

## CONTRIBUTION OF CYCLOTRON-RESONANT DAMPING TO KINETIC DISSIPATION OF INTERPLANETARY TURBULENCE

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### ABSTRACT

We examine some implications of inertial range and dissipation range correlation and spectral analyses extracted from 33 intervals of *Wind* magnetic field data. When field polarity and signatures of cross helicity and magnetic helicity are examined, most of the data sets suggest some role of cyclotron-resonant dissipative processes involving thermal protons. We postulate that an active spectral cascade into the dissipation range is balanced by a combination of cyclotron-resonant and noncyclotron-resonant kinetic dissipation mechanisms, of which only the former induces a magnetic helicity signature. A rate balance theory, constrained by the data, suggests that the ratio of the two mechanisms is of order unity. While highly simplified, this approach appears to account for several observed features and explains why complete cyclotron absorption, and the corresponding pure magnetic helicity signature, is usually not observed.

*Subject headings:* MHD — turbulence

### 1. INTRODUCTION

The solar wind plasma displays many characteristics that can be reasonably well described by a magnetohydrodynamic (MHD) fluid model (Tu & Marsch 1995; Burlaga 1995), including features that appear to be related to fluid turbulence (Coleman 1968) and MHD wave activity (Belcher & Davis 1971). Within the context of a simple nonlinear MHD theory, one expects that a key feature is the spectral cascade of energy from larger, energy-containing scales through an inertial range and ultimately into a dissipative range (von Kármán & Howarth 1938; Batchelor 1970; Martínez et al. 1997). An MHD description of the solar wind or other collisionless plasmas in astrophysics is a drastic oversimplification, and it is therefore significant that solar wind observations support the general picture of a turbulent MHD cascade from large to small scales (Jokipii 1973; Matthaeus & Goldstein 1982; Goldstein, Roberts, & Matthaeus 1995). Indeed, unless turbulent transfer and decay are invoked, it is difficult to explain the proton temperatures at 1 AU (Coleman 1968), the solar wind's general non-adiabatic temperature profile (Freeman 1988; Richardson et al. 1995), and the radial variation of the fluctuation levels (Zank, Matthaeus, & Smith 1996).

There is an appealing simplicity in an explanation of these features in terms of a cascade controlled by larger scale energy-containing fluctuations (Matthaeus et al. 1994; Zank et al. 1996) and mediated by a self-similar inertial range (Tu, Pu, & Wei 1984; Verma, Roberts, & Goldstein 1995). This model is in direct analogy with hydrodynamic turbulence (Batchelor 1970).

Nevertheless, there is an essential and theoretically challenging piece that is missing in this picture. The cascade must terminate at small scales, possibly in a dissipation range in which processes occur that convert MHD fluctuation energy into plasma thermal energy. The possible involvement of ion cyclotron activity in the observed onset of the steepening of solar wind magnetic field spectra (at  $\approx 1$  Hz) (Behannon 1976; Denskat, Beinroth, & Neubauer 1983) has been discussed for some time. Generally, discussion of the collisionless damping of interplanetary fluctuations has concentrated on Landau

damping (Barnes 1966, 1979). It is only recently (Goldstein, Roberts, & Fitch 1994; Leamon et al. 1998) that attention has begun to focus on a broader framework for explaining dissipation processes. A theoretical perspective that invokes kinetic theory to convert fluid scale energy to heat is needed, taking into account spectral transfer that continually resupplies the dissipation range through broadband nonlinear couplings. This paper provides a simple description of this process based on the assumption of a steady cascade, with the goal of explaining recently described features of the dissipation.

### 2. MHD TURBULENCE PARAMETERS

It is useful to adopt a leading-order description based on incompressible turbulence, in view of the low level of interplanetary density fluctuations (Roberts et al. 1987), the observed density spectrum (Montgomery, Brown, & Matthaeus 1987), and the low average turbulent Mach number (Matthaeus, Goldstein, & Roberts 1990). This perspective is also consistent with the persistence of the  $k^{-5/3}$  signature of the Kolmogoroff cascade spectrum. Neglecting small internal energy fluctuations, the turbulent energy per unit mass,  $E$ , consists of contributions from the turbulent (ion) velocity  $\mathbf{v}$  and the fluctuating component of the magnetic field  $\mathbf{b}$ , scaled to Alfvén units. For an appropriately defined ensemble average  $\langle \dots \rangle$ , the contribution to the energy from velocity fluctuations  $E_v$  and from magnetic fluctuations  $E_b$  is

$$E = E_v + E_b = \frac{\langle |\mathbf{v}|^2 \rangle}{2} + \frac{\langle |\mathbf{b}|^2 \rangle}{2}. \quad (1)$$

