

Physics 262

Homework #9

Show your work.

In error analysis, we distinguish between *systematic* and *random* error. Systematic error is predictable, the same every time. If a measurement of some quantity x includes a series of systematic errors ($\epsilon_1, \epsilon_2, \epsilon_3, \dots$), the result is $x' = x + \sum_n \epsilon_n$. Random errors are uncorrelated (different every time, with a mean of zero). When errors are uncorrelated, they add in quadrature: $x' = x \pm \sqrt{\sum_n \epsilon_n^2}$. Sometimes (as in this homework problem), we have a series of errors with specified magnitude but unknown sign. Even if in principle they are systematic, we can't add these errors linearly, because the signs are unknown. If we regard the sign as random (with equal probability of $+$ or $-$), these errors are uncorrelated. An honest error estimate is then calculated by summing the error magnitudes in quadrature, as with true random error.

Consider an analog tape deck. It reads in a 0-3 V signal and records it to magnetic tape, with an error of $\pm 10 \mu\text{V}$. On playback, an additional error of $\pm 10 \mu\text{V}$ applies. Two such tape decks may be used together (one playback, one record) to copy a tape.

Consider also a digital recording system. This system has an input range of 0-3 V, a 16-bit ADC, and a recording error of $\pm 10 \mu\text{V}$. Let us assume that our playback system, with 16-bit DAC, also contributes $\pm 10 \mu\text{V}$ error.

1. Find the step size and quantization error of the digital recording system.
2. What is the total error involved in recording the digital signal and then playing it back? Calculate and compare the error of the analog system.
3. Is there a quantization error in digital playback as well as in recording? Explain why or why not.
4. The digital recording is sent to a friend, who copies it digitally and sends it to another friend, and so on. When the N th copy is played back, what is the error compared to the original signal?
5. The analog tape is sent to a friend, who makes a copy and sends it to another friend, and so on. What is the error when the N th tape is played back? At what N does digital copying give less error?