EIGENFUNCTIONS AND DUAL NOTATION, E.G.

**Hermitian Operators**

\( \langle y, x \rangle = \langle x, y \rangle \)

\( \langle y_i, x \rangle = \delta_{i,0} \)

**WAY WE LIKE HERMITIAN OPERATORS**

\( \langle y_i, x \rangle = \delta_{i,0} \)

\( \lambda_i \in \mathbb{R} \)

**FOR DEGENERACIES**

\( \lambda_i = \lambda_j \)

\( \langle x, y_i \rangle = \delta_{i,j} \)

**WE CAN USE GRAM-SCHMIDT ORTHOGONALIZATION**

**TO ENSURE THAT**

\( * \) is satisfied

**GREEN'S FUNCTIONS FOR HERMITIAN OPERATORS**

**STURM-LIOUVILLE EQUATIONS**

**TODAY:**

**COMING UP:**

**RHODES BROADEN THE DEFINITION OF THE INNER PRODUCT TO**

**RHODES DISTINGUISH BETWEEN SELF-ADJOINT AND HERMITIAN. IF**

**THE AMOUNT IS DEFINED AS ABOVE, THERE IS NO DISCIPLINE.**
\[ c_i = \frac{\int_a^b \frac{1}{y} \, dx}{\int_a^b y \, dx} = \frac{\int_a^b \frac{1}{y} \, dx}{\int_a^b y \, dx} \]

Now multiply by \( y \) and integrate:

\[ \int_a^b \frac{1}{y} \, dy = \int_a^b f(x) \, dx \]

\[ \left[ \ln |y| \right]_a^b = \int_a^b f(x) \, dx \]

\[ \ln |b| - \ln |a| = \int_a^b f(x) \, dx \]

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**THE Y**

Our original inhomogeneous ODE in terms of Hit Equation with \( \beta \), and we get

\[ y_n'(x) = (x) \]

**TO DO: FIND THE C's** (here!)

\[ y_n = (x) \]

**THE SOLUTION**: Since the \( y_i \) are an orthogonal set, write

\[ \int_a^b y_i \, dx = \delta_{ij} \]

Frist, solve the eigenvalue problem for \( \phi \):

\[ \phi_n = \int_a^b f(x) \phi_n(x) \]

Suppose we want to solve

**Green's Functions**
Exercise: Prove that $g(x, y)$ is a solution.

Expression of $g(x, y) = 5 \cdot (x, x)$

Mathematically, we can see from the above

An impulse: The response of a physical system $A$ physically, it must be real since $1$ is a question: Is $g(x, x)$ real?

Question: Is $g(x, x)$ function?

\[ g(x, x) = \int x \cdot p(x) \, dx \cdot \left( \int x^2 \, dx \right) \frac{1}{Z} Z = (x) Y \]

Relevant:

\[ \langle \psi | x \rangle \langle x | \psi \rangle \frac{1}{Z} Z = \langle \psi | \psi \rangle \]

Returning to Equation...
\[ a \cdot (x) \cdot 6 = \langle 6 \mid x \rangle = \langle 6 \mid x \rangle \cdot \langle x \mid 2 \rangle = \]

\[ \langle 6 \mid x \rangle \cdot \langle x \mid 2 \rangle = \langle 6 \mid x \rangle \cdot \langle x \mid 2 \rangle = \]

\[ \langle 6 \mid x \rangle \cdot \langle x \mid 2 \rangle = \]

\[ \begin{align*}
\left( x - x \right)^2 &= 0 \\
\langle x \rangle^2 &= 0
\end{align*} \]