In its idealized definition, the turbulent energy includes contributions from all wavenumbers and frequencies. However, in some circumstances one might consider only contributions from certain scales, so that, for example, the spectral decomposition of magnetic energy,  $E_b = \int d^3k E_b(\mathbf{k})$  might include only a certain range of wavenumbers. One might choose to

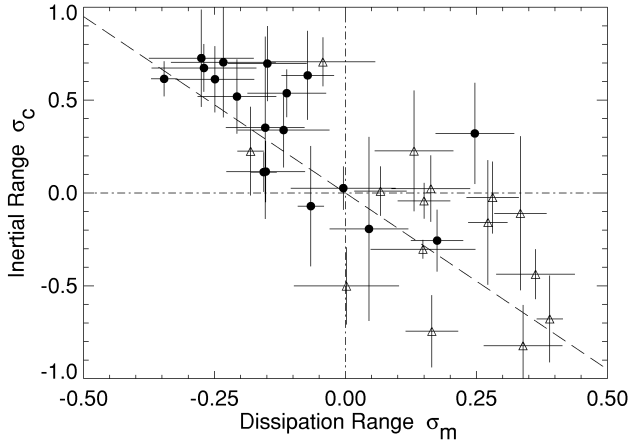


FIG. 1.—Scatter plot, for 33 *Wind* data intervals, of the normalized cross helicity in the inertial range,  $\sigma_c$ , vs. the normalized magnetic helicity in the dissipation range,  $\sigma_m$ . Triangles are intervals with outward-directed mean magnetic field, and filled circles have inward-directed mean fields. The dashed line corresponds to the best-fit line through the origin,  $\sigma_c = -1.90\sigma_m$ .

look at the energy in a finite band of wavenumbers or frequencies, for example, when the physics of the inertial or dissipation range is discussed.

Apart from energy, other quantities of importance for MHD turbulence are the magnetic helicity  $H_m = \langle \mathbf{b} \cdot \mathbf{a} \rangle$ , where  $\mathbf{b} = \nabla \times \mathbf{a}$ , the cross helicity  $H_c = \langle \mathbf{v} \cdot \mathbf{b} \rangle$ , and the respective spectral decompositions (Matthaeus & Goldstein 1982). The amounts of cross helicity and magnetic helicity relative to the energy are conveniently measured by the following dimensionless parameters. The normalized cross helicity,

$$\sigma_c = \frac{E_+ - E_-}{E_+ + E_-}, \quad (2)$$

is defined in terms of the Elsässer energies  $E_{\pm} \equiv \langle |\mathbf{v} \pm \mathbf{b}|^2 \rangle$  (Marsch & Mangeney 1987), and lies between  $-1$  and  $+1$ . Normalized magnetic helicity,

$$\sigma_m = \frac{E_L - E_R}{E_L + E_R}, \quad (3)$$

is written here in terms of  $E_L$ , the magnetic energy in left-handed (positive helicity) spatial structures, and  $E_R$ , the magnetic energy in right-handed (negative helicity) spatial structures. Note that  $E_b = E_L + E_R$ . We use the following sense of circular polarization: right-handed means a sense of rotation from the  $x$ -direction toward the  $y$ -direction as one samples in the positive  $z$ -direction for a right-handed  $(x, y, z)$  coordinate system. In terms of the integrated magnetic helicity spectrum,

$$\begin{aligned} E_L^b &= \frac{1}{2} \left[ E_b + \int d^3k |\mathbf{k}| H_m(\mathbf{k}) \right], \\ E_R^b &= \frac{1}{2} \left[ E_b - \int d^3k |\mathbf{k}| H_m(\mathbf{k}) \right]. \end{aligned} \quad (4)$$

The magnetic helicity is important in the present context because spatial handedness is related to resonance conditions with charged particles. Cross helicity relates to the direction

of propagation of large-amplitude Alfvén waves with respect to a uniform or slowly varying background magnetic field  $\mathbf{B}_0$  (Belcher & Davis 1971; Matthaeus & Goldstein 1982). Both together determine the polarization of the waves in the plasma frame (Smith et al. 1984).

