

Power Spectrum Units

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Since power spectra are used in many practical measurement contexts, it is worth understanding the typical units, how to interpret them, and how to arrive at RMS values by integrating the spectra. It is easy to become confused since real measurements take a finite period of time (one hopes), while the Fourier transform is defined over infinite limits.

Consider an accelerometer time series $a(t)$, measuring vibration in units of $g = 10 \text{ m/s}^2$. If the data are taken over a time interval T , then the mean square acceleration may be written

$$\langle a^2 \rangle = \frac{1}{T} \int_0^T |a|^2 dt, \quad \text{units: } g^2. \quad (1)$$

The Fourier transform of the acceleration data, of course, must likewise be calculated by integrating from time $t = 0$ to $t = T$:

$$\tilde{a}(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^T a(t) e^{-i\omega t} dt, \quad \text{units: } g \text{ s}. \quad (2)$$

If this bothers you, imagine that a square window function is used to define $a(t)$, and that the integral is over infinite limits. We now define a finite-time vibration spectrum,

$$S_a(\omega) = \frac{\tilde{a}^* \tilde{a}}{T}, \quad \text{units: } g^2 \text{ s}. \quad (3)$$

Using Parseval's theorem, we may relate the quantities in equations 1 and 3:

$$\langle a^2 \rangle = \int_{-\infty}^{\infty} S_a(\omega) d\omega. \quad (4)$$

Note the infinite limits on frequency, since the transform integral (equation 2) is defined for all ω . It also follows that a mean-square accelerations can be unambiguously associated with any *frequency interval* by modifying the limits of this integration.

We have used angular frequencies in keeping with typical notation in theoretical physics. However, now it should be evident to you why engineering vibration power spectra are commonly given in g^2/Hz (*cf.* equation 3), or amplitude spectra in $g/\sqrt{\text{Hz}}$.