Physics 331 Problem 3 — due Oct. 21, 2004

Gravitational force law

In the first problem you were asked to calculate the distance between the planets Earth and Mars on a given day \( d \) of year 2000

\[
\text{distance} = |\mathbf{x}_m(d) - \mathbf{x}_e(d)|,
\]

where \( \mathbf{x}_m \) and \( \mathbf{x}_e \) are two-dimensional vectors giving the locations of Mars and the Earth. In preparation for a calculation of orbital trajectories we will now calculate the force due to the combined gravitational pulls of the Earth, Mars and the Sun. The acceleration at a point \( \mathbf{x} \) is

\[
\mathbf{A}(\mathbf{x}, d) = -GM_s \frac{\mathbf{x}}{|\mathbf{x}|^3} + GM_e \frac{\mathbf{x}_e(d) - \mathbf{x}}{|\mathbf{x}_e(d) - \mathbf{x}|^3} + GM_m \frac{\mathbf{x}_m(d) - \mathbf{x}}{|\mathbf{x}_m(d) - \mathbf{x}|^3},
\]

where \( G = 6.67 \times 10^{-11} \text{ m}^3 \text{ sec}^{-2} \text{ kg}^{-1} \) and \( M_i \) is the Mass of object \( i \). When calculating orbits for Mars-bound space-craft please be sure to use consistent units!

- Sun 1.99 \times 10^{30} \text{ kg}
- Earth 5.94 \times 10^{24} \text{ kg}
- Mars 6.42 \times 10^{23} \text{ kg}

The problem:

Write a program which takes as input arguments day \( d \) and a 2-component position vector \( \mathbf{x} \), and returns as output value the acceleration vector \( \mathbf{A} \). Use this function to compute the following accelerations.

a. \( d = 100, \mathbf{x} = [1.111, 1.001] \times 10^{11} \text{ m} \)

b. \( d = 140, \mathbf{x} = [1.111, 1.001] \times 10^{11} \text{ m} \)

c. \( d = 262, \mathbf{x} = [1.763, 1.443] \times 10^{11} \text{ m} \)

Please e-mail the following to my address kankel@solar.physics.montana.edu:

1. The acceleration \( \mathbf{A} \) for each case above.

2. A copy of the program you wrote.

3. A copy of the command(s) used to run your program.