### 3. OBSERVATIONS

In a recent study, Leamon et al. (1998) described properties of the interplanetary dissipation range at 1 AU. Their analysis included spectra and other parameters computed for 33 intervals of high time resolution (up to 22 vectors  $s^{-1}$ ) *Wind* magnetic field data, along with plasma data at a much lower sampling rate (either 46 or 92 s per measurement). In this analysis, the magnetic field data provide information about  $E_b$ ,  $E_L$ , and  $E_R$  in the dissipation range and in the inertial range. For the samples in the Leamon et al. study, the inertial and dissipation ranges were distinguished according to spectral slope. The average inertial range spectral index corresponded to a one-dimensional spectral law in good agreement with the Kolmogoroff value  $E_b(k_r) \sim k_r^{-1.67}$ , for radial wavenumber  $k_r = 2\pi f/V_{SW}$ , with solar wind speed  $V_{SW}$  and  $f$  the spacecraft-frame frequency. The dissipation range spectra were steeper, averaging  $E_b(k_r) \sim k_r^{-3.01}$ , with a break point between the two ranges at an average frequency of about 0.5 Hz.

Leamon et al. noted that most of the intervals they examined showed a signature in the magnetic helicity at dissipation range frequencies, as had been reported previously by Goldstein et al. (1994). In contrast, typical inertial range magnetic helicity spectra oscillate randomly as a function of frequency (Matthaeus & Goldstein 1982). Leamon et al. found as much as 90% of the energy to be carried by waves propagating at highly oblique angles or quasi-two-dimensional turbulence rather than parallel-propagating Alfvén waves. Nevertheless, in almost all of the intervals examined, the dissipation range  $H_m$  values were consistent with the absorption of outward-propagating Alfvén waves by resonant coupling to thermal protons.

Here we examine in greater detail the data underlying the latter conclusion. In Figure 1 we show the normalized cross helicity  $\sigma_c$  computed from inertial range data, plotted versus the normalized magnetic helicity  $\sigma_m$  in the dissipation range, for the 33 data intervals previously analyzed.  $H_c$  can be computed only in the inertial range because of limited sampling rates for plasma data; we use the inertial range  $H_c$  as a proxy for the same quantity that is unmeasurable in the dissipation range. In effect, we are assuming that the direction of propagation of fluctuations in the dissipation range is the same as the direction of propagation of fluctuations in the inertial range.

It is apparent from the data in Figure 1 that most intervals for which the mean magnetic field is outwardly directed have  $\sigma_m > 0$  and  $\sigma_c < 0$ . On the other hand, inward-directed  $\mathbf{B}_0$  is associated with  $\sigma_m < 0$  and  $\sigma_c > 0$ . This implies a predominance of outward-propagating waves. One can readily see that this is consistent with cyclotron-resonant absorption of outward-propagating fluctuations by thermal protons, as follows. A proton moving outward along the magnetic field executes a left-handed helical trajectory. Waves propagating outward at the Alfvén speed will overtake most thermal particles (at  $\beta \approx 1$ ), and therefore, on average, the thermal protons will be in resonance with such waves that have a right-handed spatial handedness (negative  $H_m$ ). If the energy of these waves is assumed to be damped by the resonant protons, the energy that remains will preferentially reside in the undamped fluctuations, which have a left-handed structure and positive  $H_m$  (see, for example, Moffatt

1978). Consequently, outward  $\mathbf{B}_0$  should be associated with  $\sigma_c < 0$  (outward propagation) and  $\sigma_m > 0$ . Reversing the direction of  $\mathbf{B}_0$  but maintaining the assumption of outward propagating waves (now  $\sigma_c > 0$ ) produces the conclusion that  $\sigma_m < 0$  in the dissipation range by the same argument.

#### 4. CASCADE AND DISSIPATION

The above argument explains the clustering of the observational points in the upper left and lower right quadrants. However, there are questions that arise. First, if kinetic processes are assumed to be very rapid, why is the signature in the magnetic helicity not pure ( $\pm 1$ ), as one would expect for complete cyclotron absorption? Second, how is the above argument modified if instead of pure cyclotron-resonant absorption processes, there is also a contribution due to Landau resonance or nonresonant absorption? Finally, since the observed cross helicities are not “pure,” what is the effect of relaxing the assumption of purely outward traveling Alfvén waves?

