1. In class, we solved
\[ \frac{d^2y}{dx^2} = w(x), \quad \text{subject to} \quad y(0) = y(l) = 0. \]

Find \( y(x) \) if the boundary conditions are \( y(a) = y(-a) \) and \( y'(a) = y'(-a) \) in the limit \( a \to \infty \).

2. Consider the electrostatic problem in spherical symmetry,
\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\Phi}{dr} \right) = -\frac{\rho(r)}{\epsilon_0} = F(r). \]

The electric potential \( \Phi \) falls to zero at infinity.

(a) Show explicitly that the operator is Hermitian.

(b) Put the eigenvalue equation for this operator in Sturm-Liouville form. Does this match any of the possibilities in table 17.1?

(c) Now let \( \Phi(r) = r^{-1/2}u(r) \), and change variables to \( x = kr \). Show that the result is Bessel’s equation of order \( l + \frac{1}{2} \).

(d) Express the Green’s Function for the original differential equation as an expansion in terms of the spherical Bessel functions \( j_l(kr) \) and \( n_l(kr) \) (see §18.6).