In class, we derived Rayleigh-Sommerfeld diffraction using Dirichlet boundary conditions:

\[
\psi_D(\mathbf{r}) = \frac{1}{2i\pi} \int_A \frac{e^{ikR}}{R} \left( k + \frac{i}{R} \right) \psi_I(\mathbf{r}') dA'.
\] (1)

where the aperture \(A\) is in the plane \(z' = 0\). We define

\[
R = \sqrt{(x - x')^2 + (y - y')^2 + z^2}.
\] (2)

Your task is to derive an elementary result of Fourier optics and then apply it to single slit diffraction.

1. Rewrite equation 1 by replacing \(R \to z\), except in the complex exponential. Explain why this corresponds to the small angle approximation. Why would it be inappropriate to substitute \(\exp(ikz)\) for \(\exp(ikR)\)?

2. Now take the far field limit by removing the term that falls off as \(z^{-2}\).

3. For the complex exponential, we may assume that \(x'\) and \(y'\) are small compared to \(x, y,\) and \(z\); but they are not negligible. Show that to first order,

\[
R = r - \frac{xx'}{r} - \frac{yy'}{r},
\] (3)

where the spherical coordinate \(r\) is defined as usual.

4. Now define new variables

\[
\zeta = \frac{kr}{x}, \quad \eta = \frac{kr}{y}.
\] (4)

Express \(\psi_D\) in terms of the 2D Fourier transform\(^1\) \(\mathcal{F}_{x'y'}[A(x', y') \psi_I(x', y', 0)]\), where \(A = 1\) in the aperture \(A\), and \(A = 0\) elsewhere.

5. Using the Fourier formulation, calculate the intensity pattern \(I(x, y, z) = \psi_D^* \psi_D\) due to a plane wave \(\psi_I(\mathbf{r}') = E_0 \exp(ikz')\), incident upon a single, infinitely long slit of width \(d\).

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\(^1\)Note that the variables \(\zeta, \eta\) are spatial coordinates in the far field, but they are also frequencies conjugate to \(x', y'\).