It turns out that these questions can be addressed, in at least a preliminary fashion, by postulating a cascade and associated dissipation processes that are described by a set of energy balance equations, as follows:

$$\begin{aligned} \frac{dE_L}{dt} &= \frac{S}{2} - \gamma_0 E_L - \gamma_r E_L, \\ \frac{dE_R}{dt} &= \frac{S}{2} - \gamma_0 E_R. \end{aligned} \quad (5)$$

The energies in left- and right-handed spatial structures are respectively designated as  $E_L$  and  $E_R$  following our earlier discussion (in this case the integration over the spectrum now includes, by assumption, only the dissipation range). The rate of supply of energy (per unit mass) transferred into the dissipation range from the inertial range is designated by  $S$ . This supply rate is equally apportioned to  $L$  and  $R$  fluctuations since inertial range  $H_m$  is random. We assume that the only external contribution to  $dE_{L,R}/dt$  is due to the cascade term  $S$  and that in the dissipation range there is no exchange between  $E_L$  and  $E_R$ , or exchange between kinetic and magnetic energies. The quantity  $\gamma_r$  represents a decay rate due to cyclotron-resonant absorption by thermal protons, and it appears only in the  $L$  equation under the assumption that fluctuations are outward propagating and  $\mathbf{B}_0$  is inward. (This would also occur for inward propagation and outward  $\mathbf{B}_0$ .) The remaining damping term,  $\gamma_0$ , appears in both  $L$  and  $R$  equations and represents decay processes that produce no signature in the magnetic helicity. Included in  $\gamma_0$  are contributions from Landau damping and other mechanisms that do not involve cyclotron resonance, as well as mechanisms that are fully nonresonant.

#### 5. CYCLOTRON-RESONANT AND OTHER FORMS OF DISSIPATION

We can now proceed to estimate a typical relative strength of cyclotron-resonant and noncyclotron-resonant processes. Suppose the cascade is steady, so  $dE_{L,R}/dt = 0$ , and we may equate the right-hand sides of equation (5). From the data, we take a typical value of magnetic helicity to be  $\sigma_m \approx -\frac{1}{3}$ . This corresponds in equation (3) to  $E_R = 2E_L$  in the dissipation range. Then for consistency with equations (5) we must have  $\gamma_0 \approx \gamma_r$ , indicating that cyclotron and noncyclotron absorption mechanisms are approximately of equal strength.

Since the observed values of  $H_c$  are not pure, the above

argument should be refined to account for a distribution of propagation directions relative to the slower thermal protons. Assume, then, that there is a probability  $P(L)$  that fluctuations are propagating outward, which produces a resonance between left-handed structures and thermal protons and implies the appearance of  $\gamma_r$  in the  $E_L$  equation. Assigning the probability of inward propagation to be  $P(R) = 1 - P(L)$  implies that resonance between right-handed structures and thermal protons is weighted accordingly. Therefore, the cascade balance equations become

$$\begin{aligned} \frac{dE_L}{dt} &= \frac{S}{2} - \gamma_0 E_L - P(L)\gamma_r E_L, \\ \frac{dE_R}{dt} &= \frac{S}{2} - \gamma_0 E_R - P(R)\gamma_r E_R. \end{aligned} \quad (6)$$

According to the Elsässer representation, fluctuations with energy  $E_-$  tend to propagate along the mean field  $\mathbf{B}_0$  whereas fluctuations having energy  $E_+$  tend to propagate antiparallel to  $\mathbf{B}_0$ . We assume for simplicity that the probability that, at any location in the plasma, a typical thermal proton will “see” outward propagation is proportional to the average outward-propagating energy. Thus,

$$P(L) = \frac{E_-}{E_- + E_+} = \frac{1 + \sigma_c}{2}, \quad (7)$$

and therefore  $P(R) = (1 - \sigma_c)/2$ .

