61   1.76

Butterworth:

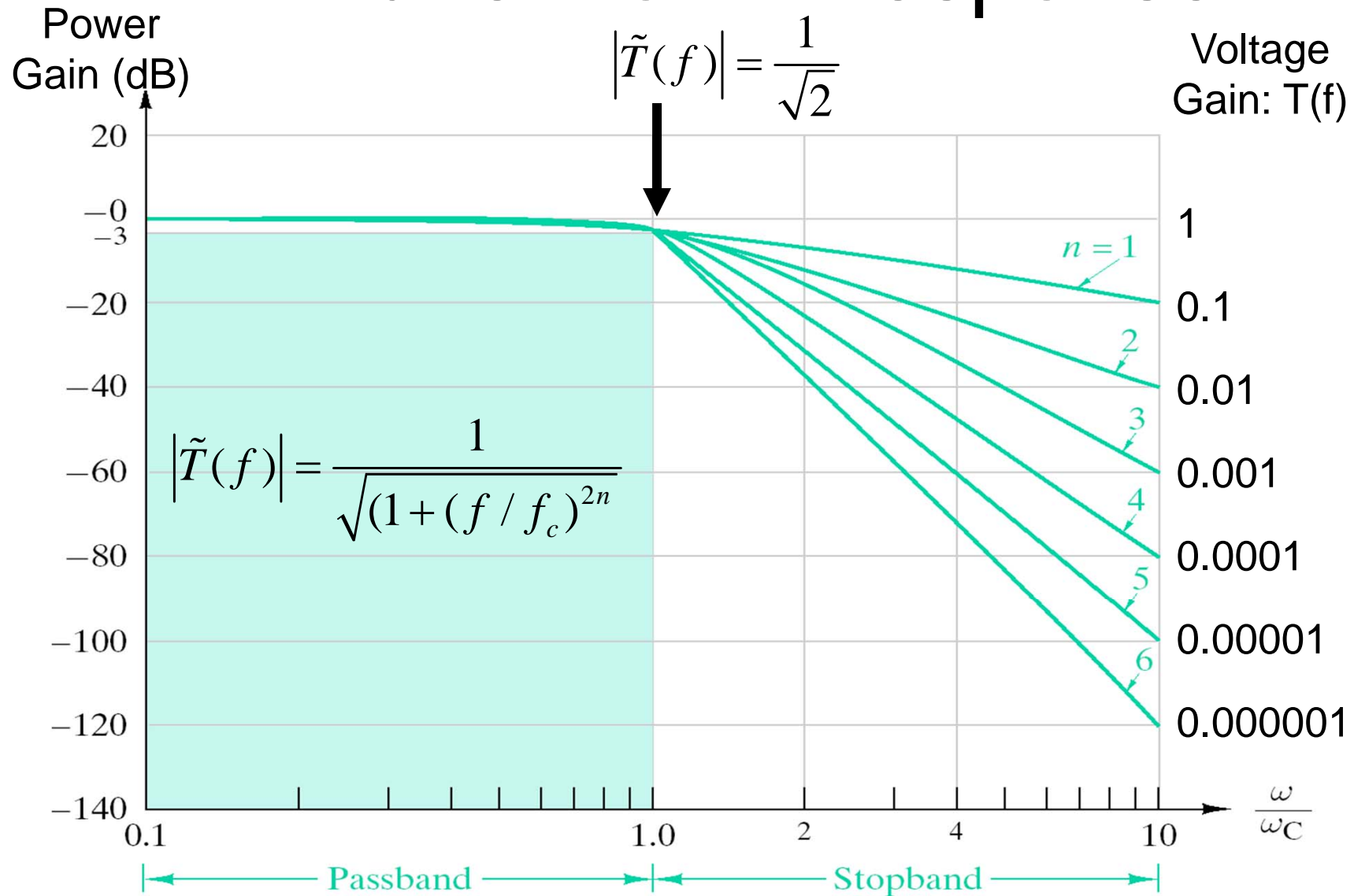
$$R_1 C_1 = R_2 C_2 = 1/(2\pi f_c)$$

$$R_{a1} = (K_1 - 1)R_{b1} = (1.15 - 1)R_{b1}$$

