

## Rocket Science

An unmanned rocket has engines which burn fuel at a rate of

$$R_B = -\frac{dM}{dt} = 2 \text{ kg/sec} . \quad (1)$$

Note that  $dM/dt < 0$  since the fuel mass is decreasing as it burns away. The rocket begins with 60 kg of fuel and an unfueled mass 30 kg, making the combined mass

$$M(t) = (90 - 2t) \text{ kg} , \quad (2)$$

for  $t \leq 30$  sec until all fuel is burned. The engines generate a force (thrust) of

$$F_t = 1000 \text{ Newtons} , \quad (3)$$

during burn (an exhaust speed of  $\sim 500$  m/sec). The aerodynamic drag is

$$F_D = 0.075 v^2 , \quad (4)$$

due to an area  $A = 0.25 \text{ m}^2$  and drag coefficient  $C_D = 0.5$ . Setting the rate of change of upward momentum,  $M(t)v$ , equal to the sum of upward thrust, downward drag and downward gravity we find the force-law

$$M(t) \frac{dv}{dt} = F_t - F_D - M(t)g .$$

Dividing by  $M(t)$  gives the differential equation for  $v(t)$

$$\frac{dv}{dt} = \frac{1000 - 0.075v^2}{90 - 2t} - 9.8 , \quad (5)$$

which applies for  $0 \leq t \leq 30$  sec.

The rocket begins from rest,  $v = 0$ , at time  $t = 0$ . Solve the differential equation, (5), to find the rocket's velocity at  $t = 15$  sec and  $t = 30$  sec. Please check that your answer is reliable (i.e. solve with two different time steps).

Please e-mail the following to my address `kankel@solar.physics.montana.edu`:

1. The velocities  $v(15)$  and  $v(30)$ .
2. A copy of the program you wrote.
3. A copy of the command(s) used to run your program, which should let me know what size time-step you have taken, and which solution algorithm you used.
4. A comment on the reliability of 1. i.e. tell me the change in  $v(30)$  resulting from two different time steps.