With this interpretation, we can make use of the data in Figure 1 to constrain our model and arrive at further insights about the dissipation processes. We invoke the steady form of equation (6) along with the definitions equations (2), (3), and (7) and assume that  $\gamma_0$  and  $\gamma_r$  are independent of  $\sigma_c$ ,  $\sigma_m$ , and other plasma turbulence parameters. Eliminating  $E_L$  and  $E_R$ , we conclude that

$$\sigma_c = - \left( 1 + 2 \frac{\gamma_0}{\gamma_r} \right) \sigma_m. \quad (8)$$

The best-fit line forced through the origin is  $\sigma_c = -1.90\sigma_m$ , while the best-fit straight line through the data is  $\sigma_c = -1.80\sigma_m + 0.10$ . Considering either 32 or 31 degrees of freedom accordingly, the reduced  $\chi^2$  values of the two fits are  $\chi_r^2 = 1.78$  and 1.55. Putting  $\sigma_c = -1.90\sigma_m$  in equation (8) implies that  $\gamma_r = 2.22\gamma_0$ . The other important consequence of equation (8) is that only when  $\gamma_0 = 0$  do pure Alfvén waves lead to purely helical states.

#### 6. INTERPRETATION AND DISCUSSION

This preliminary attempt to understand the observed interplanetary dissipation range spectra, while clearly oversimplified, appears to contain some suggestive features. We postulated an equation that balances cyclotron-resonant and noncyclotron-resonant dissipation effects of kinetic origin with steady spectral transfer into the dissipation range due to MHD-scale cascade processes. This formal structure evidently has been able to account for some of the observed properties of the distribution of inertial range cross helicity and dissipation range magnetic helicities. We note that several important approxi-

mations are implicit in our treatment. For example, we do not account in any way for the energy in the velocity field,  $E_v$ , in the dissipation range. This might be an acceptable approximation in either of two cases: if  $E_v$  and its dissipation rate are much smaller than  $E_b$  and its cascade rate  $S$ , or if a proportionality or approximate equality exists between  $E_v$  and  $E_b$  in the dissipation range. In the absence of better theoretical guidance, as well as plasma data at the requisite frequencies, we prefer the latter explanation at present. We can suppose, for example, that the “Alfvén effect” attempts to enforce near equipartition of velocity and magnetic fields, or that the phenomenological dissipation rates we assumed in fact include some contributions that are mediated by couplings to the velocity field, which would be expected to be heavily damped at scales near the thermal proton gyroradius.

Motivated by the typical values of magnetic helicity in the dissipation range (Fig. 1), and assuming that all fluctuations propagate in one direction, we estimated the near equality of cyclotron resonant contributions represented by  $\gamma_r$  and other dissipative mechanisms represented by  $\gamma_o$ . Using the inertial range cross helicity to estimate the relative likelihood of propagation direction produced a refined estimate  $\gamma_r \approx 2\gamma_o$ .

In this development we have been forced, because of limitations of spacecraft instrumentation, to use the inertial range cross helicity to compute a proxy for the propagation direction in the dissipation range. The most notable limitation of this substitution derives from the possibility that preferential dissipation may lead to different cross helicity values in the dissipation range, although we are not aware of any observational evidence for this. Indeed, the connection between  $\sigma_c$  and direction of propagation may be complicated in the dissipation range by various kinetic wave modes such as whistlers. On the other hand, lower frequency observations of  $\sigma_c$  (see Matthaeus & Goldstein 1982) often indicate that a single direction of

propagation is dominant over several orders of magnitude of scale, which would tend to support our extrapolation into the higher dissipation range frequencies. In any case, the correlation evident in Figure 1 appears encouraging with regard to use of this proxy.

The present results provide some preliminary insights into the structure of the interplanetary dissipation range, but additional work needs to be done to better understand the physics of the kinetic dissipation mechanisms represented by  $\gamma_o$  and  $\gamma_r$ . For example, we expect on general grounds, that Landau and nonresonant processes should make a contribution to dissipation of three-dimensional, MHD turbulent fluctuations, but an acceptable large-amplitude theory of such processes has not yet been developed as far as we are aware. Similarly, resonant dissipation, generally evaluated by linear Vlasov theory, requires improvement for the same reasons. In addition, linear theory makes no prediction about damping of purely transverse “two-dimensional” turbulence, which appears to be favored by MHD in the presence of a moderately strong mean magnetic field (Matthaeus, Bieber, & Zank 1995). In this regard, one would expect that MHD turbulence would be accompanied by a turbulent induced electric field  $\mathbf{E} = -\mathbf{v} \times \mathbf{b}$  that would produce stochastic acceleration of suprathermal particles and associated damping of the fluctuations. Further developments in kinetic theory are required to describe dissipation that is nonlinear, anisotropic, and driven by an MHD cascade.

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