$$R_{a2} = (K_2 - 1)R_{b1} = (2.24 - 1)R_{b2}$$



# Butterworth Response



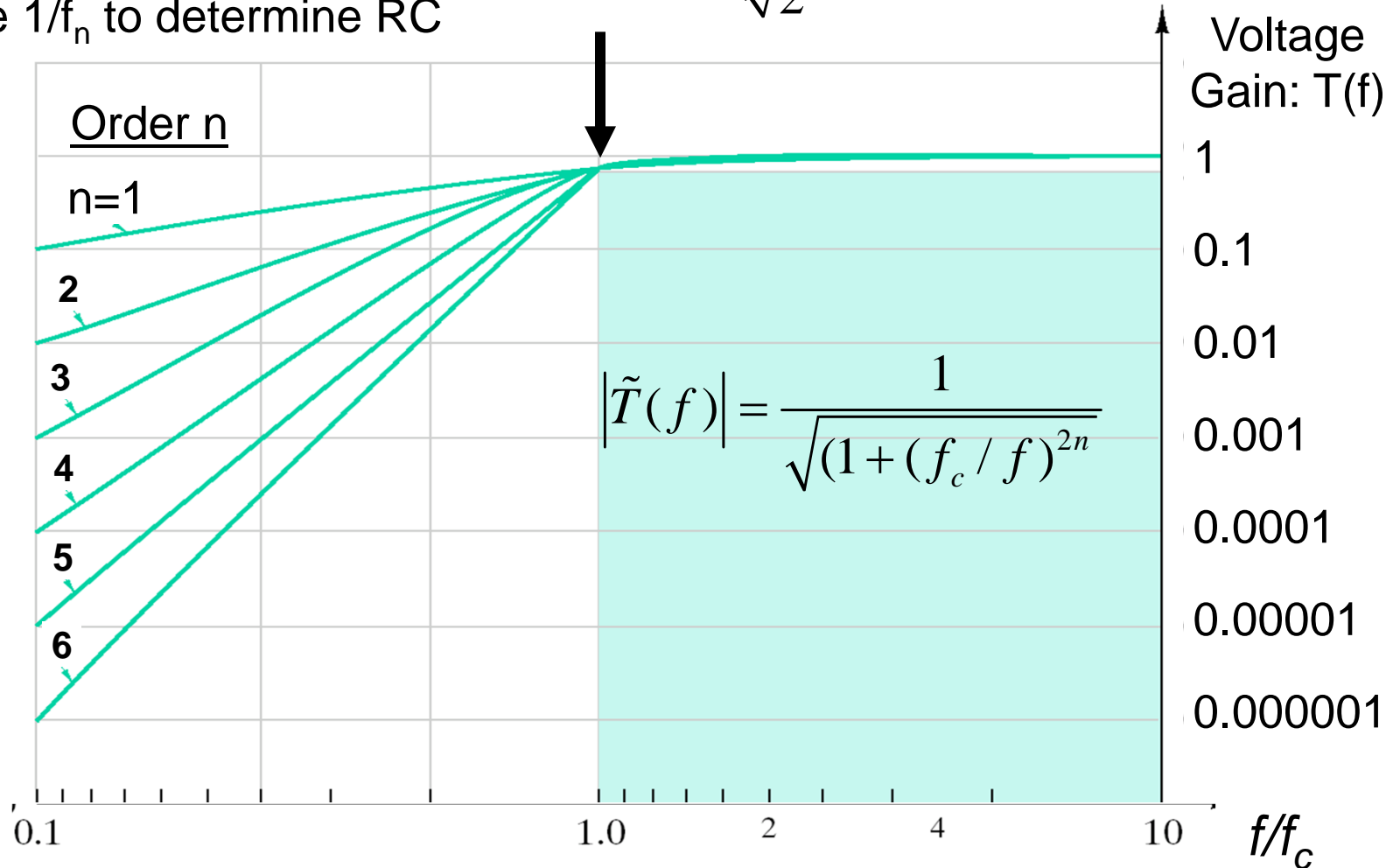
# Butterworth High Pass Filter Response

Same design table except:

