

## Ph 566 Homework 1

Due in class Wednesday, September 5, 2007

1. The Fourier transform of a gaussian is another gaussian. If you choose a gaussian of exactly the right width, it is its own Fourier transform,  $f(x) = \tilde{f}(x)$ .
  - (a) Using an arbitrary  $g(x)$  whose Fourier transform is known, construct a function  $f(x)$  that is its own Fourier transform. Apparently, this property is not uncommon!
  - (b) Show that if  $f(x) = \tilde{f}(x)$  is real, it is an even function.

2. Show that the Fourier basis functions are orthonormal in the following sense:

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega t} e^{-i\omega' t} dt = \delta(\omega - \omega').$$

Work from first principles, *i.e.* without using the inverse Fourier transform. You may find it helpful to rewrite the integral this way:

$$\lim_{a \rightarrow \infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-|t|/a} e^{i\omega t} e^{-i\omega' t} dt.$$

3. Do Exercise 13.5 (Riley, Hobson and Bence, p. 466). Describe a physical system that obeys the same differential equation ( $x$  could be time rather than position). Put the solution in this form:

$$\tilde{\phi}(k) = A e^{ik\alpha} \tilde{f}(k).$$

Sketch the phase response,  $\alpha(k)$ , and the amplitude response,  $A(k)$ . Briefly describe the significance of these response functions for your physical system.