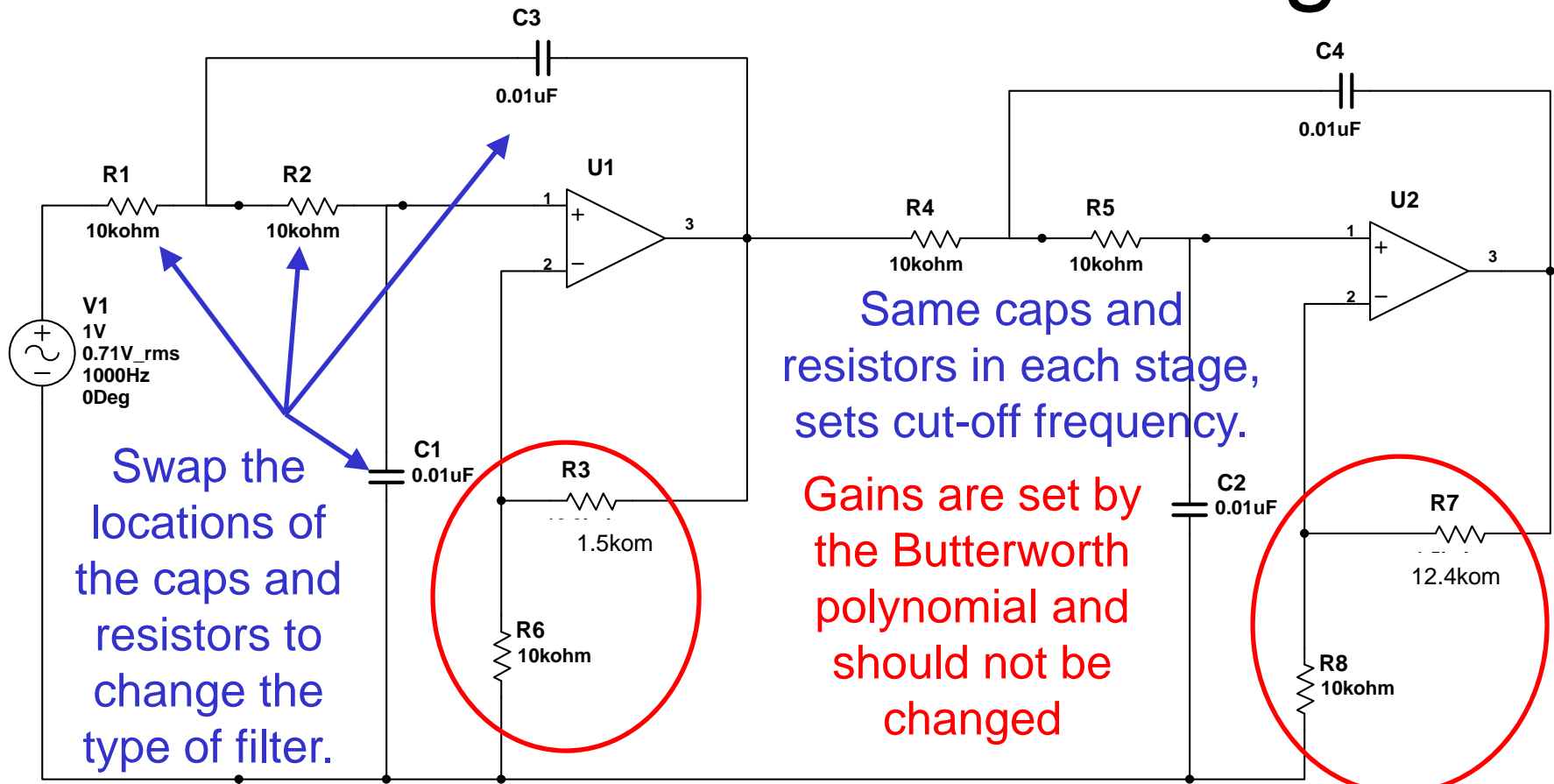
Use high pass VCVS

Use  $1/f_n$  to determine RC

$$|\tilde{T}(f)| = \frac{1}{\sqrt{2}}$$



# Review Butterworth Design



Swap the locations of the caps and resistors to change the type of filter.

Same caps and resistors in each stage, sets cut-off frequency.

Gains are set by the Butterworth polynomial and should not be changed

Choose caps and resistors to adjust cut-off frequency. Use resistor in RC that is equal to or close to  $R_b$

Different gains in each stage, as per design table.